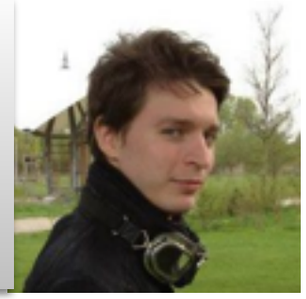
Choreographies, Logically

Marco Carbone

carbonem@itu.dk

joint work with:

Fabrizio Montesi



Carsten Schürmann

London, 16 May 2014

IT University of Copenhagen

This talk

A (linear) logical characterisation of choreographies

This talk

A (linear) logical characterisation of choreographies

Disclaimer:
I’m not a logician!

Choreographies?

Choreographic (or global) programming is a paradigm for programming distributed systems.

Choreographies?

Choreographic (or global) programming is a paradigm for programming distributed systems.

Example. Consider a standard pi-calculus terms:

$$\underbrace{\overline{x}(tea);\ x(tr);\ \overline{tr}(p)}_{P_{\mathsf{client}}} \qquad \underbrace{x(tea);\ \overline{x}(tr);\ tr(p);\ \overline{b}(m)}_{P_{\mathsf{server}}} \qquad \underbrace{b(m)}_{P_{\mathsf{bank}}}$$

Choreographies?

$$\underbrace{\overline{x}(tea);\ x(tr);\ \overline{tr}(p)}_{P_{\mathsf{client}}} \qquad \underbrace{x(tea);\ \overline{x}(tr);\ tr(p);\ \overline{b}(m)}_{P_{\mathsf{server}}} \qquad \underbrace{b(m)}_{P_{\mathsf{bank}}}$$

A global program (or **choreography**) for the system above, would be the following description of its execution flow:

Choreographies?

$$\underbrace{\overline{x}(tea);\ x(tr);\ \overline{tr}(p)}_{P_{\mathsf{client}}} \qquad \underbrace{x(tea);\ \overline{x}(tr);\ tr(p);\ \overline{b}(m)}_{P_{\mathsf{server}}} \qquad \underbrace{b(m)}_{P_{\mathsf{bank}}}$$

A global program (or **choreography**) for the system above, would be the following description of its execution flow:

1. client $\rightarrow$ server : $x(tea)$;
2. server $\rightarrow$ client : $x(tr)$;
3. client $\rightarrow$ server : $tr(p)$;
4. server $\rightarrow$ bank : $b(m)$

Choreographies?

Implementation

$$\underbrace{\overline{x}(tea);\ x(tr);\ \overline{tr}(p)}_{P_{\mathsf{client}}} \qquad \underbrace{x(tea);\ \overline{x}(tr);\ tr(p);\ \overline{b}(m)}_{P_{\mathsf{server}}} \qquad \underbrace{b(m)}_{P_{\mathsf{bank}}}$$

A global program (or **choreography**) for the system above, would be the following description of its execution flow:

1. client $\rightarrow$ server : $x(tea)$;
2. server $\rightarrow$ client : $x(tr)$;
3. client $\rightarrow$ server : $tr(p)$;
4. server $\rightarrow$ bank : $b(m)$

Global Specification

Different, but equivalent representations of a distributed system

$$\overline{x}(tea);\ x(tr);\ \overline{\overline{tr}}(p) \quad | \quad x(tea);\ \overline{x}(tr);\ tr(p);\ \overline{b}(m) \quad | \quad b(m)$$

1. client $\rightarrow$ server : $x(tea)$;
2. server $\rightarrow$ client : $x(tr)$;
3. client $\rightarrow$ server : $tr(p)$;
4. server $\rightarrow$ bank : $b(m)$

Different, but equivalent representations of a distributed system

$$\overline{x}(tea);\ x(tr);\ \overline{tr}(p) \quad | \quad x(tea);\ \overline{x}(tr);\ tr(p);\ \overline{b}(m) \quad | \quad b(m)$$

1. client $\rightarrow$ server : $x(tea)$;
2. server $\rightarrow$ client : $x(tr)$;
3. client $\rightarrow$ server : $tr(p)$;
4. server $\rightarrow$ bank : $b(m)$

Different, but equivalent representations of a distributed system

$$\overline{x}(tea);\ x(tr);\ \overline{\overline{tr}}(p) \quad | \quad x(tea);\ \overline{x}(tr);\ tr(p);\ \overline{b}(m) \quad | \quad b(m)$$

1. client $\rightarrow$ server : $x(tea)$;
2. server $\rightarrow$ client : $x(tr)$;
3. client $\rightarrow$ server : $tr(p)$;
4. server $\rightarrow$ bank : $b(m)$

client $\rightarrow$ server : $x(tea);\ \Big(x(tr);\ \overline{\overline{tr}}(p) \quad | \quad \overline{x}(tr);\ tr(p);\ \overline{b}(m) \quad | \quad b(m)\Big)$

Different, but equivalent representations of a distributed system

$$\overline{x}(tea);\ x(tr);\ \overline{\overline{tr}}(p) \quad | \quad x(tea);\ \overline{x}(tr);\ tr(p);\ \overline{b}(m) \quad | \quad b(m)$$

1. client $\rightarrow$ server : $x(tea)$;
2. server $\rightarrow$ client : $x(tr)$;
3. client $\rightarrow$ server : $tr(p)$;
4. server $\rightarrow$ bank : $b(m)$

client $\rightarrow$ server : $x(tea);\ \left(x(tr);\ \overline{\overline{tr}}(p) \quad | \quad \overline{x}(tr);\ tr(p);\ \overline{b}(m) \quad | \quad b(m)\right)$

client $\rightarrow$ server : $x(tea);$ server $\rightarrow$ client : $x(tr);\ \left(\overline{\overline{tr}}(p) \quad | \quad tr(p);\ \overline{b}(m) \quad | \quad b(m)\right)$

Different, but equivalent representations of a distributed system

$$\overline{x}(tea);\ x(tr);\ \overline{\overline{tr}}(p) \quad | \quad x(tea);\ \overline{x}(tr);\ tr(p);\ \overline{b}(m) \quad | \quad b(m)$$

