# Modeling and Analysis of Two-Way Relay Non-Orthogonal Multiple Access Systems

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Abstract-A two-way relay non-orthogonal multiple access (TWR-NOMA) system is investigated, where two groups of NOMA users exchange messages with the aid of one half-duplex (HD) decode-and-forward (DF) relay. Since the signal-plusinterference-to-noise ratios (SINRs) of NOMA signals mainly depend on effective successive interference cancellation (SIC) schemes, imperfect SIC (ipSIC) and perfect SIC (pSIC) are taken into account. In order to characterize the performance of TWR-NOMA systems, we first derive closed-form expressions for both exact and asymptotic outage probabilities of NOMA users' signals with ipSIC/pSIC. Based on the derived results, the diversity order and throughput of the system are examined. Then we study the ergodic rates of users' signals by providing the asymptotic analysis in high SNR regimes. Lastly, numerical simulations are provided to verify the analytical results and show that: 1) TWR-NOMA is superior to TWR-OMA in terms of outage probability in low SNR regimes; 2) Due to the impact of interference signal (IS) at the relay, error floors and throughput ceilings exist in outage probabilities and ergodic rates for TWR-NOMA, respectively; and 3) In delay-limited transmission mode, TWR-NOMA with ipSIC and pSIC have almost the same energy efficiency. However, in delay-tolerant transmission mode, TWR-NOMA with pSIC is capable of achieving larger energy efficiency compared to TWR-NOMA with ipSIC.

*Index Terms*—Imperfect SIC, non-orthogonal multiple access (NOMA), two-way relay

## I. INTRODUCTION

With the purpose to improve system throughput and spectrum efficiency, the fifth generation (5G) mobile communication networks are receiving a great deal of attention. The requirements of 5G networks mainly contain key performance indicator (KPI) improvement and support for new radio (NR) scenarios [2], including enhanced mobile broadband (eMBB), massive machine type communications (mMTC), and ultra-reliable and low latency communications (URLLC). Apart from crux technologies, such as massive multiple-input multiple-output (MIMO), millimeter wave and heterogeneous networks, the design of novel multiple access (MA) techniques is significant to make the contributions for 5G networks. Driven by these, non-orthogonal multiple access (NOMA) has been viewed as one of promising technologies to increase system

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Until now, point-to-point NOMA has been discussed extensively in many research contributions [8–11]. In [8], the authors have investigated the outage performance and ergodic rate of downlink NOMA with randomly deployed users by invoking stochastic geometry. Considering the secrecy issues of NOMA against external eavesdroppers, the authors in [9] investigated secrecy outage behaviors of NOMA in largerscale networks for both single-antenna and multiple-antenna transmission scenarios. Explicit insights for understanding the asynchronous NOMA, a novel interference cancellation scheme was proposed in [10], where the bit error rate and throughput performance were analyzed. By the virtue of available CSI, the performance of NOMA based multicast cognitive radio scheme (MCR-NOMA) was evaluated [11], in which outage probability and diversity order are obtained for both secondary and primary networks. Very recently, the application of cooperative communication [12] to NOMA is an efficient way to offer enhanced spectrum efficiency and spatial diversity. Hence the integration of cooperative communication with NOMA has been widely discussed in many treaties [13–16]. Cooperative NOMA has been proposed in [13], where the user with better channel condition acts as a decode-and-forward (DF) relay to forward information. Furthermore, in [14], the authors studied the ergodic rate of DF relay for a NOMA system. With the objective of improving energy efficiency, the application of simultaneous wireless information and power transfer (SWIPT) to the nearby user was investigated where the locations of NOMA users were modeled by stochastic geometry [15]. Considering the impact of imperfect channel state information (CSI), the authors in [16] investigated the performance of amplify-and-forward (AF) relay for downlink NOMA networks, where the exact and tight bounds of outage probability were derived. Moreover, in [17], the outage behavior and ergodic sum rate of NOMA for AF relay was analyzed under Nakagami-m fading channels. To further enhance spectrum efficiency, the performance of fullduplex (FD) cooperative NOMA was characterized in terms

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of outage probability [18].

Above existing treaties on cooperative NOMA are all based on one-way relay scheme, where the messages are delivered in only one direction, (i.e., from the BS to the relay or user destinations). As a further advance, two-way relay (TWR) technique introduced in [19] has attracted remarkable interest as it is capable of boosting spectral efficiency. The basic idea of TWR systems is to exchange information between two nodes with the help of a relay, where AF or DF protocol can be employed. With the emphasis on user selection, in [20], the authors analyzed the performance of multi-user TWR channels for half-duplex (HD) AF relays. By applying physical-layer network coding (PNC) schemes, the performance of twoway AF relay systems was investigated in terms of outage probability and sum rate [21]. It was shown that two time slots PNC scheme achieves a higher sum rate compared to four time slot transmission mode. In [22], the authors studied the outage behaviors of DF relay with perfect and imperfect CSI conditions, where a new relay selection scheme was proposed to reduce the complexity of TWR systems. In terms of CSI and system state information, the system outage behavior was investigated for two-way full-duplex (FD) DF relay on different multi-user scheduling schemes [23]. In [24], the authors investigated the performance of multi-antenna TWR networks in which both AF and DF protocols are examined, respectively. Taking residual self-interference into account, the tradeoffs between the outage probability and ergodic rate were analyzed in [25] for FD TWR systems. In addition, the authors in [26] studied the performance of cooperative spectrum sharing by utilizing TWR over general fading channels. It was worth mentioning that the effective spectrum sharing is achieved by restraining additional cooperative diversity order.

#### A. Motivations and Contributions

While the aforementioned theoretical researches have laid a solid foundation for the understanding of NOMA and TWR techniques in wireless networks, the TWR-NOMA systems are far from being well understood. Obviously, the application of TWR to NOMA is a possible approach to improve the spectral efficiency of systems. To the best of our knowledge, there is no contributions to investigate the performance of TWR for NOMA systems. Moreover, the above contributions for NOMA have been comprehensively studied under the assumption of perfect SIC (pSIC). In practical scenarios, there still exist several potential implementation issues with the use of SIC (i.e., complexity scaling and error propagation). More precisely, these unfavorable factors will lead to errors in decoding. Once an error occurs for carrying out SIC at the nearby user, the NOMA systems will suffer from the residual interference signal (IS). Hence it is significant to examine the detrimental impacts of imperfect SIC (ipSIC) for TWR-NOMA. Motivated by these, we investigate the performance of TWR-NOMA with ipSIC/pSIC in terms of outage probability, ergodic rate and energy efficiency, where two groups of NOMA users exchange messages with the aid of a relay node using DF protocol.

The essential contributions of our paper are summarized as follows:

- We derive the closed-form expressions of outage probability for TWR-NOMA with ipSIC/pSIC. Based on the analytical results, we further derive the corresponding asymptotic outage probabilities and obtain the diversity orders. Additionally, we discuss the system throughput in delay-limited transmission mode.
- 2) We show that the outage performance of TWR-NOMA is superior to TWR-OMA in the low signal-to-noise ratio (SNR) regime. We observe that due to the effect of IS at the relay, the outage probabilities for TWR-NOMA converge to error floors in the high SNR regime. We confirm that the use of pSIC is incapable of overcoming the zero diversity order for TWR-NOMA.
- 3) We study the ergodic rate of users' signals for TWR-NOMA with ipSIC/pSIC. To gain more insights, we discuss one special case that when there is no IS between a pair of antennas at the relay. On the basis of results derived, we obtain the zero high SNR slopes for TWR-NOMA systems. We demonstrate that the ergodic rates for TWR-NOMA converge to throughput ceilings in high SNR regimes.
- 4) We analyze the energy efficiency of TWR-NOMA with ipSIC/pSIC in both the delay-limited and tolerant transmission modes. We confirm that TWR-NOMA with ipSIC/pSIC in delay-limited transmission mode has almost the same energy efficiency. Furthermore, in delaytolerant transmission mode, the energy efficiency of system with pSIC is higher than that of system with ipSIC.

## B. Organization and Notation

The remainder of this paper is organised as follows. In Section II, the system mode for TWR-NOMA is introduced. In Section III, the analytical expressions for outage probability, diversity order and system throughput of TWR-NOMA are derived. Then the ergodic rates of users' signals for TWR-NOMA are investigated in Section IV. The system energy efficiency is evaluated in Section V. Analytical results and numerical simulations are presented in Section VI, which is followed by our conclusions in Section VII.

The main notations of this paper is shown as follows:  $\mathbb{E}\{\cdot\}$  denotes expectation operation;  $f_X(\cdot)$  and  $F_X(\cdot)$  denote the probability density function (PDF) and the cumulative distribution function (CDF) of a random variable X.

