

To focus on winning one competition or try to win more: Using a Bayesian network to help decide the optimum strategy for a football club

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In mid-February 2016 – two thirds of the way through the English Premier League season - Tottenham Hotspur (Spurs) found themselves in a position that was unprecedented in the modern era. They were on such a strong run of form that they were the clear bookmakers' favourites to win the league title (at odds of 9 to 4). To get a feel for how unlikely a position this was, they had started the season at 100 to 1; the last time Spurs won the title was in 1961 and they have failed to finish higher than 3rd since 1963. Since 1990 their highest position was 4th – which they have managed just (2010, 2012).

Winning the league would constitute a once-in-a-lifetime achievement for Spurs and its supporters. The glory and financial rewards would be enormous. It would also give their supporters 'bragging rights' over their richer rivals Arsenal for many years to come as this was the season when it was expected that Arsenal would win the title for the first time in 10 years having come close on most of the intervening seasons (and never finishing below 4th for 20 years).

It is also important to note that even finishing in the top 4 of the league – which ensures a place in the following season's European Champions League (ECL) (bringing enormous prestige and huge financial reward) would have been considered a great achievement by Spurs at the start of the season. The odds on them achieving it then were 5-1. But in February 2016 the odds were now just 1-10.

However, the complication was that Spurs were also still involved in two knock-out competitions – the FA Cup and the Europa League (EL). It is widely assumed that playing many matches in these competitions is detrimental to league form. For this reason teams who are challenging for the title typically rest many of their key players in those competitions and hence are less likely to progress. But there are notable differences between the two competitions:

- The FA Cup is limited to English teams and still has great history and prestige associated with it. As Spurs were already in the last 16 it would be possible to win the trophy playing only 4 extra matches.
- In contrast the EL involves teams from all over Europe and there is little glory in winning it. The main incentive to win it is that the winners qualify for the ECL – but with Spurs looking very likely to qualify anyway, this incentive was diminished. Spurs were in the last 32 and with each match before the final requiring a home and away leg, seven extra matches (four of which would require foreign travel) would need to be played to win the trophy.

So the question for the Spurs manager was which of the following strategic decisions was optimal:

1. Focus on league
2. Focus on league and FA Cup
3. Focus on league and EL
4. Focus on all three competitions

This is a classic decision utility problem which can be solved with a Bayesian network (BN). The BN¹ is shown in Figure 1.