1. client $\rightarrow$ server : $x(tea)$;
2. server $\rightarrow$ client : $x(tr)$;
3. client $\rightarrow$ server : $tr(p)$;
4. server $\rightarrow$ bank : $b(m)$

$$\text{client} \rightarrow \text{server} : x(tea);\ \left(x(tr);\ \overline{\overline{tr}}(p) \quad | \quad \overline{x}(tr);\ tr(p);\ \overline{b}(m) \quad | \quad b(m)\right)$$

$$\text{client} \rightarrow \text{server} : x(tea);\ \text{server} \rightarrow \text{client} : x(tr);\ \left(\overline{\overline{tr}}(p) \quad | \quad tr(p);\ \overline{b}(m) \quad | \quad b(m)\right)$$

Mixed language of choreographies and processes:
Compositional Choreographies

Different, but equivalent representations of a distributed system

$$\overline{x}(tea);\ x(tr);\ \overline{\overline{tr}}(p) \quad | \quad x(tea);\ \overline{x}(tr);\ tr(p);\ \overline{b}(m) \quad | \quad b(m)$$

1. client $\rightarrow$ server : $x(tea)$;
2. server $\rightarrow$ client : $x(tr)$;
3. client $\rightarrow$ server : $tr(p)$;
4. server $\rightarrow$ bank : $b(m)$

.....

QUESTION:
Is there a semantic relationship between these representations?

.....

clien $m)\Big)$

client $\rightarrow$ server : $x(tea)$; server $\rightarrow$ client : $x(tr)$; $\Big(\overline{\overline{tr}}(p) \quad | \quad tr(p);\ \overline{b}(m) \quad | \quad b(m)\Big)$

Mixed language of choreographies and processes:
Compositional Choreographies

Different, but equivalent representations of a distributed system

$$\overline{x}(tea);\ x(tr);\ \overline{tr}(p) \quad | \quad x(tea);\ \overline{x}(tr);\ tr(p);\ \overline{b}(m) \quad | \quad b(m)$$

1. client $\rightarrow$ server : $x(tea)$;
2. server $\rightarrow$ client : $x(tr)$;
3. client $\rightarrow$ server : $tr(p)$;
4. server $\rightarrow$ bank : $b(m)$

.....

QUESTION:
Is there a semantic relationship between these representations?

1) Choreography —> Processes: Endpoint Projection (YES)

2) Processes —> Choreography: Choreography Extraction (??)

.....

Mixed language of choreographies and processes:
Compositional Choreographies

In this work…

- The proof theory of *Linear Compositional Choreographies (LCC)* whose proofs are correspond to choreography and process language terms;
- Logically-Derived Semantics for Compositional Choreographies
- *Projection&Extraction* derived from our proof theory

In this work…

- The proof theory of *Linear Compositional Cl*
  co
  lan

- Lo
  Ch

- *Pr*

A Curry-Howard Approach…

Programs as Proofs

Types as Propositions/Formulas

Normalisation (Reductions) as Cut Elimination (Cut Reductions)

LCC Proof Theory

Our starting point…

L. Caires, F. Pfenning. Session Types as Intuitionistic Linear Propositions. [Concur 2010]

…a Curry-Howard correspondence between ILL and a fragment of the pi-calculus…

Sessions-based communication and Session Types

$$\overline{x}(tea);\ x(tr);\ \overline{tr}(p) \quad | \quad x(tea);\ \overline{x}(tr);\ tr(p);\ \overline{b}(m) \quad | \quad b(m)$$

$$\text{client} \rightarrow \text{server} : x(tea);\ \Big(x(tr);\ \overline{tr}(p) \quad | \quad \overline{x}(tr);\ tr(p);\ \overline{b}(m) \quad | \quad b(m)\Big)$$

- Channels represent a binary session
- Channels are linearly used (no races)
- Each channel is typed with a session type describing how it will be used at run-time (client's side here):

Sessions-based communication and Session Types

$$\overline{x}(tea);\ x(tr);\ \overline{tr}(p) \quad | \quad x(tea);\ \overline{x}(tr);\ tr(p);\ \overline{b}(m) \quad | \quad b(m)$$

$$\text{client} \rightarrow \text{server} : x(tea);\ \Big(x(tr);\ \overline{tr}(p) \quad | \quad \overline{x}(tr);\ tr(p);\ \overline{b}(m) \quad | \quad b(m)\Big)$$

- Channels represent a binary session
- Channels are linearly used (no races)
- Each channel is typed with a session type describing how it will be used at run-time (client's side here):

$$x \ : \ \mathbf{string} \otimes (\mathbf{int} \multimap \mathbf{end}) \multimap \mathbf{end}$$ [Caires and Pfenning '10]

Sessions-based communication and Session Types

$$\overline{x}(tea);\ x(tr);\ \overline{tr}(p) \quad | \quad x(tea);\ \overline{x}(tr);\ tr(p);\ \overline{b}(m) \quad | \quad b(m)$$

$$\text{client} \rightarrow \text{server} : x(tea);\ \Big(x(tr);\ \overline{tr}(p) \quad | \quad \overline{x}(tr);\ tr(p);\ \overline{b}(m) \quad | \quad b(m)\Big)$$

- Channels represent a binary session
- Channels are linearly used (no races)
- Each channel is typed with a session type describing how it will be used at run-time (client's side here):

$$x \ : \ \mathbf{string} \otimes (\mathbf{int} \multimap \mathbf{end}) \multimap \mathbf{end}$$

[Caires and Pfenning '10]

Sessions-based communication and Session Types

$$\overline{x}(tea);\ x(tr);\ \overline{tr}(p) \quad | \quad x(tea);\ \overline{x}(tr);\ tr(p);\ \overline{b}(m) \quad | \quad b(m)$$

$$\text{client} \rightarrow \text{server} : x(tea);\ \Big(x(tr);\ \overline{tr}(p) \quad | \quad \overline{x}(tr);\ tr(p);\ \overline{b}(m) \quad | \quad b(m)\Big)$$

- Channels represent a binary session
- Channels are linearly used (no races)
- Each channel is typed with a session type describing how it will be used at run-time (client's side here):

$$x \ : \ \mathbf{string} \otimes (\mathbf{int} \multimap \mathbf{end}) \multimap \mathbf{end}$$

[Caires and Pfenning '10]

Linear Logic Judgements

Dual intuitionistic linear logic judgements [Caires and Pfenning ’10]:

$$P \triangleright\ y_1 : A_1, \ldots, y_m : A_m \vdash x : B$$

which reads: “If composed with other processes communicating on channels $y_1, \ldots, y_m$ with types $A_1, \ldots, A_m$, then P can interact on x with type B.

