Modeling and Analysis of NOMA Enabled CRAN with Cluster Point Process

Francisco J. Martin-Vega*, Yuanwei Liu+, Gerardo Gomez*, Mari Carmen Aguayo* and Maged Elkashan+

*Dpt. Communication Engineering, University of Málaga, Spain

+School of Electronic Engineering and Computer Science, Queen Mary University of London, UK

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1. Introduction
2. Proposed Scheme
3. System Model
4. Performance Analysis
5. Numerical Results
6. Conclusions

Introduction

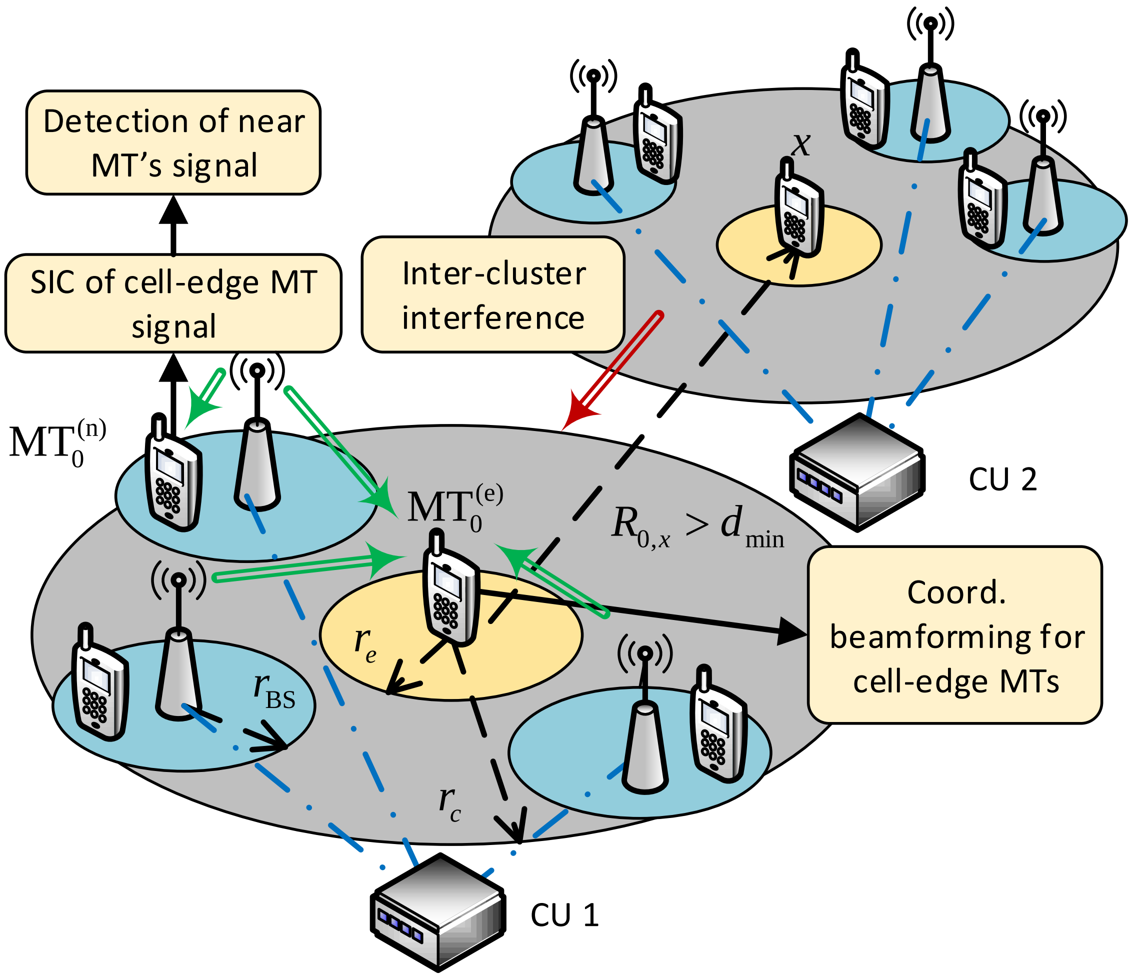
Cloud Radio Access Networks (CRANs) are composed of cheap Radio Remote Heads (RRHs) that establish wireless connections with Mobile Terminals (MTs); Central Units (CUs) that centralizes baseband processing for a cluster of RRHs; and a fronthaul network that allow fast connectivity between RHHs and CUs.

Coordinated BF aims at improving the performance of a single user who is served from a cluster of cooperating BSs.

NOMA it allows to achieve a higher sum spectral efficiency by scheduling several non-orthogonal transmissions in the same resource block

Outline

Proposed Scheme



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System Model (I)

Spatial Modeling: locations of BSs are modelled according to a Cluster Point Process (CPP). The parent point process is a Matérn Hard Core Point Process of Type II, $\tilde{\Phi}_C$, whose density is λ_C and its minimum distance is $d_{\min}$.

The daughter point process is represented as $\Phi_{\mathrm{BS}}^{(x)} = \Phi_{\mathrm{BS}}^{(o)} + x$, where *o* = *(0,0)* represents the origin and $\Phi_{\mathrm{BS}}^{(o)}$ follows a binomial point process (BPP) of n_{BS} points inside a ring whose inner radius is *re* and outer radius is *rc*. Cells are modelled as disk whose radius is r_{BS}.