## II. SYSTEM MODEL

## A. System Description

We focus our attentions on a two-way relay NOMA communication scenario which consists of one relay R, two pairs of NOMA users  $G_1 = \{D_1, D_2\}$  and  $G_2 = \{D_3, D_4\}^1$ . To reduce the complexity of systems, many research contributions on NOMA have been proposed to pair two users for the

<sup>&</sup>lt;sup>1</sup>The geographical dimensions of clusters  $G_1$  and  $G_2$  are to ensure that there is a certain distance difference from distant user and nearby user to R.

application of NOMA protocol<sup>2</sup> [27, 28]. As shown in Fig. 1, we assume that  $D_1$  and  $D_3$  are the nearby users in groups  $G_1$  and  $G_2$ , respectively, while  $D_2$  and  $D_4$  are the distant users in groups  $G_1$  and  $G_2$ , respectively. It is worth noting that the nearby user and distant user are distinguished based on the distance from the users to R [29]. For example,  $D_1$ and  $D_3$  are near to R, while  $D_2$  and  $D_4$  are far away from R. The exchange of information between user groups  $G_1$  and  $G_2$  is facilitated via the assistance of a decode-andforward (DF) relay with two antennas, namely  $A_1$  and  $A_2^3$ . User nodes are equipped with single antenna and transmit the signals by utilizing superposition coding scheme. In practical communication process, the complexity of DF protocol is too high to implement. To facilitate analysis, we focus our attention on a idealized DF protocol, where R is capable of decoding the users' information correctly. Relaxing this idealized assumption can make system mode close to the practical scenario, but this is beyond the scope of this treatise. Additionally, to evaluate the impact of error propagation on TWR-NOMA, ipSIC operation is employed at relay R and nearby users. It is assumed that the direct links between two pairs of users are inexistent due to the effect of strong shadowing. Without loss of generality, all the wireless channels are modeled to be independent quasi-static block Rayleigh fading channels and disturbed by additive white Gaussian noise with mean power  $N_0$ . Furthermore,  $h_1$ ,  $h_2$ ,  $h_3$  and  $h_4$  are denoted as the complex channel coefficient of  $D_1 \leftrightarrow R$ ,  $D_2 \leftrightarrow R$ ,  $D_3 \leftrightarrow R$  and  $D_4 \leftrightarrow R$  links, respectively. We assume that the channels from user nodes to R and the channels from R to user nodes are reciprocal. In other words, the channels from user nodes to R have the same fading impact as the channels from R to the user nodes [25, 30, 31]. The channel power gains  $|h_1|^2$ ,  $|h_2|^2$ ,  $|h_3|^2$  and  $|h_4|^2$  are assumed to be exponentially distributed random variables (RVs) with the parameters  $\Omega_i$ ,  $i \in \{1, 2, 3, 4\}$ , respectively. Note that the perfect CSIs of NOMA users are available at R for signal detection.

## B. Signal Model

During the first slot, the pair of NOMA users in  $G_1$  transmit the signals to R just as uplink NOMA. Since R is equipped with two antennas, when R receives the signals from the pair of users in  $G_1$ , it will suffer from interference signals from the pair of users in  $G_2$ . More precisely, the observation at Rfor  $A_1$  is given by

$$y_{R_{A_1}} = h_1 \sqrt{a_1 P_u x_1} + h_2 \sqrt{a_2 P_u x_2} + \varpi_1 I_{R_{A_2}} + n_{R_{A_1}},$$
(1)

where  $I_{R_{A_2}}$  denotes IS from  $A_2$  with  $I_{R_{A_2}} = (h_3\sqrt{a_3P_u}x_3 + h_4\sqrt{a_4P_u}x_4)$ .  $\varpi_1 \in [0,1]$  denotes the impact levels of IS at R.  $P_u$  is the transmission power at user nodes.  $x_1, x_2$  and  $x_3, x_4$  are the signals of  $D_1, D_2$  and  $D_3, D_4$ , respectively, i.e,

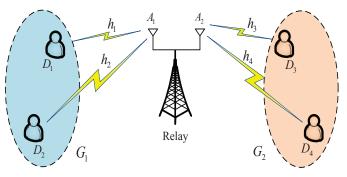


Fig. 1: An illustration of TWR-NOMA systems, in which two groups of users exchange messages with the aid of one relay node.

 $\mathbb{E}\{x_1^2\} = \mathbb{E}\{x_2^2\} = \mathbb{E}\{x_3^2\} = \mathbb{E}\{x_4^2\} = 1. a_1, a_2 \text{ and } a_3, a_4$ are the corresponding power allocation coefficients. Note that the efficient uplink power control is capable of enhancing the performance of the systems considered, which is beyond the scope of this paper.  $n_{R_{A_j}}$  denotes the Gaussian noise at R for  $A_j, j \in \{1, 2\}$ .

Similarly, when R receives the signals from the pair of users in  $G_2$ , it will suffer from interference signals from the pair of users in  $G_1$  as well and then the observation at R is given by

$$y_{R_{A_2}} = h_3 \sqrt{a_3 P_u x_3} + h_4 \sqrt{a_4 P_u x_4} + \varpi_1 I_{R_{A_1}} + n_{R_{A_2}},$$
(2)

where  $I_{R_{A_1}}$  denotes the interference signals from  $A_1$  with  $I_{R_{A_1}} = (h_1 \sqrt{a_1 P_u} x_1 + h_2 \sqrt{a_2 P_u} x_2).$ 

Applying the NOMA protocol, R first decodes  $D_l$ 's information  $x_l$  by the virtue of treating  $x_t$  as IS. Hence the received signal-to-interference-plus-noise ratio (SINR) at R to detect  $x_l$ is given by

$$\gamma_{R \to x_l} = \frac{\rho |h_l|^2 a_l}{\rho |h_t|^2 a_t + \rho \varpi_1 (|h_k|^2 a_k + |h_r|^2 a_r) + 1}, \quad (3)$$

where  $\rho = \frac{P_u}{N_0}$  denotes the transmit signal-to-noise ratio (SNR),  $(l,k) \in \{(1,3), (3,1)\}, (t,r) \in \{(2,4), (4,2)\}.$ 

After SIC is carried out at R for detecting  $x_l$ , the received SINR at R to detect  $x_t$  is given by

$$\gamma_{R \to x_t} = \frac{\rho |h_t|^2 a_t}{\varepsilon \rho |g|^2 + \rho \varpi_1 (|h_k|^2 a_k + |h_r|^2 a_r) + 1}, \quad (4)$$

where  $\varepsilon = 0$  and  $\varepsilon = 1$  denote the pSIC and ipSIC employed at R, respectively. Due to the impact of ipSIC, the residual IS is modeled as Rayleigh fading channels [32] denoted as gwith zero mean and variance  $\Omega_I$ .

In the second slot, the information is exchanged between  $G_1$  and  $G_2$  by the virtue of R. Therefore, just like the downlink NOMA, R transmits the superposed signals  $(\sqrt{b_1P_r}x_1 + \sqrt{b_2P_r}x_2)$  and  $(\sqrt{b_3P_r}x_3 + \sqrt{b_4P_r}x_4)$  to  $G_2$ and  $G_1$  by  $A_2$  and  $A_1$ , respectively.  $b_1$  and  $b_2$  denote the power allocation coefficients of  $D_1$  and  $D_2$ , while  $b_3$  and  $b_4$ are the corresponding power allocation coefficients of  $D_3$  and  $D_4$ , respectively.  $P_r$  is the transmission power at R and we assume  $P_u = P_r$ . In particular, to ensure the fairness between

<sup>&</sup>lt;sup>2</sup>Note that increasing the number of paired users, i,e,. *N* pairs of users, will not affect the performance of TWR-NOMA system. It is worth pointing that within each group, superposition coding and SIC are employed, and across the groups, transmissions are orthogonal.

 $<sup>^{3}</sup>$ For the practical scenario, we can assume that the relay is located on a mountain, where the user nodes on both sides of the mountain are capable of exchanging the information between each other.

users in  $G_1$  and  $G_2$ , a higher power should be allocated to the distant user who has the worse channel condition. Hence we assume that  $b_2 > b_1$  with  $b_1 + b_2 = 1$  and  $b_4 > b_3$  with  $b_3 + b_4 = 1$ . Note that the fixed power allocation coefficients for two groups' NOMA users are considered. Relaxing this assumption will further improve the performance of systems and should be concluded in our future work.

According to NOMA protocol, SIC is employed and the received SINR at  $D_k$  to detect  $x_t$  is given by

$$\gamma_{D_k \to x_t} = \frac{\rho |h_k|^2 b_t}{\rho |h_k|^2 b_l + \rho \varpi_2 |h_k|^2 + 1},$$
(5)

where  $\varpi_2 \in [0, 1]$  denotes the impact level of IS at the user nodes. Then  $D_k$  detects  $x_l$  and gives the corresponding SINR as follows:

$$\gamma_{D_k \to x_l} = \frac{\rho |h_k|^2 b_l}{\varepsilon \rho |g|^2 + \rho \varpi_2 |h_k|^2 + 1}.$$
(6)

Furthermore, the received SINR at  $D_t$  to detect  $x_r$  can be given by

$$\gamma_{D_r \to x_t} = \frac{\rho |h_r|^2 b_t}{\rho |h_r|^2 b_l + \rho \varpi_2 |h_r|^2 + 1}.$$
(7)

From above process, the exchange of information is achieved between the NOMA users for  $G_1$  and  $G_2$ . More specifically, the signal  $x_1$  of  $D_1$  is exchanged with the signal  $x_3$  of  $D_3$ . Furthermore, the signal  $x_2$  of  $D_2$  is exchanged with the signal  $x_4$  of  $D_4$ .

#### III. OUTAGE PROBABILITY

In this section, the performance of TWR-NOMA is characterized in terms of outage probability. Due to the channel's reciprocity, the outage probability of  $x_l$  and  $x_t$  are provided in detail in the following part.

1) Outage Probability of  $x_l$ : In TWR-NOMA system, the outage events of  $x_l$  are explained as: i) R cannot decode  $x_l$  correctly; ii) The information  $x_t$  cannot be detected by  $D_k$ ; and iii)  $D_k$  cannot detect  $x_l$ , while  $D_k$  can first decode  $x_t$  successfully. To simplify the analysis, the complementary events of  $x_l$  are employed to express its outage probability. As a consequence, the outage probability of  $x_l$  with ipSIC for TWR-NOMA system can be given by

$$P_{x_{l}}^{ipSIC} = 1 - \Pr\left(\gamma_{R \to x_{l}} > \gamma_{th_{l}}\right) \\ \times \Pr\left(\gamma_{D_{k} \to x_{t}} > \gamma_{th_{t}}, \gamma_{D_{k} \to x_{l}} > \gamma_{th_{l}}\right), \quad (8)$$

where  $\varepsilon = 1$ ,  $\varpi_1 \in [0, 1]$  and  $\varpi_2 \in [0, 1]$ .  $\gamma_{th_l} = 2^{2R_l} - 1$  with  $R_l$  being the target rate at  $D_k$  to detect  $x_l$  and  $\gamma_{th_t} = 2^{2R_t} - 1$  with  $R_t$  being the target rate at  $D_k$  to detect  $x_t$ .

