A new Bayesian Network approach to Reliability modelling

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Abstract

We present a new, effective and flexible event-based hybrid BN modelling method for reliability assessment that scales up to large, complex dynamic systems. By incorporating a recent powerful approximate inference algorithm for hybrid BNs, involving dynamically discretising the domain of all continuous variables, approximated solutions for both static and dynamic constructs are obtained simultaneously rendering unnecessary the use of modularisation techniques. Continuous and discrete nodes can be included in the model to represent the continuous failure times of system components and discrete reliabilities of the system (or any subsystem) for a given target requirement, respectively. Unlike other approaches (which tend to be restricted to using exponential distributions), our new approach is able to solve any configuration of static and dynamic gates with general parametric or empirical time-to-failure distributions, without recourse to numerical integration techniques or simulation methods. Furthermore, the diagnostic analysis capabilities of the BN combined with the dynamic discretisation algorithm allow also to obtain estimates of the parameterised marginal failure distribution (for the root nodes), either using available raw failure data or as prior information according to expert knowledge. No exact expression for the marginal is needed and no conditional probability tables need to be completed. Our BN framework allows a compact representation of the event-dependent failure behaviours characteristic of fault-tolerant systems, avoiding the state space explosion problem of the Markov Chain based approaches. Our BN framework is mathematically sound and at the same time simple enough to allow the interaction with domain experts and decision makers. Sensitivity, uncertainty, diagnosis, common cause failures, and warranty analysis can also be easily performed within this framework.

1. Introduction

The increasing complexity of the component dependencies and failure behaviours (e.g., sequence-dependent failures, functional dependencies, stand-by spares, etc.) of today's real-time safety-critical systems has led to an increasing interest in flexible modelling frameworks for reliability analysis. Space state based approaches such as dynamic fault trees (DFTs), (Dugan et al. 1992, 1993), have shown to increase the modeling power of traditional combinatorial models, like static Fault Trees (FTs), (Watson 1961, Schneeweiss 1999), by taking into account not only the combinations but also the sequential ordering of occurrence of component failures' that led to system failure. However, in practice, DFTs have severe limitations, such as the problem of space state explosion and the inability to handle non-exponential failure distributions.

A number of recent studies have attempted to use Bayesian Networks (BNs), (Pearl 1993, Jensen 2001), and their extension for time-series modelling known as Dynamic Bayesian Networks (DBNs), (Ghahramani 1998, Murphy 2002), to provide a unified framework for reliability modelling and analysis of complex systems. On one hand, BNs have been shown to increase both the modelling capabilities and analysis power of combinatorial based models, by including new modelling features - like multi-state variables, noisy gates, common cause failures, and simple sequentially dependent failures - and general a posteriori diagnostic analysis (Torres-Toledano and Sucar 1998, Portinale and Bobbio 1999, Bobbio *et al.* 2001, Langseth and Portinale 2006). On the other hand, the DBN framework allows a compact representation of the temporal (and functional) dependencies among the system components and event-dependent failure behaviours, characteristic of fault-tolerant systems, avoiding the state space explosion problem of the Markov Chain based approaches to DFT analysis (Montani *et al.* 2005, Weber and Jouffe 2003). We have applied BNs to a range of real-world dependability-type problems (Neil et al. 2001, 2003). In particular, in the area of software system reliability, we have shown the advantages of BNs over traditional methods for predictive and diagnostic reasoning (Fenton and Neil 1999, Fenton *et al.* 1998, 2001, 2002).

Another important benefit of BNs is that they enable us to integrate information from different sources, including experimental data, historical data, and prior expert opinion. This is particularly useful for the reliability assessment of fault tolerant systems, where failures in test and field operations, traditionally used as a source of information for system evaluation, is prohibitively expensive or even impossible because the state-of-knowledge about large complex systems is a collection of heterogeneous and diverse source of information, comprising generally sparse data on individual subcomponents

Despite the advances summarised above, the previous application of BNs as mainstream technology for reliability modelling problems remains modest. One of the main barriers to applying BN more widely in reliability analysis is that previous attempts to apply BN models to reliability assessment have not adequately handled the necessary 'hybrid' models required in real real-world applications, i.e. models containing both continuous and discrete variables, with general static and time-dependent failure distributions. To date the Bayesian Network (BN) framework has only partially addressed these limitations.

