

Bayesian networks for unbiased assessment of referee bias in Association Football

Anthony Costa Constantinou^{*,1,2}, Norman Elliott Fenton¹, Liam Jackson Hunter Pollock³

1. Risk and Information Management Research Group, School of Electronic Engineering and Computer Science.
2. Forensic Mental Health Research Group, Centre for Psychiatry, Wolfson Institute of Preventive Medicine, Barts and The London School of Medicine and Dentistry.
3. Biological and Experimental Psychology Research Group, The School of Biological and Chemical Sciences.

Queen Mary, University of London, London, UK, E1 4NS

* Corresponding author. E-mail address: anthony@constantinou.info

ABSTRACT

We present a novel Bayesian network model for assessing referee bias with respect to fouls and penalty kicks awarded. Unlike previous studies, our model takes into consideration explanatory factors which, if ignored, can lead to biased assessments of referee bias. For example, a team may be awarded more penalties simply because it attacks more, not because referees are biased in its favour. Hence, we incorporate causal factors such as possession, time spent in the opposition penalty box, etc. prior to estimating the degree of penalty kicks bias. The model is applied to the 2011-12 English Premier League season. Among our conclusions are that, in contrast to previous studies, being the home team does not in itself result in positive referee bias. Conversely Arsenal, a team of similar popularity and wealth and who finished third, benefited least of all 20 teams from referee bias at home with respect to penalty kicks awarded.

Keywords: causal modelling, crowd effect, home advantage, officiating bias, soccer

1 INTRODUCTION

The notion that referees in Association Football (hereafter referred to simply as *football*) are biased towards certain teams or in certain contexts is widely accepted by football pundits and supporters. In fact, whether or not such bias exists is an area of increasing interest that attracts the attention of researchers from the domains of sport science, psychology, statistics and computer science.

Irrespective of the true underlying causes, there is no doubt that 'playing at home' has a significant impact on a team's success. This home advantage effect has been extensively studied (Courneya & Carron, 1992; Nevill & Holder, 1999; Hirotsu & Wright, 2003; Pollard & Pollard, 2005; Pollard, 1986; 2006; Poulter, 2009; Anders & Rothhoff, 2012; Constantinou

37 & Fenton, 2013). Numerous explanatory factors have been proposed for home advantage.
38 The crowd effect is normally suggested as one of the most important factors (Agnew &
39 Carron, 1994; Nevill et al., 1996; Nevill et al., 1999; 2002; Downward & Jones, 2007;
40 Dohmen, 2008; Goumas, 2012) and is said to occur to a greater extent in leagues in which
41 home crowds are more hostile and vociferous (Anders & Rotthoff, 2012). Other proposed
42 factors include the travelling effect (Clarke & Norman, 1995), the familiarity with the playing
43 grounds (Neave & Wolfson, 2003; Pollard, 2006), as well as referees themselves who are
44 said to favour home teams on the basis of penalty kicks, free kicks, yellow/red cards and/or
45 extra time data (Nevill et al., 1996; Nevill et al., 1999; 2002; Sutter & Kocher, 2004; Boyko
46 et al., 2007; Downward & Jones, 2007; Dawson et al., 2007; Dohmen, 2008; Buraimo et al.,
47 2010; Goumas, 2012). However, the degree of influence of referee decisions relative to the
48 overall home advantage effect has not been extensively studied.

49 It is apparent that the literature tends to indicate with strong belief that referee
50 decisions favour the home team. However, some researchers (Page & Page, 2010) have
51 questioned this outcome and expressed their uncertainty as "*it could be the case that these*
52 *biases do not manifest themselves into significant differences in terms of the overall*
53 *performance of a team*" (Page & Page, 2010); the increased number of fouls, yellow cards,
54 red cards, penalties and so on in favour of the home team might simply be the result of the
55 home team performing better than the away team. For example, if the home team is in control
56 of the ball (possession) more often than not, then we would expect it to be awarded more
57 fouls and penalties, and less yellow and red cards relative to the opponent, on the basis that
58 its control of possession will lead to it being on the receiving end of more tackles. We should
59 also expect a higher proportion of these to be committed nearer to the opponent's goal, as
60 greater possession also tends to correspond to a marked territorial advantage. We agree that
61 the kind of explanatory causal factors proposed in (Page & Page, 2010) must be incorporated
62 into any study of referee bias.

63 Hence, in this paper we present a novel Bayesian network (BN) model developed for
64 referee bias analysis in football. It is the most comprehensive attempt to date to include
65 within-game explanatory variables in order to justify the observed discrepancies between
66 fouls and penalty kicks awarded between adversaries prior to formulating beliefs about
67 referee bias. Although previous attempts have been made to control within-game events such
68 as shots, fouls and corners (Dohmen, 2008; Goumas, 2012), this paper integrates a number of
69 important additional variables which are required for formulating a causal network model, .

70 The paper is organised as follows: Section 2 describes the BN model, Section 3 discusses the
71 results and Section 4 provides our concluding remarks.

