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Risk and confidence analysis for fuzzy multicriteria decision making

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Abstract

Recent research has recognised that multicriteria decision making (MCDM) should take account of uncertainty, risk and confidence. This paper takes this research forward by using linguistic variables and triangular fuzzy numbers to model the decision maker's (DM) risk and confidence attitudes in order to define a more complete MCDM solution. To illustrate the computation process and demonstrate the feasibility of the results we use a travel problem that has been used previously to assess MCDM techniques. The results show that the method is useful for tackling imprecision and subjectivity in complex, ill-defined and human-oriented decision problems.

Keywords: Multicriteria decision making; Triangular fuzzy numbers; Linguistic variables

1. Introduction

Multicriteria decision making (MCDM) refers to screening, prioritising, ranking, or selecting a set of *alternatives* (also referred to as "candidates" or "actions") under usually independent, incommensurate or conflicting *criteria* [2,16,28]. We will use the following example (also used in [15,31]) to illustrate the concepts and methods throughout:

Example. We have to reach the airport from our home to catch an airplane. The MCDM problem here is to select an appropriate travel type from three alternatives: *Car*, *Taxi* and *Train*. Our criteria are *price*, *journey time*, and *comfort*.

An MCDM problem is characterized by (a) the *ratings* of each alternative with respect to each criteria and (b) the *weights* given to each criteria. Classical MCDM methods assume that the ratings of alternatives and the weights of criteria are *crisp* numbers. Increasingly, this is recognized as unrealistic. In the above example, the decision maker (DM) will be unable to assign a crisp number for the journey time of a car since this value is influenced by many factors. Generally, uncertainties arise from: unquantifiable information, incomplete information, unobtainable information, and partial ignorance [8].

Since classical MCDM methods cannot handle problems with such imprecise information, the representation and interpretation of "uncertainty" and human-related subjective preference is needed [40]. The use of probabilistic methods for this purpose in MCDM has been explored in [15,31], but fuzzy set theory [38] seems to have been the most commonly used method. The general use of fuzzy set theory in MCDM is explored in [3,24,25,37], while specific fuzzy MCDM methods can be found in [4,6,8–12,14,27,32–34]. Fuzzy decision making with partial preference information has been explored in [5,18,22,25,30]. In [35–37], Yager included fuzzy methods, probabilistic information as well as the DM's attitudes and preferences for decision-making under uncertainty.

In this paper, we first introduce the general fuzzy MCDM approach (Section 2). Then we focus on the two dimensions where we believe the DM's attitude is most subjective: *risk* (Section 3) and *confidence* (Section 4). We handle risk by extending the so-called linguistic approach [1,13,17,21,39] that has previously been explored with fuzzy MCDM in [7,10–12,20,32–34]. The linguistic approach is

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an approximate way to represent natural words or sentences used in human judgment and perception. Linguistic decision analysis [4,17,18,26] transforms the linguistic description of the DM into a mathematical model to provide a flexible framework for solving decision problems. To handle confidence we use the fuzzy α cut concept [19] in addition to a linguistic approach. Our method for ranking the performance of alternatives is based on the kind of two-phase approach adopted in [8,25,40]. The first phase is to aggregate performance of the ratings of the alternatives under the criteria. The second phase is to rank alternatives with respect to aggregated performances.

2. General fuzzy MCDM approach

First we describe the general approach to fuzzy MCDM without considering risk attitudes and confidence.

2.1. Problem formulation and definitions

A general multicriteria decision problem with m alternatives A_i ($i = 1, \ldots, m$) and n criteria C_j ($j = 1, \ldots, n$) can be concisely expressed as:

$$D = [x_{ij}] \text{ and } W = (w_j), \text{ where } i = 1, \ldots, m \text{ and } j = 1, \ldots, n. \quad (1)$$

Here D is referred to as the *decision matrix* (where the entry x_{ij} represents the rating of alternative A_i with respect to criterion C_j), and W as the *weight vector* (where w_j represents the weight of criterion C_j). In general we classify criteria as either:

- *benefit criteria* (where the higher the value of x_{ij} the better it is for the DM) or
- *cost criteria* (where the lower the value of x_{ij} the better it is for the DM).

