

# Proofs of some pointer programs

## Part 2: Heap lists

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Heap lists are sequences of pointers to records linked by a named field. I define two sorts of heap lists.<sup>1</sup>

### 2.1. Rules

$A \Rightarrow E \Rightarrow B$  runs from  $A$  up to, but not including,  $B$ ;  $A \Rightarrow E \Rightarrow +B$  includes  $B$  as well.

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RULE IS    A ⇒ E ⇒ B ≐ if A=B then () else (A)@A.E ⇒ E ⇒ B fi
RULE IS    A ⇒ E ⇒ +B ≐ A ⇒ E ⇒ B@(B)
```

### 2.2. Proofs

THEOREM IS  $A \Rightarrow E \Rightarrow A = ()$

1: if  $A=A$  then()else(A)@A.E ⇒ E ⇒ A fi=() FROM A INFER if A then B else C fi≐B

2:  $A \Rightarrow E \Rightarrow A = ()$   $A \Rightarrow E \Rightarrow B \equiv$  if  $A=B$  then()else(A)@A.E ⇒ E ⇒ B fi 1

FORMULAE 0 (), 1 A, 2 A=A, 3 (A)@A.E ⇒ E ⇒ A, 4 if A=A then()else(A)@A.E ⇒ E ⇒ A fi, 5 xx1, 6 xx1=(), 7 E, 8  $A \Rightarrow E \Rightarrow A$ , 9 xx, 10 xx=()

LAYOUT "A ⇒ E ⇒ B ≐ if A=B then()else(A)@A.E ⇒ E ⇒ B fi" ALL

("rewrite≐"«8,9,4,10/A,xx,B,P»)

(LAYOUT HIDEROOT

("A ⇒ E ⇒ B ≐ if A=B then()else(A)@A.E ⇒ E ⇒ B fi"«1,1,7/A,B,E»))

(LAYOUT "FROM A INFER if A then B else C fi≐B" ALL

("rewrite≐"«4,5,0,6/A,xx,B,P»)

(LAYOUT HIDEROOT

("FROM A INFER if A then B else C fi≐B"«2,0,3/A,B,C»)

(LAYOUT HIDEROOT

("reflexive="«1/A»))

(LAYOUT HIDEROOT

("reflexive="«0/A»))

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DERIVED RULE IS FROM  $A=B$  INFER  $A \Rightarrow E \Rightarrow B = ()$

1:  $A=B$   $A=B$

2: if  $A=B$  then()else(A)@A.E ⇒ E ⇒ B fi=() FROM A INFER if A then B else C fi≐B 1

3:  $A \Rightarrow E \Rightarrow B = ()$   $A \Rightarrow E \Rightarrow B \equiv$  if  $A=B$  then()else(A)@A.E ⇒ E ⇒ B fi 2

Given:

$A=B$

FORMULAE 0 (), 1 A=B, 2 (A)@A.E ⇒ E ⇒ B, 3 if A=B then()else(A)@A.E ⇒ E ⇒ B fi, 4 xx1, 5 xx1=(), 6 A, 7 B, 8 E, 9  $A \Rightarrow E \Rightarrow B$ , 10 xx, 11 xx=()

<sup>1</sup> If you look at the source files you will find there are about eight forms, with proofs about all of them. But not all of them are relevant to the proofs in this document, so I've cut them out.

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LAYOUT "A⇒E⇒B⇐if A=B then()else(A)@A.E⇒E⇒B fi" ALL
("rewrite⇐"«9,10,3,11/A,xx,B,P»)
(LAYOUT HIDEROOT
("A⇒E⇒B⇐if A=B then()else(A)@A.E⇒E⇒B fi"«6,7,8/A,B,E»))
(LAYOUT "FROM A INFER if A then B else C fi⇐B" ALL
("rewrite⇐"«3,4,0,5/A,xx,B,P»)
(LAYOUT HIDEROOT
("FROM A INFER if A then B else C fi⇐B"«1,0,2/A,B,C»)
(GIVEN 0))
(LAYOUT HIDEROOT
("reflexive⇐"«0/A»)))
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DERIVED RULE IS FROM  $A \neq B \text{ INFER } A \Rightarrow E \Rightarrow B = (A) @ A.E \Rightarrow E \Rightarrow B$

1: $A \neq B$	$A \neq B$
2: $\neg(A=B)$	$\neg(A=B) \hat{=} A \neq B$ 1
3: if $A=B$ then()else(A)@A.E⇒E⇒B fi=(A)@A.E⇒E⇒B	FROM $\neg A$ INFER if A then B else C fi⇐C 2
4: $A \Rightarrow E \Rightarrow B = (A) @ A.E \Rightarrow E \Rightarrow B$	$A \Rightarrow E \Rightarrow B \hat{=} \text{if } A=B \text{ then}() \text{else}(A) @ A.E \Rightarrow E \Rightarrow B \text{ fi}$ 3

Given:  
 $A \neq B$

FORMULAE 0 (A)@A.E⇒E⇒B, 1 A, 2 B, 3  $\neg(A=B)$ , 4 xx2, 5  $A \neq B$ , 6  $A=B$ , 7 (),  
8 if A=B then()else(A)@A.E⇒E⇒B fi, 9 xx1, 10  $xx1=(A)@A.E \Rightarrow E \Rightarrow B$ , 11 E, 12  $A \Rightarrow E \Rightarrow B$ , 13 xx,  
14  $xx=(A)@A.E \Rightarrow E \Rightarrow B$

```
LAYOUT "A⇒E⇒B⇐if A=B then()else(A)@A.E⇒E⇒B fi" ALL
("rewrite⇐"«12,13,8,14/A,xx,B,P»)
(LAYOUT HIDEROOT
("A⇒E⇒B⇐if A=B then()else(A)@A.E⇒E⇒B fi"«1,2,11/A,B,E»))
(LAYOUT "FROM ¬A INFER if A then B else C fi⇐C" ALL
("rewrite⇐"«8,9,0,10/A,xx,B,P»)
(LAYOUT HIDEROOT
("FROM ¬A INFER if A then B else C fi⇐C"«6,7,0/A,B,C»)
(LAYOUT "¬(A=B)⇐A≠B" ALL
("rewrite⇐"«3,4,5,4/A,xx,B,P»)
(LAYOUT HIDEROOT
("¬(A=B)⇐A≠B"«1,2/A,B»)
(GIVEN 0)))
(LAYOUT HIDEROOT
("reflexive⇐"«0/A»)))
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DERIVED RULE IS FROM  $A \Rightarrow E \Rightarrow B = ()$  INFER  $A = B$

