Joint Subchannel and Power Allocation for NOMA Enhanced D2D Communications

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Abstract—In this paper, a novel non-orthogonal multiple access (NOMA) enhanced device-to-device (D2D) communication scheme is considered. Our objective is to maximize the system sum rate by optimizing subchannel and power allocation. We propose a novel solution that jointly assigns subchannels to D2D groups and allocates power to receivers in each D2D group. For the subchannel assignment, a novel algorithm based on the many-to-one two-sided matching theory is proposed for obtaining a suboptimal solution. Since the power allocation problem is non-convex, sequential convex programming is adopted to transform the original power allocation problem to a convex one. The power allocation vector is obtained by iteratively tightening the lower bound of the original power allocation problem until convergence. Numerical results illustrate that: i) the proposed joint subchannel and power allocation algorithm is an effective approach for obtaining near-optimal performance with acceptable complexity; and ii) the NOMA enhanced D2D communication scheme is capable of achieving promising gains in terms of network sum rate and number of accessed users, compared to traditional orthogonal multiple access (OMA) based D2D communication scheme.

Index Terms—Device-to-device (D2D), matching theory, non-orthogonal multiple access (NOMA), power allocation, subchannel assignment, 5G.

I. INTRODUCTION

As the rocket increasing number of powerful smart mobile devices (e.g., smartphones, tablets, and intelligent vehicle-to-vehicle devices) and upsurge growth of varied multimedia applications (e.g., high-definition videos, massive open online courses, and virtual reality games), an explosion of wireless network services has been leaded accordingly. Driven by meeting those flood data demands, several promising technologies are identified by researchers for the coming fifth generation (5G) networks recently [2], [3]. Particularly, device-to-device (D2D) communications is considered as one of the pieces of the 5G jigsaw puzzle for improving spectral efficiency [2], [4]. In D2D enabled cellular networks, devices are allowed to communicate directly without the assistance of the base station (BS). D2D communications enable low-power transmission of proximity services to improve the energy efficiency and allow to reuse the frequency of the cellular networks in an effort to increase the spectral efficiency. Moreover, D2D communications also have potentials to facilitate new types of peer-to-peer (P2P) services [5]. Motivated by the potential benefits aforementioned, many works have been prompted recently under different scenarios [6]–[9]. Solution approaches that allowed cellular devices and D2D pairs to share spectrum resources were proposed in [6], thereby improved the spectral efficiency of traditional cellular networks. In [7], the functions to facilitate D2D session setup and mechanisms were illustrated, with the aim of controlling and limiting the interference of D2D communications to the cellular network. The authors in [8] jointly studied the D2D spectrum sharing and mode selection under a hybrid network model. From the perspective of security issue, the performance of secure D2D communication was investigated in energy harvesting large-scale cognitive radio networks in [9].

Apart from invoking D2D technique to improve the spectral efficiency of the wireless networks, another emerging technique—non-orthogonal multiple access (NOMA)—is able to address both the massive connectively and spectral efficiency enhancement issue, on the standpoint of realizing a new power dimension for multiple access [10], [11]. Having been included in 3GPP long term evolution (LTE) [12], NOMA is regarded as one of the promising candidates in future 5G networks for its potential ability to significantly improve the spectral efficiency and provide ultra high connectivity [13], [14]. Different from the conventional orthogonal multiple access (OMA) technique, NOMA is capable of supporting multiple users to share the same resource (e.g., time/frequency/code), via using different power level. It is worth mentioning that due to the employment of superposition coding transmission scheme, the power allocation is an eternal problem to be investigated in NOMA, especially in multiple subchannels/subcarriers/clusters scenarios. Somewhat related power allocation and subchannel/subcarrier/cluster assignment problems have been studied in the context of NOMA [15]–[17]. More particularly, in [15], with formulating NOMA resource allocation problems under several practical constraints, the tractability of the formulated problem was analytically characterized. Regarding the multiple carrier NOMA resource allocation problem for the full-duplex NOMA communication scenarios, the monotonic optimization approach was employed in [16] for investigating an optimal solution for the formulated problem. Considering resource allocation in cluster based multiple-input multiple-output (MIMO) NOMA networks, the absolute max-min fairness issue was addressed in [17], with using the bi-section search approach for power allocation and three efficient heuristic algorithms for cluster scheduling.
A. Motivation and Related Works

Inspired by the aforementioned potential benefits of D2D and NOMA, it is natural to investigate the promising application of NOMA technology in the D2D communications for further performance improvement, in term of both spectrum efficiency and massive connectivity. More specifically, we develop a NOMA enhanced D2D communication scheme. In this new scheme, we propose the concept of “D2D group”. Unlike the traditional concept of “D2D pair” [7], [18], one D2D transmitter is able to communicate with several D2D receivers via NOMA protocol. With OMA, transmitting contents to different D2D receivers requires multiple bandwidth channels; however, NOMA can serve these receivers in a single channel use. For example, in a D2D group, the transmitter transmits contents to three receivers requiring video, audio and text messages, respectively. If the video and audio users are with good channel conditions, they can perform successive interference cancellation (SIC) for two or three times to remove their partners’ messages completely and therefore achieve high data rates. For text users, although they will experience strong co-channel interference, this is not an issue since they need to be served only with small data rates. This new concept is fundamentally different from the previous common used concept of “D2D pair”. The main advantages of implementing NOMA enhanced D2D communications are the enhanced system sum rate and the increased number of accessed D2D receivers which simultaneously served by one D2D transmitter. In this work, we consider two receivers in each D2D group, which can be extended to the case that there are multiple receivers in each group.

Recall that although D2D promises unprecedented increase in spectrum efficiency, it brings in interference to the cellular network [7], [19], [20]. Similarly, the application of NOMA into D2D communications brings intra-“D2D group” interference among receivers in the same group as well as inter-“D2D group” interference among groups occupying the same subchannel, which makes the interference management problem more complicated. As such, whether NOMA is capable of enhancing D2D communications underlaying cellular networks still remains unknown and investigating effective resource allocation strategies is more than necessary, which is one of the motivations of this work. To the best of our knowledge, there is no existing work investigating the joint subchannel and power allocation problem of NOMA enhanced D2D communications scenarios, which motivates us to develop this treatise. We attempt to explore the potential of the NOMA enhanced D2D communications in underlay cellular networks and identify the key influence factors on system performance. More particularly, we strive to investigate the answers of questions which are listed as follows:

- **Question 1:** Will NOMA enhanced D2D communications bring sum rate gains compared to the conventional OMA based scheme?
- **Question 2:** Will NOMA enhanced D2D communications significantly improve the number of accessed users compared to the conventional OMA based scheme?
- **Question 3:** How the interference constraints of cellular users influence the sum rate and number of accessed D2D groups?