Because we wish to consider *fuzzy*, as opposed to *crisp*, values in D and W we shall use the notation:

$$\tilde{D} = [\tilde{x}_{ij}] \text{ and } \tilde{W} = (\tilde{w}_j), \quad (2)$$

whereby $\tilde{x}_{ij}$ represents the fuzzy rating of alternative A_i with respect to criterion C_j, and $\tilde{w}_j$ represents the fuzzy weight of criterion C_j. In particular, an intuitively easy and effective approach to capturing the expert's uncertainty about the value of an unknown number is a triangular fuzzy number:

Definition. A triangular fuzzy number $\tilde{a}$ is defined by a triplet (a_1, a_2, a_3). The membership function is defined as [19]:

$$\mu_{\tilde{a}}(x) = \begin{cases} (x - a_1)/(a_2 - a_1), & a_1 \leqslant x \leqslant a_2, \\ (a_3 - x)/(a_3 - a_2), & a_2 \leqslant x \leqslant a_3, \\ 0, & \text{otherwise}. \end{cases} \quad (3)$$

The triangular fuzzy number is based on a three-value judgment: the minimum possible value a_1, the most possible value a_2 and the maximum possible value a_3.

Table 1
Decision matrix and weight vector

	Price(Pounds; 0.3)	Journey time(min; 0.5)	Comfort([1,10]; 0.2)
Car	(9, 10, 12)	(70, 100, 120)	(4, 5, 6)
Taxi	(20, 24, 25)	(60, 70, 100)	(7, 8, 10)
Train	(15, 15, 15)	(70, 80, 90)	(1, 4, 7)

Example. Table 1 shows the decision matrix and weight vector for the travel problem introduced in Section 1. In this example the criteria *price* and *journey time* are cost criteria measured in pounds and minutes respectively. The criterion *comfort* is a value criterion measured on a scale from 1 to 10. The ratings in the decision matrix are expressed as triangular fuzzy numbers (so, for example, the car journey to the airport most typically costs 10 pounds but it can be as low as 9 and as high as 12). For simplicity the weights are crisp numbers summing to 1 (usually the DM is able to express the weights in this way).

2.2. Normalization

To deal with criteria on different scales, we apply a normalization process. Specifically, we normalize the fuzzy numbers in the decision matrix as the performance matrix:

$$\tilde{P} = [\tilde{p}_{ij}], \quad (4)$$

where

$$\tilde{p}_{ij} = \begin{cases} \left(\frac{x_{ij1}}{M}, \frac{x_{ij2}}{M}, \frac{x_{ij3}}{M}\right), & M = \max_i x_{ij3}, \quad Cj \text{ is benefit criterion} \\ \left(\frac{N - x_{ij3}}{N}, \frac{N - x_{ij2}}{N}, \frac{N - x_{ij1}}{N}\right), & N = \max_i x_{ij3}, \quad Cj \text{ is cost criterion} \end{cases}$$

This method preserves the ranges of normalized triangular fuzzy numbers to $[0, 1]$.

Example. The performance matrix for the decision matrix of Table 1 is calculated by (4) and shown in Table 2.

2.3. Weighting the criteria

We construct the weighted performance matrix by multiplying the weight vector by the decision matrix as:

$$\tilde{P}^w = [\tilde{p}^w_{ij}], \quad (5)$$

where $p^w_{ij1} = w_{j1} p_{ij1}$, $p^w_{ij2} = w_{j2} p_{ij2}$, $p^w_{ij3} = w_{j3} p_{ij3}$, $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, n$.

Example. The running example is shown in Table 3.

Table 2
Performance matrix

	Price (Pounds; 0.3)	Journey time (min; 0.5)	Comfort ([1, 10]; 0.2)
Car	(0.520, 0.600, 0.640)	(0.000, 0.167, 0.417)	(0.400, 0.500, 0.600)
Taxi	(0.000, 0.040, 0.200)	(0.167, 0.417, 0.500)	(0.700, 0.800, 1.000)
Train	(0.400, 0.400, 0.400)	(0.250, 0.333, 0.417)	(0.100, 0.400, 0.700)