Signal Modelling:

1. The path gain includes path loss (bounded model) and Rayleigh fading: $G_{x,y} = \dfrac{H_{x,y}}{\sqrt{1+R_{x,y}^{\alpha}}}$
2. Transmitted signal:

$$X_{\mathrm{BS}_{i,q}} = \sqrt{p_{\mathrm{BS}} a_n} \cdot S_{\mathrm{MT}_{i,q}^{(n)}} + \sqrt{p_{\mathrm{BS}} a_e} \cdot W_{\mathrm{BS}_{i,q},\mathrm{MT}_q^{(e)}} \cdot S_{\mathrm{MT}_q^{(e)}}$$

System Model (II)

Weights: $W_{x,y} = G_{x,y}^{*} \Xi_{y}^{-1/2}$

BF gain: $\Xi_{\mathrm{MT}_0^{(e)}} = \sum_{\mathrm{BS}_{k,o} \in \Phi_{\mathrm{BS}}^{(o)}} \left| G_{\mathrm{BS}_{k,o}, \mathrm{MT}_0^{(e)}} \right|^2$

Cell-edge SINR:

$$\mathrm{SINR}_{\mathrm{MT}_0^{(e)}} = \frac{\Xi_{\mathrm{MT}_0^{(e)}} a_e \rho_{\mathrm{BS}}}{\Xi_{\mathrm{MT}_0^{(e)}} a_n \rho_{\mathrm{BS}} + I_{\mathrm{inter}} \rho_{\mathrm{BS}} + 1}$$

Assumption 1: The locations cluster centers can be approximated by a thinned version of a PPP with the same density as the HCPP.

$$I_{\mathrm{inter}} = \sum_{C_q \in \tilde{\Phi}_C \setminus \{C_0\}} \sum_{\mathrm{BS}_{k,q} \in \Phi_{\mathrm{BS}}^{(C_q)}} \frac{\left| H_{\mathrm{BS}_{k,q}, \mathrm{MT}_0^{(e)}} \right|^2}{1 + R^{\alpha}_{\mathrm{BS}_{k,q}, \mathrm{MT}_0^{(e)}}} \times \left(a_n + a_e \left| W_{\mathrm{BS}_{k,q}, \mathrm{MT}_q^{(e)}} \right|^2 \right)$$

$$I_{\mathrm{inter}}\left(\mathrm{MT}_0^{(e)}\right) = I_{\mathrm{inter}} = \sum_{C_q \in \Phi_C \setminus \{C_0\}} \sum_{\mathrm{BS}_{k,q} \in \Phi_{\mathrm{BS}}^{(C_q)}} \frac{\left| H_{\mathrm{BS}_{k,q}, \mathrm{MT}_0^{(e)}} \right|^2}{1 + R^{\alpha}_{\mathrm{BS}_{k,q}, \mathrm{MT}_0^{(e)}}} \times \left(a_n + a_e \left| W_{\mathrm{BS}_{k,q}, \mathrm{MT}_q^{(e)}} \right|^2 \right) \mathbf{1}\left(\left\| C_q \right\| > d_{\min} \right)$$

System Model (III)

Near MTs. Successive Interference Cancellation:

1) Near MTs decode message intended for cell-edge UE:

$$\mathrm{SINR}_{\mathrm{MT}_0^{(n)}}^{(e)} = \frac{\left|G_{\mathrm{BS}_{i,0},\mathrm{MT}_{i,0}^{(n)}}\right|^2 \left|W_{\mathrm{BS}_{i,0},\mathrm{MT}_0^{(e)}}\right|^2 a_e \rho_{\mathrm{BS}}}{\left|G_{\mathrm{BS}_{i,0},\mathrm{MT}_{i,0}^{(n)}}\right|^2 a_n \rho_{\mathrm{BS}} + I_{\mathrm{intra}} \rho_{\mathrm{BS}} + I_{\mathrm{inter}} \rho_{\mathrm{BS}} + 1}$$

2) Near MTs cancel-out the interference of the transmission intended for cell-edge MT and decode their own signal:

$$\mathrm{SINR}_{\mathrm{MT}_{0,0}^{(n)}}^{(n)} = \frac{\left|G_{\mathrm{BS}_{k,0},\mathrm{MT}_{0,0}^{(n)}}\right|^2 a_n \rho_{\mathrm{BS}}}{I_{\mathrm{intra}}\left(\mathrm{MT}_0^{(n)}\right)\rho_{\mathrm{BS}} + I_{\mathrm{inter}}\left(\mathrm{MT}_0^{(n)}\right)\rho_{\mathrm{BS}} + 1}$$

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Performance Analysis (I)

