

Proofs of some pointer programs

Part 3: Hoare logic and list reversal

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3.1. Hoare logic

3.1.1. Rules

RULES allows concise definition of anonymous alternatives.

```
RULES "assign" (OBJECT xx) ARE
  FROM Q→R«f⊕A→E/xx» INFER {Q} (A.f:=E) {R«f/xx»}
  AND FROM Q→R«E/xx» INFER {Q} (v:=E) {R«v/xx»}
END
```

```
RULE "skip" IS INFER {Q} skip {Q}
```

```
RULE semicolon(Q) IS FROM {P}P1{Q} AND {Q}P2{R} INFER {P}(P1;P2){R}
```

```
RULE "while-do-od"(P, OBJECT x, OBJECT vt) WHERE FRESH vt IS
  FROM Q→ P AND {P∧B} S {P} AND P∧¬B→R AND P∧B→(∃x:(T>x)) AND {P∧B∧vt=T} S {T<vt}
  INFER { Q } while B do S od { R}
```

```
RULE "if-then-else-fi" IS FROM {Q∧B}S1{R} AND {Q∧¬B}S2{R} INFER {Q}if B then S1 else S2 fi{R}
```

The normal rule for postconditions (the precondition rule is built into the assignment rule, which is convenient in the proofs I've done but wouldn't survive a more careful treatment).

```
RULE "strengthen postcondition" IS FROM {Q}S{R'} AND R'→R INFER {Q}S{R}
```

Semicolon is associative.

```
RULE "(A;B);C≡A;(B;C)" IS A;B;C ≡ A;(B;C)
```

The universal get-out for arithmetical dead ends. I don't think I used it, but in case I did it's recorded here.

```
RULE "****assert****" IS INFER P
```

3.1.2. Tactics

Assignment, as defined in the assign step above, sometimes requires an implication step and sometimes doesn't. This tactic deals with the alternatives. It also eliminates mappings

```
TACTIC "!=" IS
  (LETGOALPATH G
    (ALT assign (Fail "not an assignment statement"))
    (elimsimplemaps SKIP)
    (LAYOUT HIDEROOT "→-|")
    (LETGOALPATH G1
      (ALT (SEQ hyp (GOALPATH G) (LAYOUT "!=" ()) (GOALPATH G1) NEXTGOAL)
        (SEQ (GOALPATH G )(LAYOUT "!=") (GOALPATH G1))))))
```

The elimsimplemaps tactic eliminates trivial maps, which otherwise is a tedious necessity following many assignment steps.

```
TACTIC elimsimplemaps(fail) IS
  iterateL2R "rewrite=" "symmetric=" (QUOTE (_A.(_B@_A→_E))) "A.(B@A→E)=E" fail
```

3.2. List reversal

At last a proof of a program.

3.2.1. Global variables

```
INITIALISE foldformulae true
INITIALISE hideuselesscuts true
INITIALISE reasonstyle short
INITIALISE boxlinedisplay left
```

Foldformulae true folds long formulae. Hideuselesscuts true does something to the display of some cut steps which I've forgotten ... Reasonstyle short strips phrases like "Derived Rule" "Conjectured Rule" and so on. Boxlinedisplay left puts reasons on the left (forgotten I'd done that: now I've seen it again, I think the result looks rather nice).

3.2.2. Rules

As I apologised above, I simulate formula-abbreviations by using constant names PP, QQ, RR and TT. SS is a genuine constant, the sequence of pointers given as input (which is independent of the heap, of course).

```
RULE "QQ ≐ ..." IS QQ ≐ list(p⇒tl⇒nil) ∧ p⇒tl⇒nil = SS
RULE "PP ≐ ..." IS PP ≐ list(r⇒tl⇒nil) ∧ list(p⇒tl⇒nil) ∧ r⇒tl⇒nil¬∩p⇒tl⇒nil ∧
  rev (r⇒tl⇒nil) @ p⇒tl⇒nil = rev SS
RULE "RR ≐ ..." IS RR ≐ list(p⇒tl⇒nil) ∧ p⇒tl⇒nil = rev SS
RULE "TT ≐ ..." IS TT ≐ length(r⇒tl⇒nil)
```

3.2.3. Proofs

THEOREM IS {QQ} (r:=p; p := nil) {PP}

assumption	1:	QQ
(1) QQ \triangleq ...	2:	list(p \Rightarrow tl \Rightarrow nil) \wedge p \Rightarrow tl \Rightarrow nil=SS
(2) \wedge -E	3:	list(p \Rightarrow tl \Rightarrow nil)
(2) \wedge -E	4:	p \Rightarrow tl \Rightarrow nil=SS
S \neg ()	5:	p \Rightarrow tl \Rightarrow nil \neg ()
S@() \triangleq S	6:	rev(p \Rightarrow tl \Rightarrow nil)@() \triangleq rev(p \Rightarrow tl \Rightarrow nil)
reflexive=	7:	rev SS=rev SS
(4,7) rewrite=	8:	rev(p \Rightarrow tl \Rightarrow nil)=rev SS
(6,8) S@() \triangleq S	9:	rev(p \Rightarrow tl \Rightarrow nil)@() \triangleq rev SS
A \Rightarrow E \Rightarrow A=()	10:	nil \Rightarrow tl \Rightarrow nil=()
(3) \wedge -I	11:	list(p \Rightarrow tl \Rightarrow nil) \wedge list()
(11,5) \wedge -I	12:	list(p \Rightarrow tl \Rightarrow nil) \wedge list() \wedge p \Rightarrow tl \Rightarrow nil \neg ()
(12,9) \wedge -I	13:	list(p \Rightarrow tl \Rightarrow nil) \wedge list() \wedge p \Rightarrow tl \Rightarrow nil \neg () \wedge rev(p \Rightarrow tl \Rightarrow nil)@() \triangleq rev SS
(10,13) A \Rightarrow E \Rightarrow A=()	14:	list(p \Rightarrow tl \Rightarrow nil) \wedge list(nil \Rightarrow tl \Rightarrow nil) \wedge p \Rightarrow tl \Rightarrow nil \neg nil \Rightarrow tl \Rightarrow nil \wedge rev(p \Rightarrow tl \Rightarrow nil)@nil \Rightarrow tl \Rightarrow nil=rev SS
(1-14) \Rightarrow	15:	{QQ}(r:=p){list(r \Rightarrow tl \Rightarrow nil) \wedge list(nil \Rightarrow tl \Rightarrow nil) \wedge r \Rightarrow tl \Rightarrow nil \neg nil \Rightarrow tl \Rightarrow nil \wedge rev(r \Rightarrow tl \Rightarrow nil)@nil \Rightarrow tl \Rightarrow nil=rev SS}
\Rightarrow	16:	{list(r \Rightarrow tl \Rightarrow nil) \wedge list(nil \Rightarrow tl \Rightarrow nil) \wedge r \Rightarrow tl \Rightarrow nil \neg nil \Rightarrow tl \Rightarrow nil \wedge rev(r \Rightarrow tl \Rightarrow nil)@nil \Rightarrow tl \Rightarrow nil=rev SS}(p:=nil){list(r \Rightarrow tl \Rightarrow nil) \wedge list(p \Rightarrow tl \Rightarrow nil) \wedge r \Rightarrow tl \Rightarrow nil \neg pp \Rightarrow tl \Rightarrow nil \wedge rev(r \Rightarrow tl \Rightarrow nil)@p \Rightarrow tl \Rightarrow nil=rev SS}
(16) PP \triangleq ...	17:	{list(r \Rightarrow tl \Rightarrow nil) \wedge list(nil \Rightarrow tl \Rightarrow nil) \wedge r \Rightarrow tl \Rightarrow nil \neg nil \Rightarrow tl \Rightarrow nil \wedge rev(r \Rightarrow tl \Rightarrow nil)@nil \Rightarrow tl \Rightarrow nil=rev SS}(p:=nil){PP}
(15,17) sequence	18:	{QQ}(r:=p;p:=nil){PP}

