Dynamic Adjustment of Sparsity Upper Bound in Wideband Compressive Spectrum Sensing

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***Abstract*—Compressive sensing (CS) techniques play a key role for fast spectrum sensing in cognitive radio (CR) as it allows perfect signal reconstruction at sub-Nyquist sampling rates. However, for traditional compressing sampling approaches, the sparsity level of a signal is normally assumed as static and known, is impossible in practice. Traditionally, a statistical value of sparsity level upper bound is used as the sparsity level for signal reconstruction. In this paper, we proposed a dynamic adjustment scheme to estimate signal sparsity accurately and recover signals efficiently. In the proposed scheme, a Shrink Algorithm and Enlargement Algorithm are designed to adaptively adjust the value of sparsity level upper bound. Simulation results show that if sparsity level is too large or too small, our proposed scheme can adjust it to an proper value.**

I. Introduction

Cognitive Radio (CR) system is a spectrum agile wireless network, which further enhances spectrum efficiency without causing unacceptable interference to primary users (PUs). Specifically, unoccupied channels, named as spectrum holes, can be allocated to unlicensed secondary users (SUs) [1]. Spectrum sensing is the first step of a CR system to find spectrum holes. However, there is a considerable implementation challenge of spectrum sensing in the wideband wireless environment: e.g. traditional spectrum estimation methods operating at or above the Nyquist rate, which is a challenge for fast wideband spectrum sensing [2].

As spectrum of interest is normally underutilized in reality [3][4], spectrum has a sparse property in frequency domain. Compressive sensing (CS) is a novel technology for efficient data acquisition at sub-Nyquist sampling rates by utilizing the sparsity property of spectrum [5]. By utilizing CS, sparse signals can be reconstructed from a small number of compressing samples which are collected from linear productions of signals [6][7]. More specifically, a time domain signal $s \in R^N$ can be transferred into a S_{nz}-sparse signal in frequency domain by discrete Fourier transform (DFT) matrix Ψ, in the form of $s = \Psi\theta$, where the vector of expansion coefficients $\theta \in R^n$ represent the signal in frequency domain with S_{nz} ($S_{nz} << N$) nonzero entries [8]. Essentially, the main task of CS is to recover θ from a reduced set of samples $x \in R^M$ as follows:

$$\mathrm{x} = \Phi s = \Phi\Psi\theta \tag{1}$$

In the CS process, it is indicated that if $\Phi\Psi$ satisfies the restricted isometry property (RIP), the sparse signal $\theta \in R^n$ can be recovered exactly from the compressed samples x [6]and [9]. For Gaussian and Bernoulli random matrices, a log-like expression $M \geq \alpha S_{\mathrm{nz}} \log(N/S_{nz})$ holds for some constant α and can be used to decide the minimum number of samples for exact signal reconstruction [6]. As a result, the minimum sampling rates necessary for exact signal reconstruction can be described as $R = R_{nq}M/N$, where R_{nq} refers to the Nyquist sampling rates. Therefore, the necessary sampling rates R get lower with lower sparsity level S_{nz}. However, in most of the existing algorithms, sparsity level S_{nz} is assumed to be known beforehand [10] [11], but it is normally unknown or dynamically changing in practice. The maximum sparsity level $S_{\max}$ is normally used as a statistical upper bound of S_{nz} in practice. This may introduce a problem that the sampling rates might be unnecessarily high as $S_{\max} > S_{nz}$.

In [12], a two-step compressive spectrum sensing (TS-CSS) approach was proposed to remedy this wastage. This algorithm estimated the sparsity level S_{nz} of the unknown wideband spectrum by using a small number of samples collected at the first step, and then the estimated S_{nz} was used to decide the number of compressed samples required for exact signal reconstruction. The problem of this approach is that if $S_{\max}$ is too large compared with S_{nz}, this approach will be ineffective as the minimum number of samples M, which is determined by $S_{\max}$ for signal reconstruction, would be much larger than the real required number of samples. Additionally, if the sparsity level keeps varying, $S_{\max}$ may not become suitable for sparsity level estimation as $S_{\max}$ became a fixed value.