Linear Logic Judgements

There is a problem with parallel composition…

$$\frac{P \triangleright \Delta_1 \vdash x : A \qquad Q \triangleright \Delta_2, x : A \vdash y : B}{(\boldsymbol{\nu} x)\,(P \mid Q) \triangleright \Delta_1, \Delta_2 \vdash y : B}\ \text{Cut}$$

Linear Logic Judgements

There is a problem with parallel composition…

$$\frac{P \triangleright \Delta_1 \vdash x : A \qquad Q \triangleright \Delta_2, x : A \vdash y : B}{(\boldsymbol{\nu} x)\,(P \mid Q) \triangleright \Delta_1, \Delta_2 \vdash y : B}\ \text{Cut}$$

- *Ok for endpoint programs*: we just write two programs using the shared channel and then compose

- *Bad for choreographies.* A choreography describes both sides of a session simultaneously.

Linear Logic Judgements

$$\text{client} \rightarrow \text{server} : x(tea); \quad (\ x(tr); P \quad | \quad \overline{x}(tr); Q\ )$$

Linear Logic Judgements

client $\rightarrow$ server : $x(tea);\quad (\ x(tr); P \quad | \quad \overline{x}(tr); Q\ )$

$$\dfrac{x(tr); P \ \triangleright\ \Delta_1 \vdash x : A \qquad \overline{x}(tr); Q \ \triangleright\ \Delta_2, x : A \vdash y : B}{(\boldsymbol{\nu} x)\,(\ x(tr); P \mid \overline{x}(tr); Q\ ) \triangleright\ \Delta_1, \Delta_2 \vdash y : B}\ \text{Cut}$$

$$\dfrac{(\boldsymbol{\nu} x)\,(\ x(tr); P \mid \overline{x}(tr); Q\ ) \triangleright\ \Delta_1, \Delta_2 \vdash y : B}{??}$$

Linear Logic Judgements

$$\text{client} \rightarrow \text{server} : x(tea);\quad (\ x(tr); P \quad | \quad \overline{x}(tr); Q\ )$$

$$\dfrac{\dfrac{x(tr); P \ \triangleright\ \Delta_1 \vdash x : A \qquad \overline{x}(tr); Q \ \triangleright\ \Delta_2, x : A \vdash y : B}{(\boldsymbol{\nu} x)\,(\ x(tr); P \mid \overline{x}(tr); Q\ ) \triangleright\ \Delta_1, \Delta_2 \vdash y : B}\ \text{Cut}}{??}$$

We need to find a way to retain information about cuts

Hypersequents

We need new judgements, retaining more information:

$$P \triangleright \Delta_1 \vdash^{p_1} x_1 : A_1 \quad | \quad \ldots \quad | \quad \Delta_n \vdash^{p_n} x_n : A_n$$

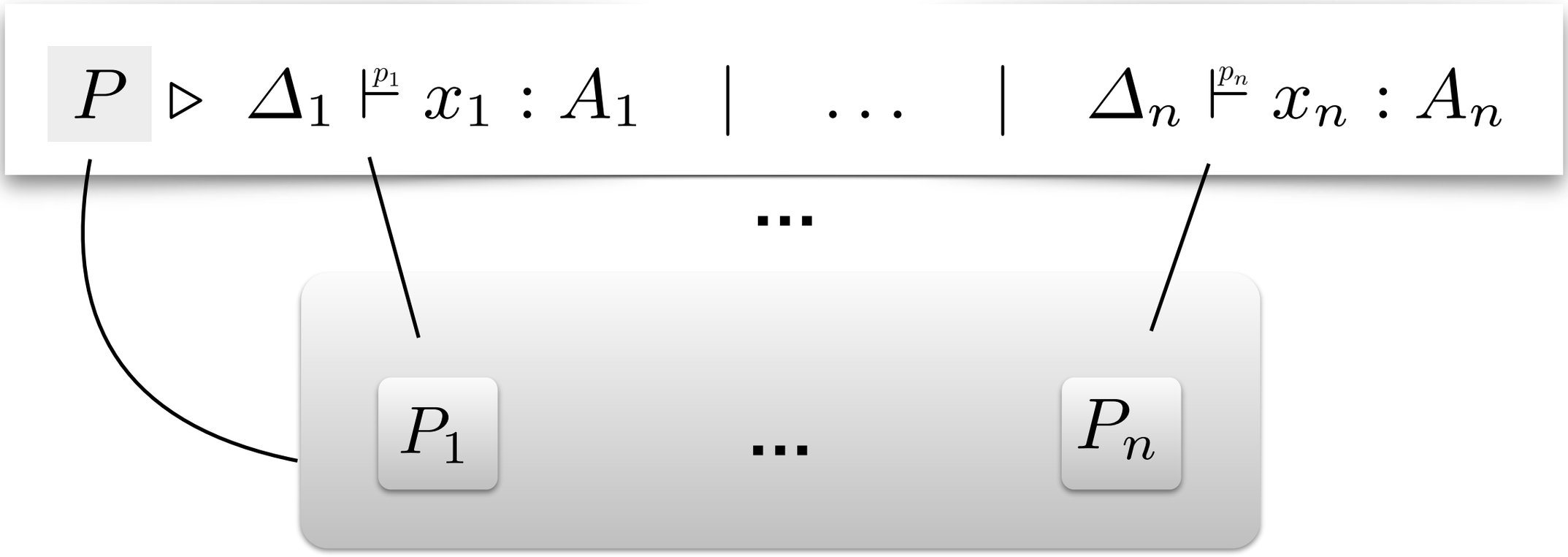
Hypersequents

We need new judgements, retaining more information:

$$P \triangleright \Delta_1 \overset{p_1}{\vdash} x_1 : A_1 \quad | \quad \ldots \quad | \quad \Delta_n \overset{p_n}{\vdash} x_n : A_n$$