1: $A = B \vee \neg(A = B)$	$A = B \vee \neg(A = B)$
2: $A = B$	assumption
3: $\neg(A = B)$	assumption
4: $A \neq B$	$\neg(A = B) \triangleq A \neq B$ 3
5: $A \Rightarrow E \Rightarrow B = (A) @ A.E \Rightarrow E \Rightarrow B$	Derived Rule FROM $A \neq B$ INFER $A \Rightarrow E \Rightarrow B = (A) @ A.E \Rightarrow E \Rightarrow B$ 4
6: $A \Rightarrow E \Rightarrow B = ()$	$A \Rightarrow E \Rightarrow B = ()$
7: $() = (A) @ A.E \Rightarrow E \Rightarrow B$	rewrite= 6,5
8: $() @ (A) @ A.E \Rightarrow E \Rightarrow B \neq ()$	$R @ (A) @ T \neq ()$
9: $(A) @ A.E \Rightarrow E \Rightarrow B \neq ()$	$() @ S \triangleq S$ 8
10: $\neg((A) @ A.E \Rightarrow E \Rightarrow B = ())$	$\neg(A = B) \triangleq A \neq B$ 9
11: $\perp$	$\neg$ -E 7,10
12: $A = B$	$\perp$ -E 11
13: $A = B$	$\vee$ -E 1,2-2,3-12

Given:

$A \Rightarrow E \Rightarrow B = ()$

FORMULAE 0 A, 1 (), 2 A.E  $\Rightarrow E \Rightarrow B$ , 3 (A), 4 ()@(A), 5 xx3, 6 xx3@A.E  $\Rightarrow E \Rightarrow B \neq ()$ , 7 (A)@A.E  $\Rightarrow E \Rightarrow B$ , 8  $\neg((A)@A.E \Rightarrow E \Rightarrow B = ())$ , 9 xx1, 10 (A)@A.E  $\Rightarrow E \Rightarrow B \neq ()$ , 11  $() = (A) @ A.E \Rightarrow E \Rightarrow B$ , 12 (A)@A.E  $\Rightarrow E \Rightarrow B = ()$ , 13 A=B, 14  $A \Rightarrow E \Rightarrow B = (A) @ A.E \Rightarrow E \Rightarrow B$ , 15  $A \Rightarrow E \Rightarrow B$ , 16 xx2, 17 xx2=(A)@A.E  $\Rightarrow E \Rightarrow B$ , 18  $A \neq B$ , 19 B, 20 E, 21  $\neg(A = B)$ , 22 xx, 23  $A = B \vee \neg(A = B)$

SEQ

```
(cut«23,13/B,C»)
("A=B∨¬(A=B)"«0,19/A,B»)
("∨-E"«13,21,13/A,B,C»)
(hyp«23/A»)
(hyp«13/A»)
(cut«18,13/B,C»)
(LAYOUT "¬(A=B)≐A≠B" ALL
  ("rewrite≐"«18,22,21,22/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("symmetric≐"«21,18/A,B»)
    (LAYOUT HIDEROOT
      ("¬(A=B)≐A≠B"«0,19/A,B»)))
  (hyp«21/A»)
(cut«14,13/B,C»)
("FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B"«0,19,20/A,B,E»)
(hyp«18/A»)
(cut«11,13/B,C»)
("rewrite="«1,16,15,17/A,xx,B,P»)
(LAYOUT HIDEROOT
  ("symmetric="«15,1/A,B»)
  (GIVEN 0))
(hyp«14/A»)
("⊥-E"«13/A»)
("¬-E"«12/B»)
(LAYOUT HIDEROOT
  ("symmetric="«1,7/A,B»)
  (hyp«11/A»))
(LAYOUT "¬(A=B)≐A≠B" ALL
  ("rewrite≐"«8,9,10,9/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("¬(A=B)≐A≠B"«7,1/A,B»))
  (LAYOUT "()@S≐S" ALL
    ("rewrite="«3,5,4,6/A,xx,B,P»))
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(LAYOUT HIDEROOT
 ("symmetric≐"«4,3/A,B»)
 (LAYOUT HIDEROOT
 ("()@S≐S"«3/S»)))
 ("R@(A)@T≐()"«0,1,2/A,R,T»))
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DERIVED RULE IS FROM  $A \Rightarrow E \Rightarrow B = (C) @ S$  INFER  $A \neq B \wedge A = C \wedge A.E \Rightarrow E \Rightarrow B = S$

1: $A \Rightarrow E \Rightarrow B = (C) @ S$	$A \Rightarrow E \Rightarrow B = (C) @ S$
2: $A = B$	assumption
3: $() = (C) @ S$	FROM $A = B$ INFER $A \Rightarrow E \Rightarrow B = ()$ 2,1
4: $() @ (C) @ S \neq ()$	$R @ (A) @ T \neq ()$
5: $(C) @ S \neq ()$	$() @ S \neq S$ 4
6: $\neg ( () = (C) @ S )$	$\neg (A = B) \neq A \neq B$ 5
7: $\perp$	$\neg E$ 3,6
8: $\neg (A = B)$	$\neg I$ 2-7
9: $A \neq B$	$\neg (A = B) \neq A \neq B$ 8
10: $(A) @ A.E \Rightarrow E \Rightarrow B = (C) @ S$	FROM $A \neq B$ INFER $A \Rightarrow E \Rightarrow B = (A) @ A.E \Rightarrow E \Rightarrow B$ 9,1
11: $A = C \wedge A.E \Rightarrow E \Rightarrow B = S$	$(A) @ R = (B) @ S \neq A = B \wedge R = S$ 10
12: $A = C$	$\wedge E$ 11
13: $A.E \Rightarrow E \Rightarrow B = S$	$\wedge E$ 11
14: $A \neq B \wedge A = C$	$\wedge I$ 9,12
15: $A \neq B \wedge A = C \wedge A.E \Rightarrow E \Rightarrow B = S$	$\wedge I$ 14,13

Given:

$A \Rightarrow E \Rightarrow B = (C) @ S$

FORMULAE 0  $A.E \Rightarrow E \Rightarrow B = S$ , 1  $A = C$ , 2  $A \neq B$ , 3  $A \neq B \wedge A = C$ , 4  $A \neq B \wedge A = C \wedge A.E \Rightarrow E \Rightarrow B = S$ , 5  $A = C \wedge A.E \Rightarrow E \Rightarrow B = S$ , 6  $(A) @ A.E \Rightarrow E \Rightarrow B = (C) @ S$ , 7  $A$ , 8  $C$ , 9  $A.E \Rightarrow E \Rightarrow B$ , 10  $S$ , 11  $xx5$ , 12  $A \Rightarrow E \Rightarrow B = (C) @ S$ , 13  $B$ , 14  $E$ , 15  $A \Rightarrow E \Rightarrow B$ , 16  $(A) @ A.E \Rightarrow E \Rightarrow B$ , 17  $xx4$ , 18  $xx4 = (C) @ S$ , 19  $()$ , 20  $(C)$ , 21  $() @ (C)$ , 22  $xx3$ , 23  $xx3 @ S \neq ()$ , 24  $(C) @ S$ , 25  $\neg ( () = (C) @ S )$ , 26  $xx2$ , 27  $() \neq (C) @ S$ , 28  $() = (C) @ S$ , 29  $A = B$ , 30  $xx1$ , 31  $xx1 = (C) @ S$ , 32  $\perp$ , 33  $\neg (A = B)$ , 34  $xx$

SEQ