72

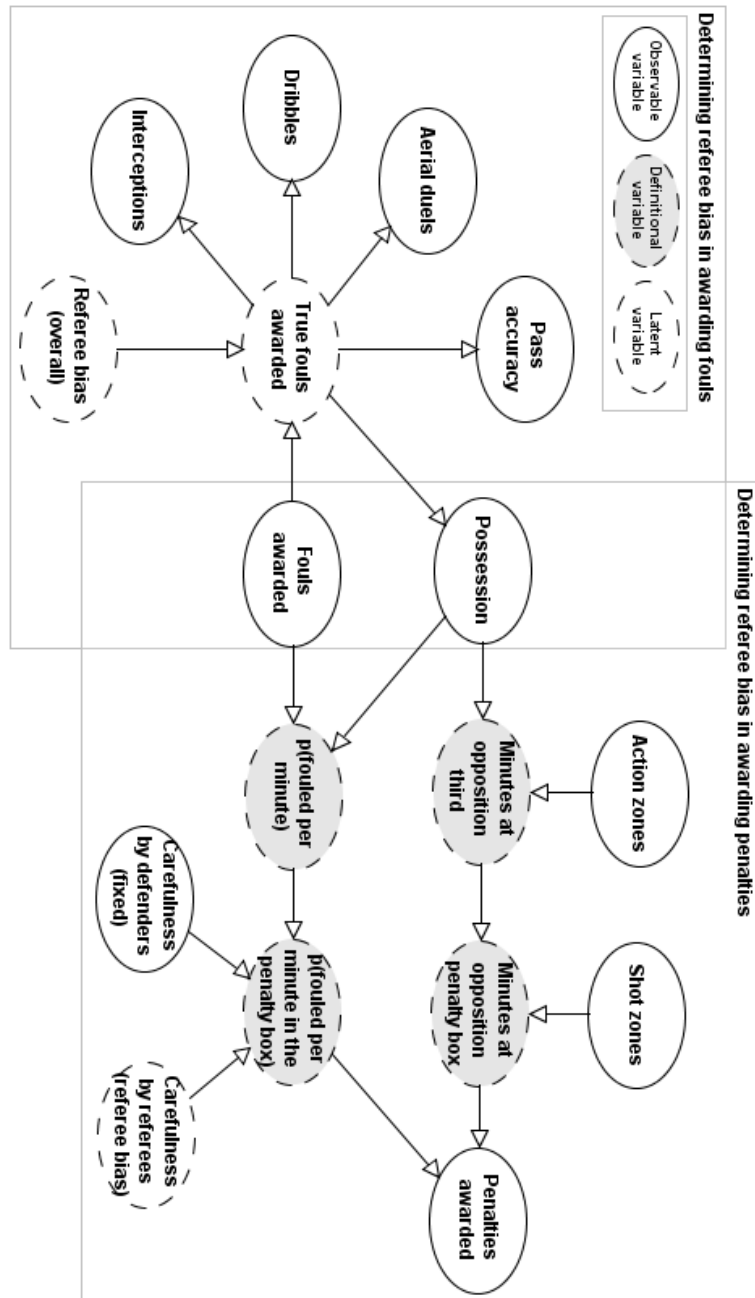
73

74 **2 THE MODEL**

75

76 In this section we describe the BN model which was developed using the AgenaRisk BN tool
77 (Agena Ltd., 2013). Details about the role of qualitative judgments and how inference is
78 done are provided in (Fenton et al., 2007; Neil et al., 2010; Fenton & Neil, 2012).

79



80
81
82
83
84
85
86
87
88
89
90
91

Figure 1. Bayesian network model topology; Components 1 and 2.

The data used to inform priors and provide observations for each of the teams is available online at (WhoScored?.com, 2012), although the data for number of penalties awarded was manually recorded by a member of the research team from bbc.co.uk/football.

The model is constructed on the basis of two components as illustrated by the model topology in Figure 1. Component 1 (described in Section 2.1) measures the referee bias over all fouls awarded, while Component 2 (described in Section 2.2) measures the referee bias over fouls awarded within the opposition penalty box (effectively penalty kicks).

The model is used to assess the referee bias for each case at home, away, and overall.

2.1. COMPONENT 1

This component simply assumes that the fouls awarded in a game are a consequence of a team's ability with respect to the following attributes (each corresponding to a node in the model):

1. *Possession*: percentage of time the team is in control of the ball ;
2. *Pass accuracy*: the percentage of successful passes (i.e. those that reach a team mate, ;
3. *Aerial duels*: the percentage of aerial duels won ;
4. *Dribbles*: the average number of times, per match instance, a player manoeuvres the ball around a player of the opposing team ;
5. *Interceptions*: the average number of times, per match instance, a player intercepts a pass made by a player of the opposing team .

Subsequently, the referee bias is simply inferred by measuring the discrepancy¹ in distributions between *predicted* (*True fouls awarded* node) and observed (*Fouls awarded* node) fouls awarded, with the bias level set to $\geq 10\%$ in terms of variability between the two distributions.

Figure A.1 presents a BN example of this component with the observations of QPR and Arsenal as inputs when playing at away grounds. The comparison in Figure A.1 shows that even though Arsenal generated superior statistics for all of the five explanatory parameters, they were still awarded 1.1 fouls less per match instance compared to QPR. As a result, the *Referee bias* distribution provides weak evidence of *Bias For* for QPR and *Bias Against* for Arsenal.

2.2. COMPONENT 2

123 The second component represents the key process of determining referee bias given penalties
124 awarded. The steps can be enumerated as follows:

125

126 1. We convert the possession rate into time spent (in minutes) holding the ball, and we
127 use the positional statistics of *Action Zones*² and *Shot Zones*³ to estimate the time
128 spent respectively at a) opposition third, and subsequently at b) opposition penalty
129 box. Essentially, we are only interested in (b), since there is where the penalties are
130 awarded.