FORMULAE 0 list(r \Rightarrow tl \Rightarrow nil) \wedge list(nil \Rightarrow tl \Rightarrow nil) \wedge r \Rightarrow tl \Rightarrow nil \neg nil \Rightarrow tl \Rightarrow nil \wedge rev(r \Rightarrow tl \Rightarrow nil)@nil \Rightarrow tl \Rightarrow nil=rev SS, 1 xx1, 2 nil,
3 list(r \Rightarrow tl \Rightarrow nil) \wedge list(xx1 \Rightarrow tl \Rightarrow nil) \wedge r \Rightarrow tl \Rightarrow nil \neg xx1 \Rightarrow tl \Rightarrow nil \wedge rev(r \Rightarrow tl \Rightarrow nil)@xx1 \Rightarrow tl \Rightarrow nil=rev SS, 4 p, 5 PP, 6 xx,
7 list(r \Rightarrow tl \Rightarrow nil) \wedge list(p \Rightarrow tl \Rightarrow nil) \wedge r \Rightarrow tl \Rightarrow nil \neg pp \Rightarrow tl \Rightarrow nil \wedge rev(r \Rightarrow tl \Rightarrow nil)@p \Rightarrow tl \Rightarrow nil=rev SS,
8 {list(r \Rightarrow tl \Rightarrow nil) \wedge list(nil \Rightarrow tl \Rightarrow nil) \wedge r \Rightarrow tl \Rightarrow nil \neg nil \Rightarrow tl \Rightarrow nil \wedge rev(r \Rightarrow tl \Rightarrow nil)@nil \Rightarrow tl \Rightarrow nil=rev SS}(p:=nil){xx},
9 rev(p \Rightarrow tl \Rightarrow nil)@() \triangleq rev SS, 10 p \Rightarrow tl \Rightarrow nil \neg (), 11 list(), 12 list(p \Rightarrow tl \Rightarrow nil), 13 list(p \Rightarrow tl \Rightarrow nil) \wedge list(),
14 list(p \Rightarrow tl \Rightarrow nil) \wedge list() \wedge p \Rightarrow tl \Rightarrow nil \neg (), 15 tl, 16 nil \Rightarrow tl \Rightarrow nil, 17 xx3, 18 (),
19 list(p \Rightarrow tl \Rightarrow nil) \wedge list xx3 \wedge p \Rightarrow tl \Rightarrow nil \neg xx3 \wedge rev(p \Rightarrow tl \Rightarrow nil)@xx3=rev SS, 20 rev SS, 21 p \Rightarrow tl \Rightarrow nil=SS,
22 rev x=rev SS, 23 p \Rightarrow tl \Rightarrow nil, 24 x, 25 SS, 26 rev(p \Rightarrow tl \Rightarrow nil), 27 rev(p \Rightarrow tl \Rightarrow nil)@(), 28 xx5, 29 xx5=rev SS,
30 list(p \Rightarrow tl \Rightarrow nil) \wedge list(nil \Rightarrow tl \Rightarrow nil) \wedge p \Rightarrow tl \Rightarrow nil \neg (nil \Rightarrow tl \Rightarrow nil) \wedge rev(p \Rightarrow tl \Rightarrow nil)@(nil \Rightarrow tl \Rightarrow nil)=rev SS,
31 list(p \Rightarrow tl \Rightarrow nil) \wedge p \Rightarrow tl \Rightarrow nil=SS, 32 QQ, 33 xx4,
34 list(p \Rightarrow tl \Rightarrow nil) \wedge list(nil \Rightarrow tl \Rightarrow nil) \wedge p \Rightarrow tl \Rightarrow nil \neg nil \Rightarrow tl \Rightarrow nil \wedge rev(p \Rightarrow tl \Rightarrow nil)@nil \Rightarrow tl \Rightarrow nil=rev SS, 35 xx2,
36 list(xx2 \Rightarrow tl \Rightarrow nil) \wedge list(nil \Rightarrow tl \Rightarrow nil) \wedge xx2 \Rightarrow tl \Rightarrow nil \neg nil \Rightarrow tl \Rightarrow nil \wedge rev(xx2 \Rightarrow tl \Rightarrow nil)@nil \Rightarrow tl \Rightarrow nil=rev SS, 37 r, 38 r:=p,
39 p:=nil

LAYOUT COMPRESS "sequence" ALL
(semicolon«0,32,38,39,5/Q,P,P1,P2,R») (LAYOUT ":= " ALL
(assign'1«35,4,32,36,37/xx,E,Q,R,v») (LAYOUT HIDEROOT
("→-I"«32,34/A,B») (cut«31,30/B,C») (LAYOUT "QQ \triangleq ..." ALL
("rewrite \triangleq "«31,33,32,33/A,xx,B,P»)

```

(LAYOUT HIDEROOT
  ("symmetric≐"«32,31/A,B»)
  (LAYOUT HIDEROOT
    ("QQ ≐ ..."))
    (hyp«32/A»)
  (cut«12,30/B,C»)
  (LAYOUT "∧-E" ALL
    ("∧-E(L)"«21,12/B,A»)
    (hyp«31/A»)
  (cut«21,30/B,C»)
  (LAYOUT "∧-E" ALL
    ("∧-E(R)"«12,21/A,B»)
    (hyp«31/A»)
  (cut«12,30/B,C»)
  (hyp«12/A»)
  (cut«11,30/B,C»)
  (LAYOUT HIDEROOT
    ("list()"))
  (cut«10,30/B,C»)
  ("S¬∩()"«23/S»)
  (cut«9,30/B,C»)
  (LAYOUT "S@()≐S" ALL
    ("rewrite≐"«27,28,26,29/A,xx,B,P»)
    (LAYOUT HIDEROOT
      ("S@()≐S"«26/S»)
      ("rewrite="«22,23,24,25/P,A,xx,B»)
      (hyp«21/A»)
    (LAYOUT HIDEROOT
      ("reflexive="«20/A»)))
  (LAYOUT "A⇒E⇒A=()" ALL
    ("rewrite="«16,17,18,19/A,xx,B,P»)
    (LAYOUT HIDEROOT
      ("A⇒E⇒A=()"«2,15/A,E»))
  (LAYOUT COMPRESS "∧-I" ALL
    ("∧-I"«14,9/A,B»)
    (LAYOUT COMPRESS "∧-I" ALL
      ("∧-I"«13,10/A,B»)
      (LAYOUT COMPRESS "∧-I" ALL
        ("∧-I"«12,11/A,B»)
        (hyp«12/A»)
        (hyp«11/A»)
        (hyp«10/A»)
        (hyp«9/A»))))
  (LAYOUT "PP ≐ ..." ALL
    ("rewrite≐"«5,6,7,8/A,xx,B,P»)
    (LAYOUT HIDEROOT
      ("PP ≐ ..."))
  (LAYOUT ":-" ()
    (assign'1«1,2,0,3,4/xx,E,Q,R,v»)
    (LAYOUT HIDEROOT
      ("→-I"«0,0/A,B»)
      (hyp«0/A»))))