```
(cut«12,4/B,C»)
(GIVEN 0)
(cut«2,4/B,C»)
(LAYOUT "¬(A=B)≐A≠B" ALL
 ("rewrite≐"«2,34,33,34/A,xx,B,P»)
 (LAYOUT HIDEROOT
 ("symmetric≐"«2,33/B,A»)
 (LAYOUT HIDEROOT
 ("¬(A=B)≐A≠B"«7,13/A,B»)))
 ("¬I"«29/A»)
 (cut«28,32/B,C»)
 (LAYOUT "FROM A=B INFER A⇒E⇒B=()" ALL
 ("rewrite="«19,30,15,31/A,xx,B,P»)
 (LAYOUT HIDEROOT
 ("symmetric="«15,19/A,B»)
 (LAYOUT HIDEROOT
 ("FROM A=B INFER A⇒E⇒B=()"«7,13,14/A,B,E»)
 (hyp«29/A»)))
 (hyp«12/A»)
 ("¬E"«28/B»)
 (hyp«28/A»)
 (LAYOUT "¬(A=B)≐A≠B" ALL
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("rewrite $\hat{=}$ " $\langle$ 25,26,27,26/A,xx,B,P $\rangle$ )
(LAYOUT HIDEROOT
  ("-(A=B) $\hat{=}$ A $\neq$ B" $\langle$ 19,24/A,B $\rangle$ ))
(LAYOUT HIDEROOT
  ("symmetric $\neq$ " $\langle$ 24,19/A,B $\rangle$ )
  (LAYOUT "() $\hat{=}$ S" ALL
    ("rewrite $\hat{=}$ " $\langle$ 20,22,21,23/A,xx,B,P $\rangle$ )
    (LAYOUT HIDEROOT
      ("symmetric $\hat{=}$ " $\langle$ 21,20/A,B $\rangle$ )
      (LAYOUT HIDEROOT
        ("() $\hat{=}$ S" $\langle$ 20/S $\rangle$ )))
      ("R@(A)@T $\neq$ ()" $\langle$ 8,19,10/A,R,T $\rangle$ ))))))
(cut $\langle$ 6,4/B,C $\rangle$ )
(LAYOUT "FROM A $\neq$ B INFER A $\Rightarrow$ E $\Rightarrow$ B=(A)@A.E $\Rightarrow$ E $\Rightarrow$ B" ALL
  ("rewrite $\hat{=}$ " $\langle$ 16,17,15,18/A,xx,B,P $\rangle$ )
  (LAYOUT HIDEROOT
    ("symmetric $\hat{=}$ " $\langle$ 15,16/A,B $\rangle$ )
    (LAYOUT HIDEROOT
      ("FROM A $\neq$ B INFER A $\Rightarrow$ E $\Rightarrow$ B=(A)@A.E $\Rightarrow$ E $\Rightarrow$ B" $\langle$ 7,13,14/A,B,E $\rangle$ )
      (hyp $\langle$ 2/A $\rangle$ )))
    (hyp $\langle$ 12/A $\rangle$ ))
  (cut $\langle$ 5,4/B,C $\rangle$ )
  (LAYOUT "(A)@R=(B)@S $\hat{=}$ A=B $\wedge$ R=S" ALL
    ("rewrite $\hat{=}$ " $\langle$ 5,11,6,11/A,xx,B,P $\rangle$ )
    (LAYOUT HIDEROOT
      ("symmetric $\hat{=}$ " $\langle$ 6,5/A,B $\rangle$ )
      (LAYOUT HIDEROOT
        ("(A)@R=(B)@S $\hat{=}$ A=B $\wedge$ R=S" $\langle$ 7,8,9,10/A,B,R,S $\rangle$ )))
      (hyp $\langle$ 6/A $\rangle$ ))
    (cut $\langle$ 1,4/B,C $\rangle$ )
    (LAYOUT " $\wedge$ -E" ALL
      (" $\wedge$ -E(L)" $\langle$ 0,1/B,A $\rangle$ )
      (hyp $\langle$ 5/A $\rangle$ ))
    (cut $\langle$ 0,4/B,C $\rangle$ )
    (LAYOUT " $\wedge$ -E" ALL
      (" $\wedge$ -E(R)" $\langle$ 1,0/A,B $\rangle$ )
      (hyp $\langle$ 5/A $\rangle$ ))
    (cut $\langle$ 1,4/B,C $\rangle$ )
    (hyp $\langle$ 1/A $\rangle$ )
    (cut $\langle$ 0,4/B,C $\rangle$ )
    (hyp $\langle$ 0/A $\rangle$ )
    (LAYOUT COMPRESS " $\wedge$ -I" ALL
      (" $\wedge$ -I" $\langle$ 3,0/A,B $\rangle$ )
      (LAYOUT COMPRESS " $\wedge$ -I" ALL
        (" $\wedge$ -I" $\langle$ 2,1/A,B $\rangle$ )
        (hyp $\langle$ 2/A $\rangle$ )
        (hyp $\langle$ 1/A $\rangle$ )
        (hyp $\langle$ 0/A $\rangle$ ))
      (hyp $\langle$ 0/A $\rangle$ ))
  )

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DERIVED RULE IS FROM  $A \Rightarrow E \Rightarrow B = S @ (C)$  AND  $\text{list}(A \Rightarrow E \Rightarrow B)$  INFER  $A \neq B \wedge A \Rightarrow E \Rightarrow C = S \wedge C.E = B$ 