The following theorem provides the outage probability of  $x_l$  for TWR-NOMA.

**Theorem 1.** The closed-form expression for the outage probability of  $x_l$  for TWR-NOMA with ipSIC is given by

$$P_{x_{l}}^{ipSIC} = 1 - e^{-\frac{\beta_{l}}{\Omega_{l}}} \prod_{i=1}^{3} \lambda_{i} \left( \frac{\Phi_{1}\Omega_{l}}{\Omega_{l}\lambda_{1} + \beta_{l}} - \frac{\Phi_{2}\Omega_{l}}{\Omega_{l}\lambda_{2} + \beta_{l}} + \frac{\Phi_{3}\Omega_{l}}{\Omega_{l}\lambda_{3} + \beta_{l}} \right) \left( e^{-\frac{\theta_{l}}{\Omega_{k}}} - \frac{\varepsilon\tau_{l}\rho\Omega_{I}}{\Omega_{k} + \varepsilon\rho\tau_{l}\Omega_{I}} e^{-\frac{\theta_{l}(\Omega_{k} + \varepsilon\rho\tau_{l}\Omega_{I})}{\varepsilon\tau_{l}\rho\Omega_{I}\Omega_{k}} + \frac{1}{\varepsilon\rho\Omega_{I}}} \right)$$

$$\tag{9}$$

**Corollary 1.** Based on (9), for the special case  $\varepsilon = 0$ , the outage probability of  $x_1$  for TWR-NOMA with pSIC is given by

$$P_{x_{l}}^{pSIC} = 1 - e^{-\frac{\beta_{l}}{\Omega_{l}} - \frac{\theta_{l}}{\Omega_{k}}} \prod_{i=1}^{3} \lambda_{i} \left( \frac{\Phi_{1}\Omega_{l}}{\Omega_{l}\lambda_{1} + \beta_{l}} - \frac{\Phi_{2}\Omega_{l}}{\Omega_{l}\lambda_{2} + \beta_{l}} + \frac{\Phi_{3}\Omega_{l}}{\Omega_{l}\lambda_{3} + \beta_{l}} \right).$$
(10)

2) Outage Probability of  $x_t$ : Based on NOMA principle, the complementary events of outage for  $x_t$  have the following cases. One of the cases is that R can first decode the information  $x_l$  and then detect  $x_t$ . Another case is that either of  $D_k$  and  $D_r$  can detect  $x_t$  successfully. Hence the outage probability of  $x_t$  can be expressed as

$$P_{x_t}^{ipSIC} = 1 - \Pr\left(\gamma_{R \to x_t} > \gamma_{th_t}, \gamma_{R \to x_l} > \gamma_{th_l}\right) \\ \times \Pr\left(\gamma_{D_k \to x_t} > \gamma_{th_t}\right) \Pr\left(\gamma_{D_r \to x_t} > \gamma_{th_t}\right), (11)$$

where  $\varepsilon = 1$ ,  $\varpi_1 \in [0, 1]$  and  $\varpi_2 \in [0, 1]$ .

The following theorem provides the outage probability of  $x_t$  for TWR-NOMA.

**Theorem 2.** The closed-form expression for the outage probability of  $x_t$  with ipSIC is given by

$$P_{x_{t}}^{ipSIC} = 1 - \frac{e^{-\frac{\beta_{l}}{\Omega_{l}} - \beta_{t}\varphi_{t} - \frac{\xi}{\Omega_{k}} - \frac{\xi}{\Omega_{r}}}}{\varphi_{t}\Omega_{t}\left(1 + \varepsilon\beta_{t}\rho\varphi_{t}\Omega_{I}\right)\left(\lambda_{2}^{\prime} - \lambda_{1}^{\prime}\right)}\prod_{i=1}^{2}\lambda_{i}^{\prime}} \times \left(\frac{\Omega_{l}}{\beta_{l} + \beta_{t}\Omega_{l}\varphi_{t} + \Omega_{l}\lambda_{1}^{\prime}} - \frac{\Omega_{l}}{\beta_{l} + \beta_{t}\Omega_{l}\varphi_{t} + \Omega_{l}\lambda_{2}^{\prime}}\right), \quad (12)$$
here,  $\epsilon = 1$ ,  $\lambda_{1}^{\prime} = -\frac{1}{2}$ , and  $\lambda_{1}^{\prime} = -\frac{1}{2}$ ,  $\beta_{l} = -\frac{\gamma_{th_{t}}}{2}$ 

where  $\varepsilon = 1$ .  $\lambda_1 = \frac{1}{\rho \varpi_1 a_k \Omega_k}$  and  $\lambda_2 = \frac{1}{\rho \varpi_1 a_r \Omega_r}$ .  $\beta_t = \frac{\gamma_{th_t}}{\rho a_t}$ ,  $\varphi_t = \frac{\Omega_t + \rho \beta_t a_t \Omega_t}{\Omega_t \Omega_t}$ . *Proof: See Appendix B.* 

**Corollary 2.** For the special case, substituting  $\varepsilon = 0$  into (12), the outage probability of  $x_2$  for TWR-NOMA with pSIC is given by

$$P_{x_{t}}^{pSIC} = 1 - \frac{e^{-\frac{\beta_{l}}{\Omega_{l}} - \beta_{t}\varphi_{t} - \frac{\xi}{\Omega_{k}} - \frac{\xi}{\Omega_{r}}}}{\varphi_{t}\Omega_{t}\left(\lambda_{2}^{\prime} - \lambda_{1}^{\prime}\right)} \prod_{i=1}^{2} \lambda_{i}^{\prime}} \times \left(\frac{\Omega_{l}}{\beta_{l} + \beta_{t}\Omega_{l}\varphi_{t} + \Omega_{l}\lambda_{1}^{\prime}} - \frac{\Omega_{l}}{\beta_{l} + \beta_{t}\Omega_{l}\varphi_{t} + \Omega_{l}\lambda_{2}^{\prime}}\right).$$
(13)

3) Diversity Order Analysis: In order to gain deeper insights for TWR-NOMA systems, the asymptotic analysis are presented in high SNR regimes based on the derived outage probabilities. The diversity order is defined as [33]

$$d = -\lim_{\rho \to \infty} \frac{\log\left(P_{x_i}^{\infty}\left(\rho\right)\right)}{\log\rho},\tag{14}$$

where  $P_{x_i}^{\infty}$  denotes the asymptotic outage probability of  $x_i$ .

**Proposition 1.** Based on the analytical results in (9) and (10), when  $\rho \to \infty$ , the asymptotic outage probabilities of  $x_l$  for ipSIC/pSIC with  $e^{-x} \approx 1 - x$  are given by

$$P_{x_{l},\infty}^{ipSIC} = 1 - \prod_{i=1}^{3} \lambda_{i} \left( \frac{\Phi_{1}\Omega_{l}}{\Omega_{l}\lambda_{1} + \beta_{l}} - \frac{\Phi_{2}\Omega_{l}}{\Omega_{l}\lambda_{2} + \beta_{l}} + \frac{\Phi_{3}\Omega_{l}}{\Omega_{l}\lambda_{3} + \beta_{l}} \right) \\ \times \left[ 1 - \frac{\theta_{l}}{\Omega_{k}} - \frac{\varepsilon\tau_{l}\rho\Omega_{I}}{\Omega_{k} + \varepsilon\rho\tau_{l}\Omega_{I}} \left( 1 - \frac{\theta_{l}\left(\Omega_{k} + \varepsilon\tau_{l}\rho\Omega_{I}\right)}{\varepsilon\rho\tau_{l}\Omega_{I}\Omega_{k}} \right) \right],$$
(15)

and

$$P_{x_l,\infty}^{pSIC} = 1 - \prod_{i=1}^{3} \lambda_i \left( \frac{\Phi_1 \Omega_l}{\Omega_l \lambda_1 + \beta_l} - \frac{\Phi_2 \Omega_l}{\Omega_l \lambda_2 + \beta_l} + \frac{\Phi_3 \Omega_l}{\Omega_l \lambda_3 + \beta_l} \right),\tag{16}$$

respectively. Substituting (15) and (16) into (14), the diversity orders of  $x_l$  with ipSIC/pSIC are equal to zeros.

**Remark 1.** An important conclusion from above analysis is that due to impact of residual interference, the diversity order of  $x_l$  with the use of ipSIC is zero. Additionally, the communication process of the first slot similar to uplink NOMA, even though under the condition of pSIC, diversity order is equal to zero as well for  $x_l$ . As can be observed that there are error floors for  $x_l$  with ipSIC/pSIC.