131

132 2. We then measure the probability of being awarded a foul for each minute spent while
133 in possession of the ball, at any part of the pitch, given the following two parameters:
134 a) the rate of observed fouls awarded from Component 1, and b) time spent holding
135 the ball (from step 1 above).

136

137 Similar to step 2, we measure the probability of being awarded a foul for each minute spent
138 while in possession of the ball in the opposition penalty box given the following two
139 parameters: a) number of penalties awarded, and b) time spent holding the ball while in the
140 opposition penalty box (from step 1b above). For the analysis we assume that fouls awarded
141 within the penalty box are penalty kicks (there are examples of indirect free kicks in the
142 penalty area but these are rare).

143 After steps 2 and 3, we can compare the two inferred probability distributions and measure
144 how the probability of *fouls awarded per minute* varies with *fouls awarded per minute while*
145 *in opposition penalty box*. In doing so, the model takes account of the extra sensitivity of
146 fouls committed inside the penalty area since a penalty kick awarded is very often decisive⁵
147 on the final outcome. As a result, for this analysis we take into consideration the following
148 widely accepted observations that a) when a player is defending in his own penalty box he is

² The positional statistical information regarding action zones (i.e. where the ball is played) is distributed in *Own Third*, *Middle* and *Opposition Third*. This information is used to estimate the time spent at each third of the pitch, while in possession of the ball.

³ The positional statistical information regarding shot zones (i.e. where do the shots come from) is distributed in *6 Yards Box*, *18 Yards Box*, and *Outside of Box*. This information is used to estimate time spent at opposition penalty box while in possession of the ball (both the *6 Yards Box* and the *18 Yards Box* information contribute to time spent at opposition penalty box).

⁵ In particular, during our period of analysis, 72% of penalties awarded were converted by the attacking team, while a single goal would have been sufficient to decisively alter the balance of the overall result in 61% of matches played.

149 extra careful not to commit a foul, and b) the referee is also extra careful when awarding such
150 fouls. Accordingly, the next step is:

151

152 4. To let the model explain the discrepancies between the two inferred probability
153 distributions (from steps 2 and 3) into the following two explanatory variables: 1)
154 *Carefulness by defenders*, and 2) *Carefulness by referees*. Since we are only interested
155 in inferring the referee bias, we assume that the level of carefulness by defenders is
156 fixed and identical for all teams (the assumption we use is 'double careful'). The
157 model then explains the residual variation in node *Carefulness by referees*; effectively
158 referee bias (i.e. the less careful referees are the more penalties a team is awarded and
159 thus, the higher the positive referee bias). In particular, we assume that if referees are
160 more (respectively less) likely - all factors being equal - to award penalties to a
161 particular team than to the other teams, then there is a degree of positive (respectively
162 negative) bias towards that team.