```

65 xx15, 66 rev xx15@p⇒tl⇒nil=rev SS, 67 r.tl⇒tl⇒nil¬r¬p⇒tl⇒nil, 68 r.tl⇒tl⇒nil¬r, 69 xx12,
 70 r.tl⇒tl⇒nil¬r)∧r.tl⇒tl⇒nil¬r¬p⇒tl⇒nil, 71 xx13, 72 list(r), 73 list(r)∧list(p⇒tl⇒nil), 74 xx11,
 75 list(r)∧list(p⇒tl⇒nil)∧r¬r¬p⇒tl⇒nil, 76 (r)¬r¬p⇒tl⇒nil∧r.tl⇒tl⇒nil¬r¬p⇒tl⇒nil, 77 (r)@r.tl⇒tl⇒nil¬r¬p⇒tl⇒nil,
 78 xx10, 79 r⇒tl⇒nil¬r¬p⇒tl⇒nil, 80 xx9, 81 xx9¬r¬p⇒tl⇒nil, 82 list(r⇒tl⇒nil),
 83 list(r.tl⇒(tl⊗r⇒p)⇒nil)∧list(r⇒(tl⊗r⇒p)⇒nil)∧(r.tl⇒(tl⊗r⇒p)⇒nil)¬r⇒(tl⊗r⇒p)⇒nil∧rev(r.tl⇒(tl⊗r⇒p)⇒nil)@r⇒
 (tl⊗r⇒p)⇒nil=rev SS, 84 list(r⇒tl⇒nil)∧list(p⇒tl⇒nil)∧r⇒tl⇒nil¬r¬p⇒tl⇒nil∧rev(r⇒tl⇒nil)@p⇒tl⇒nil=rev SS,
 85 list(r⇒tl⇒nil)∧list(p⇒tl⇒nil)∧r⇒tl⇒nil¬r¬p⇒tl⇒nil, 86 list(r⇒tl⇒nil)∧list(p⇒tl⇒nil), 87 PP, 88 xx5,
 89 PP∧r≠nil, 90 xx4,
 91 list(r.tl⇒(tl⊗xx4⇒p)⇒nil)∧list(xx4⇒(tl⊗xx4⇒p)⇒nil)∧r.tl⇒(tl⊗xx4⇒p)⇒nil¬r¬xx4⇒(tl⊗xx4⇒p)⇒nil∧rev(r.tl⇒(tl
 ⊗xx4⇒p)⇒nil)@xx4⇒(tl⊗xx4⇒p)⇒nil=rev SS, 92 q:=r, 93 r:=r.tl, 94 q:=r;r:=r.tl, 95 q.tl:=p, 96 q:=r;r:=r.tl;q.tl:=p,
 97 p:=q, 98 xx, 99 {PP∧r≠nil}(q:=r;r:=r.tl;q.tl:=p;p:=q){xx}

LAYOUT "PP ≐ ..." ALL

("rewrite≐"«87,98,84,99/A,xx,B,P»)

(LAYOUT HIDEROOT

("PP ≐ ..."))

(LAYOUT COMPRESS "sequence" ALL

(semicolon«0,89,96,97,84/Q,P,P1,P2,R»)

(LAYOUT COMPRESS "sequence" ALL

(semicolon«5,89,94,95,0/Q,P,P1,P2,R»)

(LAYOUT COMPRESS "sequence" ALL

(semicolon«9,89,92,93,5/Q,P,P1,P2,R»)

(LAYOUT ":= " ALL

(assign'1«90,13,89,91,2/xx,E,Q,R,v»)

(LAYOUT HIDEROOT

("→-l"«89,50/A,B»)

(cut«87,50/B,C»)

(LAYOUT "∧-E" ALL

("∧-E(L)"«33,87/B,A»)

(hyp«89/A»)

(cut«33,50/B,C»)

(LAYOUT "∧-E" ALL

("∧-E(R)"«87,33/A,B»)

(hyp«89/A»)

(LAYOUT HIDE CUT

(cut«84,50/B,C»)

(LAYOUT "PP ≐ ..." ALL

("rewrite≐"«84,88,87,88/A,xx,B,P»)

(LAYOUT HIDEROOT

("symmetric≐"«87,84/A,B»)

(LAYOUT HIDEROOT

("PP ≐ ..."))

(hyp«87/A»)

(LAYOUT HIDE CUT

(cut«85,50/B,C»)

("∧-E(L)"«62,85/B,A»)

(hyp«84/A»)

(LAYOUT HIDE CUT

(cut«86,50/B,C»)

("∧-E(L)"«79,86/B,A»)

(hyp«85/A»)

(cut«82,50/B,C»)

(LAYOUT "∧-E" ALL

("∧-E(L)"«21,82/B,A»)

(hyp«86/A»)

(cut«21,50/B,C»)

(LAYOUT "∧-E" ALL

("∧-E(R)"«82,21/A,B»)

(hyp«86/A»)

(cut«79,50/B,C»)

(LAYOUT "∧-E" ALL

("∧-E(R)"«86,79/A,B»)

(hyp«85/A»)

(cut«62,50/B,C»)

(LAYOUT "∧-E" ALL

("∧-E(R)"«85,62/A,B»)