**Proposition 2.** Similar to the resolving process of  $x_l$ , the asymptotic outage probabilities of  $x_t$  with ipSIC/pSIC in high SNR regimes are given by

$$P_{x_{t},\infty}^{ipSIC} = 1 - \frac{\lambda_{1}'\lambda_{2}'}{\varphi_{t}\Omega_{t}\left(1 + \varepsilon\rho\beta_{t}\varphi_{t}\Omega_{I}\right)\left(\lambda_{2}' - \lambda_{1}'\right)} \times \left(\frac{\Omega_{l}}{\beta_{l} + \beta_{t}\Omega_{1}\varphi_{t} + \Omega_{l}\lambda_{1}'} - \frac{\Omega_{l}}{\beta_{l} + \beta_{t}\Omega_{1}\varphi_{t} + \Omega_{l}\lambda_{2}'}\right), \quad (17)$$

and

$$P_{x_{t},\infty}^{pSIC} = 1 - \frac{\lambda_{1}\lambda_{2}}{\varphi_{t}\Omega_{t}\left(\lambda_{2}^{\prime} - \lambda_{1}^{\prime}\right)} \times \left(\frac{\Omega_{l}}{\beta_{l} + \beta_{t}\Omega_{1}\varphi_{t} + \Omega_{l}\lambda_{1}^{\prime}} - \frac{\Omega_{l}}{\beta_{l} + \beta_{t}\Omega_{l}\varphi_{t} + \Omega_{l}\lambda_{2}^{\prime}}\right), \quad (18)$$

respectively. Substituting (17) and (18) into (14), the diversity orders of  $x_t$  for both ipSIC and pSIC are zeros.

**Remark 2.** Based on above analytical results of  $x_l$ , the diversity orders of  $x_t$  with ipSIC/pSIC are also equal to zeros. This is because residual interference is existent in the total communication process.

4) Throughput Analysis: In delay-limited transmission scenario, the BS transmits message to users at a fixed rate, where system throughput will be subject to wireless fading channels. Hence the corresponding throughput of TWR-NOMA with ipSIC/pSIC is calculated as [15, 34]

$$R_{dl}^{\psi} = \left(1 - P_{x_1}^{\psi}\right) R_{x_1} + \left(1 - P_{x_2}^{\psi}\right) R_{x_2} \\ + \left(1 - P_{x_3}^{\psi}\right) R_{x_3} + \left(1 - P_{x_4}^{\psi}\right) R_{x_4}, \qquad (19)$$

where  $\psi \in (ipSIC, pSIC)$ .  $P_{x_1}^{\psi}$  and  $P_{x_3}^{\psi}$  with ipSIC/pSIC can be obtained from (9) and (10), respectively, while  $P_{x_2}^{\psi}$  and  $P_{x_4}^{\psi}$  with ipSIC/pSIC can be obtained from (12) and (13), respectively.

## IV. ERGODIC RATE

In this section, the ergodic rate of TWR-NOMA is investigated for considering the influence of signal's channel fading to target rate.

1) Ergodic Rate of  $x_l$ : Since  $x_l$  can be detected at the relay as well as at  $D_k$  successfully. By the virtue of (3) and (6), the achievable rate of  $x_l$  for TWR-NOMA is written as  $R_{x_l} = \frac{1}{2} \log (1 + \min (\gamma_{R \to x_l}, \gamma_{D_k \to x_l}))$ . In order to further calculate the ergodic rate of  $x_l$ , using  $X = \min (\gamma_{R \to x_l}, \gamma_{D_k \to x_l})$ , the corresponding CDF  $F_X$  is presented in the following lemma.

**Lemma 1.** The CDF 
$$F_X$$
 for  $x_l$  is given by (20) at the top  
of the next page, where  $f_W(w) = \frac{\tilde{\lambda}_1 \tilde{\lambda}_2}{\tilde{\lambda}_2 - \tilde{\lambda}_1} \left( e^{-\tilde{\lambda}_1 w} - e^{-\tilde{\lambda}_2 w} \right)$   
and  $f_Z(z) = \prod_{i=1}^3 \lambda_i \left( \Phi_1 e^{-\lambda_1 z} - \Phi_2 e^{-\lambda_2 z} + \Phi_3 e^{-\lambda_3 z} \right), \quad \tilde{\lambda}_1 = \frac{1}{\varepsilon \rho}, \quad \tilde{\lambda}_2 = \frac{1}{\rho \varpi_2}, \quad \varphi = \frac{a_l(w+1)\Omega_l + b_l(z+1)\Omega_k}{a_l(w+1)\Omega_l \Omega_k} \quad and \quad \vartheta = \frac{a_l(w+1)\Omega_l + b_l(z+1)\Omega_k}{b_l(z+1)\Omega_k \Omega_l}.$   
Proof: See Appendix C.

Substituting (20), the corresponding ergodic rate of  $x_l$  is given by

$$R_{x_{l}}^{erg} = \frac{1}{2\ln 2} \int_{0}^{\infty} \frac{1 - F_{X}(x)}{1 + x} dx,$$
 (21)

where  $X = \min(\gamma_{R \to x_l}, \gamma_{D_k \to x_l})$  and  $\varepsilon = 1$ . Unfortunately, it is difficult to obtain the closed-form expression from (21). However, it can be evaluated by applying numerical approaches. To further obtain analytical results, we consider the special cases of  $x_l$  with ipSIC/pSIC for TWR-NOMA where there is no IS between the pair of antennas at the relay in the following part.

Based on the above analysis, for the special case that substituting  $\varpi_1 = \varpi_2 = 0$  into (21), the ergodic rate of  $x_l$  with ipSIC can be obtained in the following theorem.

**Theorem 3.** The closed-form expression of ergodic rate for  $x_1$  with ipSIC for TWR-NOMA is given by

$$R_{x_{l},erg}^{ipSIC} = \frac{-1}{2\ln 2} \left[ Ae^{\Psi} \operatorname{Ei}\left(-\Psi\right) + \frac{Be^{\frac{\Psi}{\Lambda_{1}}}}{\Lambda_{1}} \operatorname{Ei}\left(\frac{-\Psi}{\Lambda_{1}}\right) + \frac{Ce^{\frac{\Psi}{\Lambda_{2}}}}{\Lambda_{2}} \operatorname{Ei}\left(\frac{-\Psi}{\Lambda_{2}}\right) \right], \qquad (22)$$

where  $\Lambda_1 = \frac{\varepsilon \Omega_I}{b_l \Omega_k}$ ,  $\Lambda_2 = \frac{a_t \Omega_t}{a_l \Omega_l}$  and  $\Psi = \frac{a_l \Omega_l + b_l \Omega_k}{\rho a_l b_l \Omega_l \Omega_k}$ ;  $A = \frac{1}{\Lambda_1 \Lambda_2 - \Lambda_2 - \Lambda_1 + 1}$ ,  $B = \frac{A(\Lambda_1 - \Lambda_1 \Lambda_2) - \Lambda_1}{(\Lambda_2 - \Lambda_1)}$  and C = 1 - A - B. Ei (·) is the exponential integral function [35, Eq. (8.211.1)]. Proof: See Appendix D.

**Corollary 3.** Based on (22), the ergodic rate of  $x_l$  for pSIC with  $\varepsilon = 0$  can be expressed in the closed form as

$$R_{x_l,erg}^{pSIC} = \frac{-1}{2\ln 2} \left[ Ae^{\Psi} \text{Ei}\left(-\Psi\right) + \frac{Ce^{\frac{\Psi}{\Lambda_2}}}{\Lambda_2} \text{Ei}\left(-\frac{\Psi}{\Lambda_2}\right) \right].$$
(23)

2) Ergodic Rate of  $x_t$ : On the condition that the relay and  $D_l$  are capable of detecting  $x_t$ ,  $x_t$  can be also detected by  $D_t$  successfully. As a consequence, combining (4), (5) and (7), the achievable rate of  $x_t$  is written as  $R_{x_t}$  =

$$F_X(x) = \int_0^\infty \int_0^\infty \frac{f_W(w) f_Z(z)}{\varphi \Omega_k} \left(1 - e^{-\frac{x(w+1)\varphi}{\rho b_l}}\right) dz dw + \int_0^\infty \int_0^\infty \frac{f_W(w) f_Z(z)}{\vartheta \Omega_l} \left(1 - e^{-\frac{x(z+1)\vartheta}{\rho a_l}}\right) dz dw.$$
(20)

 $\frac{1}{2}\log(1+\min(\gamma_{R\to x_t},\gamma_{D_k\to x_t},\gamma_{D_r\to x_t})))$ . The corresponding ergodic rate of  $x_t$  can be expressed as

$$R_{x_{t}}^{erg} = \frac{1}{2\ln 2} \int_{0}^{\infty} \frac{1 - F_{Y}(y)}{1 + y} dy,$$
 (24)

where  $Y = \min(\gamma_{R \to x_t}, \gamma_{D_k \to x_t}, \gamma_{D_r \to x_t})$  with  $\varpi_1 = \varpi_2 =$ 1 and  $\varepsilon = 1$ . To the best of authors' knowledge, (24) does not have a closed form solution. We also consider the special cases of  $x_t$  by the virtue of ignoring IS between the pair of antennas at the relay.

For the special case that substituting  $\varpi_1 = \varpi_2 = 0$  into (24) and after some manipulations, the ergodic rates of  $x_t$ with ipSIC/pSIC is given by

$$R_{x_{t},erg}^{ipSIC} = \frac{1}{2\ln 2} \int_{0}^{\frac{b_{t}}{b_{l}}} \frac{e^{-\frac{x}{\rho a_{t}\Omega_{t}} - \frac{x}{\rho(b_{t} - xb_{l})\Omega_{k}} - \frac{x}{\rho(b_{t} - xb_{l})\Omega_{r}}}{(1 + x)(1 + x\Lambda_{3})} dx,$$
(25)

and

$$R_{x_t,erg}^{pSIC} = \frac{1}{2\ln 2} \int_0^{\frac{b_t}{b_l}} \frac{e^{-\frac{x}{\rho a_t}\Omega_t} - \frac{x}{\rho(b_t - xb_l)\Omega_k} - \frac{x}{\rho(b_t - xb_l)\Omega_r}}{1 + x} dx,$$
(26)

respectively, where  $\Lambda_3 = \frac{\varepsilon \Omega_I}{a_t \Omega_t}$  with  $\varepsilon = 1$ . As can be seen from the above expressions, the exact analysis of ergodic rates require the computation of some complicated integrals. To facilitate these analysis and provide the simpler expression for the ergodic rate of  $x_t$  with ipSIC/pSIC, the following theorem and corollary provide the high SNR approximations to evaluate the performance.