163

164

165 3 RESULTS AND DISCUSSION

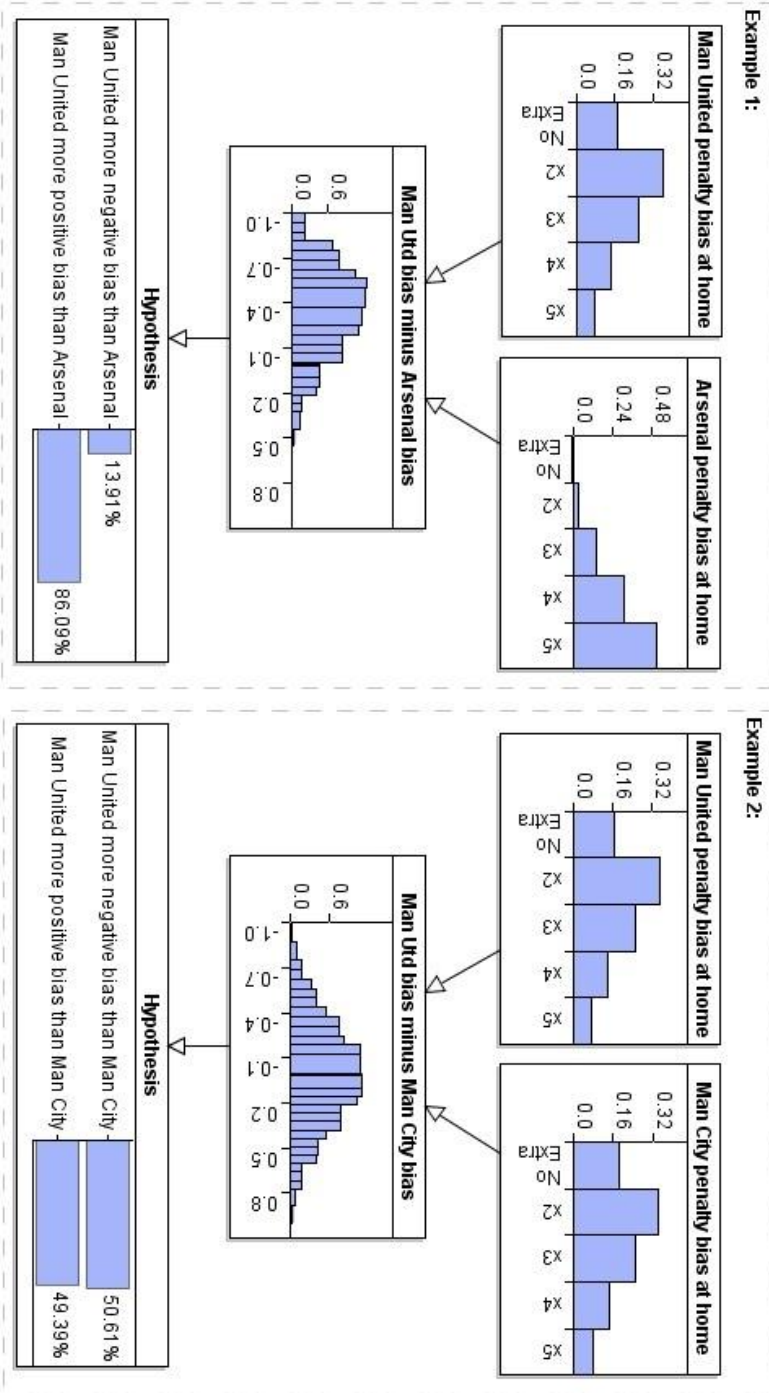
166

167 Using data from the full English Premier League (EPL) season 2011-12, we have compared
168 a) the referee carefulness given fouls awarded between teams (Component 1), b) the referee
169 carefulness given penalties awarded between teams (Component 2), and c) the association of
170 (a) and (b) inferences with crowd attendance and crowd density for each team.

171 Table C.1. presents the relative percentage increase in performance, for each of the
172 explanatory variables, a team gained when playing at home relative to the away match
173 performances. As expected, the average team demonstrated increases in possession (5.94%),
174 pass accuracy (0.94%), aerial duels won (4.57%), successful dribbles (13.24%) and fouls
175 awarded (3.04%). When it comes to interception, the average team demonstrated a decrease
176 of 1.34%, although this is not surprising since in order to be able to intercept the ball the
177 opponent has to be in possession. However, the variability between teams for this particular
178 factor is very high.

179 .

180



181
 182
 183
 184
 185
 186
 187
 188
 189
 190

3.1. Referee bias given fouls awarded (Component 1)

--	--	--

191

192 3.2. *Referee bias given penalties awarded (Component 2)*

193

194 Conversely, neither of the teams appear to have received similar benefit when playing away
195 from home. But, what makes this result particularly interesting is that these two teams were
196 the only teams fighting for the EPL title and until the very last league match (i.e. each
197 accumulated 89 league points; an impressive 19 points more than Arsenal who finished 3rd).
198 Taking into consideration both home and away match instances, however, Manchester United
199 is still ranked 1st in positive penalty kicks bias whereas Manchester City 4th and Arsenal last.

200

--	--	--

201

202

203

204 3.3. *Referee bias and match attendance*

205
206
207
208
209
210
211
212
213
214
215
216

with their respective average crowd attendance and average crowd density. Crowd density is the attendance size divided by home stadium attendance. It has been suggested in the literature as a significant predictor of home referee bias (e.g. Boyko et al., 2007; Goumas, 2012). In contrast to previous studies, our results do not demonstrate any strong positive relationship between crowd attendance (or crowd density) and positive referee bias. For example Arsenal, with the second largest average attendance as well as the second largest average crowd density, were ranked last in terms of positive referee bias for penalties awarded.

Table 3. Average home attendance and crowd density for all teams, ranked by home H_{PO} .

	Team	Average crowd attendance	Average crowd density
	Man United	75,387	99.06%
	Man City	47,044	98.01%
	Swansea	19,946	96.35%
	Blackburn	22,551	70.12%
	Stoke	27,225	95.92%
	Fulham	25,293	98.50%
	Norwich	26,605	97.74%
	QPR	18,923	94.25%
	Chelsea	41,477	99.14%
	Liverpool	44,253	97.55%
	Bolton	23,669	82.40%
	Everton	33,228	81.90%
	Wigan	18,633	74.46%
	Aston Villa	33,873	79.17%
	Newcastle	49,939	95.30%
	West Brom	24,773	93.48%
	Wolves	25,684	81.02%
	Sunderland	39,095	79.78%
	Tottenham	36,026	99.31%
	Arsenal	60,000	99.28%

217
218

4 CONCLUDING REMARKS AND FUTURE WORK

219
220
221
222
223
224
225

Any credible attempt to determine referee bias in football matches must take account of causal explanatory factors. We have presented a novel Bayesian network model for this purpose. The model enables us to account for the observed discrepancies in fouls and penalty kicks awarded between teams by taking into consideration causal factors such as possession, time spent in the opposition penalty box while in control of the ball, pass accuracy, the ability

226 to win aerial duels in the air, the ability to dribble the ball and the ability to intercept the
227 opponent's pass.

228

229 However, this did not extend to away games (Manchester City, in fact benefited less than
230 any other team away from home) nor to free kicks generally. The two Manchester clubs were,
231 however, the only serious title contenders in an extremely close title-race. While popular lay
232 theories suggest that referees have a tendency to favour elite clubs in general and Manchester
233 United in particular, at their home stadiums, it is possible that the combination of home
234 advantage and being a title-favourite team (which Manchester United have been since the
235 Premier League inception) in a close-title race is what is more predictive of positive referee
236 bias for penalty kicks awarded. To test such hypothesis properly would require applying the
237 model over multiple seasons.