Table 3
Weighted performance matrix

	Price (Pounds; 0.3)	Journey time (min; 0.5)	Comfort ([1, 10]; 0.2)
Car	(0.1560, 0.1800, 0.1920)	(0.000, 0.0835, 0.2084)	(0.0800, 0.1000, 0.1200)
Taxi	(0.0000, 0.0120, 0.0600)	(0.0835, 0.2084, 0.2500)	(0.1400, 0.1600, 0.2000)
Train	(0.1200, 0.1200, 0.1200)	(0.1250, 0.1665, 0.2084)	(0.0200, 0.0800, 0.1400)

Table 4
Performance index

Car		Taxi		Train	
P	Order	P	Order	P	Order
0.1294	1	0.1275	2	0.1248	3

2.4. Performance of alternatives

We use the *vertex method* [7] to calculate alternatives' performance index with reference to ideal solutions [16]. The most preferred alternative should have the shortest distance from the positive ideal solution and the longest distance from the negative ideal solution.

Definition. Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be two positive triangular fuzzy numbers, then the *vertex method* defines the distance between them as:

$$d(\tilde{a}, \tilde{b}) = \{[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2]/3\}^{1/2}. \quad (6)$$

For the normalized fuzzy performance matrix, we define the positive ideal solution $\tilde{p}_j^* = (1, 1, 1)$ and the negative ideal solutions $\tilde{p}_j^- = (0, 0, 0)$ under the criteria as references to measure alternatives' performance [7]. By the *vertex method*, the distance between each alternative and the positive ideal solution and the negative ideal solution is calculated as:

$$d_i^* = \sum_{j=1}^{n} d(\tilde{p}_{ij}^w, \tilde{p}_j^*) \quad (7)$$

$$d_i^- = \sum_{j=1}^{n} d(\tilde{p}_{ij}^w, \tilde{p}_j^-) \quad (8)$$

where $i = 1, \ldots, m$ and $j = 1, \ldots, n$.

We calculate performance index for each alternative as:

$$p_i = d_i^- + n - d_i^*/2n \quad (9)$$

where $i = 1, \ldots, m$, and n is the number of criteria. The nearer p_i gets to 1 the better the alternative's performance.

Example. The alternative's distance to the positive ideal solution and negative ideal solutions is calculated by (7) and (8). The alternatives' performance index is calculated by (9) and shown in Table 4 together with ranking orders.

3. Fuzzy MCDM by incorporating risk attitudes

The general approach can provide a basic ranking of the alternatives, but it cannot deal with the DM's attitudes towards risk and uncertainty. In this section, we explain how to incorporate the DM's risk attitudes into the general fuzzy MCDM approach. The linguistic approach to modeling risk attitudes in fuzzy MCDM [32–34] uses the notion of "optimism" and "pessimism". The key issue for us is to be able to use natural language to describe an appropriate range of attitudes between the extremes of "optimism" and "pessimism". The number of terms needs to be small enough so as not to impose pointless precision, yet rich enough to allow proper discrimination of the assessments [17]. Based on Miller's theory of cognitive retention [23] we use nine as the maximum number of terms for the DM's assessments.

3.1. Modeling risk attitudes

For *benefit criteria*, the DM expects a maximum value as the best value. For *cost criteria*, the DM expects a minimum value as the best value. To incorporate the DM's risk attitude to the triangular fuzzy number (a_1, a_2, a_3), we regard (a_1, a_2, a_3) as the *neutral* attitudes, (a_1, a_3, a_3) and (a_1, a_1, a_3) as *absolutely optimistic* (*AO*) (*absolutely pessimistic* (*AP*)) and *absolutely pessimistic* (*AP*) (*absolutely optimistic* (*AO*)) for *benefit* (*cost*) *criteria*. In general we use an ordered structure to incorporate other risk attitudes in (a_1, a_2, a_3) according to *benefit* (*cost*) *criteria* as shown in Table 5. The first column of Table 5 shows the set of linguistic terms. The case of *benefit criteria* in the second column and *cost criteria* in the third column shows the associated triangular fuzzy numbers derived from the triangular fuzzy number (a_1, a_2, a_3). The approach described here is easily generalized for the case where there are n as opposed to 9 linguistic terms.