B. Contributions

In this paper, we consider the setting of an uplink single-cell cellular network communications, where multiple D2D groups are allowed to reuse the same subchannel cellular user to improve the spectrum utilization. We recognize that the spectrum allocation can be regarded as a many-to-one matching process between the D2D groups and subchannels. Due to the co-channel interference among D2D groups occupying the same subchannel, D2D groups have peer effects with the interdependencies among each other. We then formulate the spectrum allocation as a many-to-one matching problem with peer effects [21], [22]. Appropriate power allocation among receivers in the same D2D group is also taken into consideration. Note that allocating D2D groups to orthogonal subchannels with considering power allocation generally turns out to be a combinatorial non-convex problem. Therefore, we decouple the subchannel assignment of D2D groups and the power allocation for each D2D group. The main contributions of this work can be summarized as follows:

- We propose two approaches to jointly consider the subchannel assignment and the power allocation for each D2D group. The main contributions of this work can be summarized as follows:
  - We develop a novel NOMA enhanced D2D scheme that introduces the concept of “D2D group”, where each D2D transmitter is enabled to communicate with multiple D2D receivers simultaneously via NOMA protocol. Based on this scheme, we design a mechanism that jointly performs subchannel assignment to D2D groups and power allocation in each D2D group.
  - For the subchannel assignment, we first give the fixed power allocation in each D2D group, and then formulate the subchannel assignment as a many-to-one matching problem. To maximize the system sum rate, we propose a matching algorithm where the peer effects among the D2D groups are taken into consideration. It is analytically proved that the proposed algorithm is capable of improving the system sum rate and converging to a stable state within limited rounds of interactions.
  - Based on the proposed subchannel assignment algorithm, the power allocation problem for each D2D group is formulated as a non-convex problem because of the existence of intra-group interference. We apply the sequential convex programming to iteratively update the power allocation vector by solving the approximate convex problem. We prove that the proposed algorithm is convergent and the solution satisfies the Karush-Kuhn-Tucker (KKT) conditions.
  - We propose two approaches to jointly consider the subchannel and power allocation problems. The iterative joint subchannel and power allocation algorithm (I-JSPA) enables the power allocation under each given case of the matching between D2D groups and subchannels. Because of the high complexity of I-JSPA, we propose a low-complexity joint subchannel and power allocation algorithm (LC-JSPA). The result of LC-JSPA is shown to closely approach to that of the I-JSPA.
  - We show that the proposed joint subchannel and power allocation algorithm can achieve the near performance
to the exhaustive-searching method at a low computational complexity. We also demonstrate that the NOMA enhanced D2D communications achieve higher system sum rate and larger number of accessed users than the OMA based D2D scheme.

C. Organization and Notation

The rest of the paper is organized as follows. In Section II, the network model for studying NOMA enhanced D2D communications is presented. Section III formulates the sum rate optimization problem, and studies the computational complexity. In Section IV and Section V, the formulated problem is decoupled to the subchannel assignment and power allocation problems, respectively. Numerical results are presented in Section VI, which is followed by conclusions in Section VII.

The notation of this paper is shown in Table I.

<table>
<thead>
<tr>
<th>$C$, $D$</th>
<th>Set of cellular users and D2D groups</th>
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<tbody>
<tr>
<td>$M$, $N$</td>
<td>Number of cellular users and D2D groups</td>
</tr>
<tr>
<td>$DT_n$</td>
<td>The $n$-th D2D transmitter</td>
</tr>
<tr>
<td>$DR_{n,k}$</td>
<td>The $k$-th receiver in the $n$-th D2D group</td>
</tr>
<tr>
<td>$L_n$</td>
<td>Number of receivers in each D2D group</td>
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<tr>
<td>$\eta$</td>
<td>Spectrum sharing indicator</td>
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<tr>
<td>$\alpha$</td>
<td>Power allocation coefficient</td>
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<tr>
<td>$f$, $h$, $g$</td>
<td>Channel coefficients</td>
</tr>
<tr>
<td>$P_c$, $P_d$</td>
<td>Transmit power of cellular users and D2D transmitters</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Additive white Gaussian noise with variance $\sigma^2$</td>
</tr>
<tr>
<td>$I_{m,n}$</td>
<td>Interference power at D2D receiver from the same group</td>
</tr>
<tr>
<td>$I_{n,m}$</td>
<td>Interference power at D2D receiver from other D2D groups</td>
</tr>
<tr>
<td>$I_n$</td>
<td>Interference power at D2D receiver from the cellular user</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Signal-to-interference-plus-noise ratio</td>
</tr>
</tbody>
</table>

II. NETWORK MODEL

A. System Description

We focus on a single-cell uplink transmission scenario, as illustrated in Fig. 1(a). We consider that $M$ cellular users, i.e. $C = \{C_1, \ldots, C_m, \ldots, C_M\}$, communicate with one BS in the traditional cellular mode. Each cellular user $C_m$ is allocated in one subchannel $SC_m \in SC$, $SC = \{SC_1, \ldots, SC_m, \ldots, SC_M\}$ and all the subchannels are orthogonal with each other. There are $N$ D2D groups $D = \{D_1, \ldots, D_n, \ldots, D_N\}$ communicating underlaying cellular networks. Unlike the traditional D2D-pair communications, each D2D transmitter can send the superimposed mixture containing the required messages for the receivers in the D2D group by applying NOMA transmission protocol, which introduces the concept of “D2D group” (as shown in Fig. 1(a)). It is worth noting that, when the number of receivers in each D2D group drops to 1, our model is the same as the conventional “D2D pair” case. In this paper, we assume that there are two NOMA receivers in each D2D group to limit the multiuser interference and to ensure low hardware complexity and low processing delay.

In Fig. 1(b), the interference received at receiver 2 of the $n$-th D2D group is illustrated as follows:

- The intra-group interference (the black dashed line) refers to the interference of superposition signals to receiver 1 in the same D2D group;
- The inter-group interference (the red dashed line) indicates the interference from the D2D transmitters of other D2D groups that reuse the same subchannel;
- Last, the cellular interference (the blue dashed line) represents the interference from the cellular user reusing the same subchannel.

It is assumed that the cellular users and D2D transmitters are uniformly distributed in the cell. The receivers in each D2D group are uniformly distributed within a disc with radius $d_{max}$, and the origin of the disc is the corresponding $DT_n$. All channels are assumed to undergo quasi-static Rayleigh fading, where the channel coefficients are constant for each channel.

1Considering subchannel assignment to cellular UEs is beyond the scope of this paper [23], [24]. We may take this into consideration in our future work and model it as a bipartite graph and perfect matching.

2The multiuser interference in each D2D group increases as more receivers are multiplexed on the same subchannel which can degrade the performance of individual users.

3NOMA requires SIC at the receivers. In practice, a user performing SIC has to demodulate and decode the signals intended for other users in addition to its own signal. Thus, hardware complexity and processing delay increase with the number of NOMA receivers multiplexed on the same subchannel.
B. Channel Model

We assume that each subchannel which is occupied by a cellular user can be reused by multiple D2D groups. As a consequence, the received signal at the BS corresponding to subchannel \( SC_m \) is given by

\[
y_m = \sqrt{P_e}h_m x_m + \sum_n \eta_{n,m} \sqrt{P_d} g_n t_n + \zeta_m, \tag{1}
\]

where \( x_m \) and \( t_n \) are the transmit signals of \( C_m \) and \( DT_n \), respectively, \( \zeta_m \) is the additive white Gaussian noise (AWGN) at the BS on subchannel \( SC_m \) with variance \( \sigma^2 \). The matrix \( \eta \in \mathbb{R}^{N \times M} \) with the elements \( \eta_{n,m} \) represents the subchannel allocation indicator for D2D groups, i.e., if \( SC_m \) is assigned to \( D_n \), \( \eta_{n,m} = 1 \); otherwise, \( \eta_{n,m} = 0 \). \( P_e \) and \( P_d \) are the transmit power of the cellular users and D2D transmitters, respectively. In this paper, we assume that all the cellular users have the same transmit power and so do all the D2D transmitters for simplicity, \( h_m \) and \( g_n \) are the channel coefficients including small-scale fading and path-loss between \( C_m \) and the BS, and that between \( DT_n \) and the BS, respectively.

Based on (1), we obtain the received signal-to-interference-plus-noise ratio (SINR) at the BS corresponding to \( C_m \) as

\[
\gamma_m = \frac{P_e |h_m|^2}{\sum_n \eta_{n,m} P_d |g_n|^2 + \sigma^2}, \tag{2}
\]

where \( |h_m|^2 = |\hat{h}_m|^2 |d_m|^{-\alpha} \) and \( |g_n|^2 = |\hat{g}_n|^2 |d_n|^{-\alpha} \). Here, \( \hat{h}_m \) and \( \hat{g}_n \) are small-scale fading with \( \hat{h}_m \sim \mathcal{CN}(0,1) \) and \( \hat{g}_n \sim \mathcal{CN}(0,1) \). \( d_m \) is the distance from \( C_m \) to the BS, and \( d_n \) is the distance from \( DT_n \) to the BS. \( \alpha \) is the path-loss exponent.