To solve this problem, we proposed a scheme to adjust the upper bound value $S_{\max}$ dynamically in this paper. This scheme includes two algorithms, named as Shrink Algorithm and Enlargement Algorithm, which are controlled by the proposed stopping rules. Stopping rules, the core of this scheme, is designed to obtain an proper value for $S_{\max}$.

II. Sparsity Level Estimation

The sparsity estimation means the estimation of the knowledge of S_{nz} in (1). The number of samples M required for signal reconstruction are decided by N, S_{nz} and the matrices Φ and Ψ. Here, N is the dimension of the signal, S_{nz} is the sparsity level of the actual signal, and S_{nz} represents the nonzero elements in actual signal θ which can be described by l_o-norm $S_{nz} = ||\theta||_0$. We use DFT matrix $\Psi = F^{-1}$ or the identity matrix as representing matrix $\Psi = I$. In order to satisfy the RIP and minimize the number of samples for signal reconstruction, Φ is taken as the random sampling matrix, e.g. Gaussian, Bernoulli or Analog-to-Information Conversion

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(AIC) sampling matrix [13], which can be randomly generated by certain probability distributions [6].

To validate sparsity estimation concept, a quantification methodology based on an experimental design is proposed in [12] to derivate the expression of minimum number of samples which is required to estimating sparsity level and signal reconstruction. The process to deduce the closed-form expressions for measurements is also adopted in this paper and described as follows.

A. Trail Steps

To deduce the closed-form expressions for minimum number of samples of sparsity level estimation and signal reconstruction, the trail steps are designed as follows

1) Generate an S_{nz}-sparse signal vector $\theta \in R^{\mathrm{n}}$.
2) Collect samples by random sampling matrix Φ.
3) Recover the S_{nz}-sparse signal vector θ by minimizing the l_1-norm of θ, which subjects to a linear constraint shown as follows:

$$\hat{\theta} = \arg\min ||\theta||_1 \quad s.t. \quad x = \Phi\Psi\theta. \tag{2}$$

4) Estimate the sparsity level by:

$$\hat{S}_{nz} = \sum_{i=1}^{N} (|\theta\,[i]| \geq \lambda). \tag{3}$$

where λ is the threshold value for spectrum decision. (For the noise free case λ is chosen as 0.5.)

When $\hat{\theta} = \theta$, it is a successful reconstruction. For $\hat{S}_{nz} = S_{nz}$, it is successful sparsity estimation. A range of values are tested for M_1 and M in success rate of 99% among 500 times Monte Carlo trial tests in order to get the minimum number of samples M_1 and M for sparsity level estimation and signal reconstruction respectively.

B. Curve Fitting Results

The simulation results and curve fitting techniques are used to find the values of the constant α in $M \geq \alpha S_{\mathrm{nz}} \log(N/S_{nz}) + C$. In this process, the values of N and S_{nz} are fixed to test the expression by different matrices [14].

Three results from the curve fitting process of Monte Carlo trials are obtained as follows:

Result 1: When M_1 is larger than the value in (4), we declare a successful sparsity estimation with 99% probability [12].

$$M_1 = 1.3S_{\max}\log(N/S_{\max}) + 4.6 \tag{4}$$

Result 2: When M is lager than the value in (4), we declare a successful signal reconstruction with 99% probability [12].

$$M = 1.7S_{nz}\log(N/S_{nz} + 1) \tag{5}$$

III. The Proposed Dynamic Sparsity Upper Bound Adjustment Scheme

A. Problem Statement

For the traditional TS-CSS approach and our proposed scheme, they all separately collect the all M measurements in two steps(collect M_1 and M_2 measurements separately). However, In the TS-CSS approach we find the gap between S_{nz} and $S_{\max}$ cannot be too large. For example, as shown