1: $A \Rightarrow E \Rightarrow B = S @ (C)$	$A \Rightarrow E \Rightarrow B = S @ (C)$
2: $\text{list}(A \Rightarrow E \Rightarrow B)$	$\text{list}(A \Rightarrow E \Rightarrow B)$
3: $\text{list}(S @ (C))$	rewrite= 1,2
4: $\text{list } S \wedge \text{list}(C) \wedge S \neg \neg (C)$	$\text{list}(R @ S) = \text{list } R \wedge \text{list } S \wedge R \neg \neg S$
5: $\text{list } S$	$\wedge\text{-E } 4$
6: $\text{list}(C)$	$\wedge\text{-E } 4$
7: $S \neg \neg (C)$	$\wedge\text{-E } 4$
8: finitesequence $S$	FROM list $S$ INFER finitesequence $S$
9: $\text{list}(cc \Rightarrow E \Rightarrow B)$	assumption
10: $cc \Rightarrow E \Rightarrow B = () @ (C)$	assumption
11: $cc \Rightarrow E \Rightarrow B = (C)$	$() @ S = S$ 10
12: $cc \Rightarrow E \Rightarrow B = (C) @ ()$	$S @ () = S$ 11
13: $cc \neq B \wedge cc = C \wedge cc.E \Rightarrow E \Rightarrow B = ()$	Derived Rule FROM $A \Rightarrow E \Rightarrow B = (C) @ S$ INFER $A \neq B \wedge A = C \wedge A.E \Rightarrow E \Rightarrow B = S$ 12
14: $cc \neq B$	$\wedge\text{-E } 13$
15: $cc = C$	$\wedge\text{-E } 13$
16: $cc.E \Rightarrow E \Rightarrow B = ()$	$\wedge\text{-E } 13$
17: $cc \Rightarrow E \Rightarrow C = ()$	Derived Rule FROM $A = B$ INFER $A \Rightarrow E \Rightarrow B = ()$ 15
18: $cc.E = B$	Derived Rule FROM $A \Rightarrow E \Rightarrow B = ()$ INFER $A = B$ 16
19: $C = cc$	symmetric= 15
20: $C.E = B$	rewrite= 19,18
21: $cc \neq B \wedge cc \Rightarrow E \Rightarrow C = ()$	$\wedge\text{-I } 14,17$
22: $cc \neq B \wedge cc \Rightarrow E \Rightarrow C = () \wedge C.E = B$	$\wedge\text{-I } 21,20$
23: $cc \Rightarrow E \Rightarrow B = () @ (C) \rightarrow cc \neq B \wedge cc \Rightarrow E \Rightarrow C = () \wedge C.E = B$	$\rightarrow\text{I } 10\text{-}22$
24: $\text{list}(cc \Rightarrow E \Rightarrow B) \rightarrow cc \Rightarrow E \Rightarrow B = () @ (C) \rightarrow cc \neq B \wedge cc \Rightarrow E \Rightarrow C = () \wedge C.E = B$	$\rightarrow\text{I } 9\text{-}23$
25: $\forall xA: (\text{list}(xA \Rightarrow E \Rightarrow B) \rightarrow xA \Rightarrow E \Rightarrow B = () @ (C) \rightarrow xA \neq B \wedge xA \Rightarrow E \Rightarrow C = () \wedge C.E = B)$	$\forall\text{-I } 24$
26: $\forall xA: (\text{list}(xA \Rightarrow E \Rightarrow B) \rightarrow xA \Rightarrow E \Rightarrow B = ys @ (C) \rightarrow xA \neq B \wedge xA \Rightarrow E \Rightarrow C = ys \wedge C.E = B)$	assumption
27: $\text{list}(cc1 \Rightarrow E \Rightarrow B)$	assumption
28: $cc1 \Rightarrow E \Rightarrow B = (y) @ ys @ (C)$	assumption
29: $cc1 \Rightarrow E \Rightarrow B = (y) @ (ys @ (C))$	$(R @ S) @ T = R @ (S @ T)$ 28
30: $cc1 \neq B \wedge cc1 = y \wedge cc1.E \Rightarrow E \Rightarrow B = ys @ (C)$	Derived Rule FROM $A \Rightarrow E \Rightarrow B = (C) @ S$ INFER $A \neq B \wedge A = C \wedge A.E \Rightarrow E \Rightarrow B = S$ 29
31: $cc1 \neq B$	$\wedge\text{-E } 30$
32: $cc1 = y$	$\wedge\text{-E } 30$
33: $cc1.E \Rightarrow E \Rightarrow B = ys @ (C)$	$\wedge\text{-E } 30$
34: $\text{list}(cc1.E \Rightarrow E \Rightarrow B) \rightarrow cc1.E \Rightarrow E \Rightarrow B = ys @ (C) \rightarrow cc1.E \neq B \wedge cc1.E \Rightarrow E \Rightarrow C = ys \wedge C.E = B$	$\forall\text{-E } 26$
35: $\text{list}(cc1.E \Rightarrow E \Rightarrow B)$	Derived Rule FROM $A \neq B$ AND $\text{list}(A \Rightarrow E \Rightarrow B)$ INFER $\text{list}(A.E \Rightarrow E \Rightarrow B)$ 31,27
36: $cc1.E \Rightarrow E \Rightarrow B = ys @ (C) \rightarrow cc1.E \neq B \wedge cc1.E \Rightarrow E \Rightarrow C = ys \wedge C.E = B$	$\rightarrow\text{E } 35,34$
37: $cc1.E \neq B \wedge cc1.E \Rightarrow E \Rightarrow C = ys \wedge C.E = B$	$\rightarrow\text{E } 33,36$
38: $cc1.E \neq B$	$\wedge\text{-E } 37$
39: $cc1.E \Rightarrow E \Rightarrow C = ys$	$\wedge\text{-E } 37$
40: $C.E = B$	$\wedge\text{-E } 37$
41: $(y) @ (ys @ (C)) = cc1 \Rightarrow E \Rightarrow B$	symmetric= 29
42: $\text{list}(y) @ (ys @ (C))$	rewrite= 41,27
43: $(y) \neg \neg ys @ (C)$	Conjectured Rule FROM $\text{list}(A @ S)$ INFER $(A) \neg \neg S$ 42
44: $(y) \neg \neg ys @ (C) \neq (y) \neg \neg ys \wedge (y) \neg \neg (C)$	$R \neg \neg S @ T = R \neg \neg S \wedge R \neg \neg T$
45: $(y) \neg \neg ys \wedge (y) \neg \neg (C) \neq (y) \neg \neg ys @ (C)$	symmetric= 44
46: $(y) \neg \neg ys \wedge (y) \neg \neg (C)$	$R \neg \neg S @ T = R \neg \neg S \wedge R \neg \neg T$ 45,43
47: $(y) \neg \neg ys$	$\wedge\text{-E } 46$
48: $(y) \neg \neg (C)$	$\wedge\text{-E } 46$
49: $(y) \neg \neg (C) \neq y \neq C$	$(A) \neg \neg (B) \neq A \neq B$
50: $y \neq C \neq (y) \neg \neg (C)$	symmetric= 49
51: $y \neq C$	$(A) \neg \neg (B) \neq A \neq B$ 50,48
52: $cc1 \neq C$	rewrite= 32,51
53: $cc1 \Rightarrow E \Rightarrow C = (cc1) @ cc1.E \Rightarrow E \Rightarrow C$	Derived Rule FROM $A \neq B$ INFER $A \Rightarrow E \Rightarrow B = (A) @ A.E \Rightarrow E \Rightarrow B$ 52
54: $(cc1) @ cc1.E \Rightarrow E \Rightarrow C = (y) @ ys \neq cc1 = y \wedge cc1.E \Rightarrow E \Rightarrow C = ys$	$(A) @ R = (B) @ S \neq A = B \wedge R = S$
55: $cc1 = y \wedge cc1.E \Rightarrow E \Rightarrow C = ys$	$\wedge\text{-I } 32,39$
56: $(cc1) @ cc1.E \Rightarrow E \Rightarrow C = (y) @ ys$	$(A) @ R = (B) @ S \neq A = B \wedge R = S$ 54,55
57: $cc1 \Rightarrow E \Rightarrow C = (y) @ ys$	FROM $A \neq B$ INFER $A \Rightarrow E \Rightarrow B = (A) @ A.E \Rightarrow E \Rightarrow B$ 53,56
58: $cc1 \neq B \wedge cc1 \Rightarrow E \Rightarrow C = (y) @ ys$	$\wedge\text{-I } 31,57$
59: $cc1 \neq B \wedge cc1 \Rightarrow E \Rightarrow C = (y) @ ys \wedge C.E = B$	$\wedge\text{-I } 58,40$
60: $cc1 \Rightarrow E \Rightarrow B = (y) @ ys @ (C) \rightarrow cc1 \neq B \wedge cc1 \Rightarrow E \Rightarrow C = (y) @ ys \wedge C.E = B$	$\rightarrow\text{I } 28\text{-}59$
61: $\text{list}(cc1 \Rightarrow E \Rightarrow B) \rightarrow cc1 \Rightarrow E \Rightarrow B = (y) @ ys @ (C) \rightarrow cc1 \neq B \wedge cc1 \Rightarrow E \Rightarrow C = (y) @ ys \wedge C.E = B$	$\rightarrow\text{I } 27\text{-}60$
62: $\forall xA: (\text{list}(xA \Rightarrow E \Rightarrow B) \rightarrow xA \Rightarrow E \Rightarrow B = (y) @ ys @ (C) \rightarrow xA \neq B \wedge xA \Rightarrow E \Rightarrow C = (y) @ ys \wedge C.E = B)$	$\forall\text{-I } 61$
63: $\forall xs: (\text{finitesequence } xs \rightarrow \forall xA: (\text{list}(xA \Rightarrow E \Rightarrow B) \rightarrow xA \Rightarrow E \Rightarrow B = xs @ (C) \rightarrow xA \neq B \wedge xA \Rightarrow E \Rightarrow C = xs \wedge C.E = B))$	finite sequence induction (L) 25,26-62
64: finitesequence $S \rightarrow \forall xA: (\text{list}(xA \Rightarrow E \Rightarrow B) \rightarrow xA \Rightarrow E \Rightarrow B = S @ (C) \rightarrow xA \neq B \wedge xA \Rightarrow E \Rightarrow C = S \wedge C.E = B)$	$\forall\text{-E } 63$
65: $\forall xA: (\text{list}(xA \Rightarrow E \Rightarrow B) \rightarrow xA \Rightarrow E \Rightarrow B = S @ (C) \rightarrow xA \neq B \wedge xA \Rightarrow E \Rightarrow C = S \wedge C.E = B)$	$\rightarrow\text{E } 8,64$
66: $\text{list}(A \Rightarrow E \Rightarrow B) \rightarrow A \Rightarrow E \Rightarrow B = S @ (C) \rightarrow A \neq B \wedge A \Rightarrow E \Rightarrow C = S \wedge C.E = B$	$\forall\text{-E } 65$
67: $A \Rightarrow E \Rightarrow B = S @ (C) \rightarrow A \neq B \wedge A \Rightarrow E \Rightarrow C = S \wedge C.E = B$	$\rightarrow\text{E } 2,66$
68: $A \neq B \wedge A \Rightarrow E \Rightarrow C = S \wedge C.E = B$	$\rightarrow\text{E } 1,67$