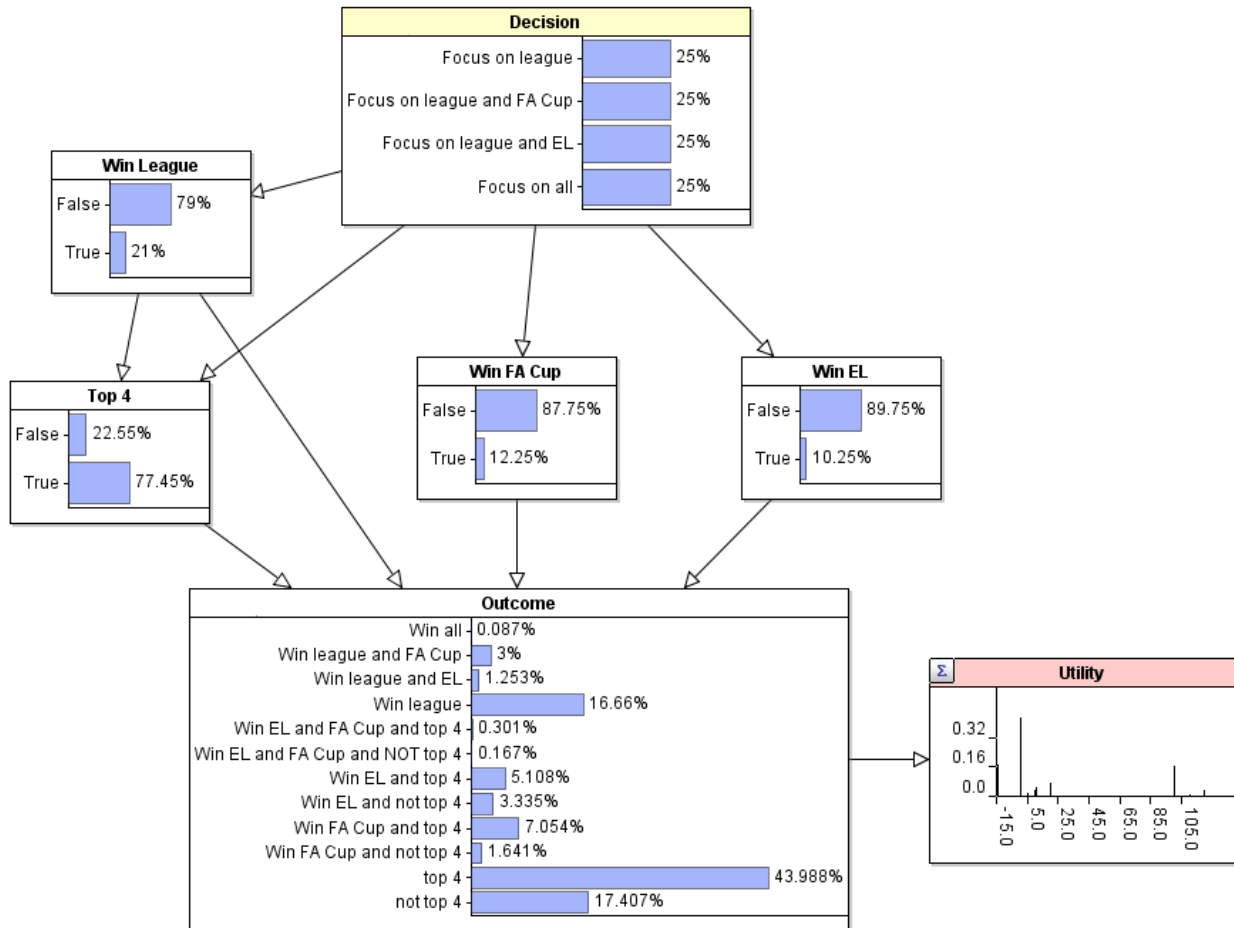


Figure 1 Bayesian network for the Spurs strategy decision problem

The BN consists of a single **decision node** (whose prior probabilities are assumed equal), four atomic **outcome nodes** which are uncertain and are conditioned on the decision, one composite outcome node, and a single **utility node**.

Before showing the conditional probability tables for the outcome nodes and utility node, it is important to note that the probability values shown in Figure 1 are the marginal probabilities that are computed when the model is run without any observations, i.e. they represent our state of uncertain knowledge when no decision is made. The reason the probabilities such as for, say Spurs winning the league, are different (lower in this case) than the bookmakers

¹ Model available to download from www.eecs.qmul.ac.uk/~norman/Models/spurs_decision_problems.cmp and can be run in free version of AgenaRisk downloadable from www.agenarisk.com/products/free_download.shtml

odds is because the bookmakers odds factor in their judgment about the different probabilities for the different decisions. So, in reality, the bookmakers assume that managers in this position would be much more likely to opt for strategy 1 (focus on league). Figure 2 shows the marginal probabilities in the case where we provide priors for the decision node that match more closely to bookmaker assumptions. In this case the marginal probabilities for the different outcomes closely match the actual bookmakers' odds as of mid-February 2016.