The NOMA protocol requires the superposition coding technique at the D2D transmitter side and SIC techniques at the receivers. The vector \( a_n = \{a_{n,1}, a_{n,2}\} \) represents the power allocation coefficients in each D2D group. The D2D transmitter \( D_n \) sends superposed messages to the destinations based on the NOMA principle, i.e., \( D_n \) sends \( a_{n,1} s_{n,1} + a_{n,2} s_{n,2} \), where \( s_{n,1} \) and \( s_{n,2} \) are the messages for receiver 1 and 2, respectively. Therefore, the received signal at receiver 1 is given by

\[
z_{n,1} = f_{n,1} \left( \sqrt{a_{n,1} P_d s_{n,1}} + \sqrt{a_{n,2} P_d s_{n,2}} \right) + \zeta_{n,1} \\
+ \sqrt{P_d} h_{m,n} x_m + \sum_{n \neq m} \eta_{n,m} \sqrt{P_d} g_{n,m} t_{n,m} + \zeta_{n,1}, \tag{3}
\]

where \( f_{n,1}, h_{m,n}, \) and \( g_{n,m} \) are the channel coefficients between \( DT_n \) and receiver 1, that between \( C_m \) and receiver 1, and that between \( DT_{n^*} \) and receiver 1, respectively. \( \zeta_{n,1} \) is the AWGN at receiver 1 with variance \( \sigma^2 \). \( \eta_{n,m} \) represents the presence of interference, i.e., if D2D group \( D_n \) and \( D_{n^*} \) reuse the same subchannel, \( \eta_{n,m} = 1 \); otherwise, \( \eta_{n,m} = 0 \).

NOMA systems exploit the power domain for multiple access, where different users are served at different power levels. For illustration, we assume that receiver 1 decodes and removes the signal to receiver 2 via SIC. The interference cancellation is successful if receiver 1’s received SINR for receiver 2’s signal is not smaller than the received SINR at receiver 2 for its own signal [10], [16], which can be expressed as

\[
\frac{|f_{n,1}|^2 P_d a_{n,2}}{I_{n,1}^{2,\text{in}} + I_{n,1}^{\text{out}} + I_{n,1}^c + \sigma^2} \geq \frac{|f_{n,2}|^2 P_d a_{n,1}}{I_{n,2}^{2,\text{in}} + I_{n,2}^{\text{out}} + I_{n,2}^c + \sigma^2}, \tag{4}
\]

where \( I_{n,1}^{2,\text{in}} = |f_{n,1}|^2 P_d a_{n,1} \) and \( I_{n,2}^{2,\text{in}} = |f_{n,2}|^2 P_d a_{n,1} \) is intra-group interference from the superimposed signals at receiver 1 and 2, respectively. \( I_{n,1}^{\text{out}} = \sum_{n \neq m} \eta_{n,m} P_d |g_{n,m,n,1}|^2 \) and \( I_{n,2}^{\text{out}} = \sum_{n \neq m} \eta_{n,m} P_d |g_{n,m,n,2}|^2 \) is inter-group interference received at receiver 1 and 2, respectively. \( I_{n,1}^c = \sum_n \eta_{n,m} P_d |h_{m,n,1}|^2 \) and \( I_{n,2}^c = \sum_n \eta_{m,n} P_d |h_{m,n,2}|^2 \) is interference from cellular users received at 1 and 2, respectively. Here, \( |f_{n,1}|^2 = |f_{n,1}|^2 |(d_{n,1})^{-\alpha} |f_{n,2}|^2 = |f_{n,2}|^2 |(d_{n,2})^{-\alpha} \), \( |g_{n,m,n,1}|^2 = |g_{n,m,n,1}|^2 |(d_{n,m,1})^{-\alpha} \), \( |g_{n,m,n,2}|^2 = |g_{n,m,n,2}|^2 |(d_{n,m,2})^{-\alpha} \), \( |h_{m,n,1}|^2 = |h_{m,n,1}|^2 |(d_{m,n,1})^{-\alpha} \), and \( |h_{m,n,2}|^2 = |h_{m,n,2}|^2 |(d_{m,n,2})^{-\alpha} \).

The inequality in (4) can be simplified and rewritten in the following:

\[
Q(\eta) = |f_{n,1}|^2 \left( I_{n,1}^{\text{out}} + I_{n,1}^c + \sigma^2 \right) - |f_{n,2}|^2 \left( I_{n,2}^{\text{out}} + I_{n,2}^c + \sigma^2 \right) \geq 0. \tag{5}
\]

Therefore, according to the received signal expressed in (3), the received SINR receiver 2 in the \( n \)-th D2D group to decode its own information is given by

\[
\gamma_{n,2}^2 = \frac{|f_{n,2}|^2 P_d a_{n,2}}{I_{n,2}^{2,\text{in}} + I_{n,2}^{\text{out}} + I_{n,2}^c + \sigma^2}. \tag{6}
\]

Since receiver 1 can decode and remove the interference from receiver 2, the received SINR at receiver 1 for decoding its own signal is expressed as

\[
\gamma_{n,1}^1 = \frac{|f_{n,1}|^2 P_d a_{n,1}}{I_{n,1}^{\text{out}} + I_{n,1}^c + \sigma^2}. \tag{7}
\]

III. PROBLEM FORMULATION

In this section, we first give the constraints of cellular users’ received interference from the D2D groups, and then introduce the network sum rate. Subsequently, we formulate the joint subchannel and power allocation problem for the NOMA enhanced D2D system.

A. Interference Constraints

One of the key challenges in D2D communications underlaying cellular networks is the co-channel interference caused by the spectrum sharing between the D2D and traditional...
cellular links. To guarantee the service qualities of cellular and D2D users, we give the interference constraints expressed in the format of SINR as follows:

\[ \gamma_m = \frac{P_m|h_m|^2}{\sum_n \eta_{n,m}P_d|g_n|^2 + \sigma^2} \geq \gamma_{m,thr}, \quad (8) \]

\[ \gamma_{n,2}^2 = \frac{|f_{n,2}|^2P_d\alpha_{n,2}}{I_{n,2}^2 + I_{n,2}^{out} + I_{n,2}^c + \sigma^2} \geq \gamma_{n,2,thr}, \quad (9) \]

\[ \gamma_{n,1} = \frac{|f_{n,1}|^2P_d\alpha_{n,1}}{I_{n,1}^2 + I_{n,1}^{out} + \sigma^2} \geq \gamma_{n,1,thr}, \quad (10) \]

where \( \gamma_{m,thr}, \gamma_{n,2,thr} \) and \( \gamma_{n,1,thr} \) are the given SINR thresholds for cellular user \( m \), and that for receiver 2 and receiver 1 in D2D group \( n \), respectively.

**B. Network Sum Rate**

Based on the expression of SINR in (2) and the Shannon formula, the data rate for the \( m \)-th cellular user \( C_m \) is given by

\[ R_m = \log_2 \left( 1 + \frac{P_m|h_m|^2}{\sum_n \eta_{n,m}P_d|g_n|^2 + \sigma^2} \right). \quad (11) \]

Similarly, the data rates for receiver 1 and 2 in D2D group \( n \) are given by

\[ R_{n,1} = \log_2 \left( 1 + \frac{|f_{n,1}|^2P_d\alpha_{n,1}}{I_{n,1}^2 + I_{n,1}^{out} + \sigma^2} \right), \quad (12) \]

and

\[ R_{n,2} = \log_2 \left( 1 + \frac{|f_{n,2}|^2P_d\alpha_{n,2}}{I_{n,2}^2 + I_{n,2}^{out} + I_{n,2}^c + \sigma^2} \right), \quad (13) \]

respectively.