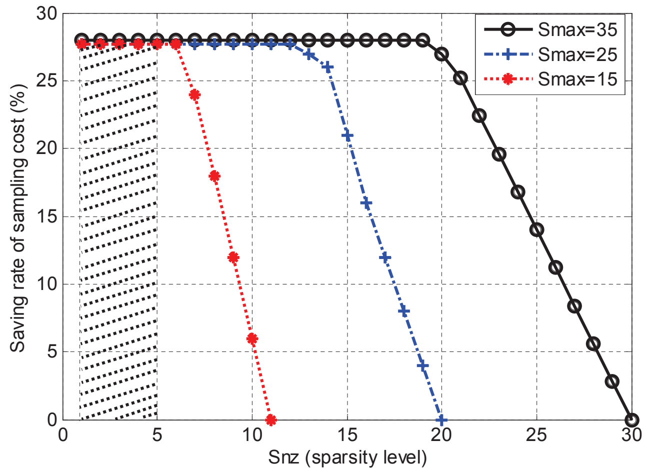

Fig. 1: TS-CSS approach: Saving rate of sampling costs with different values of $S_{\max}$ (the upper bound value of S_{nz}), N = 1024.

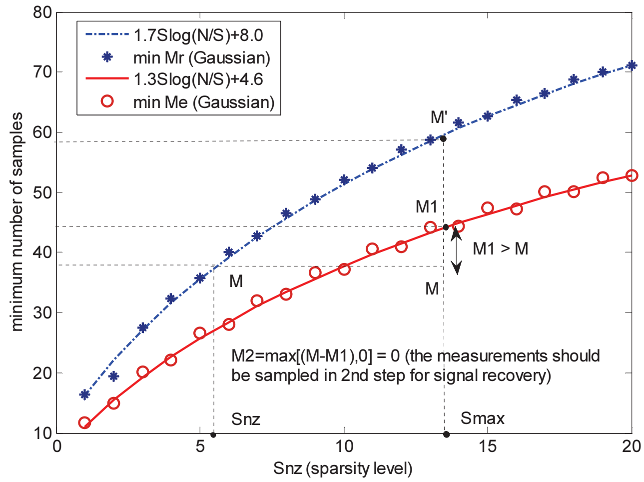

Fig. 2: Problem occurs when S_{nz} is too small compared with $S_{\max}$.

in the shadow area of Fig. 1, the saved sampling rates of TS-CSS approach keep in constant when S_{nz} is too small compared with $S_{\max}$, which leads to $M_1 \geq M$ according to (4) and (5). Under this circumstance, the second step is useless for $M_2 = 0$ as indicated in Fig. 2. In Fig. 2, it is noticed that the lower actual sparsity level, the more samples $M' - M$ saved. Here, M' is the minimum samples number for signal reconstruction without sparsity estimation in the traditional one-step compressive spectrum sensing algorithm.In the legend, min Mr is minimum number of measurements for successful signal recovery. Min Me is minimum number of measurements for successful sparsity estimation. However, if the sparsity level S_{nz} is too low compared with $S_{\max}$, it causes $M_2 = M - M_1 < 0$. In other words, M_1 samples are enough for signal recovery, so its performance degenerates to one-step compressing approach since the second step is useless. Therefore, the saving rate of sampling resources increases from one-step compressing approach, to traditional TS-CSS approach, and to the proposed scheme.

In order to solve this problem, we propose a novel dynamic

Algorithm 1 Shrink Algorithm

Input:
Received signal $r(t)$, step size Δ, sparsity level upper bound $S_{\max}$.
Initialization: $j = 1$, $M_2 = 0$.
while $M_2 < 0$ **do**
  Calculate M_1 from (4) and recover X_1 by (2);.
  Get $S_{\rm nz}$ by (3);
  Calculate $M_2 = M - M_1$;
  Update: $j = j + 1$ and $S_{\max} = S_{\max} - \Delta$.
end while
Output:
New $S_{\max}$, M_2 and X_1 (Partial recovered signal from M_1 measurements).