**Theorem 4.** The approximation expression for ergodic rate of  $x_t$  with ipSIC at high SNR is given by

$$R_{x_t,\infty}^{ipSIC} = \frac{1}{2\left(1 - \Lambda_3\right)\ln 2} \left[ \ln\left(1 + \frac{b_t}{b_l}\right) - \ln\left(1 + \frac{b_t\Lambda_3}{b_l}\right) \right].$$
(27)

**Corollary 4.** For the special case with  $\varepsilon = 0$ , the ergodic rate of  $x_t$  for pSIC can be approximated at high SNR as

$$R_{x_t,\infty}^{pSIC} = \frac{1}{2\ln 2} e^{\frac{1}{\rho a_t \Omega_t}} \left[ \operatorname{Ei}\left(\frac{-1}{\rho a_t b_l \Omega_t}\right) - \operatorname{Ei}\left(\frac{-1}{\rho a_t \Omega_t}\right) \right].$$
(28)

3) Slope Analysis: In this subsection, by the virtue of asymptotic results, we characterize the high SNR slope which is capable of capturing the influence of channel parameters on the ergodic rate. The high SNR slope is defined as

$$S = \lim_{\rho \to \infty} \frac{R_{x_i}^{\infty}(\rho)}{\log(\rho)},$$
(29)

where  $R_{x_i}^{\infty}$  denotes the asymptotic ergodic rate of  $x_i$ .

a)  $x_l$  for ipSIC/pSIC case:

**Proposition 3.** Based on the above analytical results in (22) and (23), when  $\rho \to \infty$ , by using  $\text{Ei}(-x) \approx \ln(x) + E_c$  [35, Eq. (8.212.1)] and  $e^{-x} \approx 1 - x$ , where  $E_c$  is the Euler constant, the asymptotic ergodic rates of  $x_l$  with ipSIC/pSIC in the high regime are given by

$$R_{x_{l},\infty}^{ipSIC} = \frac{-1}{2\ln 2} \left[ A \left( 1 + \Psi \right) \left( \ln \left( \Psi \right) + E_{c} \right) + \frac{B}{\Lambda_{1}} \left( 1 + \frac{\Psi}{\Lambda_{1}} \right) \right. \\ \left. \times \left( \ln \left( \frac{\Psi}{\Lambda_{1}} \right) + E_{c} \right) + \frac{E_{c}}{\Lambda_{2}} \left( 1 + \frac{\Psi}{\Lambda_{2}} \right) \left( \ln \left( \frac{\Psi}{\Lambda_{2}} \right) + E_{c} \right) \right],$$

$$(30)$$

and

$$R_{x_{l},\infty}^{pSIC} = \frac{-1}{2\ln 2} \left[ A \left( 1 + \Psi \right) \left( \ln \left( \Psi \right) + E_{c} \right) + \frac{E_{c}}{\Lambda_{2}} \left( 1 + \frac{\Psi}{\Lambda_{2}} \right) \left( \ln \left( \frac{\Psi}{\Lambda_{2}} \right) + E_{c} \right) \right], \quad (31)$$

respectively.

Substituting (30) and (31) into (29), we can see that the high SNR slopes of  $x_1$  with ipSIC/pSIC are equal to zeros.

b)  $x_t$  for ipSIC/pSIC case: Similar to (30) and (31), substituting (27) and (23) into (29), we observe that the high SNR slopes of  $x_t$  with ipSIC/pSIC are also equal to zeros.

**Remark 3.** The above analytical results demonstrate that even if there is no IS between both antennas at the relay,  $x_1$  and  $x_t$ converge to throughput ceilings and obtain zero slopes in the high SNR regime. This is due to the fact that the first phase is similar to uplink NOMA, it is suffering interference from other users which has seriously impact on the high SNR slope.

4) Throughput Analysis: In delay-tolerant transmission scenario, the system throughput is determined by evaluating the ergodic rate. Based on the above results derived, the corresponding throughput of TWR-NOMA is given by

$$R_{dt}^{\psi} = R_{x_1,erg}^{\psi} + R_{x_2,erg}^{\psi} + R_{x_3,erg}^{\psi} + R_{x_4,erg}^{\psi}, \qquad (32)$$

where  $R^{\psi}_{x_1,erg}$  and  $R^{\psi}_{x_3,erg}$  with ipSIC/PSIC can be obtained from (22) and (23), respectively, while  $R_{x_2,erg}^{\psi}$  and  $R_{x_4,erg}^{\psi}$ with ipSIC/pSIC can be obtained from and (25), (26), respectively.

## V. ENERGY EFFICIENCY

In this section, the performance of TWR-NOMA systems is characterized from the perspective of energy efficiency (EE). In particular, EE has been adopted as a efficient metric to provide quantitative analysis for 5G networks. The core idea of EE is a rate between the total data rate of all NOMA users and the total energy consumption. Therefore, the expression of EE can be given by

$$\eta_{EE} = \frac{\text{Total data rate}}{\text{Total energy consumption}}.$$
 (33)

TABLE I: Table of Parameters for Numerical Results

Monte Carlo simulations repeated	10 <sup>6</sup> iterations
Power allocation coefficients of NOMA	$b_1 = b_3 = 0.2$ $b_2 = b_4 = 0.8$
Targeted data rates	$R_1 = R_3 = 0.1$ BPCU $R_2 = R_4 = 0.01$ BPCU
Pass loss exponent	$\alpha = 2$
The distance between R and $D_1$ or $D_3$	$d_1 = 2 \text{ m}$
The distance between R and $D_2$ or $D_4$	$d_2 = 10 \text{ m}$

Based on the throughput analysis in (III-4) and (IV-4), the EE of TWR-NOMA systems is given by

$$\eta_{\Upsilon}^{EE} = \frac{2R_{\Upsilon}^{\psi}}{TP_u + TP_r},\tag{34}$$

where  $\Upsilon \in (dt, dl)$  and T denotes transmission time of the entire communication process.  $\eta_{dl}^{EE}$  and  $\eta_{dt}^{EE}$  are the system energy efficiency in delay-limited transmission mode and delay-tolerant transmission mode, respectively.

## VI. NUMERICAL RESULTS

In this section, numerical results are provide to substantiate the system performance and investigate the impact levels of IS on outage probability and ergodic rate for TWR-NOMA. Monte Carlo simulation parameters used are summarized in Table I, where BPCU is short for bit per channel use. Due to the reciprocity of channels between user groups (i.e.,  $G_1$  or  $G_2$ ) and R, the outage behaviors and ergodic rates of  $x_1$  and  $x_2$ in  $G_1$  are presented to illustrate availability of TWR-NOMA. Without loss of generality, the power allocation coefficients of  $x_1$  and  $x_2$  are set as  $a_1 = 0.8$  and  $a_2 = 0.2$ , respectively.  $\Omega_1$ and  $\Omega_2$  are set to be  $\Omega_1 = d_1^{-\alpha}$  and  $\Omega_2 = d_2^{-\alpha}$ , respectively. The performance of conventional TWR-OMA is shown as a benchmark for comparison, in which the total communication process can be finished in five slots. In the first slot, the user nodes in  $G_1$ , i,e,  $D_1$  and  $D_2$  sends signal  $x_1$  and  $x_2$  to R. Meanwhile, the user nodes in  $G_2$ , i,e,  $D_3$  and  $D_4$  sends signal  $x_3$  and  $x_4$  to R. After completing the exchange of information, R sends signal  $x_3$  and  $x_4$  to  $D_1$  and  $D_2$  in the second and third slots, respectively. Then R sends signal  $x_1$  and  $x_2$  to  $D_3$  and  $D_4$  in the fourth and fifth slots, respectively. Except power allocation coefficients, other simulation parameters of TWR-OMA is similar to that of TWR-NOMA. It is worth pointing out that the signals are transmitted at full power for TWR-OMA.

## A. Outage Probability

Fig. 2 plots the outage probabilities of  $x_1$  and  $x_2$  with both ipSIC and pSIC versus SNR for simulation setting with  $\varpi_1 = \varpi_2 = 0.01$  and  $\Omega_I = -20$  dB. The solid and dashed curves represent the exact theoretical performance of  $x_1$  and  $x_2$  for both ipSIC and pSIC, corresponding to the results derived in (9), (10) and (12), (13), respectively. Apparently, the outage probability curves match perfectly with Monte Carlo simulation results. As can be observed from the figure, the outage behaviors of  $x_1$  and  $x_2$  for TWR-NOMA are superior to TWR-OMA in the low SNR regime. This is due to the fact

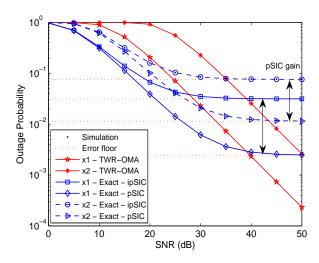


Fig. 2: Outage probability versus the transmit SNR, with  $\varpi_1 = \varpi_2 = 0.01$ ,  $R_1 = 0.1$ ,  $R_2 = 0.01$  BPCU, and  $\Omega_I = -20$  dB.

that the influence of IS is not the dominant factor at low SNR. Hence in this scenario, NOMA systems should work as much as possible at low SNR regime, such as, the wide coverage in rural areas and cell edge scenarios. Another observation is that the pSIC is capable of enhancing the performance of NOMA compare to the ipSIC. In addition, the asymptotic curves of  $x_1$  and  $x_2$  with ipSIC/pSIC are plotted according to (15), (16) and (17), (18), respectively. It can be seen that the outage behaviors of  $x_1$  and  $x_2$  converge to the error floors in the high SNR regime. The reason can be explained that due to the impact of residual interference by the use of ipSIC,  $x_1$ and  $x_2$  result in zero diversity orders. Although the pSIC is carried out in TWR-NOMA system,  $x_1$  and  $x_2$  also obtain zero diversity orders. This is due to the fact that when the relay first detect the strongest signal in the first slot, it will suffer interference from the weaker signal. This process is similar to the uplink NOMA [36]. Additionally, this observation verifies the conclusion **Remark 1** in Section III.