In this paper we present a simple event-based hybrid BN modelling method for reliability assessment that scales up to large, complex dynamic systems. The new approach incorporates a recent powerful approximate inference algorithm for hybrid BNs, based on a process of dynamic discretisation of the domain of all continuous variables in the BN, which allows it to overcome most of the limitations of both space-state based reliability models and previous BN approaches. The main significant novel research contributions provided in this work are:

- Solving any configuration of static and dynamic gates with any parametric or empirical time-to-failure distributions occurring in practical applications, without using numerical integration techniques or simulation methods (Section 2).
- Modelling system state and failure times together, because of the ability to combine discrete and continuous nodes in the BN model (Section 3).
- Offering a suitable framework for Bayesian reliability modelling and data analysis, allowing us to integrate information from multiple sources at different levels of granularity, as well as expert opinion (Section 4).

All the example models shown in this paper are built and executed using the commercial general-purpose Bayesian Network software tool AgenaRisk [1], in which our dynamic discretisation algorithm is now implemented.

2. A New BN Approach To Reliability Modelling Using Dynamic Discretisation

A Bayesian Network (BN), (Pearl 1993, Jensen 2001), encodes all relevant information contained in a full probability model. It consists of 1) a directed acyclic graph (DAG), with nodes representing random variables and directed arcs (from parent to child) representing causal or influential relationships between variables, and 2) conditional probability distributions (CPDs), which define the probabilistic relationship of each node given its respective parents. Nodes without parents, called root nodes, are described according to their marginal probability distributions.

In our BN reliability model, continuous random variables represent the time-to-failure of the components of the system. These can be either the time-to-failure of elementary components of the system (root nodes), or the time-to-failure of the fault tree constructs (non-root time-to-failure nodes). In the latter case, the nodes in the BN are connected by means of incoming arcs to several components' time-to-failures and are defined as deterministic functions of the corresponding input components' time-to-failure. Discrete random variables are also included in the model to represent the state of the system (or any subsystem) at a particular time instance. The resulting model is a hybrid BN containing both continuous as well as discrete variables.

Once the BN structure and nodes probability distributions have been defined, reliability analysis can be carried out using standard BN inference algorithms (Lauritzen *et al.* 1988, Jensen *et al.* 1990). Unfortunately, for hybrid BNs containing mixtures of discrete and continuous nodes with non-Gaussian distributions, exact inference becomes computationally intractable. The traditional approach to handling (non-Gaussian) continuous nodes is static: you have to discretise them using some pre-defined range and intervals. However, this approach is unacceptable because it assumes the analyst can identify and appropriately discretise the high-density regions for each variable in the model, and do so in advance of any inference taking place. This is cumbersome, error prone and highly inaccurate.

To overcome this problem we have developed a new and powerful approximate algorithm for performing inference in hybrid BNs. We use a process of dynamic discretisation of the domain of all continuous variables in the BN and using entropy error (Kozlov and Koller 1997) as the basis for approximation. A detailed description of the dynamic discretisation algorithm is given in (Neil *et al.* 2006). In outline, the algorithm follows these steps:

1. Convert the BN to a Junction Tree (JT) and choose an initial discretisation for all continuous variables.
2. Calculate the CPD of each node given the current discretisation.
3. Enter evidence and perform global propagation on the JT, using standard JT algorithms (Jensen *et al.* 1990).
4. Query the BN to get posterior marginals for each node, compute the approximate relative entropy error, and check if it satisfies the convergence criteria.
5. If not, create a new discretisation for the node by splitting those intervals with highest entropy error.
6. Repeat the process by recalculating the NPTs and propagating the BN, and then querying to get the marginals and then split intervals with highest entropy error.
7. Continue to iterate until the model converges to an acceptable level of accuracy.