Hypersequents

We need new judgements, retaining more information:

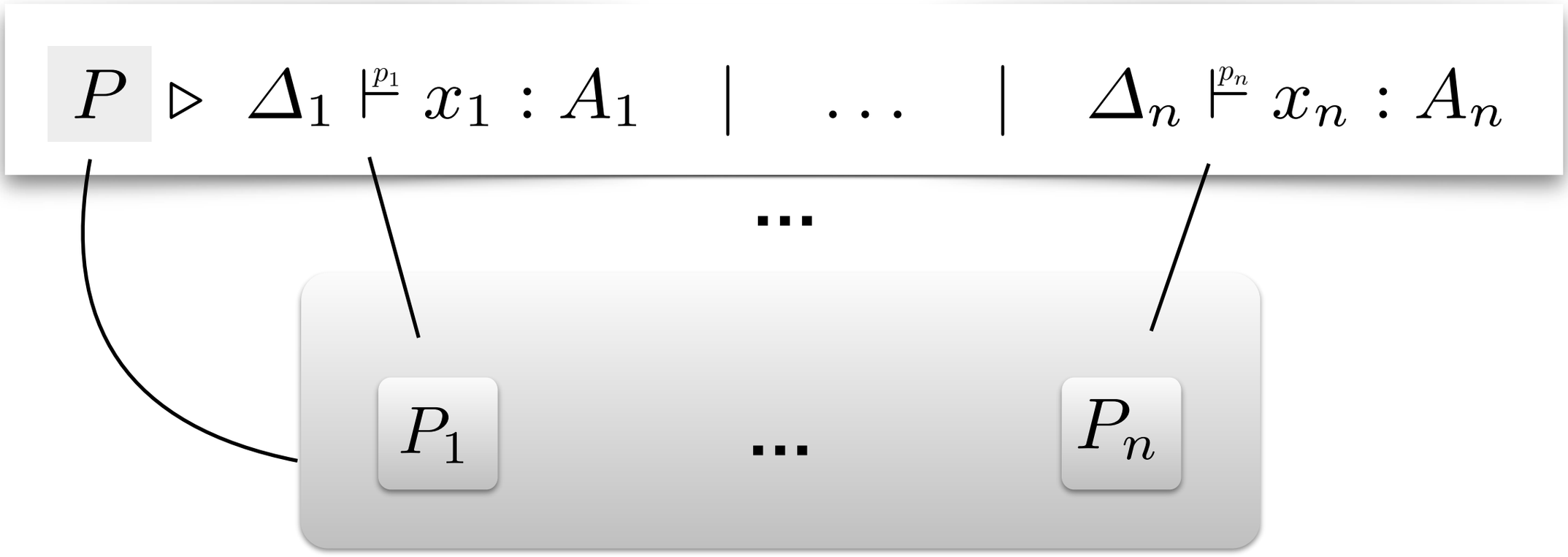


$$\textit{(Element)} \quad T ::= x : A \mid x : \bullet A \qquad \textit{(Contexts)} \quad \Delta, \Theta ::= \cdot \mid \Delta, T$$

$$\textit{(Hypersequents)} \quad \Psi ::= \Delta \vdash T \mid \Psi | \Psi$$

Hypersequents

We need new judgements, retaining more information:



$$\textit{(Element)} \quad T ::= x : A \mid x : \bullet A \qquad \textit{(Contexts)} \quad \Delta, \Theta ::= \cdot \mid \Delta, T$$

$$\textit{(Hypersequents)} \quad \Psi ::= \Delta \vdash T \mid \Psi | \Psi$$

Parallel Composition and Restriction

Our composition&restriction (Cut) is split into two rules:

$$\frac{P \triangleright \Psi_1 \mid \Delta_1 \vdash x : A \qquad Q \triangleright \Psi_2 \mid \Delta_2, x : A \vdash T}{P \mid_x Q \triangleright \Psi_1 \mid \Psi_2 \mid \Delta_1 \vdash x : \bullet A \mid \Delta_2, x : \bullet A \vdash T}\ \textsf{Conn}$$

Parallel Composition and Restriction

Our composition&restriction (Cut) is split into two rules:

$$\frac{P \triangleright \Psi_1 \mid \Delta_1 \vdash x : A \qquad Q \triangleright \Psi_2 \mid \Delta_2, x : A \vdash T}{P \mid_x Q \triangleright \Psi_1 \mid \Psi_2 \mid \Delta_1 \vdash x : \bullet A \mid \Delta_2, x : \bullet A \vdash T}\ \textsf{Conn}$$

$$\frac{P \triangleright \Psi \mid \Delta_1 \vdash x : \bullet A \mid \Delta_2, x : \bullet A \vdash T}{(\boldsymbol{\nu} x)\, P \triangleright \Psi \mid \Delta_1, \Delta_2 \vdash T}\ \textsf{Scope}$$

Parallel Composition and Restriction

Our composition&restriction (Cut) is split into two rules:

$$\frac{P \triangleright \Psi_1 \mid \Delta_1 \vdash x : A \qquad Q \triangleright \Psi_2 \mid \Delta_2, x : A \vdash T}{P \mid_x Q \triangleright \Psi_1 \mid \Psi_2 \mid \Delta_1 \vdash x : \bullet A \mid \Delta_2, x : \bullet A \vdash T}\ \textsf{Conn}$$

$$\frac{P \triangleright \Psi \mid \Delta_1 \vdash x : \bullet A \mid \Delta_2, x : \bullet A \vdash T}{(\boldsymbol{\nu} x)\, P \triangleright \Psi \mid \Delta_1, \Delta_2 \vdash T}\ \textsf{Scope}$$