Lemma 1: PDF of the distance, U, between a point placed at the origin and a randomly chosen BS that belongs to the q-th cluster, which is centered at *Cq*, is expressed as follows:

$$f_U\left(r \mid r_x\right) = \frac{g\left(u, r_c, r\right) - g\left(u, r_e, r\right)}{\pi\left(r_c^2 - r_e^2\right)}$$

$$g\left(u, r_d, d\right) = \begin{cases} 2\pi u, & \text{if } u \le r_d - d \\ \left(\dfrac{\left(d^2 - r_d^{\,2} - u^2\right)\cdot d \cdot u + r \cdot \left(d^2 - u^2 + r_d^2\right)}{\sqrt{(d+u-r_d)(d-u+r_d)(-d+u+r_d)(d+r_d+u)}} \right. \\ \left. + \dfrac{r_d u}{d\sqrt{1 - \dfrac{(d^2 - u^2 + r_d^{\,2})^2}{4d^2 r_d^{\,2}}}} + 2r \cdot \operatorname{arcsec}\left(\dfrac{2du}{d^2 + u^2 - r_d^{\,2}}\right)\right), & \text{if } \left|r_d - d\right| < u < d + r_d \\ 0, & \text{otherwise} \end{cases}$$

Performance Analysis (II)

Lemma 2: The PDF of the beamforming gain for the cell-edge MT can be expressed as a finite mixture of Erlang distributions as follows:

$$f_{\Xi}(\xi) \approx \sum_{q=1}^{n_{\mathrm{M}}} \sum_{i=1}^{n_{\mathrm{GC}}} \sum_{k=1}^{t_{q,i}} \omega_{q,i,k} f_{\mathrm{Er}}(\xi; k, \lambda) \qquad \sum_{q=1}^{n_{\mathrm{M}}} \sum_{i=1}^{n_{\mathrm{GC}}} \sum_{k=1}^{t_{q,i}} \omega_{q,i,k} = 1;$$

Proof:

$$\mathcal{L}_{\Xi}(s) = \mathbb{E} \prod_{k\in[1,n_{\mathrm{BS}}]} \exp\left(-s\Xi_{\mathrm{BS}_k^{(x)},x}\right) = \prod_{k\in[1,n_{\mathrm{BS}}]} \mathbb{E}_R \mathbb{E}_{|H|^2} \exp\left(-s\frac{\left|H_{\mathrm{BS}_k^{(x)},x}\right|^2}{1+R^{\alpha}_{\mathrm{BS}_k^{(x)},x}}\right) \overset{(a)}{=} \left(\int_{r_e\le r\le r_c} \frac{2r}{r_c^2-r_e^2}\frac{1+r^{\alpha}}{1+s+r^{\alpha}}\mathrm{d}r\right)^{n_{\mathrm{BS}}}$$

$$\mathcal{L}_{\Xi}(s) \approx \left(\frac{\pi(r_c-r_e)}{n_{\mathrm{GC}}\left(r_c^2-r_e^2\right)}\sum_{i=1}^{n_{\mathrm{GC}}}\frac{x_i\left(1+x_i^{\alpha}\right)}{1+x_i^{\alpha}+s}\left|\sin\left(\frac{2i-1}{2n_{\mathrm{GC}}}\pi\right)\right|\right)^{n_{\mathrm{BS}}}$$

$$\overset{(a)}{=} \left(\frac{\pi}{n_{\mathrm{GC}}(r_c+r_e)}\right)^{n_{\mathrm{BS}}} \sum_{t_1+t_2+\cdots+t_{n_{\mathrm{GC}}}=n_{\mathrm{BS}}} \frac{n_{\mathrm{BS}}!}{t_1!\cdots t_{n_{\mathrm{GC}}}!} \prod_{i=1}^{n_{\mathrm{BS}}}\left(\frac{x_i\left(1+x_i^{\alpha}\right)}{1+x_i^{\alpha}+s}\left|\sin\left(\frac{2i-1}{2n_{\mathrm{GC}}}\pi\right)\right|\right)^{t_i}$$

$$f_{\Xi}(\xi) \approx \left(\frac{\pi}{n_{\mathrm{GC}}(r_c+r_e)}\right)^{n_{\mathrm{BS}}} \sum_{t_1+t_2+\cdots+t_{n_{\mathrm{GC}}}=n_{\mathrm{BS}}} \frac{n_{\mathrm{BS}}!}{t_1!\cdots t_{n_{\mathrm{GC}}}!} \sum_{i=1}^{n_{\mathrm{GC}}}\sum_{k=1}^{t_i}\frac{\beta_k}{(k-1)!}\xi^{k-1}\exp\left(-\xi\left(1+x_i^{\alpha}\right)\right)$$

Performance Analysis (III)

Lemma 3: The mean and variance of the beamforming gain appear below

$$\mathbb{E}[\Xi_x] = -n_{\mathrm{BS}}\mu_{\Xi}^{(1)}(0) \qquad \mathrm{var}(\Xi_x) = n_{\mathrm{BS}}\left(\mu_{\Xi}^{(2)}(0) - \left(\mu_{\Xi}^{(1)}(0)\right)^2\right)$$

Remark 1: the mean and the variance, of the beamforming gain are linear functions with respect to n_{BS}. Additionally, its slope is always positive and only depends on r_c, r_e and α. Hence, both metrics are monotonically increasing functions with respect to n_{BS}.