This dynamic discretisation approach allows more accuracy in the regions that matter and incurs less storage space over static discretisations. Moreover, we can adjust the discretisation any time in response to new evidence to achieve greater accuracy. By efficiently integrating our iterative approximation scheme within existing robust propagation algorithms on BN architectures, such as Junction Tree, we are able to perform robust inference analysis on complex systems.

As stated before, in order to fully define our BN model, we must specify the marginal probability density functions of all root nodes and the conditional probability distributions (CPDs) of all non-root nodes. In our framework, any standard parametric density or empirical function can be used as marginal time-to-failure distributions for the root nodes. These can be either obtained as prior information according to expert knowledge, or estimated in a previous reliability data analysis step if some failure data is available. In Section 3 we explain how to perform parameter learning in our BN framework.

The CPDs for both static and dynamic gates are probability distributions of variables that are a deterministic function of its parents, and are determined according to the type of Fault Tree (FT) construct. In general, estimating the probability distribution of a variable that is a deterministic function of its parents represents a major challenge for most BN software. For some simple configurations, such as static gates or dynamic gates with exponential time-to-failure components distributions, an exact closed-form analytical expression can be derived for the CPDs. However, for general components' failure distributions, a closed-form expression for the CPDs of dynamic gates may not be feasible, so numerical approximation methods need to be applied. In our framework, once we have determined the marginal time-to failure distributions for the basic components (root nodes), the CPDs for the DFT constructs (non-root time-to-failure nodes) are automatically estimated by modelling them as an approximate mixture of Uniform distributions, and use the dynamic discretisation algorithm to fit a histogram composed of Uniform distributions. No numerical integration techniques or simulation methods are required.

From the estimated failure distributions of the DFT constructs, we also obtain estimates for the reliability of the system for any mission time. Specifically, let the continuous random variable t_S represents the time-to-failure of a system S, and the discrete child node, S_t, with an incoming arc from t_S, represents the state of the system (or any subsystem) at a particular time instance. Once we have estimated the probability density function (PDF) of t_S, denoted by f_{t_S}, the CPD for the discrete node *C*, which defines the probability distribution of the system states at a given time *t*, can be automatically computed from the system time-to-failure distribution (e.g., $P(S_t = fail) = P(t_S \le t) = \int_0^t f_{t_S}(u)\,du$). Other metrics of interest can also be automatically derived. These include mean time to failure (MTTF), and the warranty periods, for which analytical expressions might not be obtainable.

3. BN Reliability Modelling

We now illustrate how our BN formalism, which combines dynamic discretisation with robust propagation algorithms on junction tree structures, can be used to perform DFT-like modelling and reliability data analysis of a real-world fault-tolerant system. The example provided in this section is the CPU module of the Hypothetical Cardiac Assist System (HCAS), designed to treat mechanical and electrical failures of the heart. A detailed description of the system is given in (Boudali and Dugan 2006). It consists of three sub modules: a trigger (T), a Warm Standby (WSP) gate (CPU), and a Functional Dependency (FDEP) gate ($CPUT$). The trigger consists of a crossbar switch (CS) and a system-supervision (SS). The CPU unit is a warm standby configuration with primary component P and secondary B. The CPU unit is also functionally dependent on the trigger component: the failure of either CS or SS causes the failure of the CPU unit.

The BN model for the CPU module of the HCAS is shown in Figure 1. The time-to-failure of the fault tree constructs, connected in the model by means of incoming arcs to the components' time-to-failures, are defined as deterministic functions of the corresponding input components' time-to-failure. In this example, the $CPUT$ node in the BN models the time to failure of the FDEP gate with trigger T and dependent event CPU. For non-repairable systems with perfect coverage (Dugan *et al.* 1992), the FDEP gates can be modelled as OR gates, therefore the time-to failure of the Trigger and CPUT gates, t_T and t_{CPUT}, are defined as a function of the times to failure of the input components by