238 Other important results from applying our model to the 2011-12 EPL season run
239 counter to the prevailing wisdom. For example, much of the previous literature suggests that
240 the influence of home crowd is a leading factor in explaining the observed discrepancies of
241 officiating behaviour between home and away teams (Nevill et al., 1996; Nevill et al., 1999;
242 2002; Downward & Jones, 2007; Dohmen, 2008; Buraimo et al., 2010; Goumas, 2012).
243 However, we found that the home crowd alone is not associated with positive referee bias. It
244 should be acknowledged that there is some evidence that refereeing bias varies from league to
245 league in conjunction with crowd hostility (Anders & Rothhoff, 2012), and caution should
246 therefore be exercised in generalising the findings of the present study to all of world football
247 prior to the application of BN modelling to other major leagues. In order to formulate such a
248 conclusion, one has not only to understand the degree of impact of home crowd on home
249 advantage, but also to measure home advantage for individual teams before assessing referee
250 bias. After all, crowd attendance and crowd density tend to vary in conjunction with team
251 performance (i.e. teams which perform best tend to have a large fan base and thus larger
252 stadiums). In (Constantinou & Fenton, 2013) the results show how home advantage can differ
253 considerably between teams of the EPL, whereas (Clarke & Norman, 1995) reported that in
254 many cases a team can even develop a negative home advantage.

255 It is also important to note that neither crowd size nor crowd density is necessarily
256 correlated with crowd noise in the intuitive manner that might be expected. No published
257 peer-reviewed study on noise-levels within the EPL stadiums exists, but 2008 and 2011

258 attempts to measure their decibel levels by Sky Sports⁶ and fancants.com⁷ suggest little or
259 no correlation, with several clubs with smaller attendances and lower crowd densities ranking
260 above many of the elite teams in both studies. Factors such as differing stadium acoustics, fan
261 demographics, and the varying levels of organisation and coordination of the most vocal
262 elements of the home support likely play a part in this. It is also important to remember that
263 in the EPL, a league with a high ratio of visiting supporters, there is often a very substantial
264 level of vocal support for the away team also present at almost all fixtures, thus partly
265 confounding the notion that larger crowd generating higher noise levels necessarily means
266 greater vocal support for the home team.

267 Our results lead us to conclude that Page and Page (2010) were correct to question the
268 effect of the home crowd in the absence of team performance. It appears that the explanatory
269 variables taken into consideration by our model (which represent different aspects of team
270 performance) when it comes to free kicks and penalty kicks awarded between home and
271 away teams; crowd attendance and crowd density are not related with positive referee bias.

272 Whether or not there are underlying factors not yet accounted for in our model (such
273 as 'being title contenders', having 'great wealth' or even the possibility that referees secretly
274 support these clubs), as well as the relevance of those factors with other aspects of referee
275 bias (i.e. yellow and red card), is a matter for future research. If information such as
276 possession and positional statistics in combination with the ability to dribble, win aerial duels
277 and so on) also becomes available for individual match instances (rather than overall as it was
278 in our case) then we will be able to accurately determine referee bias with much higher
279 confidence. This will be achieved by also looking at how certain teams might have further
280 benefited by *negative* referee bias for their opponents in a match between them (i.e. the
281 possibility that the two Manchester clubs benefited not only from penalties awarded, but also
282 from penalties not awarded - i.e. *Bias Against* - for their opponents when playing against
283 them). We anticipate that our model now lays out a coherent and rational strategy for
284 conducting such research.

285

286 REFERENCES

287

- 288 [1] Agena Ltd. (2013). AgenaRisk Free Version. Retrieved online December 21, 2013, from Agena:
289 Bayesian network and simulation software for risk analysis and decision support:
290 http://www.agenarisk.com/products/free_download.shtml

⁶ <http://www.bbc.co.uk/dna/606/A42697579> and <http://www.stoke.vitalfootball.co.uk/article.asp?a=129620>

⁷ <http://eptalk.com/2011/05/13/top-20-loudest-football-grounds-in-premier-league/>