Lemma 4: The Laplace transform of the normalized transmit power for a randomly chosen BS can be written as follows

$$\mathcal{L}_{P_{\mathrm{BS}}}(s) \approx \mathrm{e}^{-sa_n} \int_{v=r_e}^{r_c} \frac{2v}{r_c^2 - r_e^2} \mathcal{L}_{\Xi^{-1}}\left(\frac{a_e}{1+v^{\alpha}} s\right) \mathrm{d}v \qquad P_{\mathrm{BS}_{i,q}} = \left(a_n + a_e \frac{\left|H_{\mathrm{BS}_{i,q},\mathrm{MT}_q^{(e)}}\right|^2}{1 + R^{\alpha}_{\mathrm{BS}_{i,q},\mathrm{MT}_q^{(e)}}} \frac{1}{\Xi_{\mathrm{MT}_q^{(e)}}}\right)$$

Performance Analysis (IV)

Proposition: The transmit power per BS can be modelled as a Gamma RV, hence, its Laplace transform is simplified as follows

$$\mathcal{L}_{P_{\mathrm{BS}}}\left(s;k_P,\lambda_P\right)\approx\left(1+s\lambda_P^{-1}\right)^{-k_P};\quad k_P=\frac{\left(\mathbb{E}\left[P_{\mathrm{BS}}\right]\right)^2}{\mathrm{var}\left(P_{\mathrm{BS}}\right)};\quad \lambda_P=\frac{\mathbb{E}\left[P_{\mathrm{BS}}\right]}{\mathrm{var}\left(P_{\mathrm{BS}}\right)}$$

Lemma 5: The Laplace transform of the inter-cluster interference can be approximated as

$$\mathcal{L}_{I_{\mathrm{inter}}}\left(s\right)\approx\exp\left(-2\pi\lambda_c\int_{r>d_{\min}}\left(1-\left(1+\frac{s}{\lambda_P\left(1+r^{\alpha}\right)}\right)^{-k_P n_{\mathrm{BS}}}\right)r\mathrm{d}r\right)$$

Performance Analysis (V)

Theorem 1: The outage probability of the typical cell-edge MT can be approximated as appears below

$$P_{\text{out}}^{\text{MT}_0^{(\text{e})}} \approx 1-\sum_{q=1}^{n_{\text{M}}}\sum_{i=1}^{n_{\text{GC}}}\sum_{k=1}^{t_i}\sum_{n=0}^{k-1}\omega_{q,i,k}\frac{\left(1+x_i^{\alpha}\right)^n}{n!}\nu_1^n\left(-1\right)^n\frac{\text{d}^n}{\text{d}s^n}\left[\text{e}^{-s\rho_{\text{BS}}^{-1}}\mathcal{L}_{I_{\text{inter}}}\left(s\right)\right]\Bigg|_{s=\nu_1\left(1+x_i^{\alpha}\right)}$$

Proof:

$$P_{\text{out}}^{\text{MT}_0^{(\text{e})}} = \Pr\left(\frac{\Xi_{\text{MT}_0^{(e)}}a_e\rho_{\text{BS}}}{\Xi_{\text{MT}_0^{(e)}}a_n\rho_{\text{BS}}+I_{\text{inter}}\rho_{\text{BS}}+1}\leq\gamma_e\right)=\mathbb{E}_I\left[F_{\Xi_{\text{MT}_0^{(e)}}}\left(\nu_1\left(I+\frac{1}{\rho_{\text{BS}}}\right)\right)\right]$$

$$\approx 1-\sum_{q=1}^{n_{\text{M}}}\sum_{i=1}^{n_{\text{GC}}}\sum_{k=1}^{t_i}\sum_{n=0}^{k-1}\omega_{q,i,k}\frac{\left(1+x_i^{\alpha}\right)^n}{n!}\nu_1^n\mathbb{E}_I\left[\left(I+\rho_{\text{BS}}^{-1}\right)^n e^{-\left(1+x_i^{\alpha}\right)\nu_1\left(I+\rho_{\text{BS}}^{-1}\right)}\right];$$

$$\mathbb{E}_X\left[X^n\text{e}^{-sX}\right]=\left(-1\right)^n\frac{\text{d}^n}{\text{d}s^n}\mathcal{L}_X\left(s\right); \qquad \nu_1=\frac{\gamma_e}{a_e-\gamma_e a_n}$$

Performance Analysis (VI)

Theorem 2: The outage probability of the typical near MT can be approximated as appears below

$$P_{\text{out}}^{\mathrm{MT}_o^{(n)}} \approx F_{|W|^2}\left(\frac{\gamma_1 a_n}{a_e}\right) + \frac{\pi\left(r_c - r_e\right)}{2n_{\mathrm{GC}}} \sum_{i=1}^{n_{\mathrm{GC}}} \frac{2r_i}{r_c^2 - r_e^2} \times \mathcal{L}_{\Xi}\left(\frac{\gamma_1 a_n}{a_e}\left(1+r_i^{\alpha}\right)\right) \left|\sin\left(\frac{2i-1}{n_{\mathrm{GC}}}\pi\right)\right|$$

$$\times F_Q\left(\max\left(\frac{\gamma_e}{\left(\frac{a_e}{1+r_i^{\alpha}} - \gamma_e a_n\right)\rho_{\mathrm{BS}}}, \frac{\gamma_n}{a_e \rho_{\mathrm{BS}}}\right) \middle| r_i\right);$$

$$Q_{\mathrm{MT}_0^{(n)}} = \frac{\left|G_{\mathrm{BS}_0^{(0)},\mathrm{MT}_0^{(n)}}\right|^2}{I\rho_{\mathrm{BS}} + 1}$$