As such, we can obtain the network sum rate of all the cellular and D2D users as

\[ R_{\text{sum}}(\eta, a_n) = \sum_{m=1}^{M} R_m + \sum_{n=1}^{N} \eta_{n,m} (R_{n,1} + R_{n,2}). \quad (14) \]

**C. Optimization Problem Formulation**

Now, the joint subchannel and power allocation problem for the NOMA enhanced D2D system can be formulated as the following:

\[ \max_{\eta, a_n} R_{\text{sum}}(\eta, a_n), \quad (15a) \]

subject to

\[ \gamma_m \geq \gamma_{m,thr}, \quad \forall m, \quad (15b) \]

\[ Q(\eta) \geq 0, \quad \forall m, \quad (15c) \]

\[ \gamma_{n,1} \geq \gamma_{n,1,thr}, \quad \gamma_{n,2} \geq \gamma_{n,2,thr}, \quad \forall n, \quad (15d) \]

\[ \eta_{n,m} \in \{0, 1\}, \quad \forall n, m, \quad (15e) \]

\[ \sum_m \eta_{n,m} \leq 1, \quad \forall n, \quad (15f) \]

\[ \sum_n \eta_{n,m} \leq q_{\text{max}}, \quad \forall m, \quad (15g) \]

\[ a_{n,1} \geq 0, \quad a_{n,2} \geq 0, \quad \forall n, \quad (15h) \]

\[ a_{n,1} + a_{n,2} \leq 1, \quad \forall n. \quad (15i) \]

Constraint (15b) is imposed to restrict the interference received at the cellular links from the D2D groups. Constraint (15c) is to guarantee the policy for SIC decoding order. Constraint (15d) guarantees the minimum SINR constraints for D2D users. Constraint (15e) shows that the value of \( \eta_{n,m} \) should be either 0 or 1. Constraint (15f) guarantees that at most one subchannel can be allocated to each D2D group\(^4\). Constraint (15g) introduces the maximum number of D2D groups \( q_{\text{max}} \) can be allocated to each subchannel, which is to reduce the implementation complexity and the interference on each subchannel. Constraint (15h) is a non-negative constraint for power allocation coefficients. Constraint (15i) restricts the upper bound of the D2D users’ transmit power.

The formulated problem here is a 0-1 integer program, besides, the objective function is non-convex. There is no systematic and computational efficient approach to solve this problem optimally. In addition, according to (15), the subchannel and power allocation variables are coupled. Therefore, in section IV and V, we decouple the formulated problem into two sub-problems: 1) subchannel assignment of D2D groups; and 2) power allocation to the receivers in each D2D group.

**IV. Matching Theory Based Subchannel Assignment**

In this section, we assume that the power allocated to the transmission from the transmitter to receivers in each D2D group is a fixed value, i.e., \( a_n = a_n^*, \forall n \in \{1, ..., N\} \). Thus we can obtain the sub-problem of subchannel assignment as the following:

\[ \max_{\eta} R_{\text{sum}}(\eta, a_n^*), \quad (16a) \]

subject to

\[ \text{(15b) - (15g)}, \quad (16b) \]

Note that the formulated problem is a non-convex optimization problem due to the existence of the interference term in the objective function [23]. The complexity of the exhaustive method increases exponentially with the number of D2D groups and subchannels, which makes it unpractical especially in a dense network. To describe the dynamic matching between the D2D groups and subchannels, we consider the subchannel assignment as a two-sided many-to-one matching process between the sets of D2D groups and subchannels. The D2D groups and subchannels act as two sets of players and interact with each other to maximize the sum rate. To solve this problem, we adopt the matching theory [21], [24], which provides mathematically tractable and low-complexity solutions.\(^4\)The consideration of each D2D group occupying multiple subchannels introduces power allocation problem over subchannels, which is beyond the scope of this treatise and may be considered in our future work.
solutions for the combinatorial problem of matching players in two distinct sets [25]. We then formulate the subchannel assignment problem as a many-to-one matching problem and propose an efficient algorithm to solve this problem.

A. Many-to-One Matching with Peer Effects

To proceed with proposing the subchannel assignment algorithm, we first introduce some notations and basic definitions for the proposed matching model between the sets of D2D groups and subchannels.

Definition 1. In the many-to-one matching model, a matching \( \Omega \) is a function from the set \( SC \cup D \) into the set of all subsets of \( SC \cup D \) such that (1) \( |\Omega(D_n)| = 1, \forall D_n \in D \), and \( \Omega(D_n) = \{ D_n \} \) if \( \Omega(D_n) \not\subseteq SC \); (2) \( |\Omega(SC_m)| \leq q_{\text{max}} \), \( \forall SC_m \in SC \), and \( \Omega(SC_m) = \emptyset \) if \( SC_m \) is not matched to any D2D group; 3) \( \Omega(D_n) = \{ SC_m \} \) if and only if \( D_n \in \Omega(SC_m) \).

Note that a positive integer \( q_{\text{max}} \) called quota is associated with each subchannel \( SC_m \), which indicates the maximum number of D2D groups that can be matched with each subchannel.

To better describe the competition behavior and decision process of each player, we assume that each player has preferences over the players of the other set. The preference of each player is based on the achievable utility, and we denote the set of preference lists of D2D groups and subchannels as

\[
PL = \{ P(D_1), \ldots, P(D_N), P(SC_1), \ldots, P(SC_M) \},
\]

where \( P(D_n) \) is the preference list of \( D_n \) over individual subchannels, and \( P(SC_m) \) is the preference list of \( SC_m \) over sets of D2D groups. The preference lists of players are formed in descending order with respect to the utility of each side of the players. The utility definitions of D2D groups and subchannels are defined in the following.

For a D2D group \( D_n \), the utility on a subchannel \( SC_m \) is defined as the achievable data rate of \( D_n \) when it occupies \( SC_m \), which is given by

\[
U_n(m) = \log_2 \left( 1 + \gamma_{n,m,2} \right) + \log_2 \left( 1 + \gamma_{n,m,1} \right).
\]

From (18), it is not difficult to find that the utility of a D2D group depends not only on the subchannel that it is matched with, but also on the set of D2D groups that are matched to the same subchannel, due to the existence of the co-channel interference \( I_{n,k}^{\text{out}} \). Therefore, we have the following observation:

Remark 1. The proposed matching game has peer effects [26]. That is, the D2D groups care not only where they are matched, but also which other D2D groups are matched to the same place.

This type of matching is called the matching game with peer effects, where each player has a dynamic preference list over the opposite set of players. This is different from the conventional matching games in which players have fixed preference lists [24], [27], [28]. In this matching model, the preference of players over the opposite set of players replies on the matching states. To this end, we need to define the new preference \( P'(D_n) \) of D2D group \( D_n \) on the set of possible matchings rather than the \( P(D_n) \) which is simply the preference of \( D_n \) on the subchannels. The relationship of “prefer” for a D2D group on subchannels under different matching states is expressed as

\[
(m, \Omega) \succ_n (m', \Omega') \iff U_n(m, \Omega) > U_n(m', \Omega'),
\]

where \( U_n(m, \Omega) \) is the utility of D2D group \( D_n \) when it occupies the subchannel \( SC_m \) under the matching state \( \Omega \).

We define the preference values of subchannel \( SC_m \) on a set of D2D groups \( S_D \) as the sum rate of all the D2D groups and the corresponding cellular user, which is expressed as

\[
U_m(S) = \log_2 \left( 1 + \gamma_m(S) \right) + \sum_{D_n \in S} \left( \log_2 \left( 1 + \gamma_{n,m,2} \right) + \log_2 \left( 1 + \gamma_{n,m,1} \right) \right),
\]

where \( \gamma_m(S) \) is the SINR of the cellular user \( C_m \) when it shares the subchannel with the set of D2D groups \( S \).