Algorithm 2 Enlargement Algorithm

Input:
Received signal $r(t)$, step size Δ, sparsity level upper bound $S_{\max}$.
Initialization: $j = 1$, $\hat{S}_{nz} = 0$.
while $\hat{S}_{nz}^{j} \neq \hat{S}_{nz}^{j-1}$ **do**
  Update: $j = j + 1$ and $S_{\max} = S_{\max} + \Delta$;
  Calculate M_1 from (4) and recover X_1 by (2);.
  Get ${S_{\rm nz}}^{j}$ by (3).
end while
Output:
New $S_{\max}$ and X_1 (Partial recovered signal from M_1 measurements).

sparsity upper bound adjustment scheme to adjust the maximum number of samples $S_{\max}$ for sparsity level estimation. When $S_{\max}$ is too large compared with $S_{\rm nz}$, the value of $S_{\max}$ will adjust adaptively by a step length Δ until it is suitable to current sparsity level $S_{\rm nz}$. This is achieved by the proposed Shrink Algorithm. With the Shrink Algorithm, the value of $S_{\rm nz}$ can be reduced if it is too large. There is another scenario that the available value of $S_{\max}$ is lower than $S_{\rm nz}$, which result in that M_1 is not enough for the estimation of $S_{\rm nz}$. Therefore, we need to increase $S_{\max}$ by a step length Δ. This process is achieved by the proposed Enlargement Algorithm.

B. Stopping Rules in the Proposed Two Algorithms

The core of the proposed dynamic adjustment scheme is the stopping rules for the Shrink Algorithm and the Enlargement Algorithm. In the two algorithms, the stopping rules are used to judge whether the value of $S_{\max}$ is proper or not.

The process of the proposed Shrink Algorithm is shown in Algorithm 1. Suppose that the received signal $r(t)$ with $S_{\rm nz}$ non-zero components and the upper bound $S_{\max}$ of the sparse signal are known. We get M_1 samples by (4) and recover it by (2) to decide the $S_{\rm nz}$ by (3) and get the number of M_2. In the next step, we reduce $S_{\max}$ by the step size and repeat the whole process until $M_2 > 0$. Fig. 1 and Fig. 2 show the two-step compressive sensing approach is efficient enough when $M_2 > 0$. When $M_2 = 0$ (For $M_1 \geq M$), this algorithm degenerates into one-step approach, so $S_{\max}$ is not proper.

Proposition 1: if $M_2 > 0$, $S_{\max}$ is proper with probability of 100%.

In the Enlargement Algorithm, the received signal $r(t)$ and the value of $S_{\max}$ are known in advance. The number $M_2 < 0$ can be fixed by the Shrink Algorithm when $S_{\max}$ is too large. The Enlargement Algorithm can fix the problem when $S_{\max}$ is not large enough, which is caused by the Shrink Algorithm or channel occupancy changing. The number of samples M_1 for sparsity estimation can be obtained by (4), where the sampling matrix is $X_1^j = \left[\begin{array}{cc} \Phi_1^j & 0 \end{array}\right]$ and Φ_1^j is a matrix with size ${M_1}^{\rm j} \times N/2$ (N is the dimension of sparsity signal). The original signal can be recovered by (2) where $X_1^j = \left[\begin{array}{cc} \Phi_1^j & 0 \end{array}\right] \Psi \hat{r}_f^j$. As we do in the Shrink Algorithm, $S_{\rm nz}$ can be decided by (3) and Ψ is normally taken as DFT matrix or unit matrix. In the next loop, $S_{\max}$ is increased by a step size Δ to get the updated $S_{\rm nz}$. The solutions $S_{\rm nz}$ at step $j-1$ and j is compared. If they agree, $S_{\max}$ is determined as the appropriate sparsity estimation. This whole process of the Enlargement Algorithm is shown as Algorithm 2.

Proposition 2: if $\hat{S}_{nz}^{j} = \hat{S}_{nz}^{j-1}$, then $S_{\max}$ is enough to get exact recovery with probability of 100%.

From curve fitting results, the value of $S_{\rm nz}$ can be obtained by M_1 measurements of signal by using the knowledge of $S_{\max}$. Additionally, in the situation that $S_{\max}$ is large enough to get M_1, the value of $S_{\rm nz}$ will keep in constant no matter $S_{\max}$ is increased by a step length or more.