- 291 [2] Agnew, G., & Carron, A. (1994). Crowd effects and the home advantage. *International Journal of Sport*
292 *Psychology*, 25(1), 53–62.
- 293 [3] Anders, A., & Rotthoff, W. (2012). Are Home Field Advantage and Referee Bias Driven by the Fans?
294 Evidence from Across the Ocean. Under review. Draft available at: <http://ssrn.com/abstract=2026037>
- 295 [4] Boyko, R., Boyko, A., & Boyko, M. (2007). Referee bias contributes to home advantage in English
296 premiership football. *Journal of Sports Sciences*, 25(11), 1185–1194.
- 297 [5] Buraimo, B., Forrest, D., & Simmons, R. (2010). The 12th man?: refereeing bias in English and
298 German soccer. *Journal of the Royal Statistical Society*, 173, Part 2: 431-449.
- 299 [6] Clarke, S. R., & Norman, J. M. (1995). Home ground advantage of individual clubs in English soccer.
300 *The Statistician*, 44, 509–521.
- 301 [7] Constantinou, A. & Fenton, N. (2013). Determining the level of ability of football teams by dynamic
302 ratings based on the relative discrepancies in scores between adversaries. *Journal of Quantitative*
303 *Analysis in Sports*, 0(0): 1-14.
- 304 [8] Courneya, K., & Carron, A. (1992). The home advantage in sport competitions: A literature review.
305 *Journal of Sport and Exercise Psychology*, 14(1), 13.
- 306 [9] Dawson, P., Dobson, S., Goddard, J., & Wilson, J. (2007). Are football referees really biased and
307 inconsistent?: Evidence on the incidence of disciplinary sanction in the English Premier League.
308 *Journal of the Royal Statistical Society: Series A (Statistics in Society)*, 170(1), 231–250.
- 309 [10] Dohmen, T. J. (2008). The Influence of Social Forces: Evidence from the Behavior of Football
310 Referees. *Economic Inquiry*, 46(3), 411–424.
- 311 [11] Downward, P., & Jones, M. (2007). Effects of crowd size on referee decisions: Analysis of the FA
312 Cup. *Journal of Sports Sciences*, 25(14), 1541–1545.
- 313 [12] Fenton, N. E., Neil, M. & Caballero J. G. (2007). Using Ranked nodes to model qualitative judgements
314 in Bayesian Networks. *IEEE TKDE*, 19(10), 1420-1432.
- 315 [13] Fenton, N. E. & Neil, M. (2012). *Risk Assessment and Decision Analysis with Bayesian Networks*.
316 CRC Press.
- 317 [14] Garicano, L., Palacios-Huerta, I., & Prendergast, C. (2005). Favoritism under social pressure. *Review*
318 *of Economics and Statistics*, 87(2), 208–216.
- 319 [15] Goumas, C. (2012). Home advantage and referee bias in European football. *European Journal of Sport*
320 *Science*, iFirst article: 1-7
- 321 [16] Hirotsu, N., & Wright, M. (2003). An evaluation of characteristics of teams in association football by
322 using a Markov process model. *The Statistician*, 52: 4, 591-602.
- 323 [17] Neave, N., & Wolfson, S. (2003). Testosterone, territoriality, and the 'home advantage'. *Physiology and*
324 *Behavior*, 78(2), 269–275.
- 325 [18] Neil, M., Marquez, D. & Fenton, N. E. (2010). Improved Reliability Modeling using Bayesian
326 Networks and Dynamic Discretization. *Reliability Engineering & System Safety*, 95(4), 412-425.
- 327 [19] Nevill, A., Balmer, N., & Williams, A. (2002). The influence of crowd noise and experience upon
328 refereeing decisions in football. *Psychology of Sport and Exercise*, 3(4), 261–272.
- 329 [20] Nevill, A., Balmer, N., & Williams, M. (1999). Crowd influence on decisions in association football.
330 *Lancet*, 353(9162), 1416.
- 331 [21] Nevill, A., & Holder, R. (1999). Home advantage in sport: An overview of studies on the advantage of
332 playing at home. *Sports Medicine*, 28(4), 221–236.
- 333 [22] Nevill, A., Newell, S., & Gale, S. (1996). Factors associated with home advantage in English and
334 Scottish soccer matches. *Journal of Sports Sciences*, 14(2), 181–186.
- 335 [23] Page, K. & Page, L. (2010). Alone against the crowd: Individual differences in referees' ability to cope
336 under pressure. *Journal of Economic Psychology*, 31: 192-199.
- 337 [24] Pollard, R. (1986). Home advantage in soccer: A retrospective analysis. *Journal of Sports Sciences*,
338 4(3), 237–248.
- 339 [25] Pollard, R. (2006). Worldwide regional variations in home advantage in association football. *Journal of*
340 *Sports Sciences*, 24(3), 231–240.
- 341 [26] Poulter, D. R. (2009). Home advantage and player nationality in international club football. *Journal of*
342 *Sports Sciences*, 27(8): 797-805.
- 343 [27] Pollard, R., & Pollard, G. (2005). Long-term trends in home advantage in professional team sports in
344 North America and England (1876–2003). *Journal of Sports Sciences*, 23(4), 337–350.
- 345 [28] Sutter, M., & Kocher, M. (2004). Favoritism of agents – The case of referees' home bias. *Journal of*
346 *Economic Psychology*, 25(4), 461–469.

347 [29] WhoScored?.com (2012). WhoScored?.com: Revolutionising Football Statistics. Retrieved online
348 January 17, 2014, from England: Premier League 2011/12:
349 <http://www.whoscored.com/Regions/252/Tournaments/2/Seasons/2935>