Given:

 $A \Rightarrow E \Rightarrow B = S @ (C)$  $\text{list}(A \Rightarrow E \Rightarrow B)$

FORMULAE 0 C.E=B, 1 cc1 $\Rightarrow$ E $\Rightarrow$ C=(y)@ys, 2 cc1 $\neq$ B, 3 cc1 $\neq$ B $\wedge$ cc1 $\Rightarrow$ E $\Rightarrow$ C=(y)@ys,  
 4 cc1 $\neq$ B $\wedge$ cc1 $\Rightarrow$ E $\Rightarrow$ C=(y)@ys $\wedge$ C.E=B, 5 cc1.E $\Rightarrow$ E $\Rightarrow$ C=ys, 6 cc1=y, 7 cc1, 8 y, 9 cc1.E $\Rightarrow$ E $\Rightarrow$ C, 10 ys,  
 11 (cc1)@cc1.E $\Rightarrow$ E $\Rightarrow$ C=(y)@ys, 12 xx7, 13 cc1=y $\wedge$ cc1.E $\Rightarrow$ E $\Rightarrow$ C=ys, 14 y $\neq$ C, 15 x $\neq$ C, 16 x, 17 C, 18 E,  
 19 cc1 $\Rightarrow$ E $\Rightarrow$ C, 20 xx6, 21 (cc1)@cc1.E $\Rightarrow$ E $\Rightarrow$ C, 22 xx6=(y)@ys, 23 (y) $\neg$ (C), 24 xx10, 25 (y) $\neg$ (y $\wedge$ (y) $\neg$ (C)),  
 26 (y) $\neg$ (ys), 27 (y) $\neg$ (ys@C), 28 (y), 29 (C), 30 xx8, 31 list((y)@(ys@C)), 32 ys@C, 33 list(cc1 $\Rightarrow$ E $\Rightarrow$ B),  
 34 cc1 $\Rightarrow$ E $\Rightarrow$ B=(y)@(ys@C), 35 cc1 $\Rightarrow$ E $\Rightarrow$ B, 36 (y)@(ys@C), 37 xx9, 38 list xx9,  
 39 cc1.E $\neq$ B $\wedge$ cc1.E $\Rightarrow$ E $\Rightarrow$ C=ys $\wedge$ C.E=B, 40 cc1.E $\neq$ B $\wedge$ cc1.E $\Rightarrow$ E $\Rightarrow$ C=ys, 41 cc1.E $\neq$ B,  
 42 cc1.E $\Rightarrow$ E $\Rightarrow$ B=ys@C $\rightarrow$ cc1.E $\neq$ B $\wedge$ cc1.E $\Rightarrow$ E $\Rightarrow$ C=ys $\wedge$ C.E=B, 43 cc1.E $\Rightarrow$ E $\Rightarrow$ B=ys@C,  
 44 list(cc1.E $\Rightarrow$ E $\Rightarrow$ B) $\rightarrow$ cc1.E $\Rightarrow$ E $\Rightarrow$ B=ys@C $\rightarrow$ cc1.E $\neq$ B $\wedge$ cc1.E $\Rightarrow$ E $\Rightarrow$ C=ys $\wedge$ C.E=B, 45 list(cc1.E $\Rightarrow$ E $\Rightarrow$ B), 46 B,  
 47  $\forall$ xA:(list(xA $\Rightarrow$ E $\Rightarrow$ B) $\rightarrow$ xA $\Rightarrow$ E $\Rightarrow$ B=ys@C $\rightarrow$ xA $\neq$ B $\wedge$ xA $\Rightarrow$ E $\Rightarrow$ C=ys $\wedge$ C.E=B), 48 cc1.E,  
 49 list(xA $\Rightarrow$ E $\Rightarrow$ B) $\rightarrow$ xA $\Rightarrow$ E $\Rightarrow$ B=ys@C $\rightarrow$ xA $\neq$ B $\wedge$ xA $\Rightarrow$ E $\Rightarrow$ C=ys $\wedge$ C.E=B, 50 xA,  
 51 cc1 $\neq$ B $\wedge$ cc1=y $\wedge$ cc1.E $\Rightarrow$ E $\Rightarrow$ B=ys@C, 52 cc1 $\neq$ B $\wedge$ cc1=y, 53 cc1 $\Rightarrow$ E $\Rightarrow$ B=(y)@ys@C, 54 (y)@ys@C, 55 xx4,  
 56 cc1 $\Rightarrow$ E $\Rightarrow$ B=xx4, 57 cc1 $\Rightarrow$ E $\Rightarrow$ B=(y)@ys@C $\rightarrow$ cc1 $\neq$ B $\wedge$ cc1 $\Rightarrow$ E $\Rightarrow$ C=(y)@ys $\wedge$ C.E=B,  
 58 list(xA $\Rightarrow$ E $\Rightarrow$ B) $\rightarrow$ xA $\Rightarrow$ E $\Rightarrow$ B=(y)@ys@C $\rightarrow$ xA $\neq$ B $\wedge$ xA $\Rightarrow$ E $\Rightarrow$ C=(y)@ys $\wedge$ C.E=B, 59 cc $\Rightarrow$ E $\Rightarrow$ C=(), 60 cc $\neq$ B,  
 61 cc $\neq$ B $\wedge$ cc $\Rightarrow$ E $\Rightarrow$ C=(), 62 cc $\neq$ B $\wedge$ cc $\Rightarrow$ E $\Rightarrow$ C=() $\wedge$ C.E=B, 63 cc.E=B, 64 cc=C, 65 cc, 66 xx5, 67 xx5.E=B,  
 68 cc.E $\Rightarrow$ E $\Rightarrow$ B=(), 69 cc.E, 70 cc $\neq$ B $\wedge$ cc=C $\wedge$ cc.E $\Rightarrow$ E $\Rightarrow$ B=(), 71 cc $\neq$ B $\wedge$ cc=C, 72 cc $\Rightarrow$ E $\Rightarrow$ B=C@(), 73 (),  
 74 cc $\Rightarrow$ E $\Rightarrow$ B=C, 75 (C)@(), 76 xx3, 77 cc $\Rightarrow$ E $\Rightarrow$ B=xx3, 78 cc $\Rightarrow$ E $\Rightarrow$ B=()@C, 79 ()@C, 80 xx2,  
 81 cc $\Rightarrow$ E $\Rightarrow$ B=xx2, 82 list(cc $\Rightarrow$ E $\Rightarrow$ B), 83 cc $\Rightarrow$ E $\Rightarrow$ B=()@C $\rightarrow$ cc $\neq$ B $\wedge$ cc $\Rightarrow$ E $\Rightarrow$ C=() $\wedge$ C.E=B,  
 84 list(xA $\Rightarrow$ E $\Rightarrow$ B) $\rightarrow$ xA $\Rightarrow$ E $\Rightarrow$ B=()@C $\rightarrow$ xA $\neq$ B $\wedge$ xA $\Rightarrow$ E $\Rightarrow$ C=()@C.E=B,  
 85  $\forall$ xA:(list(xA $\Rightarrow$ E $\Rightarrow$ B) $\rightarrow$ xA $\Rightarrow$ E $\Rightarrow$ B=xs@C $\rightarrow$ xA $\neq$ B $\wedge$ xA $\Rightarrow$ E $\Rightarrow$ C=xs $\wedge$ C.E=B), 86 xs, 87 S,  
 88 finitesequence xs $\rightarrow$  $\forall$ xA:(list(xA $\Rightarrow$ E $\Rightarrow$ B) $\rightarrow$ xA $\Rightarrow$ E $\Rightarrow$ B=xs@C $\rightarrow$ xA $\neq$ B $\wedge$ xA $\Rightarrow$ E $\Rightarrow$ C=xs $\wedge$ C.E=B), 89 list S,  
 90 finitesequence S, 91  $\forall$ xA:(list(xA $\Rightarrow$ E $\Rightarrow$ B) $\rightarrow$ xA $\Rightarrow$ E $\Rightarrow$ B=S@C $\rightarrow$ xA $\neq$ B $\wedge$ xA $\Rightarrow$ E $\Rightarrow$ C=S $\wedge$ C.E=B), 92 A,  
 93 list(xA $\Rightarrow$ E $\Rightarrow$ B) $\rightarrow$ xA $\Rightarrow$ E $\Rightarrow$ B=S@C $\rightarrow$ xA $\neq$ B $\wedge$ xA $\Rightarrow$ E $\Rightarrow$ C=S $\wedge$ C.E=B, 94 list(A $\Rightarrow$ E $\Rightarrow$ B),  
 95 A $\Rightarrow$ E $\Rightarrow$ B=S@C $\rightarrow$ A $\neq$ B $\wedge$ A $\Rightarrow$ E $\Rightarrow$ C=S $\wedge$ C.E=B, 96 A $\Rightarrow$ E $\Rightarrow$ B=S@C, 97 A $\neq$ B $\wedge$ A $\Rightarrow$ E $\Rightarrow$ C=S $\wedge$ C.E=B,  
 98 list S $\wedge$ list(C) $\wedge$ S $\neg$ (C), 99 list S $\wedge$ list(C), 100 S $\neg$ (C), 101 list(C), 102 list(S@C), 103 xx, 104 A $\Rightarrow$ E $\Rightarrow$ B,  
 105 S@C, 106 xx1, 107 list xx1