$$t_T = \min\{t_{CS}, t_{SS}\}$$
$$t_{CPUT} = \min\{t_{CPU}, t_T\} \quad (1)$$

where t_{CS}, t_{SS}, t_{CPU}, and t_T represent the time-to-failure of CS, SS, CPU, and T, respectively. On the other hand, for the Warm spare redundancy CPU unit, each operation mode of the spare components B is represented by its failure distribution, (with the hazard rate of the spare component lesser in standby mode than in active mode). Thus, the time-to-failure of the CPU unit, t_{CPU}, is in turn given by (see (Marquez *et al* 2007) for details):

$$t_{CPU} = \begin{cases} t_P & \text{if } t_B^{sb} < t_P \\ t_P + t_B^{act} & \text{if } t_B^{sb} > t_P \end{cases} \qquad (2)$$

where t_P, t_B^{sb}, and t_B^{act} represent the time-to-failure of the primary component, and the spare component when in standby mode and active mode, respectively. Notice that the above formula is no longer valid if the failure distribution of the spare component is not exponential, as for components that have already accumulated some operation time in its wearout region (non-exponential failure distribution), the probability of failure during the next mission time depends upon the prior operating time. In this case, we need to include in the above expression the accumulated operation time of the spare component when it becomes active, had it been operating in the active mode since the start of the mission (Marquez *et al* 2007). We also have included in the model a binary node Reliability CPUT, with an incoming arc from t_{CPUT}, representing the state of the system at a mission time *t* hours. The NPT for this discrete node give us an estimate of the reliability of the system at a given time. This is computed from the CPUT time-to-failure by $R_S(t) = P(t_{CPUT} > t)$.

Once we have defined the marginal time-to failure distributions for the basic components, the CPDs for the DFT constructs are automatically estimated by modelling them as an approximate mixture of Uniform distributions and use the dynamic discretisation algorithm to fit a histogram composed of Uniform distributions (Marquez *et al* 2007). No analytical calculation needs to be performed and no tables need to be populated. The FT-like analysis is then carried out using our new approximate algorithm for performing inference in hybrid BNs. By running the model for 25 iterations, using fictitious input data, we obtain the reliability of the system at a mission time $t = 10^6$ hours, which is 0.594, and summary statistics for time to failure, $MTTF_{HCAS} = 112$ and $s.d._{HCAS} = 67$.

4. Data Analysis

In order to fully define the model, the parameters of the marginal time-to-failure distributions of the root nodes need to be specified. In this example, we assumed that cross switch component (CS) is exponentially distributed, and the system-supervision (SS), and the primary (P) and secondary (B) components of the CPU unit follow a Weibull distribution. All the parameters of the failure distributions for the input components are assumed unknown. The values of these parameters can be either obtained as prior information according to expert knowledge, or estimated in a previous reliability data analysis step if some failure data is available. Here, we show how Bayesian data analysis can be carried out to compute the unknown parameters of the components failure distribution, using both, historical data resulting from tests conducted on similar components and expert knowledge. We then input the result into a reliability model to compute system level reliability.

We conduct the Bayesian parameter estimation using some fictitious time to failure data for each of the components of the HCAS system, which is implemented using the dynamic discretisation algorithm available in the AgenaRisk software. Furthermore given that the target reliability for HCAS is 10^6 hours, and direct testing to these reliability levels would be infeasible, we assume that failure data has been obtained by accelerated life testing. Hence the *TTF* estimates and predictions throughout the model are given in units of 10^4 hours.