(hyp«84/A»)

```

(cut«17,83/B,C»)
("FROM A≠B AND list(A⇒E⇒B) INFER list(A.E⇒E⇒B)"«13,23,8/A,B,E»)
(hyp«33/A»)
(hyp«82/A»)
(cut«38,83/B,C»)
("FROM A≠B AND list(A⇒E⇒B) INFER (A)¬∩A.E⇒E⇒B"«13,23,8/A,B,E»)
(hyp«33/A»)
(hyp«82/A»)
(cut«77,50/B,C»)
(LAYOUT "FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B" ALL
  ("rewrite="«64,80,63,81/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("symmetric="«63,64/A,B»)
    (LAYOUT HIDEROOT
      ("FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B"«13,23,8/A,B,E»)
      (hyp«33/A»)))
    (hyp«79/A»))
(LAYOUT HIDE CUT
  (cut«76,50/B,C»)
  (LAYOUT "R@S¬∩T≡R¬∩T∧S¬∩T" ALL
    ("rewrite≡"«76,78,77,78/A,xx,B,P»)
    (LAYOUT HIDEROOT
      ("symmetric≡"«77,76/A,B»)
      (LAYOUT HIDEROOT
        ("R@S¬∩T≡R¬∩T∧S¬∩T"«45,39,24/R,S,T»)))
      (hyp«77/A»))
  (cut«22,50/B,C»)
  (LAYOUT "∧-E" ALL
    ("∧-E(L)"«67,22/B,A»)
    (hyp«76/A»))
  (cut«67,50/B,C»)
  (LAYOUT "∧-E" ALL
    ("∧-E(R)"«22,67/A,B»)
    (hyp«76/A»))
  (cut«22,50/B,C»)
  (hyp«22/A»)
  (cut«17,50/B,C»)
  (hyp«17/A»)
  (cut«72,50/B,C»)
  (LAYOUT HIDEROOT
    ("list(A)"«13/A»))
  (cut«21,50/B,C»)
  (hyp«21/A»)
  (cut«22,50/B,C»)
  (hyp«22/A»)
  (cut«16,50/B,C»)
  (LAYOUT "list(R@S)≡list R∧list S∧R¬∩S" ALL
    ("rewrite≡"«16,74,75,74/A,xx,B,P»)
    (LAYOUT HIDEROOT
      ("list(R@S)≡list R∧list S∧R¬∩S"«45,24/R,S»))
    (LAYOUT HIDEROOT
      (LAYOUT COMPRESS "∧-I" ALL
        ("∧-I"«73,22/A,B»)
        (LAYOUT COMPRESS "∧-I" ALL
          ("∧-I"«72,21/A,B»)
          (hyp«72/A»)
          (hyp«21/A»))
        (hyp«22/A»))))
  (cut«68,50/B,C»)
  (LAYOUT HIDEROOT
    (LAYOUT "R¬∩S≡S¬∩R" ALL
      ("rewrite≡"«68,71,38,71/A,xx,B,P»)
      (LAYOUT HIDEROOT
        ("R¬∩S≡S¬∩R"«39,45/R,S»))
      (hyp«38/A»))
    (cut«67,50/B,C»)
    (hyp«67/A»)

```

```

(cut«15,50/B,C»)
(LAYOUT "R¬S@T≠R¬S∧R¬T" ALL
("rewrite≠"«15,69,70,69/A,xx,B,P»)
(LAYOUT HIDEROOT
("R¬S@T≠R¬S∧R¬T"«39,45,24/R,S,T»)
(LAYOUT HIDEROOT
(LAYOUT COMPRESS "∧-" ALL
("∧-"«68,67/A,B»)
(hyp«68/A»)
(hyp«67/A»))))
(cut«55,50/B,C»)
(LAYOUT "FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B" ALL
("rewrite="«64,65,63,66/A,xx,B,P»)
(LAYOUT HIDEROOT
("symmetric="«63,64/A,B»)
(LAYOUT HIDEROOT
("FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B"«13,23,8/A,B,E»)
(hyp«33/A»)))
(hyp«62/A»)
(LAYOUT HIDE CUT
(cut«57,50/B,C»)
("finitesequence(A)"«13/A»)
(cut«56,50/B,C»)
(LAYOUT HIDEROOT
("FROM list S INFER finitesequence S"«39/S»)
(hyp«17/A»)
(cut«51,50/B,C»)
(LAYOUT "FROM finitesequence R AND finitesequence S INFER rev(R@S)≠rev S@rev R"
ALL
("rewrite≠"«59,60,58,61/A,xx,B,P»)
(LAYOUT HIDEROOT
("symmetric≠"«58,59/A,B»)
(LAYOUT HIDEROOT
("FROM finitesequence R AND finitesequence S INFER rev(R@S)≠rev S@rev
R"«45,39/R,S»)
(hyp«57/A»)
(hyp«56/A»)))
(hyp«55/A»)
(cut«43,50/B,C»)
(LAYOUT "rev(A)≠(A)" ALL
("rewrite≠"«45,53,52,54/A,xx,B,P»)
(LAYOUT HIDEROOT
("symmetric≠"«52,45/A,B»)
(LAYOUT HIDEROOT
("rev(A)≠(A)"«13/A»)))
(hyp«51/A»)
(cut«14,50/B,C»)
(LAYOUT HIDEROOT
(LAYOUT "iterate" (1)
("rewrite="«47,48,46,49/A,xx,B,P»)
("symmetric="«46,47/A,B»)
(("R@S)@T≠R@(S@T)"«44,45,24/R,S,T»)
(LAYOUT HIDEROOT
(hyp«43/A»))))
(LAYOUT "FROM (C)¬A⇒E⇒B AND list(A⇒E⇒B) INFER A⇒E⇒B=A⇒(E⊕C⇒F)⇒B"
ALL
("rewrite="«40,41,39,42/A,xx,B,P»)
(LAYOUT HIDEROOT
("symmetric="«39,40/A,B»)
(LAYOUT HIDEROOT
("FROM (C)¬A⇒E⇒B AND list(A⇒E⇒B) INFER
A⇒E⇒B=A⇒(E⊕C⇒F)⇒B"«11,23,13,8,4/A,B,C,E,F»)
(hyp«38/A»)
(hyp«17/A»)))
("rewrite="«36,35,37,20/P,A,xx,B»)
("transitive="«35,32,20/A,B,C»)
("FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B"«13,23,34/A,B,E»)

```

```

(hyp«33/A»)
("transitive="«32,28,20/A,B,C»)
(LAYOUT "FROM A=C INFER A.(B⊕C→E)=E" ALL
("rewrite="«29,30,4,31/A,xx,B,P»)
(LAYOUT HIDEROOT
("FROM A=C INFER A.(B⊕C→E)=E"«13,8,13,4/A,B,C,E»)
(LAYOUT HIDEROOT
("reflexive="«13/A»)))
(LAYOUT HIDEROOT
("reflexive="«28/A»)))
(LAYOUT "FROM (C)¬∩A⇒E⇒B AND list(A⇒E⇒B) INFER A⇒E⇒B=A⇒(E⊕C→F)⇒B"
ALL
("rewrite="«25,26,24,27/A,xx,B,P»)
(LAYOUT HIDEROOT
("symmetric="«24,25/A,B»)
(LAYOUT HIDEROOT
("FROM (C)¬∩A⇒E⇒B AND list(A⇒E⇒B) INFER
A⇒E⇒B=A⇒(E⊕C→F)⇒B"«4,23,13,8,4/A,B,C,E,F»)
(hyp«22/A»)
(hyp«21/A»)))
(LAYOUT HIDEROOT
("reflexive="«20/A»)))
(LAYOUT COMPRESS "∧-I" ALL
("∧-I"«19,14/A,B»)
(LAYOUT COMPRESS "∧-I" ALL
("∧-I"«18,15/A,B»)
(LAYOUT COMPRESS "∧-I" ALL
("∧-I"«17,16/A,B»)
(hyp«17/A»)
(hyp«16/A»)
(hyp«15/A»)
(hyp«14/A»)))))))))
(LAYOUT ":@" ()
(assign'1«10,11,9,12,13/xx,E,Q,R,v»)
(LAYOUT HIDEROOT
("→-I"«9,9/A,B»)
(hyp«9/A»)))
(LAYOUT ":@" ()
(assign'0«6,2,4,5,7,8/xx,A,E,Q,R,f»)
(LAYOUT HIDEROOT
("→-I"«5,5/A,B»)
(hyp«5/A»)))
(LAYOUT ":@" ()
(assign'1«1,2,0,3,4/xx,E,Q,R,v»)
(LAYOUT HIDEROOT
("→-I"«0,0/A,B»)
(hyp«0/A»)))