3.2. Performance of alternatives on risk attitudes

Now that we have triangular fuzzy numbers that capture the DM's risk attitude we incorporate these into the decision matrix as:

$$\tilde{D}^r = [\tilde{x}_{ij}^r], \quad (10)$$

where $\tilde{x}_{ij}^r$ is the triangular fuzzy number derived from $\tilde{x}_{ij}$ under the specific risk attitude by Table 5. After normalization and weighting of criteria, we obtain the performance index with respect to risk attitudes.

Example. The alternatives' performance index and ranking orders under different risk attitudes is shown in Table 6.

Table 5
Linguistic terms of risk attitudes

Linguistic term	Triangular fuzzy number derived from (a_1, a_2, a_3) for benefit criteria	Triangular fuzzy number derived from (a_1, a_2, a_3) for cost criteria
Absolutely optimistic (AO)	(a_1, a_3, a_3)	(a_1, a_1, a_3)
Very optimistic (VO)	$(a_1, (a_2+3a_3)/4, a_3)$	$(a_1, (a_2+3a_1)/4, a_3)$
Optimistic (O)	$(a_1, (a_2+a_3)/2, a_3)$	$(a_1, (a_2+a_1)/2, a_3)$
Fairly optimistic (FO)	$(a_1, (3a_2+a_3)/4, a_3)$	$(a_1, (3a_2+a_1)/4, a_3)$
Neutral (N)	(a_1, a_2, a_3)	(a_1, a_2, a_3)
Fairly pessimistic (FP)	$(a_1, (3a_2+a_1)/4, a_3)$	$(a_1, (3a_2+a_3)/4, a_3)$
Pessimistic (P)	$(a_1, (a_2+a_1)/2, a_3)$	$(a_1, (a_2+a_3)/2, a_3)$
Very pessimistic (VP)	$(a_1, (a_2+3a_1)/4, a_3)$	$(a_1, (a_2+3a_3)/4, a_3)$
Absolutely pessimistic (AP)	(a_1, a_1, a_3)	(a_1, a_3, a_3)

Table 6
Performance index with respect to risk attitudes

	Car		Taxi		Train	
	P	Order	P	Order	P	Order
AO	0.1465	1	0.1418	2	0.1363	3
VO	0.1419	1	0.1380	2	0.1333	3
O	0.1375	1	0.1343	2	0.1303	3
FO	0.1333	1	0.1308	2	0.1275	3
N	0.1294	1	0.1275	2	0.1248	3
FP	0.1263	1	0.1233	2	0.1221	3
P	0.1235	1	0.1193	3	0.1196	2
VP	0.1208	1	0.1154	3	0.1173	2
AP	0.1183	1	0.1119	3	0.1151	2

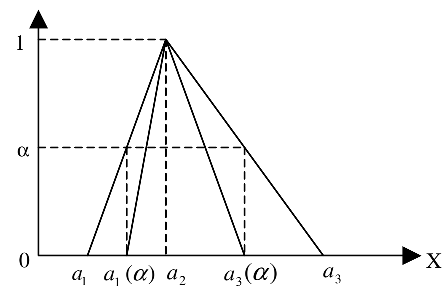

Fig. 1. A triangular fuzzy number Ã and its α-cut triangular fuzzy number.

4. Fuzzy MCDM by incorporating confidence attitudes

Table 6 is interesting in that it seems to suggest that any DM ranging from an extreme optimist to an extreme pessimist will always choose the Car as the preferred alternative (although the order of train and taxi vary). However, this result does not take account of the DM's confidence/uncertainty about the value of a rating. For example, the fuzzy value of journey time for car is (70, 100, 120) compared with (70, 80, 90) for train. Somebody who was extremely confident about the values would tend to believe that the most likely value was the true value in each case, i.e. 100 and 80, respectively. Thus, a pessimist (from the risk perspective) who was nevertheless extremely confident about the value would be more likely to favour the train than the car. In this section we formalize these notions so that we are able to complete our MCDM process by incorporating the DM's confidence on top of their risk attitudes. We draw on work from [14,32] (including the notion of the α cut concept) and [29].

4.1. Incorporating confidence levels

To assess confidence and uncertainty about a triangular fuzzy number we use the α cut concept as described in Fig. 1.