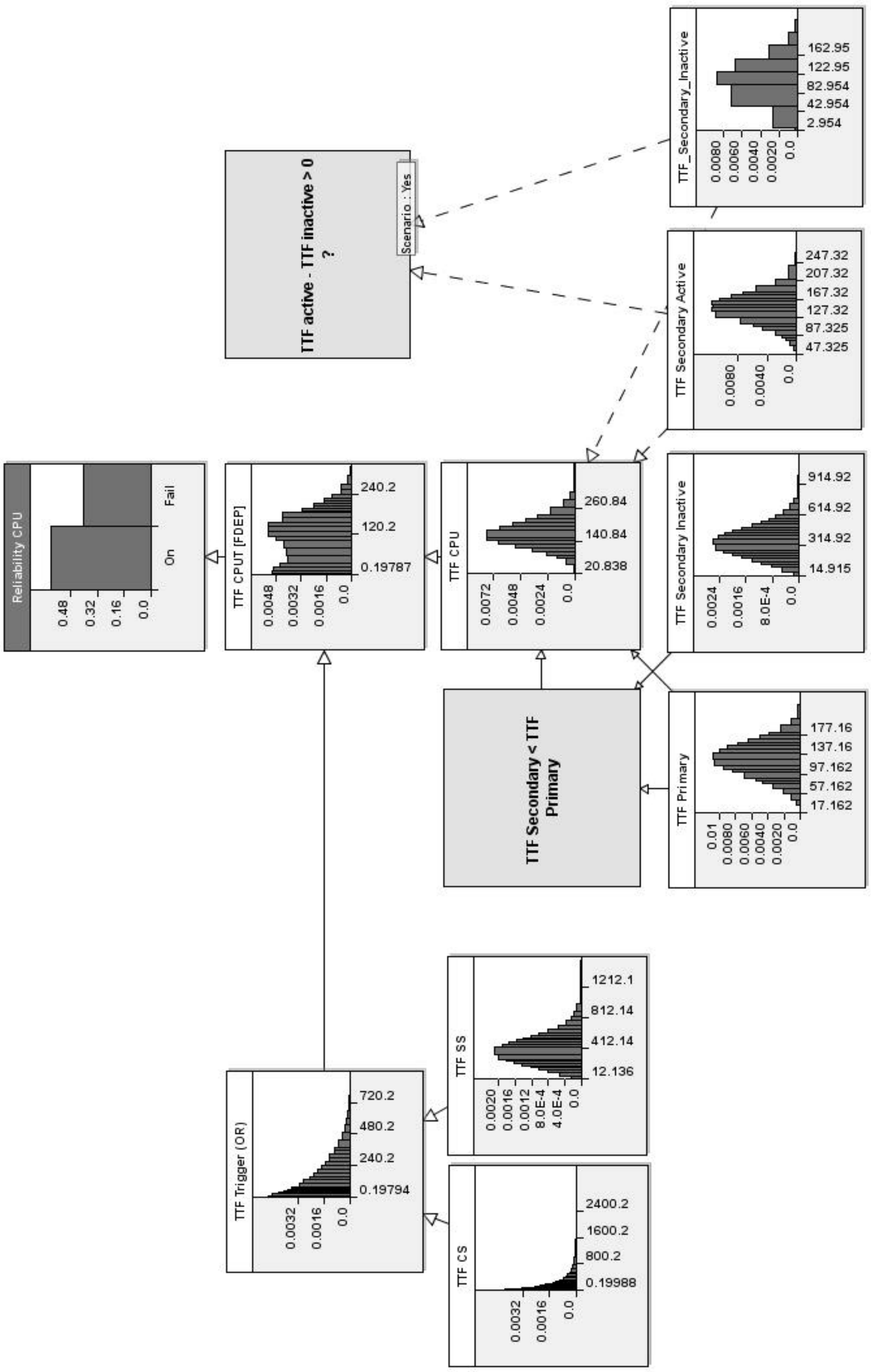

Figure 1. BN for the *HCAS* system showing marginal *TTF* , reliability distributions for primary components, gates and at system level superimposed on the BN nodes

4.1 Parameter Estimation for Crossbar Switch Component using Exponential model

Here we show how a hierarchical Bayesian data analysis can be carried out to compute the unknown hazard rate of CS using historical data gathered from test conducted on different sets of units of five types of components, $B_1 ... B_5$ with similar failure behaviour to CS. Thus the untested component CS is considered to be exchangeable with the tested components. For example, the component of interest might be a 2 litres engine and the historical data might be the time-to-failure of similar cross bar switches, but ones with slight design variations.