Based on the utility definition of the subchannel \( SC_m \), we can obtain the “prefer” relationship of \( SC_m \) on the set of D2D groups \( S \) as \( S' \) as

\[
(S, \Omega) \succ (S', \Omega') \iff U_m(S, \Omega) > U_m(S', \Omega'),
\]

where \( U_m(S, \Omega) \) is the utility of \( SC_m \) on the set of D2D groups \( S \) under the matching state \( \Omega \).

There is a growing literature studying many-to-one matchings with peer effects [29], [30]. However, these researches find that designing matching mechanisms is significantly more challenging when peer effects are considered. Motivated by the housing assignment problem in [26], we propose an extended matching algorithm for the many-to-one matching problem with peer effects in the following.

Different from the traditional deferred acceptance algorithm solution [24], the swap operations between any two D2D groups to exchange their matched subchannels is enabled. To better describe the interdependencies between the players’ preferences, we first define the concept of swap matching as follows:

\[
\Omega_n' = \{ \Omega \setminus \{ (D_n, \Omega(D_n)), (D_n', \Omega(D_n')) \} \} \cup \{ (D_n, \Omega(D_n'), (D_n', \Omega(D_n))) \},
\]

where D2D groups \( D_n \) and \( D_n' \) switch places while keeping other D2D groups and subchannels’ matchings unchanged. It is worth noticing that one of the D2D groups involved in the swap can be a “hole” representing an open spot, thus allowing for single D2D groups moving to available vacancies of subchannels. Similarly, one of the subchannels involved in the swap can be a “hole” when \( \Omega(D_n) = \emptyset \).

Based on the concept of a swap matching, the swap-blocking pair is defined in the following:

Definition 2. \((D_n, D_n')\) is a swap-blocking pair if and only if

1) \( \forall i \in \{ D_n, D_n', \Omega(D_n), \Omega(D_n') \}, U_i(\Omega_n') \geq U_i(\Omega) \) and
2) \( \exists i \in \{ D_n, D_n', \Omega(D_n), \Omega(D_n') \}, \) such that \( U_i(\Omega_n') > U_i(\Omega) \).
Note that the above definition implies that if two D2D groups want to switch between two subchannels (or a single D2D group wants to switch with a “hole”), the subchannels involved must approve the swap. The condition (1) implies that the utilities of all the involved players should not be reduced after the swap operation between the swap-blocking pair \((D_i,D_{i'})\). The condition (2) indicates that at least one of the players’ utilities is increased after the swap operation between the swap-blocking pair. This avoids looping between equivalent matchings where the utilities of all involved agents are indifferent. Through multiple swap operations, the dynamic preferences of players which depend on the entire matching of the others, and the peer effects of matchings are well handled.

As stated in [24], there is no longer a guarantee that a traditional “pairwise-stability” exists when players care about more than their own matching, and, if a stable matching does exist, it can be computationally difficult to find. The authors in [26] focused on the two-sided exchange-stable matchings, which is defined as follows:

**Definition 3.** A matching \(\mu\) is two-sided exchange-stable if there does not exist a swap-blocking pair.

The two-sided exchange stability is a distinct notion of stability compared to the traditional notion of stability of [24], but one that is relevant to our situation where agents can compare notes with each other.

**B. Proposed Subchannel Assignment Algorithm (SAA) Based on Many-to-One Matching**

To find a two-sided exchange-stable matching for the matching game, we propose a matching-theory based subchannel assignment algorithm, i.e., SAA, between D2D groups and subchannels based on multiple swap operations, as shown in Algorithm 1. The input of the proposed algorithm includes the initial list of the number of D2D groups matched to each subchannel as well as the initial matching state. To initialize the matching state, we randomly match each D2D group with a subchannel or an empty set. If a D2D group is matched to an empty set, it indicates that no subchannel is allocated to the D2D group in the initial state. The main process of the proposed algorithm is the swap operation between different D2D groups, where each D2D group keeps searching for all the other D2D groups to check whether there is a swap-blocking pair. Note that one of the D2D groups taking part in the swap operations can be an available vacancy of a subchannel. The swap operations continue until there are no more swap-blocking pairs, and the final matching state is the output.

Regarding the time scale of SAA, the signaling packet length required for the communication between the D2D groups and subchannels until the algorithm converges is very short. In particular, each D2D group is only required to send one bit to another D2D group indicating a swap-operation offer, and then the involved D2D groups each send a one-bit request to their occupying subchannels. Finally, the subchannels only need to send one bit back to the offering D2D groups indicating either accept or reject the request. The total amount of overhead from SAA thus can be quite small, which enables it to well perform in practical scenarios.

**Algorithm 1 Matching-Theory Based Subchannel Assignment Algorithm (SAA)**

1. **– Input:**
   - Initial matching \(\Omega_0\): Randomly match each D2D group with \(SC \in \{SC, \emptyset\}\) satisfying the constraint that \(q_m \leq q_{max}, \forall q_m \in Q\).
   - Initial list of the number of D2D groups matched to each subchannel \(Q = \{q_1, ..., q_M\}\).
2. **– Swap Operations:**
   3. **repeat**
   4. **for** \(\forall D_n \in D\) **do**
   5. **for** \(\forall D_{n'} \in \{D \setminus \{D_n\}, O\}\), where \(O\) is an open spot of subchannel’s available vacancies, with \(\Omega(n) = m\), and \(\Omega(n') = m'\) **do**
   6. **if** \((D_n, D_{n'})\) is a swap-blocking pair, and (15b)-(15g) are satisfied **then**
   7. **end if**
   8. Update \(Q\);
   9. **break**;
   10. **end for**
   11. **end for**
   12. **until** \(\notexists(D_n, D_{n'})\) blocks the current matching.
14. **– Output:** Final matching \(\Omega^*\).

**C. Property Analysis of SAA**

To evaluate the performance of SAA, we analyze the properties in terms of effectiveness, stability, convergence and complexity in this subsection.

**Theorem 1.** The final matching \(\Omega^*\) of SAA is a two-sided exchange-stable matching.

**Proof.** Assume that there exists a swap-blocking pair \((D_n, D_{n'})\) in the final matching \(\Omega^*\) satisfying that \(\forall i \in \{n, n', \Omega(D_n), \Omega(D_{n'})\}, U_i ((\Omega^*)^n) \geq U_i(\Omega^*)\) and \(\exists i \in \{n, n', \Omega(D_n), \Omega(D_{n'})\}, \) such that \(U_i ((\Omega^*)^n) > U_i(\Omega^*)\). According to SAA, the algorithm does not terminate until all the swap-blocking pairs are eliminated. In other words, \(\Omega^*\) is not the final matching, which causes conflict. Therefore, there does not exist a swap-blocking pair in the final matching, and thus we can conclude that the proposed algorithm reaches a two-sided exchange stability in the end of the algorithm.