Fig. 3 plots the outage probabilities of  $x_1$  and  $x_2$  versus SNR with the different impact levels of IS from  $\varpi_1 = \varpi_2 = 0$ to  $\varpi_1 = \varpi_2 = 0.1$ . The solid and dashed curves represent the outage behaviors of  $x_1$  and  $x_2$  with ipSIC/pSIC, respectively. As can be seen that when the impact level of IS is set to be  $\varpi_1 = \varpi_2 = 0$ , there is no IS between  $A_1$  and  $A_2$  at the relay, which can be viewed as a benchmark. Additionally, one can observed that with the impact levels of IS increasing, the outage performance of TWR-NOMA degrades significantly. As a consequence, it is crucial to hunt for efficient strategies for suppressing the effect of interference between antennas. Fig. 4 plots the outage probabilities versus SNR with different values of residual IS from -20 dB to 0 dB. It can be seen that the different values of residual IS affects the performance of ipSIC seriously. Similarly, as the values of residual IS increases, the preponderance of ipSIC is inexistent. When  $\Omega_I = 0$  dB, the outage probabilities of  $x_1$  and  $x_2$  will be in close proximity to one. Therefore, it is important to design effective SIC schemes for TWR-NOMA.

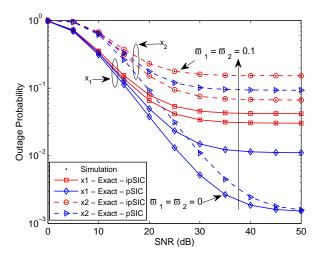


Fig. 3: Outage probability versus the transmit SNR, with the different impact levels of IS from  $\varpi_1 = \varpi_2 = 0$  to  $\varpi_1 = \varpi_2 = 0.1$ ,  $R_1 = 0.1$ ,  $R_2 = 0.01$  BPCU, and  $\Omega_I = -20$  dB.

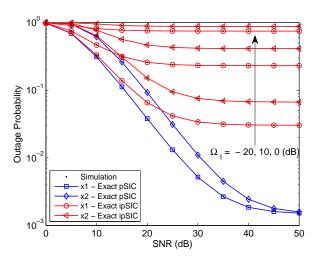


Fig. 4: Outage probability versus the transmit SNR, with different values of residual IS from -20 dB to 0 dB,  $\varpi_1 = \varpi_2 = 0$ ,  $R_1 = 0.1$ ,  $R_2 = 0.01$  BPCU.

Fig. 5 plots system throughput versus SNR in delay-limited transmission mode for TWR-NOMA with different values of residual IS from -20 dB to -10 dB. The blue solid curves represent throughput for TWR-NOMA with both pSIC and ipSIC, which can be obtained from (19). One can observe that TWR-NOMA is capable of achieving a higher throughput compared to TWR-OMA in the low SNR regime, since it has a lower outage probability. Moreover, the figure confirms that TWR-NOMA converges to the throughput ceiling in the high SNR regime. Additionally, it is worth noting that ipSIC considered for TWR-NOMA will further degrade throughput with the values of residual IS becomes larger in high SNR regimes.

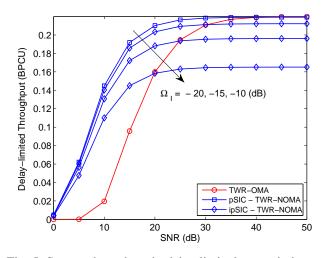


Fig. 5: System throughput in delay-limited transmission mode versus SNR with ipSIC/pSIC,  $R_1 = 0.1$ ,  $R_2 = 0.01$  BPCU,  $\varpi_1 = \varpi_2 = 0.01$ .

#### B. Ergodic Rate

Fig. 6 plots the ergodic rate of  $x_1$  and  $x_2$  for TWR-NOMA versus SNR and the values of SI are assumed to be  $\varpi_1 = \varpi_2 = 0.01$ , and  $\Omega_I = -20$  dB. The blue and red dashdotted curves represent the achievable rate of  $x_1$  and  $x_2$  with ipSIC/pSIC for TWR-NOMA, respectively, which considers IS between  $A_1$  and  $A_2$  at the relay. The blue and red solid curves represent ergodic rates of  $x_1$  and  $x_2$  with ipSIC/pSIC according to (22), (23) and (25), (26), respectively. We can observe that the ergodic rates of  $x_1$  and  $x_2$  with pSIC are larger than that of  $x_1$  and  $x_2$  with ipSIC. This is due to the fact that pSIC can provide more performance gain than ipSIC. In addition, due to the influence of interference,  $x_1$  and  $x_2$  converge to the throughput ceilings in high SNR regimes, which verifies the conclusion **Remark 3** in Section IV.

Fig. 7 plots the system throughput versus SNR in delaytolerant transmission mode for TWR-NOMA. The blue solid curves represent system throughput for TWR-NOMA with ipSIC/pSIC, which can be obtained from (19). The system throughput of IS-based is selected to be the benchmark denoted by the red dash-dotted curves. It is observed that TWR-NOMA can achieve a higher throughput in the absence of IS at the relay. Hence, we need to find an effective way to restrain IS for both antennas at the relay.

## C. Energy Efficiency

Fig. 8 plots energy efficiency of TWR-NOMA systems versus SNR with delay-limited/tolerant transmission modes. The red solid curves represent system energy efficiency for the delay-limited transmission mode with ipSIC/pSIC, respectively, which can be obtained from (19) and (34). The blue curves represent system energy efficiency for the delay-tolerant transmission mode with ipSIC/pSIC, respectively, which can be obtained from (32) and (34). It is can be seen that TWR-NOMA with ipSIC/pSIC in delay-limited transmission mode have almost the same energy efficiency. Additionally, we can

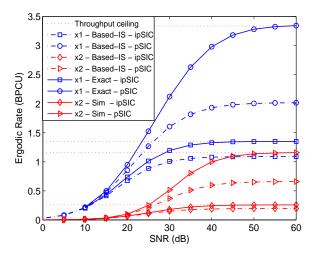


Fig. 6: Ergodic rate versus the transmit SNR with ipSIC/pSIC,  $\varpi_1 = \varpi_2 = 0.01$ , and  $\Omega_I = -20$  dB.

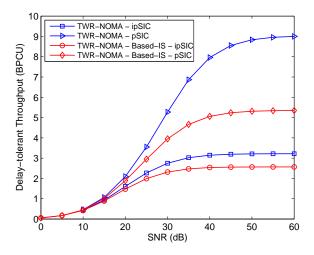


Fig. 7: System throughput in delay-tolerant transmission mode versus SNR with ipSIC/pSIC,  $\varpi_1 = \varpi_2 = 0.01$ , and  $\Omega_I = -20$  dB.

observed that the energy efficiency of TWR-NOMA with pSIC is superior to ipSIC in high SNR regimes.

## VII. CONCLUSION

This paper has investigated the application of TWR to NOMA systems, in which two pairs of users can exchange their information between each other by the virtue of a relay node. The performance of TWR-NOMA systems has been characterized in terms of outage probability and ergodic rate for both ipSIC and pSIC. The closed-form expressions of outage probability for the NOMA users' signals have been derived. Owing to the impact of IS at relay, there were the error floors for TWR-NOMA with ipSIC/pSIC in high SNR regimes and zero diversity orders were obtained. Based on the analytical results, it was shown that the performance of TWR-NOMA with ipSIC/pSIC outperforms TWR-OMA in the low

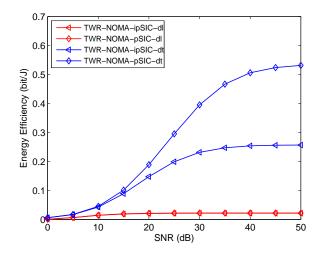


Fig. 8: System throughput in delay-limited/tolerant transmission mode versus SNR with ipSIC/pSIC, where  $P_u = P_r = 10$  W, and T = 1.

SNR regime. Furthermore, the ergodic rates of TWR-NOMA have been discussed in detail. The results have shown that TWR-NOMA with pSIC is capable of achieving a larger rate in the absence of IS at the relay. More particularly, the users' signals for TWR-NOMA converge to the throughput ceiling and gain zero high slopes in high SNR regimes. Finally, the system energy efficiencies with ipSIC/pSIC were discussed in a pair of transmission modes.