$$F_{Q_{\mathrm{MT}_0^{(n)}}|r}\left(q\right) = 1 - \frac{\pi r_{\mathrm{BS}}}{2n_{\mathrm{GC}}} \sum_{i=1}^{n_{\mathrm{GC}}} \frac{2v_i}{r_{\mathrm{BS}}^2} \mathrm{e}^{-q\left(1+v_i^{\alpha}\right)} \left|\sin\left(\frac{2i-1}{n_{\mathrm{GC}}}\pi\right)\right|$$

$$\mathcal{L}_{I_{\text{intra}}|r}\left(q\left(1+v_i^{\alpha}\right)\rho_{\mathrm{BS}}\right) \mathcal{L}_{I_{I\text{inter}}}\left(q\left(1+v_i^{\alpha}\right)\rho_{\mathrm{BS}}\right)$$

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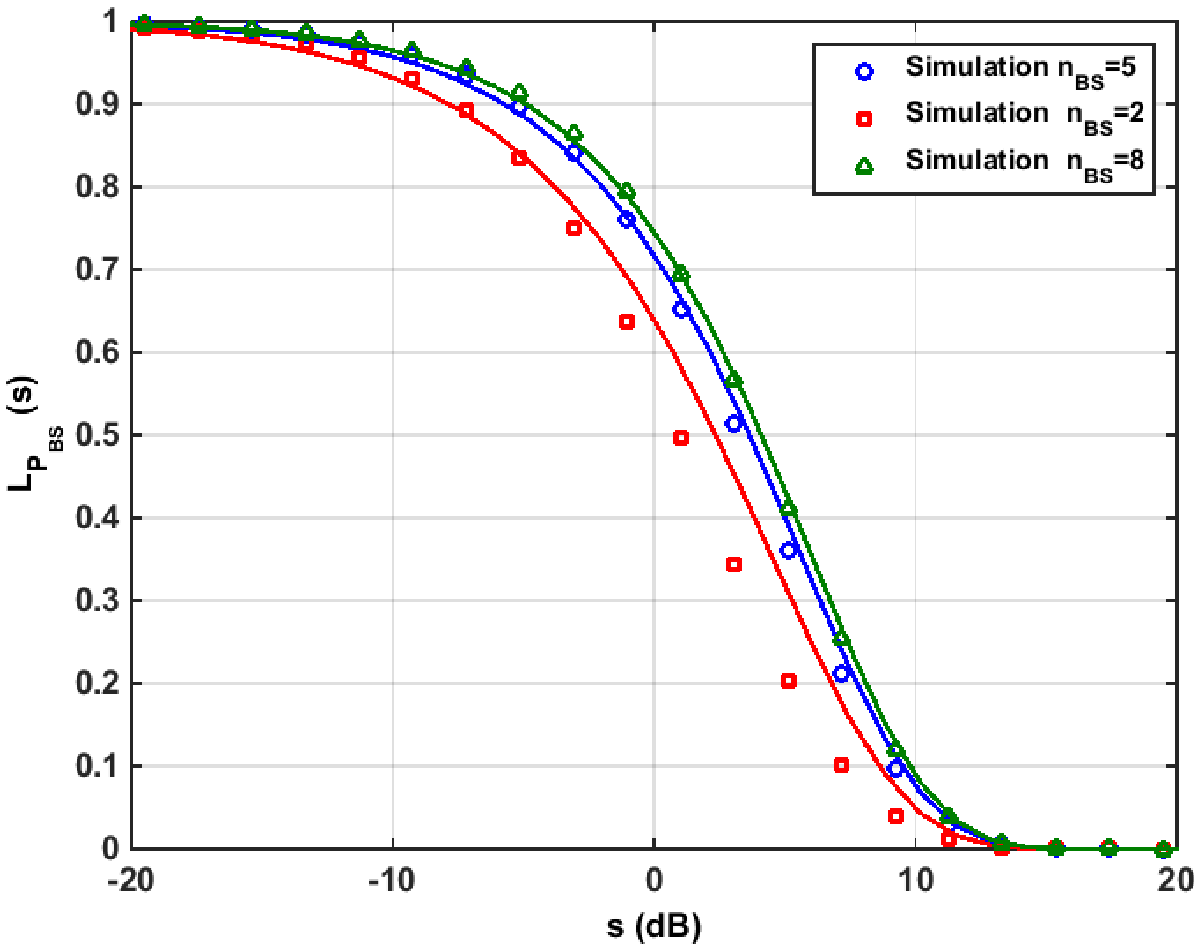
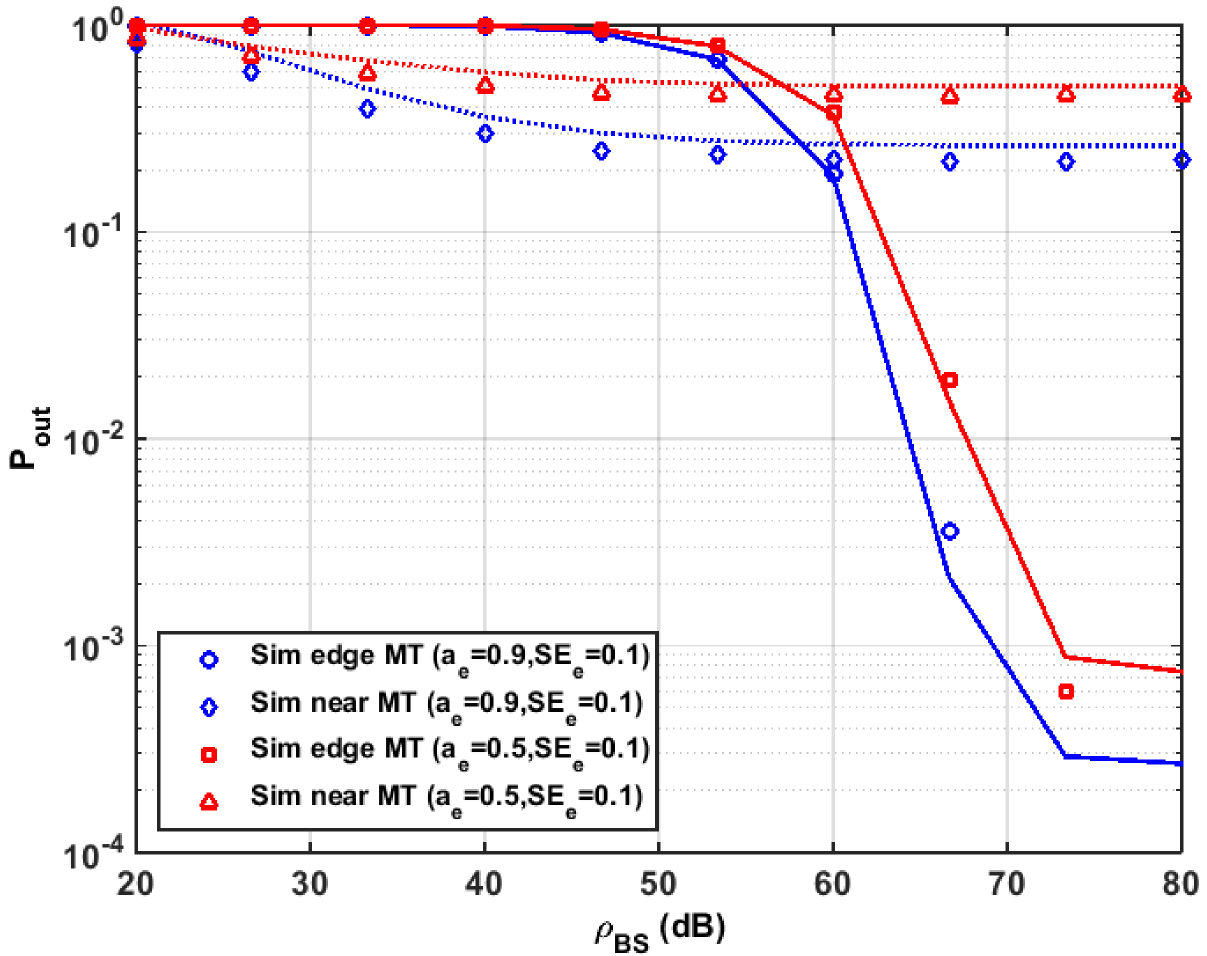
Numerical Results (I)