```

THEOREM IS $\{PP \wedge r \neq \text{nil} \wedge vt = TT\} (q := r; r := r.tl; q.tl := p; p := q) \{TT < vt\}$

	assumption	1:	$PP \wedge r \neq \text{nil} \wedge vt = TT$
	(1) \wedge -E	2:	PP
	(1) \wedge -E	3:	$r \neq \text{nil}$
	(1) \wedge -E	4:	$vt = TT$
	(2) \wedge -E	5:	$\text{list}(r \Rightarrow \text{tl} \Rightarrow \text{nil})$
(3,5) FROM $A \neq B$ AND $\text{list}(A \Rightarrow E \Rightarrow B)$ INFER $(A) \neg \neg A.E \Rightarrow E \Rightarrow B$		6:	$(r) \neg \neg r.tl \Rightarrow \text{tl} \Rightarrow \text{nil}$
(3,5) FROM $A \neq B$ AND $\text{list}(A \Rightarrow E \Rightarrow B)$ INFER $\text{list}(A.E \Rightarrow E \Rightarrow B)$		7:	$\text{list}(r.tl \Rightarrow \text{tl} \Rightarrow \text{nil})$
(7) FROM finitesequence R AND finitesequence S INFER $\text{length}(R @ (A) @ S) > \text{length}(R @ S)$		8:	$\text{length}(() @ (r) @ r.tl \Rightarrow \text{tl} \Rightarrow \text{nil}) > \text{length}(() @ r.tl \Rightarrow \text{tl} \Rightarrow \text{nil})$
(8) $A < B \neq B > A$		9:	$\text{length}(() @ r.tl \Rightarrow \text{tl} \Rightarrow \text{nil}) < \text{length}(() @ (r) @ r.tl \Rightarrow \text{tl} \Rightarrow \text{nil})$
(9) $() @ S \neq S$		10:	$\text{length}(r.tl \Rightarrow \text{tl} \Rightarrow \text{nil}) < \text{length}(() @ (r) @ r.tl \Rightarrow \text{tl} \Rightarrow \text{nil})$
(10) $() @ S \neq S$		11:	$\text{length}(r.tl \Rightarrow \text{tl} \Rightarrow \text{nil}) < \text{length}((r) @ r.tl \Rightarrow \text{tl} \Rightarrow \text{nil})$
(3,11) FROM $A \neq B$ INFER $A \Rightarrow E \Rightarrow B \neq (A) @ A.E \Rightarrow E \Rightarrow B$		12:	$\text{length}(r.tl \Rightarrow \text{tl} \Rightarrow \text{nil}) < \text{length}(r \Rightarrow \text{tl} \Rightarrow \text{nil})$
(12) $TT \neq \dots$		13:	$\text{length}(r.tl \Rightarrow \text{tl} \Rightarrow \text{nil}) < TT$
(4,13) rewrite=		14:	$\text{length}(r.tl \Rightarrow \text{tl} \Rightarrow \text{nil}) < vt$
(6,7,14) FROM $(C) \neg \neg A \Rightarrow E \Rightarrow B$ AND $\text{list}(A \Rightarrow E \Rightarrow B)$ INFER $A \Rightarrow E \Rightarrow B \neq (E @ C \Rightarrow F) \Rightarrow B$		15:	$\text{length}(r.tl \Rightarrow (tl @ r \Rightarrow p) \Rightarrow \text{nil}) < vt$
(1-15) :=		16:	$\{PP \wedge r \neq \text{nil} \wedge vt = TT\} (q := r) \{ \text{length}(r.tl \Rightarrow (tl @ q \Rightarrow p) \Rightarrow \text{nil}) < vt \}$
		17:	$\{ \text{length}(r.tl \Rightarrow (tl @ q \Rightarrow p) \Rightarrow \text{nil}) < vt \} (r := r.tl) \{ \text{length}(r \Rightarrow (tl @ q \Rightarrow p) \Rightarrow \text{nil}) < vt \}$
		18:	$\{ \text{length}(r \Rightarrow (tl @ q \Rightarrow p) \Rightarrow \text{nil}) < vt \} (q.tl := p) \{ \text{length}(r \Rightarrow \text{tl} \Rightarrow \text{nil}) < vt \}$
		19:	$\{ \text{length}(r \Rightarrow \text{tl} \Rightarrow \text{nil}) < vt \} (p := q) \{ \text{length}(r \Rightarrow \text{tl} \Rightarrow \text{nil}) < vt \}$
(16,17,18,19) sequence		20:	$\{PP \wedge r \neq \text{nil} \wedge vt = TT\} (q := r; r := r.tl; q.tl := p; p := q) \{ \text{length}(r \Rightarrow \text{tl} \Rightarrow \text{nil}) < vt \}$
(20) $TT \neq \dots$		21:	$\{PP \wedge r \neq \text{nil} \wedge vt = TT\} (q := r; r := r.tl; q.tl := p; p := q) \{ TT < vt \}$