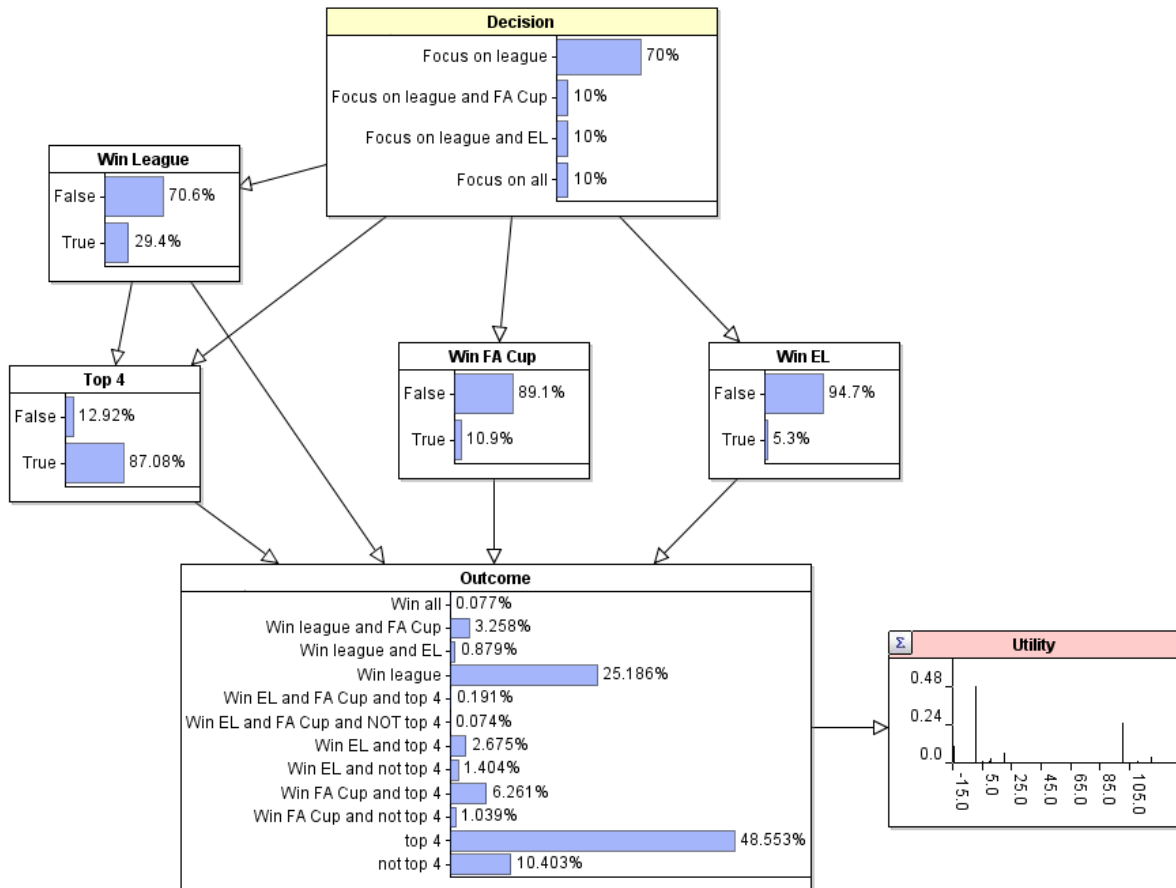


Figure 2: Non-uniform priors for the decision node based on bookmakers assumptions

The conditional probability tables for “win league”, “win FA Cup” and “win EL” are shown in Figures 3-5 respectively.

Decision	Focus on league	Focus on league and FA Cup	Focus on league and EL	Focus on all
False	0.65	0.74	0.87	0.9
True	0.35	0.26	0.13	0.1

Figure 3 Table for 'Win league'

Decision	Focus on league	Focus on league and FA Cup	Focus on league and EL	Focus on all
False	0.9	0.7	0.95	0.96
True	0.1	0.3	0.05	0.04

Figure 4 Table for 'Win FA Cup'

Decision	Focus on league	Focus on league and FA Cup	Focus on league and EL	Focus on all
False	0.98	0.99	0.8	0.82
True	0.02	0.01	0.2	0.18

Figure 5 NPT for 'win EL'

The table for 'top 4' is slightly different as it also encodes the logical certainty that 'top 4' is true when 'win league' is true.