350

351

352

353

354

355

356

357

358

359

360

361

362

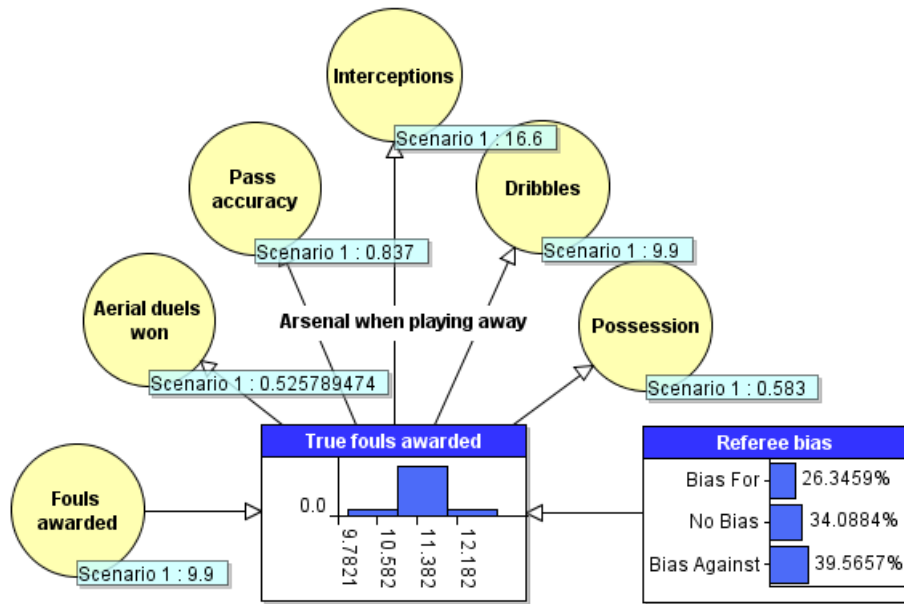
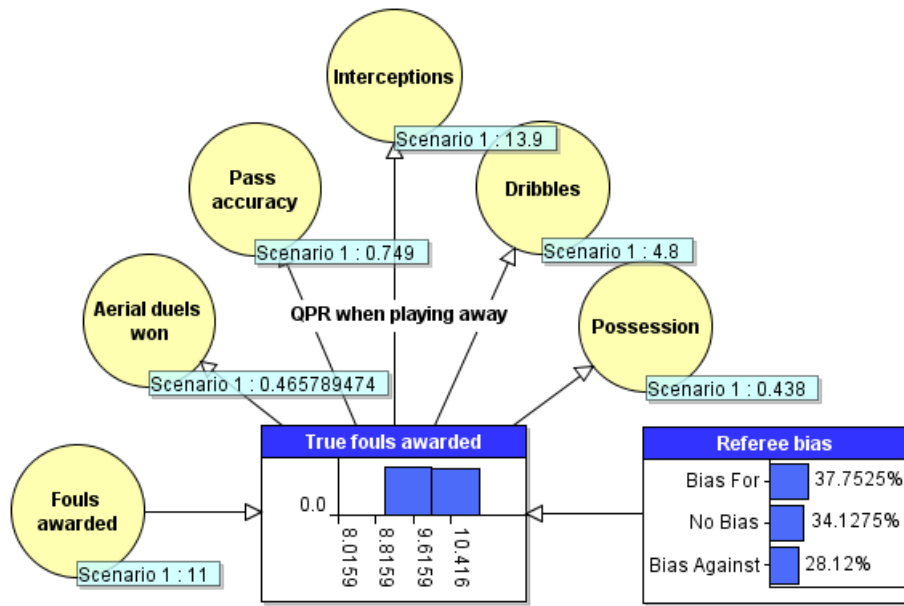
363

364 **APPENDIX A:** BN model examples with scenarios

365

366

367



368
369
370
371
372

Figure A.1. Assessing referee bias given overall fouls awarded; a Component 1 example given observations of QPR and Arsenal when playing at away grounds.