**Lemma 1.** The system sum rate increases after each swap operation.

**Proof.** Suppose a swap operation makes the matching state change from \(\Omega\) to \(\Omega^n\). According to SAA, a swap operation occurs only when \(U_m(\Omega^n) \geq U_m(\Omega)\) as well as \(U_{m'}(\Omega^n) \geq U_{m'}(\Omega)\)...
Given that $U_m(\Omega(m), \Omega) = R_m(\Omega(m), \Omega) + \sum_{n \in \Omega(m)} \sum_{k=1}^{L_n} R_{n,k}(m, \Omega)$, we have

$$\Phi_{\Omega \rightarrow \Omega'} = U_m(\Omega'(m), \Omega') - U_m(\Omega(m), \Omega)$$

$$= R_{sum}(\Omega'(m)) - R_{sum}(\Omega)$$

$$> 0,$$  \hspace{2cm} (23)$$

where $\Phi_{\Omega \rightarrow \Omega'}$ is the difference of the system sum rates under the matching state $\Omega'$ and that under the matching state $\Omega$. From (23), we conclude that the system sum rate increases after each successful swap operation. \hfill \Box

**Theorem 2.** The proposed subchannel assignment algorithm converges within limited number of iterations.

**Proof.** In the proposed matching model, the number of players is limited and the maximum number of D2D groups can be allocated to each subchannel is restricted, which indicates that the number of potential swap operations is finite. Moreover, from (23), we find that the system sum rate increases after each successful swap operation. Since the system sum rate has an upper bound due to limited spectrum resources, the swap operations stop when the system sum rate is saturated. Therefore, within limited number of rounds, the matching process converges to the final state which is stable. \hfill \Box

**Theorem 3.** The computational complexity of the proposed algorithm is of the order $O\left(\frac{M_{D2D}}{\Delta_{m,n}}\right)$ in the worst case.

**Proof.** As shown in SAA, the complexity of the proposed algorithm mainly depends on the number of iterations in the swap-matching phase. Since it is uncertain that at which step the algorithm converges to a two-sided exchange stable matching, the number of iterations cannot be given in a closed-form expression. We will analyze the number of total iterations for different numbers of D2D groups in Fig. 3, and give more detailed analysis in section VI. Here, we give an upper bound of the complexity. As proved in (23), the sum rate increases with the swap operations going on. We denote the difference of the sum rates of the final matching and the initial matching as $\Phi_{\Omega \rightarrow \Omega^*}$, and the minimum increase of each swap operation as $\Delta_{m,n}$. Thus, in the worst case, the computational complexity of the proposed algorithm is of the order $O\left(\frac{M_{D2D}}{\Delta_{m,n}}\right)$. \hfill \Box

**Theorem 4.** All local maxima of system sum rate corresponds to a two-sided exchange-stable matching.

**Proof.** Assume that the sum rate achieved by matching $\Omega$ is a local maximal value. If $\Omega$ is not a stable matching, there exists a swap-blocking pair that can improve the system sum rate, as proved in Lemma 1. However, this is inconsistent with the assumption that $\Omega$ is local optimal. Hence, we conclude that $\Omega$ is a two-sided exchange-stable matching. \hfill \Box

However, not all two-sided exchange-stable matchings obtained from SAA can achieve the local maximum of system sum rate. The reason can be shown in a simple example. D2D group $D_n$ does not approve the swap operation with $D_{n'}$ along with their current matched subchannels $SC_m$ and $SC_{m'}$, due to the fact that the utility of $D_n$ is decreased after the swap operation. However, $SC_m$ and $SC_{m'}$ can benefit a lot from this swap operation, which causes that the optimal sum rate can not be achieved by the swap operations. Of course, we can force the swap operation to happen to further improve the sum rate, but this will obtain a weaker stability [31].

**V. POWER ALLOCATION IN EACH D2D GROUP**

In this section, we solve the sub-problem of allocating power to receivers in each D2D group. For a given subchannel assignment strategy $\eta = \eta^*$, power allocation can be performed independently by the BS in a centralized way. To make the notation simplified, we drop the D2D group index $n$, thus the power allocation problem for D2D group $n$ can be expressed as

$$\max_{a_n} (R_{n,1} + R_{n,2}),$$ \hspace{2cm} (24a)

$$\text{s.t.} \ (15d), (15h), (15i).$$ \hspace{2cm} (24b)

**A. Pareto Optimal Solution**

Because of the existence of the co-channel interference, the formulated problem is a non-convex problem with respect to $a_n$. Therefore, obtaining the global optimum with affordable complexity is rather difficult. Alternatively, we apply the sequential convex programming [32], i.e., finding local optimum of (24) by solving a sequence of easier problems. In the following, we propose a low-complexity algorithm to obtain a local-optimal solution for the optimization problem.

The objective function in (24a) can be rewritten as

$$\max_{a_n} (R_{n,1} + R_{n,2}) = \left(\log_2 (1 + \gamma_{n,1}) + \log_2 (1 + \gamma_{n,2})\right).$$ \hspace{2cm} (25)

As proved in [33], the following inequality exists:

$$\log_2 (1 + \gamma_{n,1}) \geq b_1 \log_2 \gamma_{n,1} + c_1,$$ \hspace{2cm} (26)

$$\log_2 (1 + \gamma_{n,2}) \geq b_2 \log_2 \gamma_{n,2} + c_2,$$ \hspace{2cm} (27)

where $b_1$, $b_2$, $c_1$ and $c_2$ are defined as

$$b_1 = \frac{\gamma_{n,1}}{1 + \gamma_{n,1}},$$ \hspace{2cm} (28)

$$b_2 = \frac{\gamma_{n,2}}{1 + \gamma_{n,2}},$$ \hspace{2cm} (29)

$$c_1 = \log_2 (1 + \gamma_{n,1}) - \frac{\gamma_{n,1}}{1 + \gamma_{n,1}},$$ \hspace{2cm} (30)

$$c_2 = \log_2 (1 + \gamma_{n,2}) - \frac{\gamma_{n,2}}{1 + \gamma_{n,2}},$$ \hspace{2cm} (31)

respectively. The equalities in (26) and (27) are satisfied when $\gamma_{n,1} = \gamma_{n,1}$ and $\gamma_{n,2} = \gamma_{n,2}$, respectively.

It is worth noting that sequential convex programming is one efficient method to solve the power allocation problem. Other feasible methods such as gradient descent algorithm may be considered as well.
Based on the inequality functions in (26) and (27), we can obtain the lower bound of the objective function in (25) as

\[ R_{n,1} + R_{n,2} \geq \Theta_{n,1}(a_{n,1}) + \Theta_{n,2}(a_{n,2}), \]  

(32)

where \( \Theta_{n,1}(a_{n,1}) \) and \( \Theta_{n,2}(a_{n,2}) \) are defined as

\[ \Theta_{n,1}(a_{n,1}) = b_1 \log_2 \gamma_{n,1}^1 + c_1, \]

(33)

and

\[ \Theta_{n,2}(a_{n,2}) = b_2 \log_2 \gamma_{n,2}^2 + c_2, \]

(34)

respectively.

Set \( a_{n,1} = 2^{s_{n,1}} \) and \( a_{n,2} = 2^{s_{n,2}} \), and define \( s_n = \{s_{n,1}, s_{n,2}\} \). We can formulate a new optimization problem from (24) and (32) as follows:

\[
\max_{s_n} \left( \Theta_{n,1}(2^{s_{n,1}}) + \Theta_{n,2}(2^{s_{n,2}}) \right),
\]  

(35a)

s.t.

\[
\gamma_{n,1}^1 \geq \gamma_{n,1}^{thr}, \quad \gamma_{n,2}^2 \geq \gamma_{n,2}^{thr},
\]  

(35b)

\[ 2^{s_{n,1}} + 2^{s_{n,2}} \leq 1. \]

(35c)

**Remark 2.** The new formulated problem is a concave problem, which is proved as the following:

**Proof.** Rearranging \( \Theta_{n,1}(2^{s_{n,1}}) \) and \( \Theta_{n,2}(2^{s_{n,2}}) \), we can obtain

\[
\Theta_{n,1}(2^{s_{n,1}}) = b_1[s_{n,1} - \log_2(f_{n,2}^a + I_{n,2}^c + \sigma^2)] + b_2 \log_2(f_{n,2}^a + I_{n,2}^c + \sigma^2) + c_1,
\]  

(36)

\[
\Theta_{n,2}(2^{s_{n,2}}) = b_2[s_{n,2} - \log_2(f_{n,1}^a + I_{n,1}^c + \sigma^2)] + b_2 \log_2(f_{n,1}^a + I_{n,1}^c + \sigma^2) + c_2,
\]  

(37)

\( \Theta_{n,1}(2^{s_{n,1}}) \) and \( \Theta_{n,2}(2^{s_{n,2}}) \) are concave functions of \( s_{n,u} \) and \( s_{n,v} \), respectively, because of the convexity of the sum-exp function [34]. Since the objective function in (35a) is a summation of concave terms of \( s_n \), we can conclude that the problem in (35) is a standard convex optimization problem.