#### APPENDIX A: PROOF OF THEOREM 1

Substituting (3), (5) and (6) into (8), the outage probability of  $x_l$  can be further given by

$$P_{x_l}^{ipSIC} = 1$$

$$-\underbrace{\Pr\left(\frac{\rho|h_{l}|^{2}a_{l}}{\rho|h_{t}|^{2}a_{t}+\rho\varpi_{1}(|h_{k}|^{2}a_{k}+|h_{r}|^{2}a_{r})+1} > \gamma_{th_{l}}\right)}_{J_{1}} \times \Pr\left(\frac{\rho|h_{k}|^{2}b_{t}}{\rho|h_{k}|^{2}b_{l}+\rho\varpi_{2}|h_{k}|^{2}+1} > \gamma_{th_{t}}, \frac{\rho|h_{k}|^{2}b_{l}}{\varepsilon\rho|g|^{2}+\rho\varpi_{2}|h_{k}|^{2}+1} > \gamma_{th_{l}}\right), \quad (A.1)$$

where  $\varepsilon = 1$ .

To calculate the probability  $J_1$  in (A.1), let  $Z = \rho a_t |h_t|^2 + \rho \varpi_1 a_k |h_k|^2 + \rho \varpi_1 a_r |h_r|^2$ . We first calculate the PDF of Z and then give the process derived of  $J_1$ . As is known,  $|h_i|^2$  follows the exponential distribution with the parameters  $\Omega_i$ ,  $i \in (1, 2, 3, 4)$ . Furthermore, we denote that  $Z_1 = \rho a_t |h_t|^2$ ,  $Z_2 = \rho \varpi_1 a_k |h_k|^2$  and  $Z_3 = \rho \varpi_1 a_r |h_r|^2$  are also independent exponentially distributed random variables (RVs) with parameters  $\lambda_1 = \frac{1}{\rho a_t \Omega_t}$ ,  $\lambda_2 = \frac{1}{\rho \varpi_1 a_k \Omega_k}$  and  $\lambda_3 = \frac{1}{\rho \varpi_1 a_r \Omega_r}$ , respectively.

Based on [37], for the independent non-identical distributed (i.n.d) fading scenario, the PDF of Z can be given by

$$f_Z(z) = \prod_{i=1}^{3} \lambda_i \left( \Phi_1 e^{-\lambda_1 z} - \Phi_2 e^{-\lambda_2 z} + \Phi_3 e^{-\lambda_3 z} \right), \quad (A.2)$$

 $\Phi_1 = \frac{1}{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)}, \quad \Phi_2 = \frac{1}{(\lambda_3 - \lambda_2)(\lambda_2 - \lambda_1)}$ where and

 $\Phi_3 = \frac{1}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)}$ . According to the above explanations,  $J_1$  is calculated as follows:

$$J_{1} = \Pr\left(|h_{l}|^{2} > (Z+1)\beta_{l}\right) = \int_{0}^{\infty} f_{Z}(z)e^{-\frac{(z+1)\beta_{l}}{\Omega_{l}}}dz,$$
(A.3)

where  $\beta_l = \frac{\gamma_{th_l}}{\rho_{a_l}}$ . Substituting (A.2) into (A.3) and after some algebraic manipulations,  $J_1$  is given by

$$J_1 = e^{-\frac{\beta_l}{\Omega_l}} \prod_{i=1}^3 \lambda_i \left( \frac{\Phi_1 \Omega_l}{\Omega_l \lambda_1 + \beta_l} - \frac{\Phi_2 \Omega_l}{\Omega_l \lambda_2 + \beta_l} + \frac{\Phi_3 \Omega_l}{\Omega_l \lambda_3 + \beta_l} \right),$$
(A.4)

 $J_2$  can be further calculated as follows:

$$J_{2} = \Pr\left(\left|h_{k}\right|^{2} > \xi_{t}, \left|g\right|^{2} < \frac{\left|h_{k}\right|^{2} - \tau_{l}}{\varepsilon\rho\tau_{l}}, \left|h_{k}\right|^{2} > \tau_{l}\right)$$
$$= \Pr\left(\left|h_{k}\right|^{2} > \max\left(\tau_{l}, \xi_{t}\right) \stackrel{\Delta}{=} \theta_{l}, \left|g\right|^{2} < \frac{\left|h_{k}\right|^{2} - \tau_{l}}{\varepsilon\rho\tau_{l}}\right)$$
$$= \int_{\theta}^{\infty} \frac{1}{\Omega_{k}} \left(e^{-\frac{y}{\Omega_{k}}} - e^{-\frac{y-\tau_{l}}{\varepsilon\tau_{l}\rho\Omega_{I}} - \frac{y}{\Omega_{k}}}\right) dy$$
$$= e^{-\frac{\theta_{l}}{\Omega_{k}}} - \frac{\tau_{l}\varepsilon\rho\Omega_{I}}{\Omega_{k} + \varepsilon\rho\tau_{l}\Omega_{I}} e^{-\frac{\theta_{l}\left(\Omega_{k} + \rho\tau_{l}\varepsilon\Omega_{I}\right)}{\tau_{l}\varepsilon\rho\Omega_{I}\Omega_{k}} + \frac{1}{\varepsilon\rho\Omega_{I}}}, \quad (A.5)$$

where  $\xi_t = \frac{\gamma_{th_t}}{\rho(b_t - b_l \gamma_{th_t} - \varpi_2 \gamma_{th_l})}$  with  $\tau_l = \frac{\gamma_{th_l}}{\rho(b_l - \varpi_2 \gamma_{th_l})}$  with  $b_l > \varpi_2 \gamma_{th_l}$ . with  $b_t > (b_l + \varpi_2) \gamma_{th_t}$ ,

Combining (A.4) and (A.5), we can obtain (9). The proof is complete.

## **APPENDIX B: PROOF OF THEOREM 2**

Substituting (3), (4), (6) and (7) into (11), the outage probability of  $x_t$  is rewritten as

$$P_{x_{t}}^{ipSIC} = 1$$

$$-\Pr\left(\frac{\rho|h_{t}|^{2}a_{t}}{\varepsilon\rho|g|^{2} + \rho\varpi_{1}(|h_{k}|^{2}a_{k} + |h_{r}|^{2}a_{r}) + 1} > \gamma_{th_{t}}, \frac{\rho|h_{l}|^{2}a_{l}}{\rho|h_{t}|^{2}a_{t} + \rho\varpi_{1}(|h_{k}|^{2}a_{k} + |h_{r}|^{2}a_{r}) + 1} > \gamma_{th_{l}}\right)}{\Theta_{1}}$$

$$\times \underbrace{\Pr\left(\frac{\rho|h_{k}|^{2}b_{t}}{\rho|h_{k}|^{2}b_{l} + \rho\varpi_{2}|h_{k}|^{2} + 1} > \gamma_{th_{t}}\right)}{\Theta_{2}}}_{\Theta_{2}}$$

$$\times \underbrace{\Pr\left(\frac{\rho|h_{r}|^{2}b_{t}}{\rho|h_{r}|^{2}b_{l} + \rho\varpi_{2}|h_{r}|^{2} + 1} > \gamma_{th_{t}}\right)}_{\Theta_{3}}, \quad (B.1)$$

where  $\varpi_1 = \varpi_2 \in [0,1]$  and  $\varepsilon = 1$ .

Similar to (A.2), let  $Z' = \rho \varpi_1 a_k |h_k|^2 + \rho \varpi_1 a_r |h_r|^2$ , the PDF of Z' is given by

$$f_{Z'}\left(z'\right) = \prod_{i=1}^{2} \lambda'_{i} \left(\frac{e^{-\lambda'_{1}z'}}{(\lambda'_{2} - \lambda'_{1})} - \frac{e^{-\lambda'_{2}z'}}{(\lambda'_{2} - \lambda'_{1})}\right), \quad (B.2)$$

where  $\lambda_1' = \frac{1}{\rho \varpi_1 a_k \Omega_k}$  and  $\lambda_2' = \frac{1}{\rho \varpi_1 a_r \Omega_r}$ . After some variable substitutions and manipulations,

$$\Theta_{1} = \Pr\left(\left|h_{t}\right|^{2} > \beta_{t}\left(\varepsilon\rho|g|^{2} + Z' + 1\right), \\ \left|h_{l}\right|^{2} > \beta_{l}\left(\rho|h_{t}|^{2}a_{t} + Z' + 1\right)\right)$$
$$= \int_{0}^{\infty} f_{Z'}\left(z'\right)e^{-\frac{\beta_{l}\left(z'+1\right)}{\Omega_{l}}} \\ \times \int_{0}^{\infty} f_{|g|^{2}}\left(y\right)\frac{1}{\varphi_{t}\Omega_{t}}e^{-\beta_{t}\left(\varepsilon\rho y + z'+1\right)\varphi_{t}}dydz'$$
$$= \frac{1}{\varphi_{t}\Omega_{t}\left(1 + \varepsilon\rho\beta_{t}\varphi_{t}\Omega_{I}\right)}e^{-\frac{\beta_{l}}{\Omega_{1}} - \beta_{t}\varphi_{t}} \\ \times \int_{0}^{\infty} f_{Z'}\left(z'\right)e^{-\frac{\left(\beta_{l} + \beta_{t}\Omega_{l}\varphi_{t}\right)z'}{\Omega_{l}}}dz', \qquad (B.3)$$

where  $\beta_t = \frac{\gamma_{th_t}}{\rho a_t}$  and  $\varphi_t = \frac{\Omega_l + \rho \beta_l a_t \Omega_t}{\Omega_l \Omega_t}$ . Substituting (B.2) into (B.3),  $\Theta_1$  can be given by

$$\Theta_{1} = \frac{e^{-\frac{\beta_{l}}{\Omega_{l}} - \beta_{t}\varphi_{t}}}{\varphi_{t}\Omega_{t}\left(1 + \beta_{t}\varepsilon\rho\varphi_{t}\Omega_{I}\right)\left(\lambda_{2}^{\prime} - \lambda_{1}^{\prime}\right)} \times \prod_{i=1}^{2}\lambda_{i}^{\prime}\left(\frac{\Omega_{l}}{\beta_{l} + \beta_{t}\Omega_{l}\varphi_{t} + \Omega_{l}\lambda_{1}^{\prime}} - \frac{\Omega_{l}}{\beta_{l} + \beta_{t}\Omega_{l}\varphi_{t} + \Omega_{l}\lambda_{2}^{\prime}}\right).$$
(B.4)

 $\Theta_2$  and  $\Theta_3$  can be easily calculated

$$\Theta_2 = \Pr\left(|h_k|^2 > \xi_t\right) = e^{-\frac{\xi_t}{\Omega_k}},\tag{B.5}$$

and

$$\Theta_3 = \Pr\left(\left|h_r\right|^2 > \xi_t\right) = e^{-\frac{\xi_t}{\Omega_r}},\tag{B.6}$$

respectively, where  $\xi_t = \frac{\gamma_{th_t}}{\rho(b_t - b_l \gamma_{th_t} - \varpi_2 \gamma_{th_t})}$  with  $b_t$ > $(b_l + \varpi_2) \gamma_{th_t}.$ 

Finally, combining (B.4), (B.5) and (B.6), we can obtain (12) and the proof is completed.