373

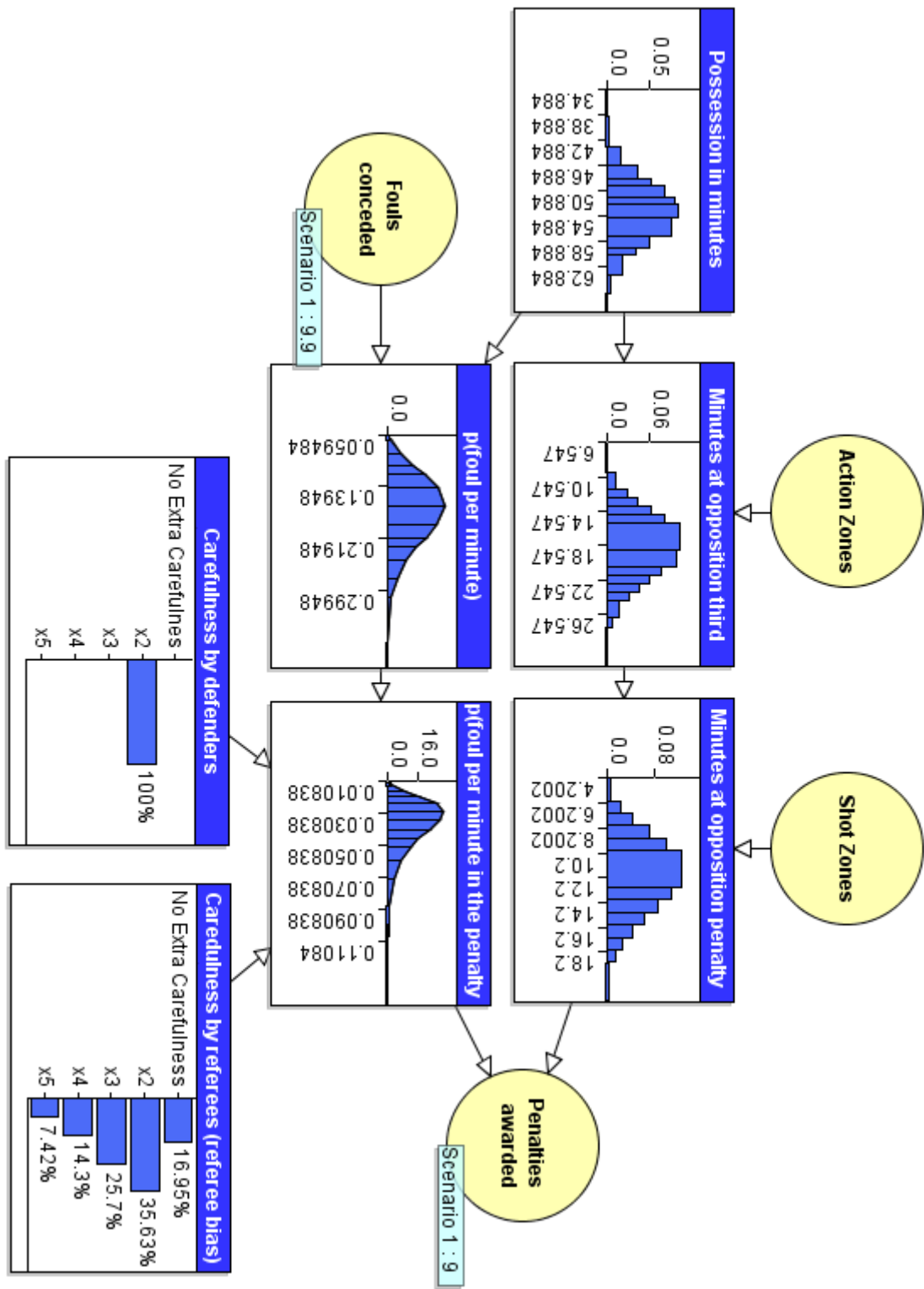


Figure A.2. Assessing referee bias given penalties awarded; a Component 2 example based on Manchester United home match data of the EPL season 2011-12.

374
 375
 376
 377
 378
 379

380 **APPENDIX B: Model description**

381

382

383

Table B.1. Description of the BN variable nodes

Node name	Node ID	Node type	Observable/ Latent	Description
Possession	POS	$\sim\text{TNormal}(\mu, \sigma^2, 0,1)$	Observable	where μ is the mean probability value observed over n match instances, and σ^2 is the variance associated with μ
Pass accuracy	PA	$\sim\text{TNormal}(\mu, \sigma^2, 0,1)$	Observable	
Aerial duels	AD	$\sim\text{TNormal}(\mu, \sigma^2, 0,1)$	Observable	
Dribbles	DR	$\sim\text{Pois}(\lambda)$	Observable	where λ is the expected value over n match instances
Interceptions	INT	$\sim\text{Pois}(\lambda)$	Observable	
Fouls Awarded	FA	$\sim\text{Pois}(\lambda)$	Observable	
True fouls awarded	TFA	$\sim\text{Pois}(\lambda)$	Latent	
Referee bias (overall)	RBO	Ranked $\sim\text{TNormal}(\mu, \sigma^2, 0,1)$	Latent	with states ‘ <i>Bias For</i> ’, ‘ <i>No Bias</i> ’, and ‘ <i>Bias Against</i> ’. Assuming ignorant prior (uniformly distributed)
Action Zones	AZ	Labelled	Observable	with states ‘ <i>Own third</i> ’, ‘ <i>Middle third</i> ’, and ‘ <i>Opposition third</i> ’.
Shot Zones	SZ	Labelled	Observable	with states ‘ <i>6 Yard Box</i> ’, ‘ <i>18 Yard Box</i> ’, and ‘ <i>Outside of Box</i> ’.
Minutes at opposition third	MOT	$\sim\text{Binomial}(n, p)$	Latent	$B\left(\begin{matrix} B(90, \text{POS}^8), \\ \text{AZ}_{\text{OppositionThird}} \end{matrix}\right)$
Minutes at opposition penalty box	MOP	$\sim\text{Binomial}(n, p)$	Latent	$B\left(\begin{matrix} \text{MOT}, \\ \text{SZ}_{6\text{YardBox}+18\text{YardBox}} \end{matrix}\right)$
p(fouled per minute)	FM	$\sim\text{Beta}(\alpha, \beta)$	Latent	$\text{Beta}\left(\begin{matrix} \text{FA}, \\ B(90, \text{POS}) - \text{FA} \end{matrix}\right)$
p(fouled per minute in the penalty box)	FMP	Arithmetic	Latent	$\frac{\text{FM}}{(\text{CD} \times \text{CR})}$
Penalties awarded	PAW	$\sim\text{Binomial}(n, p)$	Observable	$B(\text{MOP} \times g, \text{FMP})$; where $g = 19$ represents the number of gameweeks at home/away grounds (and 38 for overall assessment)
Carefulness by defenders	CD	Ranked $\sim\text{TNormal}(\mu, \sigma^2, 0,1)$	Latent	with states ‘ <i>No Extra Carefulness</i> ’, ‘ <i>x2</i> ’, ‘ <i>x3</i> ’, ‘ <i>x4</i> ’ and ‘ <i>x5</i> ’. Assuming ignorant prior (uniformly distributed).
Carefulness by referees (referee bias)	CR	Ranked $\sim\text{TNormal}(\mu, \sigma^2, 0,1)$	Latent	with states ‘ <i>No Extra Carefulness</i> ’, ‘ <i>x2</i> ’, ‘ <i>x3</i> ’, ‘ <i>x4</i> ’ and ‘ <i>x5</i> ’. Assuming ignorant prior (uniformly distributed).

384

385

386

387

⁸ Translated into minutes

397
398

399
400