Since (35) is a standard convex optimization problem, there exists many efficient numerical algorithms such as the interior-point method to obtain the optimal solution. We iteratively update the power allocation vector \( a_n \) by solving (35) to tighten the lower bound in (32) until convergence. The proposed power allocation algorithm (PAA) for each D2D group is shown in **Algorithm 2**. The algorithm contains two main steps. The first step is to initialize the power allocation vector \( a_n(0) \) to the \( n \)-th D2D group \( D_n \). The second step is the update step. In the \( i \)-th round of the update step, we set \( \gamma_{n,1}^1 = \gamma_{n,1}^1(i - 1) \) and \( \gamma_{n,2}^2 = \gamma_{n,2}^2(i - 1) \), and subsequently derive the solution \( s_n(i) \) by solving the convex-optimization problem in (35). This process continues until the gaps between the values of \( \gamma_{n,1}^1, \gamma_{n,2}^2 \) in the current round and that in the previous round, respectively, are smaller than the threshold \( \Delta \).

**Algorithm 2** Power Allocation Algorithm for Each D2D Group (PAA)

1: **Initialization Phase:**
2: Set \( i = 0 \).
3: Initialize the power allocation vector \( a_n(0) \) and the maximum number of iterations \( I_{max} \). Calculate \( \gamma_{n,1}^1(0) \) and \( \gamma_{n,2}^2(0) \) based on \( a_n(0) \).
4: Set the convergence threshold \( \Delta \).
5: **Update Phase:**
6: while \( |\gamma_{n,1}^1(i) - \gamma_{n,1}^1(i - 1)| \geq \Delta \) or \( |\gamma_{n,2}^2(i) - \gamma_{n,2}^2(i - 1)| \geq \Delta \)
7: \( \Delta \)
8: \( i = i + 1 \);
9: \( \Delta \)
10: Update \( a_n(i) \), where \( a_n(1(i)) = 2^{s_{n,1}(i)} \) and \( a_n(2(i)) = 2^{s_{n,2}(i)} \).
11: \( \Delta \)
12: Calculate \( \gamma_{n,1}^1(i) \) and \( \gamma_{n,2}^2(i) \) based on \( a_n(i) \).
13: Result: \( a_n^* = a_n(i) \).

**B. Property Analysis of PAA**

In this subsection, we give the analysis on the convergence and the local-optimal property of the proposed power allocation algorithm.

**Theorem 5.** The proposed PAA for power allocation is guaranteed to converge.

**Proof.** Assume that the optimal solution of the convex problem in (35) is \( s_n(i) \) after the \( i \)-th iteration. Set \( a_n(i) = 2^{s_n(i)} \). Then, the following inequalities can be obtained:

\[
R_{n,1}(a_{n,1}(i)) + R_{n,2}(a_{n,2}(i)) = \Theta_{n,1}^{i+1}(2^{s_{n,1}(i)}) + \Theta_{n,2}^{i+1}(2^{s_{n,2}(i)}) < \Theta_{n,1}^{i+1}(2^{s_{n,1}(i+1)}) + \Theta_{n,2}^{i+1}(2^{s_{n,2}(i+1)}) \leq R_{n,1}(a_{n,1}(i + 1)) + R_{n,2}(a_{n,2}(i + 1)),
\]  

(38)

where \( \Theta_{n,1}^{i+1} \) and \( \Theta_{n,2}^{i+1} \) are the expressions of \( \Theta_{n,1} \) and \( \Theta_{n,2} \) during the \( (i + 1) \)-th iteration, respectively. The first equality holds because \( b_1, b_2, c_1 \) and \( c_2 \) are calculated based on \( \gamma_i \), thus the bound is tight; the second inequality holds because \( s_n(i + 1) \) is the optimal solutions of (35) for the \( (i + 1) \)-th iteration; the third inequality holds because \( \Theta_{n,1}^{i+1}(s_{n,1}(i + 1)) \) and \( \Theta_{n,2}^{i+1}(s_{n,2}(i + 1)) \) are the lower bounds of \( R_{n,1}(a_{n,1}(i + 1)) \) and \( R_{n,2}(a_{n,2}(i + 1)) \), respectively. Therefore, from (38), we know that the value of \( (R_{n,1} + R_{n,2}) \) increases after each iteration in PAA. Since the value of \( (R_{n,1} + R_{n,2}) \) is upper bounded due to limited spectrum resources, PAA can finally converge and output the power allocation result \( a_n^* \).

**Theorem 6.** The convergent solution of PAA is a first-order optimal solution of the problem in (24), which satisfies the KKT conditions.

**Proof.** Denote the power allocation indicator at convergence of PAA as \( a_n^* \). Since \( a_n^* \) is the optimal solution of the
concave problem in (35), \( a_n^* \) must satisfy the KKT conditions of (35). Actually, the problems (24) and (35) share the same constraints but have different objective functions with \( (R_{n,1} + R_{n,2}) \) and \( (\Theta_{n,1}(2^{\gamma_{n,1}}) + \Theta_{n,2}(2^{\gamma_{n,2}})) \), respectively. However, when PAA converges, we have \( (R_{n,1} + R_{n,2}) = (\Theta_{n,1}(2^{\gamma_{n,1}}) + \Theta_{n,2}(2^{\gamma_{n,2}})) \). Therefore, \( a_n^* \) also satisfies the KKT conditions of the problem in (24).

C. Joint Subchannel and Power Allocation Algorithm

Based on the two proposed algorithms, i.e., SAA and PAA, it is worth considering how to jointly consider subchannel and power allocation together. In the following, we demonstrate two approaches:

1) Iterative Joint Subchannel and Power Allocation Algorithm (I-JSPA): According to the optimization problem in (15), the subchannel assignment indicator \( \eta \) and the power allocation coefficient \( a \) jointly influence the sum data rate. Therefore, we propose a joint subchannel and power allocation algorithm where the power allocation, i.e., PAA, is executed iteratively after each swap operation in SAA, as shown in Fig. 2(a). In this way, the power allocation coefficient \( a \) can be updated timely after any change of the subchannel assignment indicator \( \eta \), which improves the system performance. However, the shortcoming of this approach is the high complexity, which increases exponentially with the number of swap operations.

2) Low-Complexity Joint Subchannel and Power Allocation Algorithm (LC-JSPA): Because of the high complexity of I-JSPA, we propose an alternative low-complexity approach, which is shown in Fig. 2(b). Without knowing the subchannel assignment result, we can not decide on the SIC order in each D2D group, and thus can not complete the power allocation. Therefore, we first solve the subchannel assignment problem via SAA based on random given initial values of the power allocation coefficients \( a_n, \forall n \). After the convergence of SAA, the BS can allocate power to receivers in each D2D group via PAA.

VI. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we investigate the performance of the proposed joint subchannel and power allocation algorithm through simulations. We give the performance of the joint subchannel and power allocation algorithm in I-JSPA and LC-JSPA, respectively. The performance of the exhaustive search and the one-to-one matching based algorithm are also provided as benchmarks for comparison, in order to show the effectiveness of the proposed algorithm. More particularly, the exhaustive search enables searching for all possible subchannel allocation ways while the power allocation is also performed exhaustively for each given case. Since power is a continuous variable, it is not easy to search for all possible power allocation values. Therefore, we search approximately for the values of \( a_n = \{a_{n,1}, a_{n,2}\} \) with an interval of \( \epsilon \), through which we can obtain the approximately global optimal solution. In the one-to-one matching algorithm, one D2D group can use no more than one subchannel, and one subchannel can only be allocated to one D2D group. We give random pairing between subchannels and cellular users. The specific parameter value settings are summarized in Table II unless otherwise specified.