FORMULAE 0 $\text{length}(r \Rightarrow \text{tl} \Rightarrow \text{nil}) < vt$, 1 $xx1$, 2 q , 3 p , 4 $\text{length}(r \Rightarrow (tl @ q \Rightarrow p) \Rightarrow \text{nil}) < vt$, 5 $xx2$, 6 $\text{length}(r \Rightarrow xx2 \Rightarrow \text{nil}) < vt$, 7 tl , 8 $\text{length}(r.tl \Rightarrow (tl @ q \Rightarrow p) \Rightarrow \text{nil}) < vt$, 9 $xx3$, 10 $r.tl$, 11 $\text{length}(xx3 \Rightarrow (tl @ q \Rightarrow p) \Rightarrow \text{nil}) < vt$, 12 r , 13 $\text{list}(r.tl \Rightarrow \text{tl} \Rightarrow \text{nil})$, 14 $r.tl \Rightarrow \text{tl} \Rightarrow \text{nil}$, 15 $()$, 16 $\text{length}(() @ r.tl \Rightarrow \text{tl} \Rightarrow \text{nil})$, 17 $\text{length}(() @ (r) @ r.tl \Rightarrow \text{tl} \Rightarrow \text{nil})$, 18 $\text{length}(() @ r.tl \Rightarrow \text{tl} \Rightarrow \text{nil}) < \text{length}(() @ (r) @ r.tl \Rightarrow \text{tl} \Rightarrow \text{nil})$, 19 $xx11$, 20 $\text{length}(() @ (r) @ r.tl \Rightarrow \text{tl} \Rightarrow \text{nil}) > \text{length}(() @ r.tl \Rightarrow \text{tl} \Rightarrow \text{nil})$, 21 $() @ r.tl \Rightarrow \text{tl} \Rightarrow \text{nil}$, 22 $xx10$, 23 $\text{length}(xx10) < \text{length}(() @ (r) @ r.tl \Rightarrow \text{tl} \Rightarrow \text{nil})$, 24 (r) , 25 $() @ (r)$, 26 $xx9$, 27 $\text{length}(r.tl \Rightarrow \text{tl} \Rightarrow \text{nil}) < \text{length}(xx9 @ r.tl \Rightarrow \text{tl} \Rightarrow \text{nil})$, 28 $r \neq \text{nil}$, 29 nil , 30 $r \Rightarrow \text{tl} \Rightarrow \text{nil}$, 31 $xx8$, 32 $(r) @ r.tl \Rightarrow \text{tl} \Rightarrow \text{nil}$, 33 $\text{length}(r.tl \Rightarrow \text{tl} \Rightarrow \text{nil}) < \text{length}(xx8)$, 34 TT , 35 $xx7$, 36 $\text{length}(r \Rightarrow \text{tl} \Rightarrow \text{nil})$, 37 $\text{length}(r.tl \Rightarrow \text{tl} \Rightarrow \text{nil}) < xx7$, 38 $vt = TT$, 39 $\text{length}(r.tl \Rightarrow \text{tl} \Rightarrow \text{nil}) < x$, 40 vt , 41 x , 42 $(r) \neg \neg r.tl \Rightarrow \text{tl} \Rightarrow \text{nil}$, 43 $r.tl \Rightarrow (tl @ r \Rightarrow p) \Rightarrow \text{nil}$, 44 $xx6$, 45 $\text{length}(xx6) < vt$, 46 $\text{list}(r \Rightarrow \text{tl} \Rightarrow \text{nil})$, 47 $\text{length}(r.tl \Rightarrow (tl @ r \Rightarrow p) \Rightarrow \text{nil}) < vt$, 48 $\text{list}(r \Rightarrow \text{tl} \Rightarrow \text{nil}) \wedge \text{list}(p \Rightarrow \text{tl} \Rightarrow \text{nil}) \wedge r \Rightarrow \text{tl} \Rightarrow \text{nil} \neg \neg p \Rightarrow \text{tl} \Rightarrow \text{nil} \wedge \text{rev}(r \Rightarrow \text{tl} \Rightarrow \text{nil}) @ p \Rightarrow \text{tl} \Rightarrow \text{nil} = \text{rev } SS$, 49 $\text{list}(r \Rightarrow \text{tl} \Rightarrow \text{nil}) \wedge \text{list}(p \Rightarrow \text{tl} \Rightarrow \text{nil}) \wedge r \Rightarrow \text{tl} \Rightarrow \text{nil} \neg \neg p \Rightarrow \text{tl} \Rightarrow \text{nil}$, 50 $\text{rev}(r \Rightarrow \text{tl} \Rightarrow \text{nil}) @ p \Rightarrow \text{tl} \Rightarrow \text{nil} = \text{rev } SS$, 51 $\text{list}(r \Rightarrow \text{tl} \Rightarrow \text{nil}) \wedge \text{list}(p \Rightarrow \text{tl} \Rightarrow \text{nil})$, 52 $r \Rightarrow \text{tl} \Rightarrow \text{nil} \neg \neg p \Rightarrow \text{tl} \Rightarrow \text{nil}$, 53 $\text{list}(p \Rightarrow \text{tl} \Rightarrow \text{nil})$, 54 PP , 55 $xx5$, 56 $PP \wedge r \neq \text{nil} \wedge vt = TT$, 57 $PP \wedge r \neq \text{nil}$, 58 $xx4$, 59 $\text{length}(r.tl \Rightarrow (tl @ xx4 \Rightarrow p) \Rightarrow \text{nil}) < vt$, 60 $q := r$, 61 $r := r.tl$, 62 $q := r; r := r.tl$, 63 $q.tl := p$, 64 $q := r; r := r.tl; q.tl := p$, 65 $p := q$, 66 xx , 67 $\{PP \wedge r \neq \text{nil} \wedge vt = TT\} (q := r; r := r.tl; q.tl := p; p := q) \{xx < vt\}$

LAYOUT "TT $\neq \dots$ " ALL

("rewrite" «34,66,36,67/A,xx,B,P»)

(LAYOUT HIDEROOT

("TT $\neq \dots$ "))

(LAYOUT COMPRESS "sequence" ALL

(semicolon «0,56,64,65,0/Q,P,P1,P2,R»)

(LAYOUT COMPRESS "sequence" ALL

(semicolon «4,56,62,63,0/Q,P,P1,P2,R»)

(LAYOUT COMPRESS "sequence" ALL

(semicolon «8,56,60,61,4/Q,P,P1,P2,R»)

(LAYOUT " := " ALL

(assign'1 «58,12,56,59,2/xx,E,Q,R,v»)

(LAYOUT HIDEROOT

(" \rightarrow -!" «56,47/A,B»)

(LAYOUT HIDE CUT

(cut «57,47/B,C»)

(" \wedge -E(L)" «38,57/B,A»)

(hyp «56/A»)

(cut «54,47/B,C»)

(LAYOUT " \wedge -E" ALL

(" \wedge -E(L)" «28,54/B,A»)

(hyp «57/A»))

(cut «28,47/B,C»)

(LAYOUT " \wedge -E" ALL

(" \wedge -E(R)" «54,28/A,B»)

(hyp «57/A»))

(cut «38,47/B,C»)

(LAYOUT " \wedge -E" ALL


```

(LAYOUT HIDEROOT
  ("{}@S=S"«24/S»)))
(LAYOUT "{}@S=S" ALL
  ("rewrite="«14,22,21,23/A,xx,B,P»)
(LAYOUT HIDEROOT
  ("symmetric="«21,14/A,B»)
(LAYOUT HIDEROOT
  ("{}@S=S"«14/S»)))
(LAYOUT "A<B=B>A" ALL
  ("rewrite="«18,19,20,19/A,xx,B,P»)
(LAYOUT HIDEROOT
  ("A<B=B>A"«16,17/A,B»))
("FROM finitesequence R AND finitesequence S INFER
length(R@{A}@S)>length(R@S)"«12,15,14/A,R,S»)
(LAYOUT HIDEROOT
  ("finitesequence(!")
(LAYOUT HIDEROOT
  ("FROM list S INFER finitesequence S"«14/S»)
  (hyp«13/A»)))))))))

(LAYOUT "!=" ()
  (assign'1«9,10,8,11,12/xx,E,Q,R,v»)
(LAYOUT HIDEROOT
  ("→!"«8,8/A,B»)
  (hyp«8/A»))))

(LAYOUT "!=" ()
  (assign'0«5,2,3,4,6,7/xx,A,E,Q,R,f»)
(LAYOUT HIDEROOT
  ("→!"«4,4/A,B»)
  (hyp«4/A»))))

(LAYOUT "!=" ()
  (assign'1«1,2,0,0,3/xx,E,Q,R,v»)
(LAYOUT HIDEROOT
  ("→!"«0,0/A,B»)
  (hyp«0/A»))))