Note. We wish to keep a tree-like structure for hypersequents

Language Syntax (excerpt)

$$
\begin{array}{rcll}
P, Q, R & ::= & \overline{x}(y); (P \mid Q) & \textit{(send)} \\
& \mid & \mathsf{close}[x] & \textit{(close)} \\
& \mid & P \mid_x P & \textit{(par)} \\
& \mid & p \to q : x(y); P & \textit{(global com)} \\
& \mid & x(y); P & \textit{(recv)} \\
& \mid & \mathsf{wait}[x]; P & \textit{(wait)} \\
& \mid & (\boldsymbol{\nu} x)\, P & \textit{(res)} \\
& \mid & p \to q : \mathsf{close}(x); P & \textit{(global close)}
\end{array}
$$

Choreography

Processes

Adaptation of [Caires and Pfenning'10], for processes:

$$\frac{}{\mathsf{close}[x] \;\triangleright\; \cdot \vdash x : \mathbf{1}}\ \mathsf{1R}$$

$$\frac{P \;\triangleright\; \Psi \;\mid\; \Delta \vdash T}{\mathsf{wait}[x]; P \;\triangleright\; \Psi \;\mid\; \Delta, x : \mathbf{1} \vdash T}\ \mathsf{1L}$$

Processes

$$\frac{P \triangleright \Psi_1 \mid \Delta_1 \vdash y : A \qquad Q \triangleright \Psi_2 \mid \Delta_2 \vdash x : B}{\overline{x}(y); (P \mid Q) \triangleright \Psi_1 \mid \Psi_2 \mid \Delta_1, \Delta_2 \vdash x : A \otimes B} \otimes\mathsf{R}$$

$$\frac{P \triangleright \Psi \mid \Delta, y : A, x : B \vdash T}{x(y); P \triangleright \Psi \mid \Delta, x : A \otimes B \vdash T} \otimes\mathsf{L}$$

Choreographies

$$\frac{P \triangleright \Psi \mid \Delta_1 \vdash y : \bullet A \mid \Delta_2 \vdash x : \bullet B \mid \Delta_3, y : \bullet A, x : \bullet B \vdash T}{p \to q : x(y); P \triangleright \Psi \mid \Delta_1, \Delta_2 \vdash x : \bullet A \otimes B \mid \Delta_3, x : \bullet A \otimes B \vdash T} \otimes \mathsf{C}$$

Choreographies

$$\frac{P \triangleright \Psi \mid \Delta_1 \vdash y : \bullet A \mid \Delta_2 \vdash x : \bullet B \mid \Delta_3, y : \bullet A, x : \bullet B \vdash T}{p \to q : x(y); P \triangleright \Psi \mid \Delta_1, \Delta_2 \vdash x : \bullet A \otimes B \mid \Delta_3, x : \bullet A \otimes B \vdash T} \otimes\mathsf{C}$$

1.

$$\frac{P \triangleright \Psi_1 \mid \Delta_1 \vdash y : A \qquad Q \triangleright \Psi_2 \mid \Delta_2 \vdash x : B}{\overline{x}(y); (P \mid Q) \triangleright \Psi_1 \mid \Psi_2 \mid \Delta_1, \Delta_2 \vdash x : A \otimes B} \otimes\mathsf{R} \qquad \frac{P \triangleright \Psi \mid \Delta, y : A, x : B \vdash T}{x(y); P \triangleright \Psi \mid \Delta, x : A \otimes B \vdash T} \otimes\mathsf{L}$$

Choreographies

$$\frac{P \triangleright \Psi \mid \Delta_1 \vdash y : \bullet A \mid \Delta_2 \vdash x : \bullet B \mid \Delta_3, y : \bullet A, x : \bullet B \vdash T}{p \to q : x(y); P \triangleright \Psi \mid \Delta_1, \Delta_2 \vdash x : \bullet A \otimes B \mid \Delta_3, x : \bullet A \otimes B \vdash T} \otimes\mathsf{C}$$

1.

$$\frac{P \triangleright \Psi_1 \mid \Delta_1 \vdash y : A \qquad Q \triangleright \Psi_2 \mid \Delta_2 \vdash x : B}{\overline{x}(y); (P \mid Q) \triangleright \Psi_1 \mid \Psi_2 \mid \Delta_1, \Delta_2 \vdash x : A \otimes B} \otimes\mathsf{R} \qquad \frac{P \triangleright \Psi \mid \Delta, y : A, x : B \vdash T}{x(y); P \triangleright \Psi \mid \Delta, x : A \otimes B \vdash T} \otimes\mathsf{L}$$

2.

$$\frac{\dfrac{C_1 \vdash A \qquad C_2 \vdash B}{C_1, C_2 \vdash A \otimes B} \otimes\mathsf{R} \qquad \dfrac{A, B \vdash D}{A \otimes B \vdash D} \otimes\mathsf{L}}{C_1, C_2 \vdash D}\ \mathsf{Cut} \quad \Longrightarrow \quad \frac{C_1 \vdash A \qquad \dfrac{C_2 \vdash B \qquad A, B \vdash D}{C_2, A \vdash D}\ \mathsf{Cut}}{C_1, C_2 \vdash D}\ \mathsf{Cut}$$

Choreographies

$$\frac{P \;\triangleright\; \Psi \;\mid\; \Delta \vdash T}{p \to q : \mathsf{close}(x); P \;\triangleright\; \Psi \;\mid\; \cdot \vdash x : \bullet\mathbf{1} \;\mid\; \Delta, x : \bullet\mathbf{1} \vdash T}\ \mathsf{1C}$$