Win League	False				True			
	Focus on league	Focus on league and FA Cup	Focus on league and EL	Focus on all	Focus on l...	Focus on l...	Focus on le...	Focus on all
False	0.1	0.15	0.4	0.42	0.0	0.0	0.0	0.0
True	0.9	0.85	0.6	0.58	1.0	1.0	1.0	1.0

Figure 6 Table for "top 4"

The table for the composite outcome node is defined logically as shown in Figure 7.

Win League	False								True							
	False				True				False				True			
	False		True		False		True		False		True		False		True	
Win FA Cup	False	True	False	True	False	True	False	True	False	True	False	True	False	True	False	True
Win EL	False	True	False	True	False	True	False	True	False	True	False	True	False	True	False	True
Top 4	False	True	False	True	False	True	False	True	False	True	False	True	False	True	False	True
Win all	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0
Win league and FA Cup	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0
Win league and EL	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	0.0
Win league	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
Win EL and FA Cup and top 4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Win EL and FA Cup and NOT top 4	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Win EL and top 4	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Win EL and not top 4	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Win FA Cup and top 4	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Win FA Cup and not top 4	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
top 4	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
not top 4	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Figure 7 Table for composite outcome node

We base the utility of the composite outcomes on the following assumptions for the atomic outcomes (the opposite outcome in each case is assumed to have 0 utility).

outcome	value
Win league	100
Win FA Cup	20
Win EL	10
Not top 4	-5

So, for example, the composite outcome “win all” has a value of 130, but “win FA Cup and not top 4” has value 15. We use these utility values as arithmetic expressions after defining the table for the utility node as a ‘partitioned expression’ on parent ‘outcome’.

We can now compute the utilities of each possible decision by simply ‘observing’ each decision in turn and running the model. The results are shown in Figures 8-12. In summary we have:

Decision	(Mean) utility
1. Focus on league	36
2. Focus on league and FA Cup	30
3. Focus on league and EL	12
4. Focus on all	8

So the decision to focus on the league is the optimal strategy.