The idea is that $\alpha \in [0, 1]$ is a basic measure of our confidence about the fuzzy number. We use it to compute a refined fuzzy number that is 'closer' to the value with highest possibility as α tends to 1. Formally, assuming that the confidence in the triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$ is at level α, the refined fuzzy number is defined as:

$$\tilde{a}^\alpha = (a_1(\alpha), a_2, a_3(\alpha)) = (a_1 + \alpha(a_2 - a_1), a_2, a_3 - \alpha(a_3 - a_2)). \tag{11}$$

Having already incorporated the risk attitude in the decision matrix as in Section 3, we can now construct the decision matrix with risk attitude given confidence level α as:

$$\tilde{D}^\alpha = [\tilde{x}_{ij}^\alpha], \tag{12}$$

where $\tilde{x}_{ij}^\alpha$ is the triangular fuzzy number derived from $\tilde{x}_{ij}^r$ under the specific confidence level by (11). Suppose that there are l confidence levels. After normalization and weighting of criteria, we obtain the performance index vector given confidence levels as:

$$P_i = \left(p_{ik}^\alpha\right), \text{ where } \alpha = k - 1/l - 1, (l \geqslant 2), \quad i = 1, \ldots, m, \text{ and } k = 1, \ldots, l. \tag{13}$$

Example. By applying the above equations we get the performance index under *neutral* risk attitude with 11 confidence levels shown in Table 7.

4.2. Modeling of confidence attitudes

Instead of providing a direct value α to construct a confidence level, we next use a linguistic variable to represent the DM's qualitative assessment of confidence. As before

Table 7
Performance index under *neutral* risk attitude with 11 confidence levels

	Confidence level										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Car	0.1294	0.1269	0.1243	0.1217	0.1189	0.1160	0.1131	0.1100	0.1068	0.1035	0.1000
Taxi	0.1275	0.1264	0.1252	0.1240	0.1229	0.1217	0.1206	0.1196	0.1185	0.1176	0.1167
Train	0.1248	0.1228	0.1208	0.1188	0.1167	0.1146	0.1126	0.1105	0.1084	0.1063	0.1042

we use a nine-point linguistic scale shown in Table 8. Intuitively the membership value of confidence increases linearly as α increase from 0 to 1.

Table 8
Linguistic terms of confidence attitude

Linguistic term	Membership function
Absolutely confident (AC)	$\mu_{AC}(\alpha)=\begin{cases}1, & \alpha=1\\ 0, & \text{otherwise}\end{cases},\ \alpha\in[0,1]$
Very confident (VC)	$\mu_{VC}(\alpha)=(\mu_C(\alpha))^2=\alpha^2,\ \alpha\in[0,1]$
Confident (C)	$\mu_C(\alpha)=\alpha,\ \alpha\in[0,1]$
Fairly confident (FC)	$\mu_{FC}(\alpha)=(\mu_C(\alpha))^{0.5}=\sqrt{\alpha},\ \alpha\in[0,1].$
Neutral (N)	$\mu_U(\alpha)=1,\ \alpha\in[0,1]$
Fairly non-confident (FNC)	$\mu_{FNC}(\alpha)=(1-\mu_C(\alpha))^{0.5}=\sqrt{1-\alpha},\ \alpha\in[0,1].$
Non-confident (NC)	$\mu_{NC}(\alpha)=1-\mu_C(\alpha)=1-\alpha,\ \alpha\in[0,1]$
Very non-confident (VNC)	$\mu_{VC}(\alpha)=(1-\mu_C(\alpha))^2=(1-\alpha)^2,\ \alpha\in[0,1]$
Absolutely non-confident(ANC)	$\mu_{ANC}(\alpha)=\begin{cases}1, & \alpha=0\\ 0, & \text{otherwise}\end{cases},\ \alpha\in[0,1]$

Table 9
Performance index under *neutral* risk attitude with respect to confidence attitudes

	Car		Taxi		Train	
	P	Order	P	Order	P	Order
AC	0.1000	3	0.1167	1	0.1042	2
VC	0.1071	3	0.1189	1	0.1088	2
C	0.1085	3	0.1185	1	0.1093	2
FC	0.1110	3	0.1197	1	0.1112	2
N	0.1143	2	0.1207	1	0.1134	3
FNC	0.1189	2	0.1227	1	0.1167	3
NC	0.1202	2	0.1228	1	0.1175	3
VNC	0.1239	2	0.1252	1	0.1206	3
ANC	0.1294	1	0.1275	2	0.1248	3