```

THEOREM IS {PP} (while r ≠ nil do q:=r; r:=r.tl; q.tl:=p; p:=q od) {RR}

	{→-I,hyp}	1: PP→PP
{PP∧r≠nil}(q:=r;r:=r.tl;q.tl:=p;p:=q){PP}		2: {PP∧r≠nil}(q:=r;r:=r.tl;q.tl:=p;p:=q){PP}
assumption		3: PP∧¬(r≠nil)
(3) ∧-E		4: PP
(3) ∧-E		5: ¬(r≠nil)
(4) ∧-E		6: list(p⇒tl⇒nil)
(4) ∧-E		7: rev(r⇒tl⇒nil)@p⇒tl⇒nil=rev SS
¬(A≠B)≐A=B		8: ¬(r≠nil)≐r=nil
(8) symmetric≐		9: r=nil≐¬(r≠nil)
(9,5) ¬(A≠B)≐A=B		10: r=nil
(10) FROM A=B INFER A⇒E⇒B=()		11: r⇒tl⇒nil=()
(11) symmetric=		12: ()=r⇒tl⇒nil
(12,7) FROM A=B INFER A⇒E⇒B=()		13: rev()@p⇒tl⇒nil=rev SS
rev()≐()		14: rev()≐()
(14) symmetric≐		15: ()≐rev()
(15,13) rev()≐()		16: ()@p⇒tl⇒nil=rev SS
()@S≐S		17: ()@p⇒tl⇒nil≐p⇒tl⇒nil
(17) symmetric≐		18: p⇒tl⇒nil≐()@p⇒tl⇒nil
(18,16) ()@S≐S		19: p⇒tl⇒nil=rev SS
RR ≐ ...		20: RR≐list(p⇒tl⇒nil)∧p⇒tl⇒nil=rev SS
(6,19) ∧-I		21: list(p⇒tl⇒nil)∧p⇒tl⇒nil=rev SS
(20,21) RR ≐ ...		22: RR
(3-22) →-I		23: PP∧¬(r≠nil)→RR
assumption		24: PP∧r≠nil
(24) ∧-E		25: PP
(24) ∧-E		26: r≠nil
length(R@(A)@S)>0		27: length(()@(r)@r.tl⇒tl⇒nil)>0
(27) ()@S≐S		28: length((r)@r.tl⇒tl⇒nil)>0
(26,28) FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B		29: length(r⇒tl⇒nil)>0
(29) TT ≐ ...		30: TT>0
(30) ∃-I		31: ∃x:(TT>x)
(24-31) →-I		32: PP∧r≠nil→∃x:(TT>x)
{PP∧r≠nil∧vt=TT}(q:=r;r:=r.tl;q.tl:=p;p:=q){TT<vt}		33: {PP∧r≠nil∧vt=TT}(q:=r;r:=r.tl;q.tl:=p;p:=q){TT<vt}
(1,2,23,32,33) while-do-od		34: {PP}(while r≠nil do q:=r;r:=r.tl;q.tl:=p;p:=q od){RR}

FORMULAE 0 vt, 1 r, 2 (), 3 r.tl⇒tl⇒nil, 4 (r), 5 ()@(r), 6 xx9, 7 length(xx9@r.tl⇒tl⇒nil)>0, 8 r≠nil, 9 nil, 10 tl, 11 r⇒tl⇒nil, 12 xx8, 13 (r)@r.tl⇒tl⇒nil, 14 length xx8>0, 15 TT, 16 xx7, 17 length(r⇒tl⇒nil), 18 xx7>0, 19 0, 20 TT>x, 21 x, 22 list(r⇒tl⇒nil)∧list(p⇒tl⇒nil)∧r⇒tl⇒nil¬p⇒tl⇒nil∧rev(r⇒tl⇒nil)@p⇒tl⇒nil=rev SS, 23 list(r⇒tl⇒nil)∧list(p⇒tl⇒nil)∧r⇒tl⇒nil¬p⇒tl⇒nil, 24 rev(r⇒tl⇒nil)@p⇒tl⇒nil=rev SS, 25 ∃x:(TT>x), 26 list(r⇒tl⇒nil)∧list(p⇒tl⇒nil), 27 r⇒tl⇒nil¬p⇒tl⇒nil, 28 list(r⇒tl⇒nil), 29 list(p⇒tl⇒nil), 30 PP, 31 xx6, 32 PP∧r≠nil, 33 p⇒tl⇒nil=rev SS, 34 RR, 35 xx1, 36 list(p⇒tl⇒nil)∧p⇒tl⇒nil=rev SS, 37 ()@p⇒tl⇒nil=rev SS, 38 p⇒tl⇒nil, 39 ()@p⇒tl⇒nil, 40 xx5, 41 xx5=rev SS, 42 rev()@p⇒tl⇒nil=rev SS, 43 rev(), 44 xx4, 45 xx4@p⇒tl⇒nil=rev SS, 46 r=nil, 47 xx2, 48 rev xx2@p⇒tl⇒nil=rev SS, 49 ¬(r≠nil), 50 xx3, 51 xx, 52 PP∧¬(r≠nil), 53 q:=r;r:=r.tl;q.tl:=p;p:=q