After the Shrink Algorithm and Enlargement Algorithm are applied in the compressive spectrum sensing, the final process is as follows:

1) Collect M_2 samples from the received signal $r(t)$. Get the recovered signal X_2 from M_2 samples. Combine X_1 and X_2 to get $X = \left[\begin{array}{c} X_1 \\ X_2 \end{array}\right]$. X is the accurately recovered signal
2) Reconstruct the received signal by (2).
3) Make spectrum occupancy decision $\hat{d}$.

In summary, the whole process of the proposed dynamic sparsity upper bound adjustment scheme can be shown in Fig. 3. It includes that the start point (calculate M_1) and two loops. One loop is: calculate M_1, get $S_{\rm nz}$, update $S_{\max}$ and update M_1 (the Enlargement Algorithm). Another loop follows: calculate M_1, get $S_{\rm nz}$, calculate M, calculate M_2, update $S_{\max}$ and calculate M_1 (the Shrink Algorithm). If the value of $S_{\max}$ passes the first loop, the second loop begins to work. After the two loops, the proper value of $S_{\max}$ can be obtained.

IV. Numerical Results

We simulate a cognitive radio network with $N = 128$ channels and the identity matrix is used as representation matrix. Gaussian random matrix is taken as a sampling matrix.Theoretically, the step length Δ can be any positive integers. If it is changed slowly, large step length can help us to get proper $S_{\max}$ by less sensing times so as to get higher saving rate of sampling cost, but if spectrum occupancy situation is changed rapidly, it is difficult to get the suitable value precisely by a large step since it takes very little time to get close to the proper $S_{\max}$. Thus, the smallest value likely is not optimal for any situation, but it is conservative

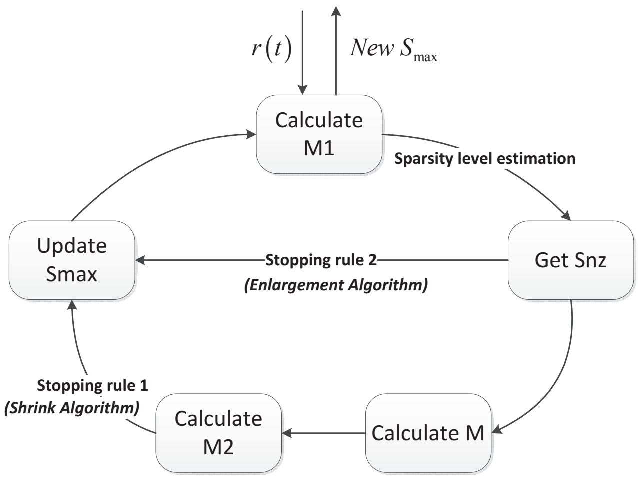

Fig. 3: Flow chart of the proposed scheme.

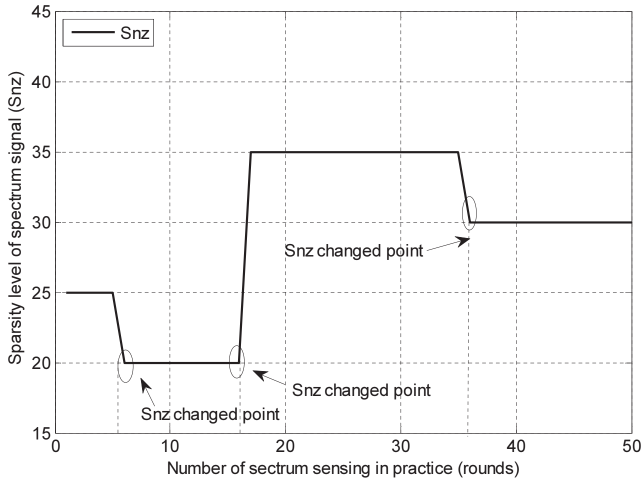

Fig. 5: $S_{\rm nz}$ changes during the detection process.