4.3. Performance of alternatives on confidence attitudes

In general, assuming a total of l ($l \geqslant 2$) confidence levels, we define the normalized confidence membership vector as:

$$C_{LT}=\left(c_k\bigg/\sum_{k=1}^{l}c_k\right), \tag{14}$$

where $c_k=\mu_{LT}(\alpha)$, $\alpha=k-1/l-1$, $k=1,\ldots,l$ and LT represents linguistic terms AC, VC, C, FC, N, FNC, NC, VNC, and ANC, respectively. Based on the confidence membership vectors (14), the performance of the ith alternative under confidence attitudes is:

$$p_i^{LT}=P_i(C_{LT})^T=\sum_{k=1}^{l}p_{ik}c_k\bigg/\sum_{k=1}^{l}c_k. \tag{15}$$

The DM can rank, prioritize, and select alternatives under different risk attitudes and confidence attitudes according to the performance index.

Example. The alternatives' performance index and ranking orders under *neutral* risk attitude with respect to different confident attitudes are shown in Table 9.

For clear evaluation and analysis, we calculate and show the alternatives' performance index under risk and confidence attitude simultaneously. Performance index and ranking orders of *Car* under different risk and confidence attitudes are shown in Tables 10 and 11, respectively. Those of *Taxi* are shown in Tables 12 and 13, respectively, and those of *Train* are shown in Tables 14 and 15, respectively.

Thus, the DM can choose the best alternative under different risk and confidence attitudes accordingly. For example, a DM who is *absolutely pessimistic* (with respect

Table 10
Performance index of *Car* under risk and confidence attitudes

Car	AO	VO	O	FO	N	FP	P	VP	AP
AC	0.0950	0.0963	0.0976	0.0988	0.1000	0.0976	0.0951	0.0926	0.0901
VC	0.1101	0.1091	0.1083	0.1076	0.1071	0.1042	0.1014	0.0987	0.0960
C	0.1138	0.1122	0.1108	0.1095	0.1085	0.1056	0.1028	0.1000	0.0973
FC	0.1179	0.1158	0.1140	0.1124	0.1110	0.1080	0.1051	0.1023	0.0995
N	0.1238	0.1211	0.1186	0.1163	0.1143	0.1113	0.1084	0.1056	0.1029
FNC	0.1314	0.1279	0.1246	0.1216	0.1189	0.1157	0.1127	0.1098	0.1071
NC	0.1338	0.1300	0.1264	0.1231	0.1202	0.1170	0.1140	0.1111	0.1084
VNC	0.1389	0.1348	0.1309	0.1272	0.1239	0.1207	0.1177	0.1148	0.1121
ANC	0.1465	0.1419	0.1375	0.1333	0.1294	0.1263	0.1235	0.1208	0.1183

Table 11
Ranking order of *Car* under risk and confidence attitudes

Car	AO	VO	O	FO	N	FP	P	VP	AP
AC	1	2	2	3	3	3	3	3	3
VC	1	2	2	2	3	3	3	3	3
C	1	1	2	2	3	3	3	3	3
FC	1	1	2	2	3	3	3	3	2
N	1	1	2	2	2	2	2	1	1
FNC	1	1	1	2	2	2	2	1	1
NC	1	1	1	2	2	2	1	1	1
VNC	1	1	1	1	2	1	1	1	1
ANC	1	1	1	1	1	1	1	1	1

Table 12
Performance index of *Taxi* under risk and confidence attitudes

Taxi	AO	VO	O	FO	N	FP	P	VP	AP
AC	0.0905	0.0989	0.1059	0.1117	0.1167	0.1103	0.1045	0.0993	0.0944
VC	0.1050	0.1094	0.1131	0.1162	0.1189	0.1129	0.1073	0.1022	0.0973
C	0.1087	0.1119	0.1145	0.1166	0.1185	0.1127	0.1073	0.1023	0.0976
FC	0.1127	0.1150	0.1169	0.1185	0.1197	0.1141	0.1088	0.1038	0.0992
N	0.1187	0.1195	0.1201	0.1204	0.1207	0.1153	0.1103	0.1055	0.1011
FNC	0.1263	0.1255	0.1245	0.1236	0.1227	0.1176	0.1128	0.1082	0.1039
NC	0.1287	0.1272	0.1257	0.1242	0.1228	0.1179	0.1132	0.1088	0.1046
VNC	0.1339	0.1316	0.1293	0.1272	0.1252	0.1204	0.1158	0.1115	0.1074
ANC	0.1418	0.1380	0.1343	0.1308	0.1275	0.1233	0.1193	0.1154	0.1119