In order to assess the failure distribution of the similar components, we assume that a series of reliability tests have been conducted under the same operational settings for the first four components $B_1 ... B_4$ but that the failure data from component B_5 is right censored. In the case of $B_1 ... B_4$ the data resulting from these tests consist in the observed times-to-failures, $\{t_{ij}\}$, $i = 1,...,4$ and $j = 1,...,n_i$, after a fixed period of testing time, of n_i items of type *i*. In the case of B_5 the data consists of time to failure intervals $\{t_{5j} > 2000\}$, and $j = 1,..., n_5$ i.e. the tests were suspended after 2000 time units. For non-repairable systems the order of the data is immaterial.

Our aim is to use the sequence of observed failure times and intervals to assess the failure distributions of each one of the similar components, from which we wish to estimate the posterior predictive failure distribution for CS. Thus, if n_i independent tests were conducted on components of type *i* for a defined period of time *T*, the data result in n_i independent time-to-failure, with underlying exponential population distribution:

$$\{t_{ij}\}_{i=1}^{n_i} \sim \exp(l_i), \quad i = 1,...,5 \tag{3}$$

The unknown failure rates, l_i, of the B_i similar components are assumed exchangeable in their joint distribution, reflecting the lack of information - other than data - about the failure distribution of the components. The parameters l_i are thus considered a sample from the conjugate gamma prior distribution, governed by unknown hyperparameters (a, b):

$$\{l_i\}_{i=1}^{5} \sim Gamma(a, b) \tag{4}$$

To complete the specification of the hierarchical model, we need to assign a prior probability distribution to the hyperparameters (a, b). Since no joint conjugate prior is available when a and b are both assumed unknown, their prior distributions are specified independently. In the absence of any additional information about the hyperparameters, we can assign them vague prior distributions, for example, by defining vague priors such as $P(a, b) = P(a)P(b) \sim Exp(1.0)Gamma(0.01,0.01)$. However, because reliability data can be sparse and heavily censored, additional information in the form of expert judgement plays an important role in the definition of statistical reliability models. Here we choose illustrative distributions for shape and scale parameters, but in practice these might be elicited from experts:

$$a \sim Triangular(0,1,10) \text{ and } \log_{10} b \sim Triangular(-6,-3,-1) \tag{5}$$

The prior distributions used for the hyperparameters might be based on past experience, and in this particular case:

a) asking experts to use a triangular distribution is relatively easier compared to using other more complex distributions, and
b) the parameters, (a, b), can be interpreted in terms of time to failure estimates respectively. So $Triangular(0,1,10)$ has a modal failure count around 3 and decreasing probability of experiencing up to 10 failures, and this might be the range of values observed in past practice.

Figure 2 shows the BN graph of the above model, with the marginal posterior distributions superimposed for each of the hyperparameter, (a, b), the failure rates of $B_1 ... B_5$, and CS, and the posterior predictive distribution of the time to failure of CS, TTF_{CS}, shown in larger size. The relevant summary statistics for TTF_{CS} are $MTTF_{CS} = 278$ and $s.d._{CS} = 411$ (the high standard deviation clearly shows the result of pooling the diverse data from similar components. Had we simply pooled the data as if it was from a common population with one single unknown parameter, the estimate of the standard deviation would have been over optimistic).