SEQ

(cut«96,97/B,C»)

(GIVEN 0)

(cut«94,97/B,C»)

(GIVEN 1)

(cut«102,97/B,C»)

("rewrite="«105,106,104,107/A,xx,B,P»)

(LAYOUT HIDEROOT

  ("symmetric="«104,105/A,B»)

  (hyp«96/A»)

(hyp«94/A»)

(cut«98,97/B,C»)

(LAYOUT "list(R@S)=list R $\wedge$ list S $\wedge$ R $\neg$ (S)" ALL

  ("rewrite="«98,103,102,103/A,xx,B,P»)

  (LAYOUT HIDEROOT

    ("symmetric="«102,98/A,B»)

    (LAYOUT HIDEROOT

      ("list(R@S)=list R $\wedge$ list S $\wedge$ R $\neg$ (S)"«87,29/R,S»))

    (hyp«102/A»)

(LAYOUT HIDEDECUT

  (cut«99,97/B,C»)

  ("^-E(L)"«100,99/B,A»)

  (hyp«98/A»)

  (cut«89,97/B,C»)

  (LAYOUT "^-E" ALL

    ("^-E(L)"«101,89/B,A»)

    (hyp«99/A»)

  (cut«101,97/B,C»)

  (LAYOUT "^-E" ALL

    ("^-E(R)"«89,101/A,B»)

    (hyp«99/A»)

  (cut«100,97/B,C»)

  (LAYOUT "^-E" ALL

    ("^-E(R)"«99,100/A,B»)

    (hyp«98/A»)

  ("→E"«96,97/A,B»)

  (hyp«96/A»)

  ("→E"«94,95/A,B»)

  (hyp«94/A»)

  ("∀E"«92,93,50/B,A,x»)