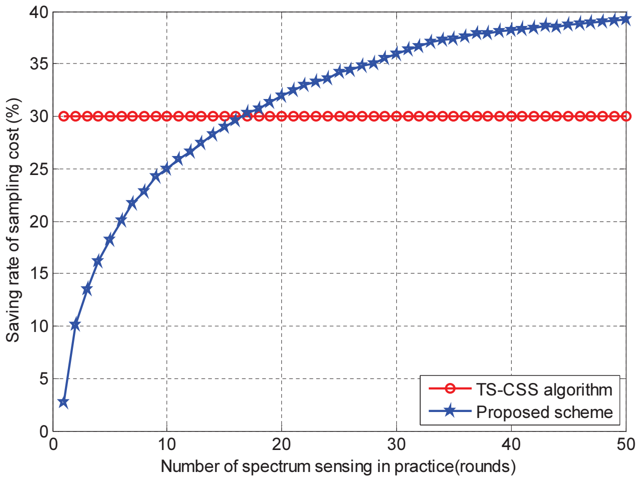

Fig. 4: Saving rate of sampling cost when spectrum occupancy keeps fixed, where $S_{\rm max}$ is much bigger than $S_{\rm nz}$.

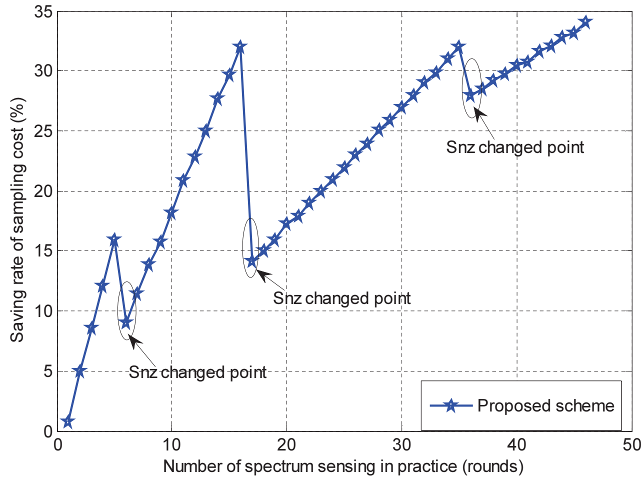

Fig. 6: Saving rate of sampling cost when spectrum occupancy experiences three changes.

value to chosen to avoid problem with the change of spectrum occupancy. So $\Delta = 1$ is chosen in the simulation.

Before we use a new $S_{\rm max}$ to perform sparsity estimation and signal reconstruction, we should adjust $S_{\rm max}$ within a transient time period. In Fig. 4, we can see that the saving rate of sampling cost of our proposed scheme is poor than the original two-step approach when we just perform the compressive spectrum sensing in a few sensing periods. This is because the process of calculating the approximate value of $S_{\rm max}$ would introduce some extra costs than the traditional TS-CSS. However, the proposed scheme will benefit when the updated $S_{\rm max}$ is used in multiple continuous sensing periods when the spectrum occupancy is not changed. The algorithm have to adjust the $S_{\rm max}$ again when the spectrum occupancy changes.

Another scenario is that the spectrum occupancy is varying, which means that the sparsity level $S_{\rm nz}$ of spectrum signal is changing. When the value of $S_{\rm nz}$ is changed as shown in Fig. 5, the performance of proposed scheme is indicated in Fig. 6. As the result of spectrum situation changing, the performance of proposed scheme can be degraded temporarily.

V. Conclusions

In CS based spectrum sensing algorithm, the sparsity level should be estimated before determining the minimum number of samples for exact signal reconstruction. Normally, a statistical maximum sparsity level $S_{\rm max}$ used for sparsity level estimation is unknown. In this paper, a dynamic adjustment scheme for sparsity upper bound is proposed for accurate sparsity level estimation. In the proposed scheme, the value of $S_{\rm max}$ is adjusted dynamically by the proposed Shrink Algorithm and Enlargement Algorithm, and the proposed stopping rules are used to determine whether the proper value of $S_{\rm max}$ is obtained. Simulation results show that our proposed algorithm is efficient and can deal with the varying spectrum occupancy.

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