Numerical Results (II)



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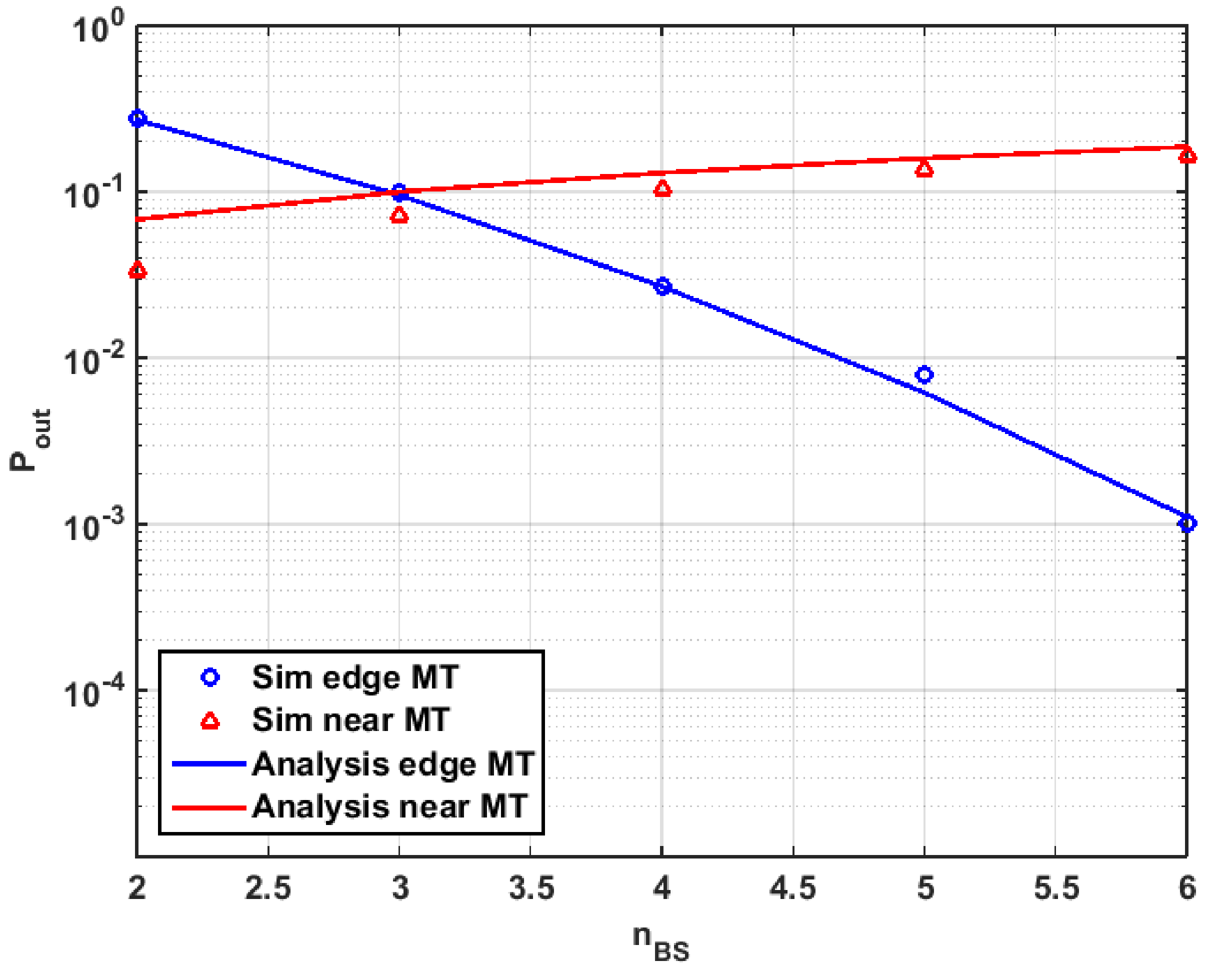