SEQ

("while-do-od"«30,21,0,8,30,34,53,15/P,x,vt,B,Q,R,S,T»)
 (LAYOUT "" ()
 ("→-I"«30,30/A,B»)

```

(hyp«30/A»)
("{PP^r≠nil}(q:=r;r:=r.tl;q.tl:=p;p:=q){PP}")
("→!"«52,34/A,B»)
(cut«30,34/B,C»)
(LAYOUT "^-E" ALL
  ("^-E(L)"«49,30/B,A»)
  (hyp«52/A»)
(cut«49,34/B,C»)
(LAYOUT "^-E" ALL
  ("^-E(R)"«30,49/A,B»)
  (hyp«52/A»)
(LAYOUT HIDE CUT
  (cut«22,34/B,C»)
  (LAYOUT "PP ≐ ..." ALL
    ("rewrite≐"«22,51,30,51/A,xx,B,P»)
    (LAYOUT HIDE ROOT
      ("symmetric≐"«30,22/A,B»)
      (LAYOUT HIDE ROOT
        ("PP ≐ ..."))
    (hyp«30/A»)
  (LAYOUT HIDE CUT
    (cut«23,34/B,C»)
    ("^-E(L)"«24,23/B,A»)
    (hyp«22/A»)
    (LAYOUT HIDE CUT
      (cut«26,34/B,C»)
      ("^-E(L)"«27,26/B,A»)
      (hyp«23/A»)
      (cut«28,34/B,C»)
      (LAYOUT "^-E" ALL
        ("^-E(L)"«29,28/B,A»)
        (hyp«26/A»)
      (cut«29,34/B,C»)
      (LAYOUT "^-E" ALL
        ("^-E(R)"«28,29/A,B»)
        (hyp«26/A»)
      (cut«27,34/B,C»)
      (LAYOUT "^-E" ALL
        ("^-E(R)"«26,27/A,B»)
        (hyp«23/A»)
      (cut«24,34/B,C»)
      (LAYOUT "^-E" ALL
        ("^-E(R)"«23,24/A,B»)
        (hyp«22/A»)
      (cut«29,34/B,C»)
      (hyp«29/A»)
      (cut«46,34/B,C»)
      (LAYOUT HIDE ROOT
        (LAYOUT "¬(A≠B)≐A=B" ALL
          ("rewrite≐"«46,50,49,50/A,xx,B,P»)
          (LAYOUT HIDE ROOT
            ("symmetric≐"«49,46/A,B»)
            (LAYOUT HIDE ROOT
              ("¬(A≠B)≐A=B"«1,9/A,B»))
            (hyp«49/A»))
          (cut«46,34/B,C»)
          (hyp«46/A»)
          (cut«42,34/B,C»)
          (LAYOUT "FROM A=B INFER A⇒E⇒B=()" ALL
            ("rewrite="«2,47,11,48/A,xx,B,P»)
            (LAYOUT HIDE ROOT
              ("symmetric="«11,2/A,B»)
              (LAYOUT HIDE ROOT
                ("FROM A=B INFER A⇒E⇒B=()"«1,9,10/A,B,E»)
                (hyp«46/A»))
              (hyp«24/A»)
            (cut«37,34/B,C»)

```

```

(LAYOUT "rev()≐()" ALL
 ("rewrite≐"«2,44,43,45/A,xx,B,P»)
 (LAYOUT HIDEROOT
 ("symmetric≐"«43,2/A,B»)
 (LAYOUT HIDEROOT
 ("rev()≐()")))
 (hyp«42/A»)
 (cut«33,34/B,C»)
 (LAYOUT "()"@S≐S" ALL
 ("rewrite≐"«38,40,39,41/A,xx,B,P»)
 (LAYOUT HIDEROOT
 ("symmetric≐"«39,38/A,B»)
 (LAYOUT HIDEROOT
 ("()"@S≐S"«38/S»)))
 (hyp«37/A»)
 (cut«33,34/B,C»)
 (hyp«33/A»)
 (LAYOUT "RR ≐ ..." ALL
 ("rewrite≐"«34,35,36,35/A,xx,B,P»)
 (LAYOUT HIDEROOT
 ("RR ≐ ...")))
 (LAYOUT COMPRESS "^-I" ALL
 ("^-I"«29,33/A,B»)
 (hyp«29/A»)
 (hyp«33/A»))))))
("→-I"«32,25/A,B»)
(cut«30,25/B,C»)
(LAYOUT "^-E" ALL
 ("^-E(L)"«8,30/B,A»)
 (hyp«32/A»)
 (cut«8,25/B,C»)
 (LAYOUT "^-E" ALL
 ("^-E(R)"«30,8/A,B»)
 (hyp«32/A»)
 (LAYOUT HIDE CUT
 (cut«22,25/B,C»)
 (LAYOUT "PP ≐ ..." ALL
 ("rewrite≐"«22,31,30,31/A,xx,B,P»)
 (LAYOUT HIDEROOT
 ("symmetric≐"«30,22/A,B»)
 (LAYOUT HIDEROOT
 ("PP ≐ ...")))
 (hyp«30/A»)
 (LAYOUT HIDE CUT
 (cut«23,25/B,C»)
 ("^-E(L)"«24,23/B,A»)
 (hyp«22/A»)
 (LAYOUT HIDE CUT
 (cut«26,25/B,C»)
 ("^-E(L)"«27,26/B,A»)
 (hyp«23/A»)
 (cut«28,25/B,C»)
 (LAYOUT "^-E" ALL
 ("^-E(L)"«29,28/B,A»)
 (hyp«26/A»)
 (cut«29,25/B,C»)
 (LAYOUT "^-E" ALL
 ("^-E(R)"«28,29/A,B»)
 (hyp«26/A»)
 (cut«27,25/B,C»)
 (LAYOUT "^-E" ALL
 ("^-E(R)"«26,27/A,B»)
 (hyp«23/A»)
 (cut«24,25/B,C»)
 (LAYOUT "^-E" ALL
 ("^-E(R)"«23,24/A,B»)
 (hyp«22/A»)

```

```

(LAYOUT "∃-I" (0)
  ("∃-I"«19,20,21/B,A,x»)
  (LAYOUT "TT ≐ ..." ALL
    ("rewrite≐"«15,16,17,18/A,xx,B,P»)
    (LAYOUT HIDEROOT
      ("TT ≐ ...")
      (LAYOUT "FROM A≐B INFER A⇒E⇒B=(A)@A.E⇒E⇒B" ALL
        ("rewrite="«11,12,13,14/A,xx,B,P»)
        (LAYOUT HIDEROOT
          ("FROM A≐B INFER A⇒E⇒B=(A)@A.E⇒E⇒B"«1,9,10/A,B,E»)
          (hyp«8/A»)
          (LAYOUT "()"@S≐S" ALL
            ("rewrite≐"«4,6,5,7/A,xx,B,P»)
            (LAYOUT HIDEROOT
              ("symmetric≐"«5,4/A,B»)
              (LAYOUT HIDEROOT
                ("()"@S≐S"«4/S»)))
            ("length(R@(A)@S)>0"«1,2,3/A,R,S»))))))
  ("{}PP∧r≠nil∧vt=TT}(q:=r;r:=r.tl;q.tl:=p;p:=q){TT<vt}"«0/vt»)

```

THEOREM IS {QQ} (r:=p; p:=nil; while r≠nil do q:=r; r:=r.tl; q.tl:=p; p:=q od) {RR}

{QQ}(r:=p;p:=nil){PP} 1: {QQ}(r:=p;p:=nil){PP}

{PP}(while r≠nil do q:=r;r:=r.tl;q.tl:=p;p:=q od){RR} 2: {PP}while r≠nil do q:=r;r:=r.tl;q.tl:=p;p:=q od{RR}

(1,2) sequence 3: {QQ}(r:=p;p:=nil;while r≠nil do q:=r;r:=r.tl;q.tl:=p;p:=q od){RR}

FORMULAE 0 PP, 1 QQ, 2 r:=p;p:=nil, 3 while r≠nil do q:=r;r:=r.tl;q.tl:=p;p:=q od, 4 RR

LAYOUT COMPRESS "sequence" ALL

(semicolon«0,1,2,3,4/Q,P,P1,P2,R»)

("{QQ}(r:=p;p:=nil){PP}")

("{PP}(while r≠nil do q:=r;r:=r.tl;q.tl:=p;p:=q od){RR}")