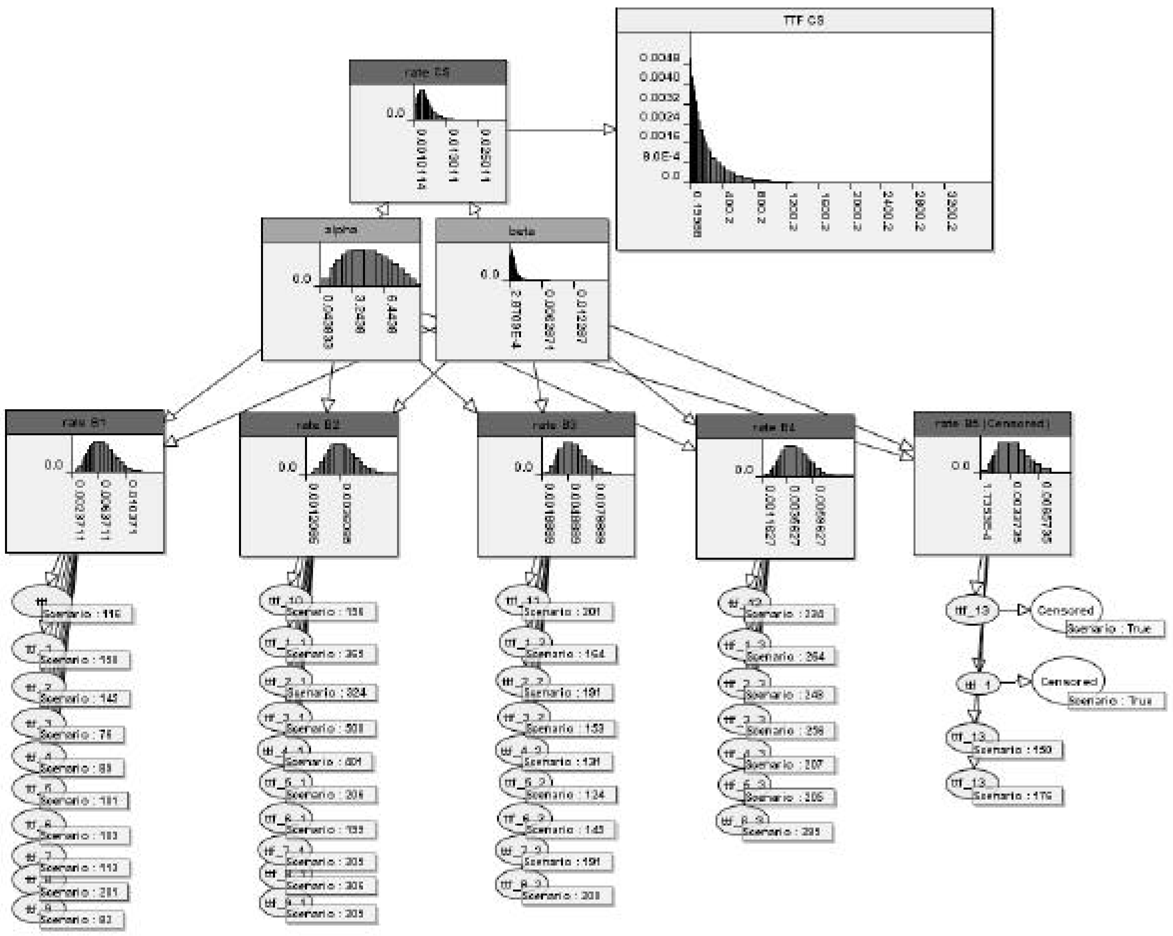

Figure 2. Parameter learning BN for the CS component showing marginal posterior distributions for rate l_i, l_{CS}, hyperparamters, (a, b), and TTF_{CS}

4.2 Estimation of System Supervision, Primary, Secondary Active and Inactive Components using Weibull model

Here we estimate parameters of the failure distribution of the System Supervision component, SS, using historical data gathered from six previous tests conducted on components with similar failure behaviour to SS. In this case, the time-to-failure data, $\{t_i\}_{i=1}^{6}$, is assumed to be a sample of independent and identically distributed observations, from a two parameter Weibull distribution:

$$\{t_i\}_{i=1}^{6} \sim Weibull(a, b) \quad (6)$$

A Bayesian inference approach is carried out, by assigning a prior distribution for the shape, a, and scale, b parameters. This is based on engineering judgement about the reliability characteristics of the component:

$$\begin{array}{l} \text{shape: } b \sim Triangular(1,2,5) \\ \text{scale: } a \sim Triangular(0,500,1000) \end{array} \quad (7)$$

The prior summary statistics for SS are $MTTF_{SS} = 449$ and $s.d._{SS} = 309$ respectively. The posterior predictive distribution of the time to failure of SS, TTF_{SS}, after learning the parameters from the data, is shown in Figure 3 superimposed over the BN model. The posterior summary statistics for SS are $MTTF_{SS} = 407$ and $s.d._{SS} = 246$ respectively.