Table 13
Ranking order of *Taxi* under risk and confidence attitudes

Taxi	AO	VO	O	FO	N	FP	P	VP	AP
AC	2	1	1	1	1	1	1	1	1
VC	2	1	1	1	1	1	1	1	1
C	2	2	1	1	1	1	1	1	2
FC	2	2	1	1	1	1	1	1	3
N	2	2	1	1	1	1	1	2	3
FNC	2	2	2	1	1	1	1	3	3
NC	2	2	2	1	1	1	2	3	3
VNC	2	2	2	2	1	2	2	3	3
ANC	2	2	2	2	2	2	3	3	3

Table 14
Performance index of *Train* under risk and confidence attitudes

Train	AO	VO	O	FO	N	FP	P	VP	AP
AC	0.0717	0.0832	0.0922	0.0991	0.1042	0.1018	0.0989	0.0953	0.0912
VC	0.0903	0.0968	0.1019	0.1059	0.1088	0.1062	0.1034	0.1002	0.0967
C	0.0954	0.1004	0.1043	0.1072	0.1093	0.1068	0.1040	0.1010	0.0977
FC	0.1002	0.1042	0.1073	0.1096	0.1112	0.1086	0.1059	0.1029	0.0988
N	0.1079	0.1102	0.1118	0.1129	0.1134	0.1108	0.1081	0.1053	0.1025
FNC	0.1172	0.1175	0.1175	0.1172	0.1167	0.1141	0.1115	0.1088	0.1062
NC	0.1204	0.1199	0.1193	0.1185	0.1175	0.1149	0.1123	0.1097	0.1072
VNC	0.1264	0.1251	0.1236	0.1221	0.1206	0.1179	0.1153	0.1128	0.1104
ANC	0.1363	0.1333	0.1303	0.1275	0.1248	0.1221	0.1196	0.1173	0.1151

Table 15
Ranking order of *Train* under risk and confidence attitudes

Train	AO	VO	O	FO	N	FP	P	VP	AP
AC	3	3	3	2	2	2	2	2	2
VC	3	3	3	3	2	2	2	2	2
C	3	3	3	3	2	2	2	2	1
FC	3	3	3	3	2	2	2	2	1
N	3	3	3	3	3	3	3	3	2
FNC	3	3	3	3	3	3	3	2	2
NC	3	3	3	3	3	3	3	2	2
VNC	3	3	3	3	3	3	3	2	2
ANC	3	3	3	3	3	3	2	2	2

to risk attitude) but *very confident* will rank *Car* as the last alternative and *Taxi* as the first, whereas the DM who is *absolutely pessimistic* and *fairly confident* will rank *Train* as first.

5. Conclusions

Since multicriteria decision problems generally involve uncertainty it is important to incorporate different types of uncertainty in any proposed solution. We have presented a novel fuzzy MCDM approach based on risk and confidence analysis, that we believe is effective in tackling complex, ill-defined and human-oriented decision problems. In summary our approach consists of the following steps:

1. Formulate the problem in terms of the (fuzzy) decision matrix and the weight vector.
2. Normalize the decision matrix as the performance matrix.
3. Construct the weighted performance matrix.
4. With reference to ideal solutions, calculate alternatives' performance index.
5. According to the DM's risk attitudes (which can be characterized linguistically), construct the performance matrix with risk attitudes. Calculate alternatives' performance index by 2, 3, and 4 under risk attitudes.
6. Construct the performance matrix with risk attitudes on confidence levels and calculate performance index vector with respect to confidence levels by 2, 3, and 4.
7. According to the DM's confidence attitudes (which can again be characterized linguistically), determine the confidence membership vectors and calculate alternatives' performance index under confidence attitudes.

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