LCC, Processes

$$\frac{P \triangleright \Psi_1 | \Delta_1 \vdash y : A \qquad Q \triangleright \Psi_2 | \Delta_2 \vdash x : B}{\overline{x}(y); (P|Q) \triangleright \Psi_1 | \Psi_2 | \Delta_1, \Delta_2 \vdash x : A \otimes B} \otimes\mathsf{R} \qquad \frac{P \triangleright \Psi | \Delta, y : A, x : B \vdash T}{x(y); P \triangleright \Psi | \Delta, x : A \otimes B \vdash T} \otimes\mathsf{L}$$

$$\frac{P \triangleright \Psi | \Delta, y : A \vdash x : B}{x(y); P \triangleright \Psi | \Delta \vdash x : A \multimap B} \multimap \mathsf{R} \qquad \frac{P \triangleright \Psi_1 | \Delta_1 \vdash y : A \qquad Q \triangleright \Psi_2 | \Delta_2, x : B \vdash T}{\overline{x}(y); (P|Q) \triangleright \Psi_1 | \Psi_2 | \Delta_1, \Delta_2, x : A \multimap B \vdash T} \multimap \mathsf{L}$$

$$\frac{}{\mathsf{close}[x] \triangleright \cdot \vdash x : \mathbf{1}} \mathsf{1R} \qquad \frac{P \triangleright \Psi | \Delta, x : A \vdash T}{x.\mathsf{inl}; P \triangleright \Psi | \Delta, x : A \& B \vdash T} \&\mathsf{L}_1 \qquad \frac{Q \triangleright \Psi | \Delta, x : B \vdash T}{x.\mathsf{inr}; Q \triangleright \Psi | \Delta, x : A \& B \vdash T} \&\mathsf{L}_2$$

$$\frac{P \triangleright \Psi | \Delta \vdash T}{\mathsf{wait}[x]; P \triangleright \Psi | \Delta, x : \mathbf{1} \vdash T} \mathsf{1L} \qquad \frac{P \triangleright \Psi | \Delta \vdash x : A}{x.\mathsf{inl}; P \triangleright \Psi | \Delta \vdash x : A \oplus B} \oplus\mathsf{R}_1 \qquad \frac{Q \triangleright \Psi | \Delta \vdash x : B}{x.\mathsf{inr}; Q \triangleright \Psi | \Delta \vdash x : A \oplus B} \oplus\mathsf{R}_2$$

$$\frac{P \triangleright \Psi | \Delta \vdash x : A \qquad Q \triangleright \Psi | \Delta \vdash x : B}{x.\mathsf{case}(P, Q) \triangleright \Psi | \Delta \vdash x : A \& B} \&\mathsf{R} \qquad \frac{P \triangleright \Psi | \Delta, x : A \vdash T \qquad Q \triangleright \Psi | \Delta, x : B \vdash T}{x.\mathsf{case}(P, Q) \triangleright \Psi | \Delta, x : A \oplus B \vdash T} \oplus\mathsf{L}$$

LCC, Choreographies

$$\frac{P \triangleright \Psi|\Delta_1 \vdash y:\bullet A|\Delta_2 \vdash x:\bullet B|\Delta_3, y:\bullet A, x:\bullet B \vdash T}{\overrightarrow{x(y)}; P \triangleright \Psi|\Delta_1, \Delta_2 \vdash x:\bullet A \otimes B|\Delta_3, x:\bullet A \otimes B \vdash T}\ \otimes\mathsf{C}$$

$$\frac{P \triangleright \Psi|\Delta_1 \vdash y:\bullet A|\Delta_2, y:\bullet A \vdash x:\bullet B|\Delta_3, x:\bullet B \vdash T}{\overrightarrow{x(y)}; P \triangleright \Psi|\Delta_2 \vdash x:\bullet A \multimap B|\Delta_1, \Delta_3, x:\bullet A \multimap B \vdash T}\ \multimap\mathsf{C}$$

$$\frac{P \triangleright \Psi|\Delta \vdash T}{\overrightarrow{\mathsf{close}[x]}; P \triangleright \Psi|\cdot \vdash x:\bullet\mathbf{1}|\Delta, x:\bullet\mathbf{1} \vdash T}\ \mathsf{1C}$$

$$\frac{P \triangleright \Psi|\Psi'|\Delta_1 \vdash x:\bullet A|\Delta_2, x:\bullet A \vdash T \qquad Q \triangleright \Psi'|\Delta_1 \vdash x:B}{\overrightarrow{x.\mathsf{l}}(P,Q) \triangleright \Psi|\Psi'|\Delta_1 \vdash x:\bullet A\&B|\Delta_2, x:\bullet A\&B \vdash T}\ \&\mathsf{C}_1$$

$$\frac{P \triangleright \Psi|\Psi'|\Delta_1 \vdash x:\bullet A|\Delta_2, x:\bullet A \vdash T \qquad Q \triangleright \Psi'|\Delta_2, x:B \vdash T}{\overrightarrow{x.\mathsf{l}}(P,Q) \triangleright \Psi|\Psi'|\Delta_1 \vdash x:\bullet A \oplus B|\Delta_2, x:\bullet A \oplus B \vdash T}\ \oplus\mathsf{C}_1$$

$$\frac{P \triangleright \Psi|\Delta_1 \vdash x:A \qquad Q \triangleright \Psi|\Psi'|\Delta_1 \vdash x:\bullet B|\Delta_2, x:\bullet B \vdash T}{\overrightarrow{x.\mathsf{r}}(P,Q) \triangleright \Psi|\Psi'|\Delta_1 \vdash x:\bullet A\&B|\Delta_2, x:\bullet A\&B \vdash T}\ \&\mathsf{C}_2$$

$$\frac{P \triangleright \Psi|\Delta_2, x:A \vdash T \qquad Q \triangleright \Psi|\Psi'|\Delta_1 \vdash x:\bullet B|\Delta_2, x:\bullet B \vdash T}{\overrightarrow{x.\mathsf{r}}(P,Q) \triangleright \Psi|\Psi'|\Delta_1 \vdash x:\bullet A \oplus B|\Delta_2, x:\bullet A \oplus B \vdash T}\ \oplus\mathsf{C}_2$$

- LCC is also a conservative extension of intuitionistic linear logic

Theorem. If $\Delta \vdash A$ in linear logic then $\Delta \vdash A$ in LCL

Semantics

Semantics = Scope Reductions

For processes:

$$(\boldsymbol{\nu} x)\left(\overline{x}(y); (P \mid Q) \mid_x x(y); R\right) \xrightarrow{x} (\boldsymbol{\nu} x)\,(\boldsymbol{\nu} y)\,\left(Q \mid_x (P \mid_y R)\right)$$