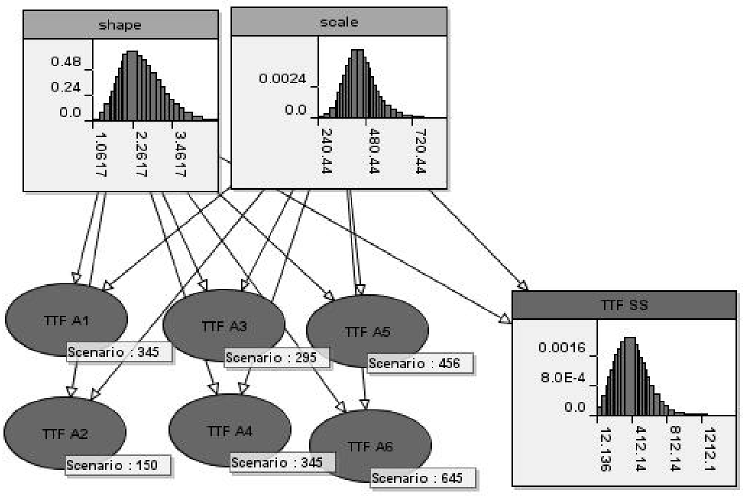

Figure 3: Parameter learning BN for the SS component showing marginal distributions for rate l_{SS}, hyperparameters, (a, b), and TTF_{SS}

The results for each of the Primary, Secondary Active and Secondary Inactive components are calculated using the same model as component SS. The failure distributions for the Primary and Spare Active components are assumed the same. The posterior summary statistics for each of the components are:

- Primary Component — $MTTF_P = 129$ and $s.d._P = 188$
- Secondary Component Active — $MTTF_{Bactive} = 129$ and $s.d._{Bactive} = 188$
- Secondary Component Inactive — $MTTF_{Binactive} = 358$ and $s.d._{Binactive} = 159$

5. Conclusions

We have provided an overview of a new approximate inference algorithm designed for a general class of hybrid BNs. This dynamic discretisation algorithm (implemented in the AgenaRisk software) finally frees BN modellers from the burden (and inaccuracies) associated with having to statically discretise continuous nodes. Continuous and discrete nodes can be included in the model to represent the continuous failure times of system components and discrete reliabilities of the system (or any subsystem) for a given target requirement, respectively. Unlike other approaches (which tend to be restricted to using exponential distributions), our new approach is able to solve any configuration of static and dynamic gates with general parametric or empirical time-to-failure distributions, without recourse to numerical integration techniques or simulation methods. Furthermore, the diagnostic analysis capabilities of the BN combined with the dynamic discretisation algorithm allow also to obtain estimates of the parameterised marginal failure distribution (for the root nodes), either using some available raw failure data or as prior information according to expert knowledge. No exact expression for the marginal is needed and no conditional probability tables need to be filled.

We have described how this approach enables us to estimate reliability of a complex system comprised of a variety of dynamic fault tree constructs, including functional dependency, OR and Warm Standby gates. Our approach provides a flexible modelling framework for reliability analysis, especially for dynamic fault trees (DFTs) and overcomes the severe limitations in competing DFT approaches, such as the problem of space state explosion and the inability to handle non-exponential failure distributions.

Likewise we have illustrated how to use the dynamic discretisation algorithm to carry out complex data analysis tasks involving hierarchical models with non conjugate priors. The most common estimation strategy for such hierarchical models, where the resulting joint distribution of the associated model parameters cannot be evaluated analytically, has been to use intensive sampling algorithms, collectively known as Markov Chain Monte Carlo (MCMC) methods, from which approximate solutions can be obtained after drawing probably ten of thousand of dependent samples. We have shown how our scheme offers a powerful alternative solution to MCMC analysis for reliability problems even in the presence of censored data.

Our BN framework is mathematically sound and at the same time simple enough and sufficiently easy to use to allow the interaction with domain experts and decision makers. Sensitivity, uncertainty, diagnosis, common cause failures, and warranty analysis can also be easily performed within this framework.

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