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("→-E"«90,91/A,B»)
("FROM list S INFER finitesequence S"«87/S»)
(hyp«89/A»)
("∀-E"«87,88,86/B,A,x»)
("finite sequence induction (L)"«8,10,85,86/y,ys,P,xs»)
("∀-I"«65,84,50/cc,A,x»)
("→-I"«82,83/A,B»)
("→-I"«78,62/A,B»)
(cut«74,62/B,C»)
(LAYOUT "()"@S≐S" ALL
  ("rewrite="«29,80,79,81/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("symmetric≐"«79,29/A,B»)
    (LAYOUT HIDEROOT
      ("()"@S≐S"«29/S»)))
  (hyp«78/A»))
(cut«72,62/B,C»)
(LAYOUT "S@()"≐S" ALL
  ("rewrite="«75,76,29,77/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("S@()"≐S"«29/S»))
  (hyp«74/A»))
(cut«70,62/B,C»)
("FROM A⇒E⇒B=(C)@S INFER A≐B∧A=C∧A.E⇒E⇒B=S"«65,46,17,18,73/A,B,C,E,S»)
(hyp«72/A»)
(LAYOUT HIDE CUT
  (cut«71,62/B,C»)
  ("∧-E(L)"«68,71/B,A»)
  (hyp«70/A»)
  (cut«60,62/B,C»)
  (LAYOUT "∧-E" ALL
    ("∧-E(L)"«64,60/B,A»)
    (hyp«71/A»))
  (cut«64,62/B,C»)
  (LAYOUT "∧-E" ALL
    ("∧-E(R)"«60,64/A,B»)
    (hyp«71/A»))
  (cut«68,62/B,C»)
  (LAYOUT "∧-E" ALL
    ("∧-E(R)"«71,68/A,B»)
    (hyp«70/A»))
  (cut«60,62/B,C»)
  (hyp«60/A»)
  (cut«59,62/B,C»)
  ("FROM A=B INFER A⇒E⇒B=()"«65,17,18/A,B,E»)
  (hyp«64/A»)
  (cut«63,62/B,C»)
  ("FROM A⇒E⇒B=() INFER A=B"«69,46,18/A,B,E»)
  (hyp«68/A»)
  (cut«0,62/B,C»)
  ("rewrite="«17,66,65,67/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("symmetric="«65,17/A,B»)
    (hyp«64/A»))
  (hyp«63/A»)
  (cut«0,62/B,C»)
  (hyp«0/A»)
  (LAYOUT COMPRESS "∧-I" ALL
    ("∧-I"«61,0/A,B»)
    (LAYOUT COMPRESS "∧-I" ALL
      ("∧-I"«60,59/A,B»)
      (hyp«60/A»)
      (hyp«59/A»))
    (hyp«0/A»)))
("∀-I"«7,58,50/cc,A,x»)
("→-I"«33,57/A,B»)
("→-I"«53,4/A,B»)

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(cut«34,4/B,C»)
(LAYOUT "(R@S)@T≠R@(S@T)" ALL
("rewrite="«36,55,54,56/A,xx,B,P»)
(LAYOUT HIDEROOT
("symmetric="«54,36/A,B»)
(LAYOUT HIDEROOT
("(R@S)@T≠R@(S@T)"«28,10,29/R,S,T»)))
(hyp«53/A»)
(cut«51,4/B,C»)
("FROM A⇒E⇒B=(C)@S INFER A≠B∧A=C∧A.E⇒E⇒B=S"«7,46,8,18,32/A,B,C,E,S»)
(hyp«34/A»)
(LAYOUT HIDEDECUT
(cut«52,4/B,C»)
("∧-E(L)"«43,52/B,A»)
(hyp«51/A»)
(cut«2,4/B,C»)
(LAYOUT "∧-E" ALL
("∧-E(L)"«6,2/B,A»)
(hyp«52/A»))
(cut«6,4/B,C»)
(LAYOUT "∧-E" ALL
("∧-E(R)"«2,6/A,B»)
(hyp«52/A»))
(cut«43,4/B,C»)
(LAYOUT "∧-E" ALL
("∧-E(R)"«52,43/A,B»)
(hyp«51/A»))
(cut«2,4/B,C»)
(hyp«2/A»)
(cut«44,4/B,C»)
("∀-E"«48,49,50/B,A,x»)
(hyp«47/A»)
(cut«45,4/B,C»)
("FROM A≠B AND list(A⇒E⇒B) INFER list(A.E⇒E⇒B)"«7,46,18/A,B,E»)
(hyp«2/A»)
(hyp«33/A»)
(cut«42,4/B,C»)
("→-E"«45,42/A,B»)
(hyp«45/A»)
(hyp«44/A»)
(cut«39,4/B,C»)
("→-E"«43,39/A,B»)
(hyp«43/A»)
(hyp«42/A»)
(LAYOUT HIDEDECUT
(cut«40,4/B,C»)
("∧-E(L)"«0,40/B,A»)
(hyp«39/A»)
(cut«41,4/B,C»)
(LAYOUT "∧-E" ALL
("∧-E(L)"«5,41/B,A»)
(hyp«40/A»))
(cut«5,4/B,C»)
(LAYOUT "∧-E" ALL
("∧-E(R)"«41,5/A,B»)
(hyp«40/A»))
(cut«0,4/B,C»)
(LAYOUT "∧-E" ALL
("∧-E(R)"«40,0/A,B»)
(hyp«39/A»))
(cut«6,4/B,C»)
(hyp«6/A»)
(cut«5,4/B,C»)
(hyp«5/A»)
(cut«31,4/B,C»)
("rewrite="«36,37,35,38/A,xx,B,P»)
(LAYOUT HIDEROOT

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("symmetric="«35,36/A,B»)
(hyp«34/A»)
(hyp«33/A»)
(cut«27,4/B,C»)
("FROM list((A)@S) INFER (A)¬∩S"«8,32/A,S»)
(hyp«31/A»)
(cut«25,4/B,C»)
(LAYOUT "R¬∩S@T≐R¬∩S∧R¬∩T" ALL
("rewrite≐"«25,30,27,30/A,xx,B,P»)
(LAYOUT HIDEROOT
("symmetric≐"«27,25/A,B»)
(LAYOUT HIDEROOT
("R¬∩S@T≐R¬∩S∧R¬∩T"«28,10,29/R,S,T»)))
(hyp«27/A»)
(cut«26,4/B,C»)
(LAYOUT "∧-E" ALL
("∧-E(L)"«23,26/B,A»)
(hyp«25/A»)
(cut«23,4/B,C»)
(LAYOUT "∧-E" ALL
("∧-E(R)"«26,23/A,B»)
(hyp«25/A»)
(cut«14,4/B,C»)
(LAYOUT "(A)¬∩(B)≐A≠B" ALL
("rewrite≐"«14,24,23,24/A,xx,B,P»)
(LAYOUT HIDEROOT
("symmetric≐"«23,14/A,B»)
(LAYOUT HIDEROOT
("(A)¬∩(B)≐A≠B"«8,17/A,B»)))
(hyp«23/A»)
(cut«1,4/B,C»)
(LAYOUT "FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B" ALL
("rewrite="«19,20,21,22/A,xx,B,P»)
(LAYOUT HIDEROOT
("FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B"«7,17,18/A,B,E»)
("rewrite="«15,7,16,8/P,A,xx,B»)
(hyp«6/A»)
(hyp«14/A»)
(LAYOUT "(A)@R=(B)@S≐A=B∧R=S" ALL
("rewrite≐"«11,12,13,12/A,xx,B,P»)
(LAYOUT HIDEROOT
("(A)@R=(B)@S≐A=B∧R=S"«7,8,9,10/A,B,R,S»))
(LAYOUT COMPRESS "∧-I" ALL
("∧-I"«6,5/A,B»)
(hyp«6/A»)
(hyp«5/A»))))
(cut«0,4/B,C»)
(hyp«0/A»)
(LAYOUT COMPRESS "∧-I" ALL
("∧-I"«3,0/A,B»)
(LAYOUT COMPRESS "∧-I" ALL
("∧-I"«2,1/A,B»)
(hyp«2/A»)
(hyp«1/A»)
(hyp«0/A»))))

