Outage Performance of Full/Half-Duplex User Relaying in NOMA Systems

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Outline

➢ Background

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Non-Orthogonal Multiple Access (NOMA) protocol can allow multiple users to share the same resource elements via different power levels.

Half-duplex (HD) NOMA provides performance gains for weak users, but it brings additional slot cost for the systems.

Full-duplex (FD) NOMA can be further realize more spectrally efficient systems.

HD/FD user relaying for NOMA are researched.
System model

➢ System model for the NOMA transmission protocol considering one source (i.e., the base station (BS)) that intends to communicate with the far user $D_2$ under the assistance of the near user $D_1$.

➢ $D_1$ works as a FD/HD decode-and-forward (DF) relaying to help the far user.

➢ Both no direct link and direct link scenarios between the BS and $D_2$ are considered.
Applying NOMA principle, successive interference cancellation (SIC) is employed at $D_1$, the signal-to-interference-plus-noise ratio (SINR) at $D_1$ to detect $D_2$'s message $x_2$ is given by

$$\gamma_{D_1 \to D_2} = \frac{|h_1|^2 a_2 \rho}{|h_1|^2 a_1 \rho + \sigma |h_{LI}|^2 \rho + 1}.$$  \hspace{1cm} (1)

where $\sigma$ denotes the switching operation factor between HD mode and FD mode with $\sigma = 0$ and $\sigma = 1$. $\rho$ is the transmit signal-to-noise radio (SNR).

After SIC, the received SNR at $D_1$ to detect its own message $x_1$ is given by

$$\gamma_{D_1} = \frac{|h_1|^2 a_1 \rho}{\sigma |h_{LI}|^2 \rho + 1}.$$ \hspace{1cm} (2)
For the direct link, the received SINR at $D_2$ to detect $x_2$ is given by

$$\gamma_{1,D_2} = \frac{|h_0|^2 a_2 \rho}{|h_0|^2 a_1 \rho + 1}. \quad (3)$$

For the relaying link, the received SNR can be given by

$$\gamma_{2,D_2} = |h_2|^2 \rho. \quad (4)$$

Assuming that the signals from the relaying link and direct link can be combined by maximal ratio combining (MRC) at $D_2$. The received SINR after MRC at $D_2$ can be given by

$$\gamma_{D_2}^{MRC} = |h_2|^2 \rho + \frac{|h_0|^2 a_2 \rho}{|h_0|^2 a_1 \rho + 1}. \quad (5)$$
Outage probability

➢ User Relaying without Direct Link

For $D_1$, according to the NOMA protocol, the complementary events of outage at $D_1$ can be explained as: $D_1$ can detect $x_2$ as well as its own message $x_1$. From the above description, the outage probability of $D_1$ for FD NOMA can be expressed as below:

$$P_{D_1, nodir}^{FD} = 1 - P_r \left( \gamma_{D_2 \rightarrow D_1} > \gamma_{th_2}^{FD}, \gamma_{D_1} > \gamma_{th_1}^{FD} \right),$$

(6)

where $\sigma = 1, \gamma_{th_1}^{FD} = 2^{R_1} - 1$ with $R_1$ being the target rate at $D_1$ to detect $x_1$ and $\gamma_{th_2}^{FD} = 2^{R_2} - 1$ with $R_2$ being the target rate at $D_1$ to detect $x_2$.

Similar to (6), the outage probability of $D_1$ for HD NOMA is given by

$$P_{D_1, nodir}^{HD} = 1 - P_r \left( \gamma_{D_2 \rightarrow D_1} > \gamma_{th_2}^{HD}, \gamma_{D_1} > \gamma_{th_1}^{HD} \right).$$

(7)

where $\sigma = 0, \gamma_{th_1}^{HD} = 2^{2R_1} - 1$ and $\gamma_{th_2}^{HD} = 2^{2R_2} - 1$ denote the target SNR at $D_1$ to detect $x_1$ and $x_2$, respectively.
Outage probability

➢ User Relaying without Direct Link

For $D_2$, the outage events can be explained for two reasons. The first is that $D_1$, cannot detect $x_2$. The second is that $D_2$ cannot detect its own message $x_2$ on the conditions that $D_1$ can detect $x_2$ successfully. Based on these, the outage probability of $D_2$ for FD NOMA can be expressed as below:

$$P_{D_2,\text{nodir}}^{FD} = P_r \left( \gamma_{D_2 \rightarrow D_1} < \gamma_{th_2}^{FD} \right) + P_r \left( \gamma_{D_2} < \gamma_{th_2}^{FD}, \gamma_{D_2 \rightarrow D_1} > \gamma_{th_2}^{FD} \right), \quad (8)$$

where $\varpi = 1$.