Semantics = Scope Reductions

For processes:

$$(\boldsymbol{\nu} x)\big(\overline{x}(y); (P \mid Q) \mid_x x(y); R\big) \xrightarrow{x} (\boldsymbol{\nu} x)\,(\boldsymbol{\nu} y)\,\big(Q \mid_x (P \mid_y R)\big)$$

$$\dfrac{\dfrac{\dfrac{\dfrac{P \triangleright \Psi_1 \mid \Delta_1 \vdash y : A \quad Q \triangleright \Psi_2 \mid \Delta_2 \vdash x : B}{\overline{x}(y); (P|Q) \triangleright \Psi_1 \mid \Psi_2 \mid \Delta_1, \Delta_2 \vdash x : A \otimes B}\otimes\mathsf{R} \quad \dfrac{R \triangleright \Psi_3 \mid \Delta_3, y : A, x : B \vdash T}{x(y); R \triangleright \Psi_3 \mid \Delta_3, x : A \otimes B \vdash T}\otimes\mathsf{L}}{\overline{x}(y); (P|Q) \mid_x x(y); R \triangleright \Psi_1 \mid \Psi_2 \mid \Psi_3 \mid \Delta_1, \Delta_2 \vdash x : \bullet A \otimes B \mid \Delta_3, x : \bullet A \otimes B \vdash T}\mathsf{Conn}^x}{(\boldsymbol{\nu} x)\,(\overline{x}(y); (P|Q) \mid_x x(y); R) \triangleright \Psi_1 \mid \Psi_2 \mid \Psi_3 \mid \Delta_1, \Delta_2, \Delta_3 \vdash T}\mathsf{Scope}^x$$

$$\xrightarrow{x}$$

$$\dfrac{\dfrac{\dfrac{P \triangleright \Psi_1 \mid \Delta_1 \vdash y : A \quad \dfrac{Q \triangleright \Psi_2 \mid \Delta_2 \vdash x : B \quad R \triangleright \Psi_3 \mid \Delta_3, y : A, x : B \vdash T}{Q \mid_x R \triangleright \Psi_2 \mid \Psi_3 \mid \Delta_2 \vdash x : \bullet B \mid \Delta_3, y : A, x : \bullet B \vdash T}\mathsf{Conn}^x}{P \mid_y (Q \mid_x R) \triangleright \Psi_1 \mid \Psi_2 \mid \Psi_3 \mid \Delta_1 \vdash y : \bullet A \mid \Delta_2 \vdash x : \bullet B \mid \Delta_3, y : \bullet A, x : \bullet B \vdash T}\mathsf{Conn}^y}{(\boldsymbol{\nu} x)\,(P \mid_y (Q \mid_x R)) \triangleright \Psi_1 \mid \Psi_2 \mid \Psi_3 \mid \Delta_1 \vdash y : \bullet A \mid \Delta_2, \Delta_3, y : \bullet A \vdash T}\mathsf{Scope}^x}{(\boldsymbol{\nu} y)\,(\boldsymbol{\nu} x)\,(P \mid_y (Q \mid_x R)) \triangleright \Psi_1 \mid \Psi_2 \mid \Psi_3 \mid \Delta_1, \Delta_2, \Delta_3 \vdash T}\mathsf{Scope}^y$$

Semantics = Scope Reductions

And for choreographies:

$$[\beta_{\otimes \mathsf{C}}] \qquad (\boldsymbol{\nu} x)\, \mathsf{p} \to \mathsf{q} : x(y);\ P \xrightarrow{\bullet x} (\boldsymbol{\nu} y)\,(\boldsymbol{\nu} x)\, P$$

$$\dfrac{\dfrac{P \triangleright \Psi \mid \Delta_1 \vdash x : \bullet B \mid \Delta_2 \vdash y : \bullet A \mid \Delta_3, y : \bullet A, x : \bullet B \vdash T}{\mathsf{p} \to \mathsf{q} : x(y); P \triangleright \Psi \mid \Delta_1, \Delta_2 \vdash x : \bullet A \otimes B \mid \Delta_3, x : \bullet A \otimes B \vdash T}\ \otimes \mathsf{C}^{x}}{(\boldsymbol{\nu} x)\, \mathsf{p} \to \mathsf{q} : x(y); P \triangleright \Psi \mid \Delta_1, \Delta_2, \Delta_3 \vdash T}\ \mathsf{Scope}^{x}$$

$$\xrightarrow{\bullet x}$$

$$\dfrac{\dfrac{P \triangleright \Psi | \Delta_1 \vdash x : \bullet B \mid \Delta_2 \vdash y : \bullet A \mid \Delta_3, y : \bullet A, x : \bullet B \vdash T}{(\boldsymbol{\nu} x)\ P \triangleright \Psi | \Delta_2 \vdash y : \bullet A \mid \Delta_1, \Delta_3, y : \bullet A \vdash T}\ \mathsf{Scope}^{x}}{(\boldsymbol{\nu} y)\,(\boldsymbol{\nu} x)\ P \triangleright \Psi \mid \Delta_1, \Delta_2, \Delta_3 \vdash T}\ \mathsf{Scope}^{y}$$

Scope Elimination

Theorem 3 (Deadlock-freedom). *$P \triangleright \Psi$ implies there exist Q restriction-free and $\tilde{t}$ such that $P \overset{\tilde{t}}{\longrightarrow\!\!\!\!\!\rightarrow} Q$ and $Q \triangleright \Psi$.*

i.e., any instance of scope can be eliminated.