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Conclusions

1. It has been demonstrated that the beamforming gain can be expressed as a mixture Erlang distribution.
2. It has been proven that the moments of the beamforming gain are linear functions with respect to the number of BS per cluster.
3. It has been derived simple expressions for the outage probability of cell-edge MTs with the aid of the k−th derivative of the Lapalce transform of the inter-cluster interference.
4. It has been shown that the proposed scheme greatly improves the performance of the cell-edge MT.
5. As a future line a OMA alternative will be proposed and compared with this NOMA scheme in terms of sum rate, rate per user and fairness.

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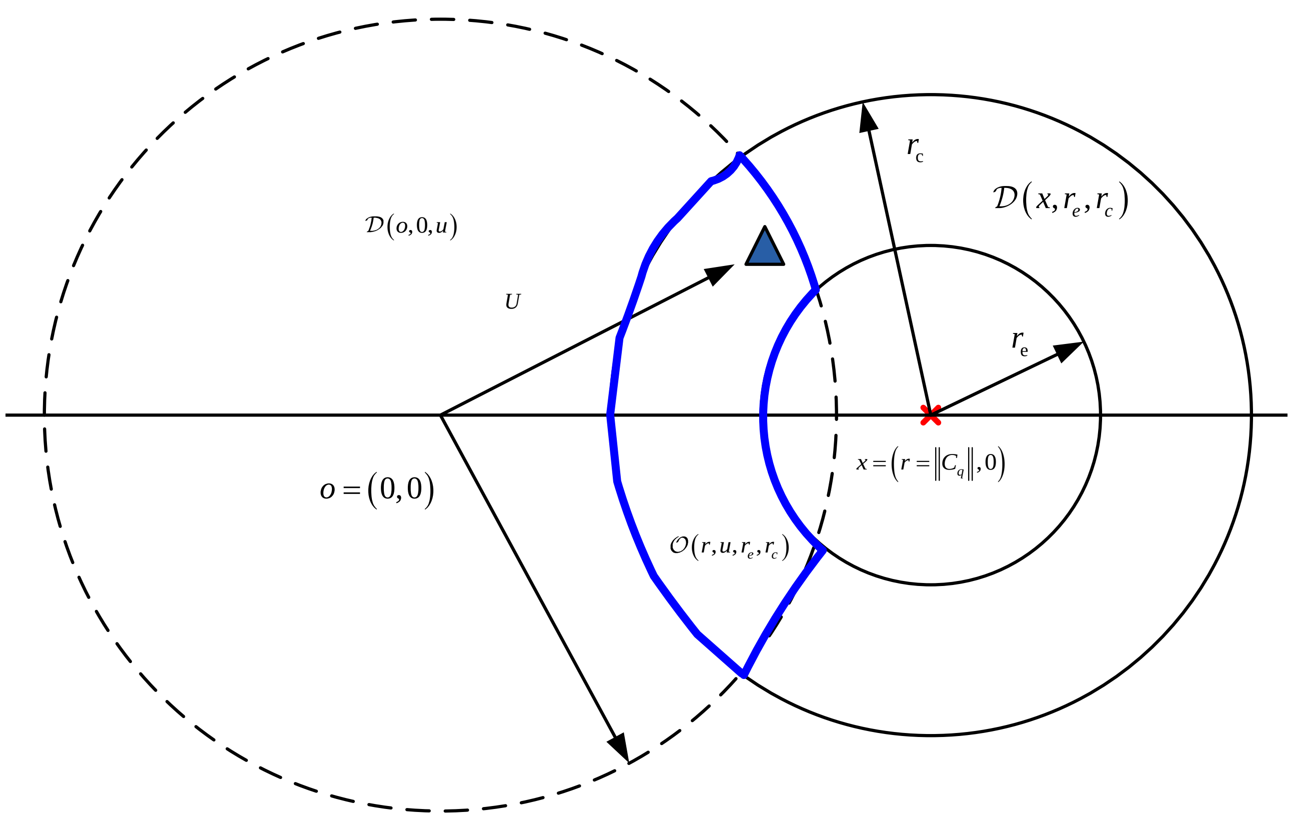
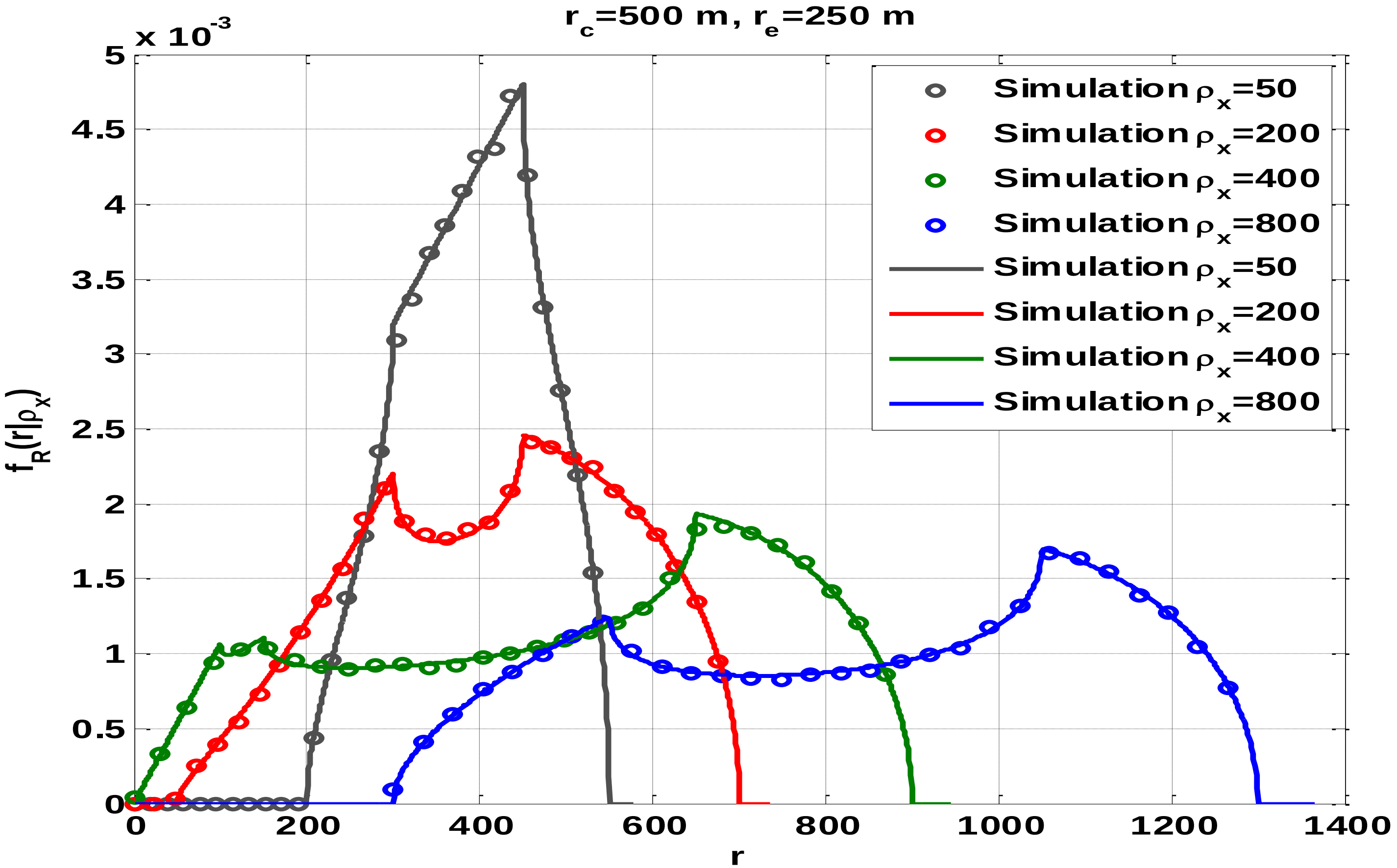
Performance Analysis (II)

Proof: $F_U\left(u \mid C_q\right) = \Pr\left(\mathrm{BS}_{j,q} \in \mathcal{D}(o,0,u) \mid C_q\right) \overset{(a)}{=} \dfrac{\left|\mathcal{D}(o,0,u) \bigcap \mathcal{D}(x,r_e,r_c)\right|}{\left|\mathcal{D}(x,r_e,r_c)\right|} \overset{(b)}{=} \dfrac{\left|\mathcal{O}(r,u,r_e,r_c)\right|}{\pi\left(r_c^2 - r_e^2\right)}$



Performance Analysis (III)



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