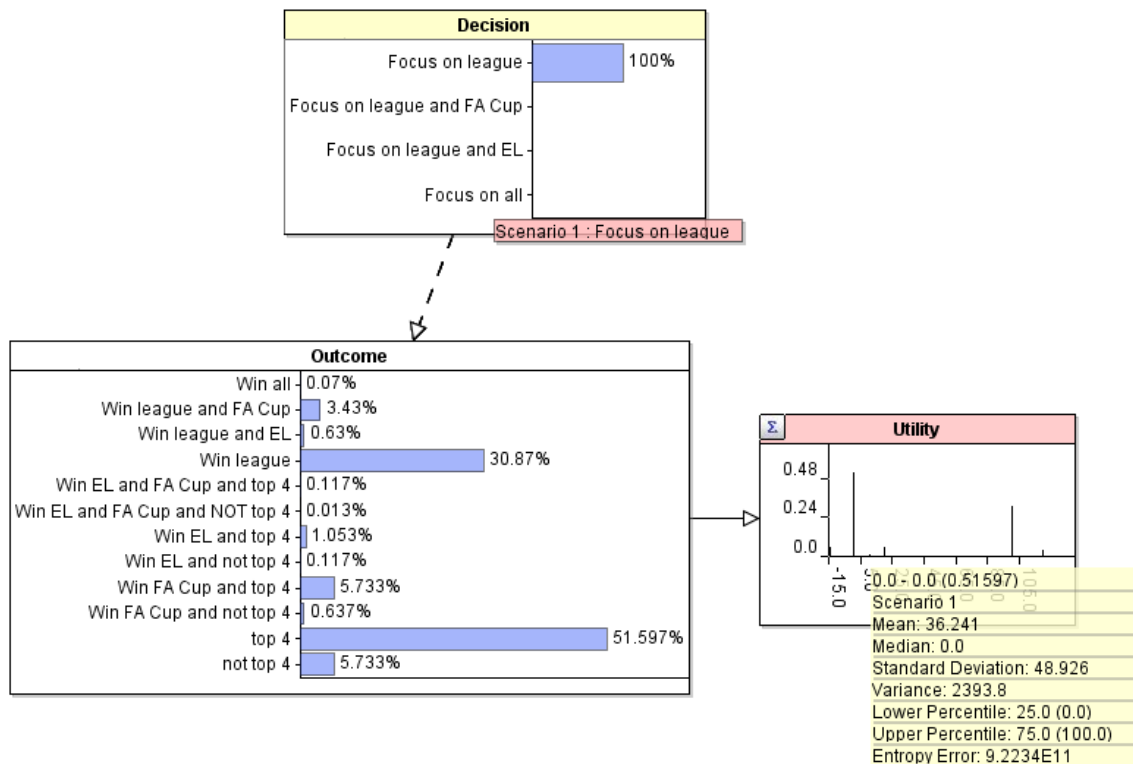


Figure 8 Results for decision 1 (focus on league)

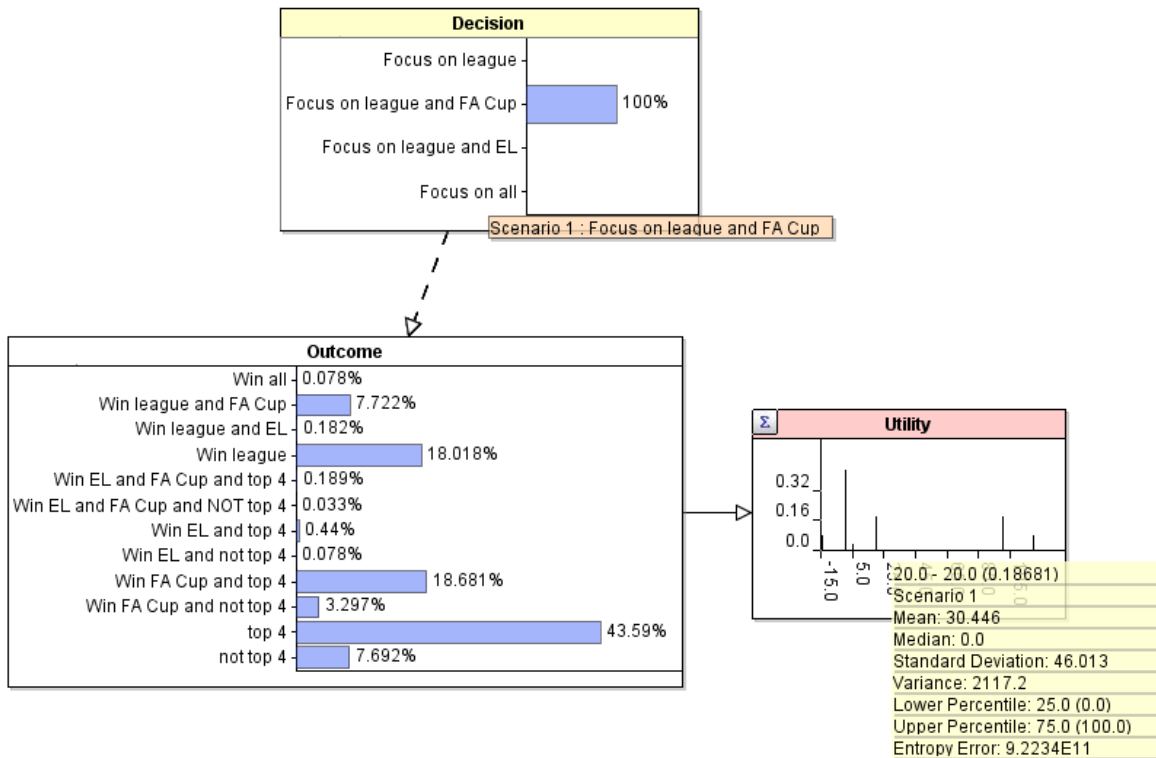


Figure 9 Results for decision 2 (focus on league and FA Cup)