Projection and Extraction

Projection and Extraction

Processes and choreographies are interconnected:

$$\frac{P \triangleright \Psi_1 \mid \Delta_1 \vdash y : A \qquad Q \triangleright \Psi_2 \mid \Delta_2 \vdash x : B}{\overline{x}(y); (P \mid Q) \triangleright \Psi_1 \mid \Psi_2 \mid \Delta_1, \Delta_2 \vdash x : A \otimes B} \otimes\mathsf{R}$$

$$\frac{P \triangleright \Psi \mid \Delta, y : A, x : B \vdash T}{x(y); P \triangleright \Psi \mid \Delta, x : A \otimes B \vdash T} \otimes\mathsf{L}$$

$$\frac{P \triangleright \Psi \mid \Delta_1 \vdash y : \bullet A \mid \Delta_2 \vdash x : \bullet B \mid \Delta_3, y : \bullet A, x : \bullet B \vdash T}{p \to q : x(y); P \triangleright \Psi \mid \Delta_1, \Delta_2 \vdash x : \bullet A \otimes B \mid \Delta_3, x : \bullet A \otimes B \vdash T} \otimes\mathsf{C}$$

Projection and Extraction

We can actually formally relate the two…

$$\dfrac{\dfrac{P \triangleright \Psi_1 | \Delta_1 \vdash y : A \qquad Q \triangleright \Psi_2 | \Delta_2 \vdash x : B}{\overline{x}(y); (P|Q) \triangleright \Psi_1 | \Psi_2 | \Delta_1, \Delta_2 \vdash x : A \otimes B}\otimes\mathsf{R} \qquad \dfrac{R \triangleright \Psi_3 | \Delta_3, y : A, x : B \vdash T}{x(y); R \triangleright \Delta_3, x : A \otimes B \vdash T}\otimes\mathsf{L}}{\overline{x}(y); (P|Q) \mid_x x(y); R \triangleright \Psi_1 | \Psi_2 | \Psi_3 | \Delta_1, \Delta_2 \vdash x : \bullet A \otimes B | \Delta_3, x : \bullet A \otimes B \vdash T}\,\mathsf{Conn}$$

$$\Downarrow x$$

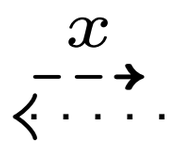
$$\dfrac{P \triangleright \Psi_1 | \Delta_1 \vdash y : A \qquad \dfrac{Q \triangleright \Psi_2 | \Delta_2 \vdash x : B \qquad R \triangleright \Psi_3 \mid \Delta_3, y : A, x : B \vdash T}{Q \mid_x R \triangleright \Psi_2 | \Psi_3 | \Delta_2 \vdash x : \bullet B | \Delta_3, y : A, x : \bullet B \vdash T}\,\mathsf{Conn}^x}{P \mid_y (Q \mid_x R) \triangleright \Psi_1 \mid \Psi_2 \mid \Psi_3 \mid \Delta_1 \vdash y : \bullet A \mid \Delta_2 \vdash x : \bullet B \mid \Delta_3, y : \bullet A, x : \bullet B \vdash T}\,\mathsf{Conn}^y$$

$$\dfrac{}{\mathsf{p} \to \mathsf{q} : x(y); (P \mid_y (Q \mid_x R)) \triangleright \Psi_1 \mid \Psi_2 \mid \Psi_3 \mid \Delta_1, \Delta_2 \vdash x : \bullet A \otimes B \mid \Delta_3, x : \bullet A \otimes B \vdash T}\otimes\mathsf{C}^x$$

Projection and Extraction

$$\frac{}{\mathsf{close}[x] \triangleright \cdot \vdash x : \mathbf{1}}\ 1\mathrm{R} \qquad \frac{P \triangleright \Psi \mid \Delta \vdash T}{\mathsf{wait}[x]; P \triangleright \Psi \mid \Delta, x : \mathbf{1} \vdash T}\ 1\mathrm{L}$$

$$\frac{}{\mathsf{close}[x] \mid_x \mathsf{wait}[x]; P \triangleright \Psi \mid \cdot \vdash x : \bullet\mathbf{1} \mid \Delta, x : \bullet\mathbf{1} \vdash T}\ \mathrm{Conn}$$



$$\frac{P \triangleright \Psi \mid \Delta \vdash T}{\mathsf{p} \longrightarrow \mathsf{q} : \ \mathsf{close}(x); P \triangleright \Psi \mid \cdot \vdash x : \bullet\mathbf{1} \mid \Delta, x : \bullet\mathbf{1} \vdash T}\ 1\mathrm{C}$$

Main Results

Theorem 4 (Extraction and Projection). *Let* $P \triangleright \Psi$. *Then:*

(choreography extraction) $P \overset{\tilde{x}}{\dashrightarrow\!\!\!\!\rightarrow} Q$ *for some* $\tilde{x}$ *and* Q *such that* $Q \triangleright \Psi$ *and* Q *does not contain subterms of the form* $R \mid_x R'$;

(endpoint projection) $P \overset{\tilde{x}}{\cdots\!\!\!\!\twoheadrightarrow} Q$ *for some* $\tilde{x}$ *and* Q *such that* $Q \triangleright \Psi$ *and* Q *does not contain choreography terms.*

Theorem 5 (Correspondence). $P \triangleright \Psi$ *implies:*

(CE) $P \xrightarrow{\tilde{x}} P'$, *with* P' *restriction-free, implies* $P \overset{\tilde{x}}{\dashrightarrow\!\!\!\!\rightarrow} Q$ *such that* $Q \xrightarrow{\bullet\tilde{x}} P'$.

(EPP) $P \xrightarrow{\bullet\tilde{x}} P'$, *with* P' *restriction-free, implies* $P \overset{\tilde{x}}{\cdots\!\!\!\!\twoheadrightarrow} Q$ *such that* $Q \xrightarrow{\tilde{x}} P'$.

Proof Idea…

- It is necessary to transform proofs so that reductions and EPP/CE can be applied

- Scope and Conn can always be commuted towards one another

- Scope and C-rules can always be commuted towards one another

- Applications of Scope can always be eliminated
- C-rules can always be eliminated
- Conn-rules can always be eliminated

Conclusions and Future Work

- Logical Characterisation of Choreographies
- Choreography Extraction
- *Future Work*: Exponentials and Iterative Behaviour
- *Future Work*: Multiparty Session Types and Choreographies?
- *Future Work*: More advanced constructs?