Similar to (8), the outage probability of $D_2$ for HD NOMA is given by

$$P_{D_2,\text{nodir}}^{HD} = P_r \left( \gamma_{D_2 \rightarrow D_1} < \gamma_{th_2}^{HD} \right) + P_r \left( \gamma_{D_2} < \gamma_{th_2}^{HD}, \gamma_{D_2 \rightarrow D_1} > \gamma_{th_2}^{HD} \right), \quad (9)$$

where $\varpi = 0$. 
Outage probability

➢ User Relaying with Direct Link

For $D_1$, the outage probability of $D_1$ will not be affected by the direct link between the BS and $D_2$.

For $D_2$, the outage events can be described below. One of the events is when $x_2$ can be detected at $D_1$, but the received SINR after MRC at $D_2$ in one slot is less than its target SNR. Another event is that neither $D_1$ nor $D_2$ can detect $x_2$. Therefore, the outage probability of $D_2$ for FD NOMA can be expressed as below:

$$P_{D_2,dir}^{FD} = P_r \left( \gamma_{D_2}^{MRC} < \gamma_{th_2}^{FD}, \gamma_{D_2 \rightarrow D_1} > \gamma_{th_2}^{FD} \right) + P_r \left( \gamma_{D_2 \rightarrow D_1} < \gamma_{th_2}^{FD}, \gamma_{1,D_2} < \gamma_{th_2}^{FD} \right), \quad (10)$$

where $\sigma = 1$.

Similar to (10), the outage probability of $D_2$ for HD NOMA is given by

$$P_{D_2,dir}^{HD} = P_r \left( \gamma_{D_2}^{MRC} < \gamma_{th_2}^{HD}, \gamma_{D_2 \rightarrow D_1} > \gamma_{th_2}^{HD} \right) + P_r \left( \gamma_{D_2 \rightarrow D_1} < \gamma_{th_2}^{HD}, \gamma_{1,D_2} < \gamma_{th_2}^{HD} \right), \quad (11)$$

where $\sigma = 0$. 
To obtain more insights, the diversity analysis is provided in terms of the outage probability investigated in the high SNR region.

\[ d = - \lim_{\rho \to \infty} \frac{\log(P_D^\infty(\rho))}{\log \rho}, \]  \hspace{1cm} (12)

where \( P_D^\infty(\rho) \) is the asymptotic outage probability of users.

- **User Relaying without Direct Link**

  **Remark 1:** The diversity order of \( D_1 \) and \( D_2 \) for FD NOMA is zero, which is the same as the conventional FD relaying. However, the diversity order of \( D_1 \) and \( D_2 \) for HD NOMA is one.

  **Remark 2:** The error floors of two users are existent at high SNR region for FD NOMA without direct link.
Diversity analysis

➢ User Relaying with Direct Link

For D₂, since the direct link exists between the BS and D₂, the diversity order of D₂ for FD NOMA is one. However, the diversity order of D₂ for HD NOMA is two.

**Remark 3:** From the above explanation, the observation is that the direct link (BS → D₂) to convey information is an effective way to overcome the problem of zero diversity order for D₂.

➢ Throughput Analysis

The delay-limited transmission mode is considered for FD/HD NOMA. On the basis of (6), (7), (8), (9), (10) and (11), the system sum throughput of FD/HD NOMA without/with direct link can be given by

\[ R_\Phi^{FD} = \left(1 - P_{D_1,\Phi}^{FD}\right) R_1 + \left(1 - P_{D_2,\Phi}^{FD}\right) R_2, \]  

(13)

and

\[ R_\Phi^{HD} = \left(1 - P_{D_1,\Phi}^{HD}\right) R_1 + \left(1 - P_{D_2,\Phi}^{HD}\right) R_2, \]  

(14)

respectively, where \( \Phi \in \{dir, nodir\} \) denote the direct/nodirect scenarios.
Numerical Results

➢ User Relaying without Direct Link

➢ As can be observed that the performance of FD NOMA exceeds the HD NOMA and OMA on the condition of low SNR region. This is because loop interference is not the dominant impact factor for FD NOMA in low SNR region.

➢ It is shown that error floors exist in FD NOMA, which verify the conclusion in Remark 2 and obtain the zero diversity order.

Fig. 1 Outage probability versus transmit SNR without direct link.
Numerical Results

➢ User Relaying with Direct Link

➢ We observe that $D_2$ obtains one diversity order by using the direct link, which overcomes the problem of zero diversity order inherent to FD cooperative system.

➢ It is observed that the superiority of FD NOMA is no longer apparent with the values of LI increasing.

Fig. 2 Outage probability versus transmit SNR for different values of LI with direct link
This paper has investigated FD/HD user relaying in cooperative NOMA systems and two cooperative relaying scenarios have been considered insightfully.

Due to the influence of residual loop interference, the diversity orders achieved by two user were zero for FD NOMA.

The direct link between the BS and the far user was utilized to convey the information and one diversity order was obtained for the far user.

It was shown that FD NOMA was superior to HD NOMA at low SNR region rather than at high SNR region.

Furthermore, the superior of FD NOMA was not apparent with the loop interference value increasing.
Thank you!