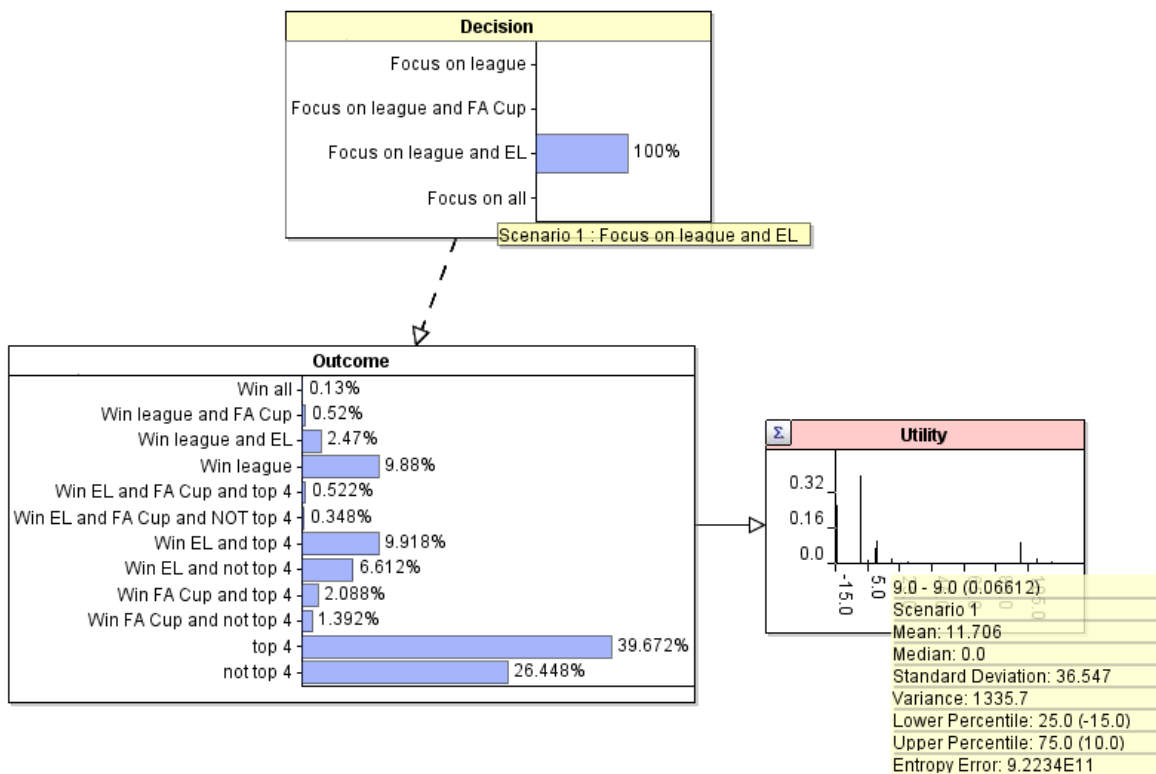


Figure 10 Results for decision 3 (focus on league and EL)