## APPENDIX C: PROOF OF LEMMA 1

To derive the CDF  $F_X$ , based on (3) and (6), we can formulate

$$F_{X}(x) = \Pr\left(\min\left(\frac{\rho|h_{l}|^{2}a_{l}}{Z+1}, \frac{\rho|h_{k}|^{2}b_{l}}{W+1}\right) < x\right),$$

$$= \Pr\left(\frac{\rho|h_{k}|^{2}b_{l}}{W+1} < \frac{\rho|h_{l}|^{2}a_{l}}{Z+1}, \frac{\rho|h_{k}|^{2}b_{l}}{W+1} < x\right)$$

$$+ \Pr\left(\frac{\rho|h_{l}|^{2}a_{l}}{Z+1} < \frac{\rho|h_{k}|^{2}b_{l}}{W+1}, \frac{\rho|h_{l}|^{2}a_{l}}{Z+1} < x\right),$$

$$Q_{2}$$
(C.1)

where  $Z = \rho a_t |h_w|^2 + \rho a_k \varpi_1 |h_k|^2 + \rho a_r \varpi_1 |h_r|^2$  and where  $A = \frac{1}{\Lambda_1 \Lambda_2 - \Lambda_2 - \Lambda_1 + 1}$ ,  $B = \frac{A(\Lambda_1 - \Lambda_1 \Lambda_2) - \Lambda_1}{(\Lambda_2 - \Lambda_1)}$  and  $C = W = \varepsilon \rho |g|^2 + \rho \varpi_2 |h_k|^2$ . For the i.n.d variable, based 1 - A - B; (D.3) can be obtained by using [35, Eq. (3.352.4)]. on (A.2) and (B.2), the PDF  $f_Z$  and  $f_W$  can be written as  $f_Z(z) = \prod_{i=1}^{3} \lambda_i \left( \Phi_1 e^{-\lambda_1 z} - \Phi_2 e^{-\lambda_2 z} + \Phi_3 e^{-\lambda_3 z} \right)$  and  $f_W(w) = \frac{\tilde{\lambda}_1 \tilde{\lambda}_2}{\tilde{\lambda}_2 - \tilde{\lambda}_1} \left( e^{-\tilde{\lambda}_1 w} - e^{-\tilde{\lambda}_2 w} \right)$ , respectively, where  $\tilde{\lambda}_1 = \frac{1}{\varepsilon \rho}$  and  $\tilde{\lambda}_2 = \frac{1}{\rho \varpi_2}$ .  $Q_1$  can be calculated as follows:

$$Q_{1} = \Pr\left(|h_{l}|^{2} > \frac{|h_{k}|^{2}b_{l}\left(Z+1\right)}{a_{l}\left(W+1\right)}, |h_{k}|^{2} < \frac{x\left(W+1\right)}{\rho b_{l}}\right)$$

$$= \int_{0}^{\infty} \int_{0}^{\infty} f_{W}\left(w\right) f_{Z}\left(z\right) \int_{0}^{\frac{x\left(w+1\right)}{\rho b_{l}}} \frac{e^{-u\varphi}}{\Omega_{k}} du dz dt$$

$$= \int_{0}^{\infty} \int_{0}^{\infty} \frac{f_{W}\left(w\right) f_{Z}\left(z\right)}{\varphi \Omega_{k}} \left(1 - e^{-\frac{x\left(w+1\right)\varphi}{\rho b_{l}}}\right) dz dw,$$
(C.2)

where  $\varphi = \frac{a_l(w+1)\Omega_l + b_l(z+1)\Omega_k}{a_l(w+1)\Omega_l\Omega_k}$ . Similar to (C.2), after some algebraic manipulations,  $Q_2$  is given by

$$Q_{2} = \int_{0}^{\infty} \int_{0}^{\infty} \frac{f_{W}(w) f_{Z}(z)}{\vartheta \Omega_{l}} \left(1 - e^{-\frac{x(z+1)\vartheta}{\rho a_{l}}}\right) dz dw,$$
(C.3)

where  $\vartheta = \frac{a_l(w+1)\Omega_l + b_l(z+1)\Omega_k}{b_l(z+1)\Omega_k\Omega_l}$ . Combine (C.2) and (C.3), we can obtain (20).

The proof is completed.

#### APPENDIX D: PROOF OF THEOREM 3

The proof starts by substituting  $\varpi_1 = \varpi_2 = 0$  into (21), the ergodic rate of  $x_l$  with ipSIC is given by

$$R_{x_{l},erg}^{ipSIC} = \frac{1}{2} \mathbb{E} \left[ \log \left( 1 + \min \left( \frac{\rho |h_{k}|^{2} b_{l}}{\varepsilon \rho |g|^{2} + 1}, \frac{\rho |h_{l}|^{2} a_{l}}{\rho |h_{t}|^{2} a_{t} + 1} \right) \right) \right]$$
$$= \frac{1}{2 \ln 2} \int_{0}^{\infty} \frac{1 - F_{U}(u)}{1 + u} du, \qquad (D.1)$$

where  $\varepsilon = 1$ .

Applying some algebraic manipulations, the CDF of U can be given by

$$F_U(u) = 1 - \frac{e^{-u\Psi}}{(1+u\Lambda_1)(1+u\Lambda_2)},$$
 (D.2)

where  $\Lambda_1 = \frac{\varepsilon \Omega_I}{b_l \Omega_k}$ ,  $\Lambda_2 = \frac{a_t \Omega_t}{a_l \Omega_l}$  and  $\Psi = \frac{a_l \Omega_l + b_l \Omega_l}{\rho a_l b_l \Omega_l \Omega_k}$ . Substituting (D.2) into (D.1), the ergodic rate of  $x_l$  with

ipSIC can be further expressed as follows:

$$\begin{aligned} R_{x_{l},erg}^{ipSIC} &= \frac{1}{2\ln 2} \int_{0}^{\infty} \frac{e^{-u\Psi}}{(1+u)\left(1+u\Lambda_{1}\right)\left(1+u\Lambda_{2}\right)} du \\ &= \frac{1}{2\ln 2} \int_{0}^{\infty} \left(\frac{Ae^{-u\Psi}}{1+u} + \frac{Be^{-u\Psi}}{1+u\Lambda_{1}} + \frac{Ce^{-u\Psi}}{1+u\Lambda_{2}}\right) du \\ &= \frac{-1}{2\ln 2} \left[Ae^{\Psi} \text{Ei}\left(-\Psi\right) + \frac{Be^{\frac{\Psi}{\Lambda_{1}}}}{\Lambda_{1}} \text{Ei}\left(\frac{-\Psi}{\Lambda_{1}}\right) \right. \\ &\left. + \frac{Ce^{\frac{\Psi}{\Lambda_{2}}}}{\Lambda_{2}} \text{Ei}\left(\frac{-\Psi}{\Lambda_{2}}\right) \right], \end{aligned}$$
(D.3)

The proof is completed.

## **APPENDIX E: PROOF OF THEOREM 4**

We can rewrite (25) as follows:

$$R_{x_{t},erg}^{ipSIC} = \frac{1}{2} \mathbb{E} \left[ \log \left( 1 + \min \left( \frac{\frac{\rho |h_{t}|^{2}a_{t}}{\varepsilon \rho |g|^{2} + 1}, \frac{\rho |h_{k}|^{2}b_{t}}{\rho |h_{r}|^{2}b_{t}}, \frac{\frac{\rho |h_{t}|^{2}b_{t}}{\rho |h_{r}|^{2}b_{t} + 1}}{\frac{\rho |h_{r}|^{2}b_{t}}{Q_{1}}} \right) \right],$$
(E.1)

where  $\varepsilon = 1$ .

At high SNR regime,  $Q_1$  can be approximated as

$$Q_1 \approx \underbrace{\min\left(\frac{|h_t|^2 a_t}{\varepsilon |g|^2}, \frac{b_t}{b_l}\right)}_{\mathbf{Y}}.$$
 (E.2)

As such, the CDF of X in (E.2) can be given by

$$F_X(x) = 1 - \frac{1}{1 + x\Lambda_3}, 0 < x < \frac{b_t}{b_l},$$
(E.3)

where  $\Lambda_3 = \frac{\varepsilon \Omega_I}{a_t \Omega_t}$ . Substituting (E.3) into (E.1) and through some manipulations, the approximation solution for ergodic rate of  $x_t$  with ipSIC at the high SNR regime can be obtained in (27).

The proof is completed.

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