The performance of the conventional OMA based D2D communications is also illustrated in an effort to demonstrate the potential benefits of the proposed NOMA enhanced D2D scheme. For OMA based D2D communications, the achievable data rates for receiver 1 and 2 in the \( n \)-th D2D group are \( \frac{1}{2} \log_2 \left( 1 + \frac{P_o|f_{n,1}|^2}{I_{n,1} + I_{n,2} + \sigma^2} \right) \) and \( \frac{1}{2} \log_2 \left( 1 + \frac{P_o|f_{n,2}|^2}{I_{n,1} + I_{n,2} + \sigma^2} \right) \), respectively, where \( \frac{1}{2} \) is due to the fact that the time/frequency resource is split among receivers 1 and 2, which is as mentioned in Section II of [12]. The many-to-one matching, one-to-one matching and exhaustive search are also applied to the OMA based D2D scenarios, respectively, with the aim of comparing the performance of the corresponding NOMA enhanced D2D scenarios with.

<table>
<thead>
<tr>
<th>TABLE II: Simulation Parameters</th>
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<tr>
<td>Cellular radius</td>
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<td>Maximum distance between D2D pairs</td>
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<td>Cellular-user SINR threshold</td>
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<tr>
<td>Transmit power of cellular users</td>
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<tr>
<td>Noise power</td>
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<tr>
<td>Path-loss exponent</td>
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<tr>
<td>Number of subchannels</td>
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</table>

Fig. 3 plots the cumulative distribution function (CDF) of the number of swap operations for the matching process, and thus demonstrates the convergence of the proposed subchannel assignment algorithm for different number of D2D groups in the network. The CDF shows that the proposed matching...
algorithm converges within a small number of iterations. For example, when there are 11 D2D groups in the network, on average a maximum of 40 iterations is required to ensure the proposed algorithm to converge. One can also observe that the number of swap operations increases with the increased number of D2D groups, which is due to the improved probability of the existence of swap-blocking pairs.

B. I-JSPA versus LC-JSPA

Fig. 4 investigates the total sum rate versus different D2D transmit signal-to-noise-ratio (SNR). The number of D2D groups is set to $N = 6$. As can be observed, the total sum rate increases with the D2D transmit SNR since the received SINR at the receivers are improved by allocating more power at the transmitters. For comparison, Fig. 4 shows the performance of the fixed power allocation algorithm, where the power allocation coefficients are set to $a_{n,1} = 0.4$, $a_{n,2} = 0.6$. It can be observed that the fixed power allocation algorithm achieves substantially lower sum rate compared to the proposed algorithm. Besides, it also shows that LC-JSPA closely approaches the performance of I-JSPA. As discussed before, since the complexity of I-JSPA increases exponentially with the number of swap operations, we adopt LC-JSPA in the remaining parts of this paper.

C. NOMA-enhanced versus OMA-based D2D Communications

Fig. 5 shows that, the number of accessed D2D groups increases as the number of D2D groups in the network increases. This is because as $N$ increases, the probability of D2D groups with less interference to the cellular UEs being assigned to them increases, which leads to larger number of accessed D2D groups that can meet the SINR constraints of cellular UEs. This phenomenon is similar to the effect of multi-user diversity. It is worth noting that with the increase of the number of D2D groups in the network, the increasing rate of the number of accessed D2D groups becomes smaller due to the enhanced co-channel interference. One can also observe that the number of accessed D2D groups can get saturated quickly in the one-to-one matching algorithm. This is due to the fact that each subchannel can be allocated to no more than one D2D group.

Fig. 6 plots the total sum rate versus different number of D2D groups in the network. One can observe that the sum rate increases with the number of D2D groups, which follows the intuition that more D2D groups contribute to a higher total sum rate. It is also observed that the proposed algorithm achieves much higher sum rate compared to the one-to-one matching algorithm. Meanwhile, the proposed algorithm is capable of reaching around 94.3% of the result of the exhaustive search. Recall the complexity of the proposed algorithm, which is much lower than the exhaustive search, unequivocally substantiates the plausibility of the proposed algorithm. Fig. 6 also demonstrates that the NOMA enhanced D2D scheme achieves larger sum rate than the conventional OMA based D2D scheme, which demonstrates the performance gains of the prior one and provides a good answer to Question 1 in Section I.

Fig. 7 plots the number of accessed receivers versus different number of D2D groups in the network. It can be seen from the figure that the number of accessed receivers
in the proposed algorithm is larger than that in the one-to-one matching algorithm. This is because more than one D2D groups are allowed to be allocated to one subchannel in the proposed algorithm, and thus the resource utilization is improved. It is also noted that the NOMA enhanced D2D communications achieves a larger number of accessed D2D receivers than the OMA based D2D communications, which further shows the merits of applying NOMA transmission protocol in D2D communications, and answers Question 2 in Section I.

D. Impact of Interference Constraints for Cellular Users

Fig. 8(a) shows the number of accessed D2D groups versus different D2D transmit SNR and different SINR constraints of cellular users. It can be observed that the number of accessed D2D groups decreases with higher SINR constraint of the cellular users. This is because the maximum allowed interference for the cellular users gets smaller with the higher SINR constraint, and therefore the number of acceptable D2D groups for each subchannel is decreased. Fig. 8(a) further shows that the number of accessed D2D groups increases with the lower D2D transmit SNR. This is due to the fact that the interference caused to the cellular users and other D2D groups occupying the same subchannels gets smaller with the lower D2D transmit SNR, and thus the acceptable number of D2D groups on each subchannel is increased.

Fig. 8(b) depicts the total sum rate versus different D2D transmit SNR and different SINR constraint of the cellular users. We can see that the total sum rate decreases with the higher SINR constraint of the cellular users. This can be easily understood because of the smaller number of accessed D2D groups, as shown in Fig. 8(a). Besides, it is easy to find that, when the D2D transmit SNR is small, the total sum rate increases with the larger SNR, which is caused by the increased transmit power. When the D2D transmit SNR increases to a certain value, the total sum rate starts to decrease with the higher value of D2D transmit SNR. This is because of the smaller number of accessed D2D groups as shown in Fig. 8(a). Fig. 8(a) and Fig. 8(b) illustrates how the interference constraints of cellular users influence the sum rate and number of accessed D2D groups, which gives an answer to Question 3 in Section I.

VII. CONCLUSIONS

In this paper, the application of non-orthogonal multiple access (NOMA) to the device-to-device (D2D) communications has been studied. With the objective of maximizing network sum rate while satisfying the interference constraints of cellular users, a joint subchannel and power allocation problem was formulated. Since the formulated problem was a mixed-integer non-convex problem, it was decoupled into two subproblems, i.e., subchannel assignment and power allocation problems. A novel algorithm invoking many-to-one matching theory was proposed for tackling the subchannel assignment problem. Based on the subchannel assignment result, the non-convex power allocation problem for receivers in each D2D group was solved by applying the sequential convex programming, which was proved to be convergent. Simulation results showed that the proposed joint subchannel and power allocation algorithm approached close to the exhaustive-searching method. It was also shown that the proposed NOMA enhanced D2D scheme outperformed the conventional OMA based D2D scheme, in terms of both sum rate and number of accessed users.

REFERENCES

(a) Number of accessed D2D groups versus different D2D transmit SNR and CU SINR constraint, with $N = 6$.

(b) Total sum rate versus different D2D transmit SNR and CU SINR constraint, with $N = 6$.

Fig. 8: Performance analysis of the proposed algorithm.


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