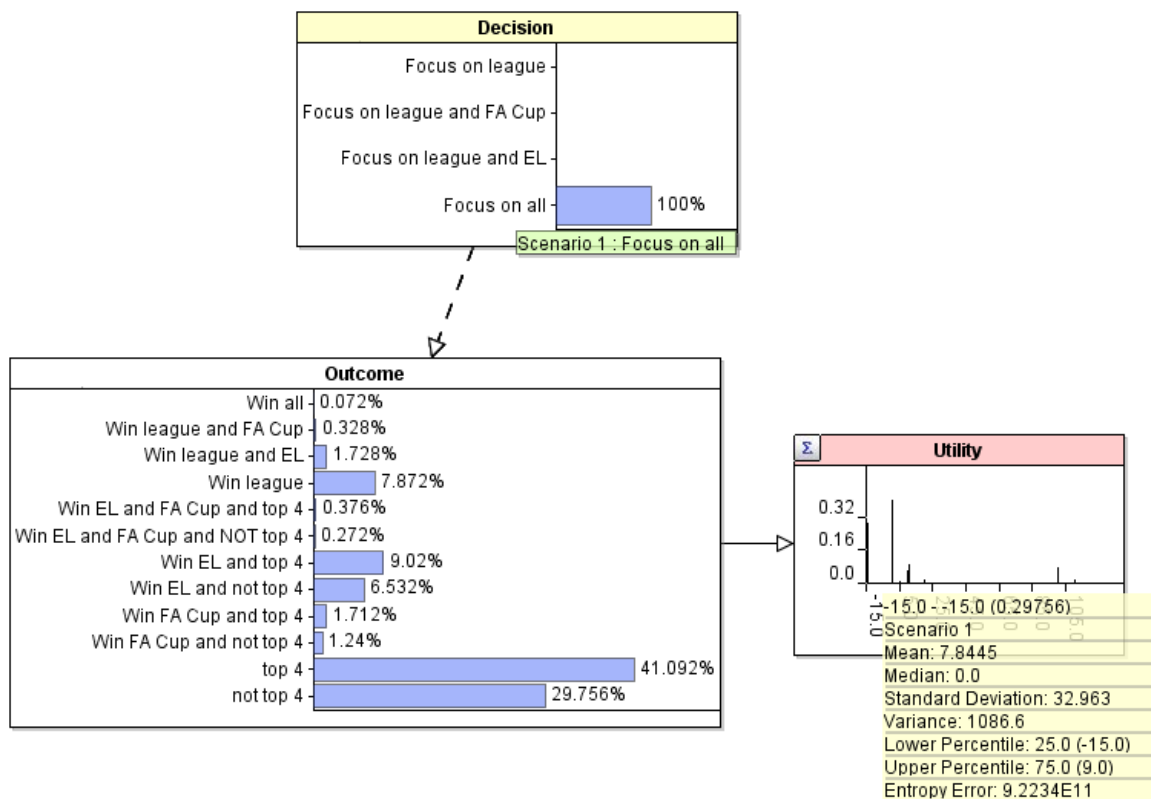


Figure 11 Results for decision 4 (focus on all)

But that is not the decision that Spurs manager Mauricio Pochettino opted for. At the time of writing this article, it was clear that he had instead opted for decision 3 “focus on league and EL”. This is evident from the following

- He played more or less his strongest team in the two legs of the EL tie against a very strong side from Italy (Fiorentina) winning 4-1 on aggregate.
- He played a very weakened side in the FA Cup tie against a poor team (Crystal Palace) at home. Spurs lost 1-0.
- He continued to play more or less his strongest team in the league matches.

Possibly as a result of the intense matches against Fiorentina Spurs league form dipped in subsequent matches and Leicester moved above Spurs as favourites for winning the league.

The reason for Pochettino’s decision is that, having played and managed for most of his career in Spain, the FA Cup has relatively little value to him, whereas winning the EL would be considered a major contribution to his CV. He would also be unaware of the enormity of what a league win would mean for Spurs. In other words Pochettino’s model must necessarily incorporate a different utility value for winning the EL. Indeed if we simply change this value to 150 in the model then decision 3 becomes the optimal strategy (with utility 39 compared to 38 for decision 1).