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DERIVED RULE IS FROM  $A \neq B$  AND  $\text{list}(A \Rightarrow E \Rightarrow B)$  INFER  $A \neq A.E$

1: $A = A.E$	assumption
2: $A \neq B$	$A \neq B$
3: $A \Rightarrow E \Rightarrow B = (A) @ A.E \Rightarrow E \Rightarrow B$	Derived Rule FROM $A \neq B$ INFER $A \Rightarrow E \Rightarrow B = (A) @ A.E \Rightarrow E \Rightarrow B$ 2
4: $A \Rightarrow E \Rightarrow B = (A) @ A \Rightarrow E \Rightarrow B$	rewrite= 1,3
5: $\text{list}(A \Rightarrow E \Rightarrow B)$	$\text{list}(A \Rightarrow E \Rightarrow B)$
6: $\text{list}((A) @ A \Rightarrow E \Rightarrow B)$	rewrite= 4,5
7: $\text{list}(A) \wedge \text{list}(A \Rightarrow E \Rightarrow B) \wedge (A) \neg \neg A \Rightarrow E \Rightarrow B$	$\text{list}(R @ S) \doteq \text{list } R \wedge \text{list } S \wedge R \neg \neg S$ 6
8: $\text{list}(A)$	$\wedge$ -E 7
9: $\text{list}(A \Rightarrow E \Rightarrow B)$	$\wedge$ -E 7
10: $(A) \neg \neg A \Rightarrow E \Rightarrow B$	$\wedge$ -E 7
11: $(A) @ A \Rightarrow E \Rightarrow B = A \Rightarrow E \Rightarrow B$	symmetric= 4
12: $(A) \neg \neg (A) @ A \Rightarrow E \Rightarrow B$	rewrite= 11,10
13: $(A) \neg \neg (A) @ A \Rightarrow E \Rightarrow B \doteq (A) \neg \neg (A) \wedge (A) \neg \neg A \Rightarrow E \Rightarrow B$	$R \neg \neg S @ T \doteq R \neg \neg S \wedge R \neg \neg T$
14: $(A) \neg \neg (A) \wedge (A) \neg \neg A \Rightarrow E \Rightarrow B \doteq (A) \neg \neg (A) @ A \Rightarrow E \Rightarrow B$	symmetric= 13
15: $(A) \neg \neg (A) \wedge (A) \neg \neg A \Rightarrow E \Rightarrow B$	$R \neg \neg S @ T \doteq R \neg \neg S \wedge R \neg \neg T$ 14,12
16: $(A) \neg \neg (A)$	$\wedge$ -E 15
17: $(A) \neg \neg A \Rightarrow E \Rightarrow B$	$\wedge$ -E 15
18: $(A) \neg \neg (A) \doteq A \neq A$	$(A) \neg \neg (B) \doteq A \neq B$
19: $A \neq A \doteq (A) \neg \neg (A)$	symmetric= 18
20: $A \neq A$	$(A) \neg \neg (B) \doteq A \neq B$ 19,16
21: $\neg(A = A) \doteq A \neq A$	$\neg(A = B) \doteq A \neq B$
22: $\neg(A = A)$	$\neg(A = B) \doteq A \neq B$ 21,20
23: $A = A$	reflexive=
24: $\perp$	$\neg$ -E 23,22
25: $\neg(A = A.E)$	$\neg$ -I 1-24
26: $A \neq A.E$	$\neg(A = B) \doteq A \neq B$ 25

Given:  
 $A \neq B$   
 $\text{list}(A \Rightarrow E \Rightarrow B)$

FORMULAE 0  $\neg(A = A)$ , 1  $A$ , 2  $A = A$ , 3  $A \neq A$ , 4  $xx7$ , 5  $\perp$ , 6  $(A) \neg \neg (A)$ , 7  $xx6$ , 8  $(A) \neg \neg (A) \wedge (A) \neg \neg A \Rightarrow E \Rightarrow B$ , 9  $(A) \neg \neg A \Rightarrow E \Rightarrow B$ , 10  $(A) \neg \neg (A) @ A \Rightarrow E \Rightarrow B$ , 11  $(A)$ , 12  $A \Rightarrow E \Rightarrow B$ , 13  $xx4$ , 14  $A \Rightarrow E \Rightarrow B = (A) @ A \Rightarrow E \Rightarrow B$ , 15  $(A) @ A \Rightarrow E \Rightarrow B$ , 16  $xx5$ , 17  $(A) \neg \neg xx5$ , 18  $\text{list}(A) \wedge \text{list}(A \Rightarrow E \Rightarrow B) \wedge (A) \neg \neg A \Rightarrow E \Rightarrow B$ , 19  $\text{list}(A) \wedge \text{list}(A \Rightarrow E \Rightarrow B)$ , 20  $\text{list}(A)$ , 21  $\text{list}(A \Rightarrow E \Rightarrow B)$ , 22  $\text{list}((A) @ A \Rightarrow E \Rightarrow B)$ , 23  $xx1$ , 24  $xx3$ , 25  $\text{list}(xx3)$ , 26  $A \Rightarrow E \Rightarrow B = (A) @ A.E \Rightarrow E \Rightarrow B$ , 27  $A = A.E$ , 28  $xx2$ , 29  $A.E$ , 30  $A \Rightarrow E \Rightarrow B = (A) @ xx2 \Rightarrow E \Rightarrow B$ , 31  $B$ , 32  $E$ , 33  $A \neq A.E$ , 34  $\neg(A = A.E)$ , 35  $xx$

LAYOUT " $\neg(A = B) \doteq A \neq B$ " ALL  
 ("rewrite="«33,35,34,35/A,xx,B,P») (LAYOUT HIDEROOT  
 ("symmetric="«33,34/B,A») (LAYOUT HIDEROOT  
 ("¬(A=B)≐A≠B"«1,29/A,B»))) ("¬-I"«27/A») (cut«26,5/B,C») ("FROM  $A \neq B$  INFER  $A \Rightarrow E \Rightarrow B = (A) @ A.E \Rightarrow E \Rightarrow B$ "«1,31,32/A,B,E») (GIVEN 0) (cut«14,5/B,C») ("rewrite="«1,28,29,30/A,xx,B,P») (hyp«27/A») (hyp«26/A») (cut«21,5/B,C»)

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(GIVEN 1)
(cut«22,5/B,C»)
("rewrite="«15,24,12,25/A,xx,B,P»)
(LAYOUT HIDEROOT
  ("symmetric="«12,15/A,B»)
  (hyp«14/A»)
(hyp«21/A»)
(cut«18,5/B,C»)
(LAYOUT "list(R@S)≐list R^list S^R¬∩S" ALL
  ("rewrite≐"«18,23,22,23/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("symmetric≐"«22,18/A,B»)
    (LAYOUT HIDEROOT
      ("list(R@S)≐list R^list S^R¬∩S"«11,12/R,S»)))
    (hyp«22/A»)
(LAYOUT HIDE CUT
  (cut«19,5/B,C»)
  ("^E(L)"«9,19/B,A»)
  (hyp«18/A»)
  (cut«20,5/B,C»)
  (LAYOUT "^E" ALL
    ("^E(L)"«21,20/B,A»)
    (hyp«19/A»)
  (cut«21,5/B,C»)
  (LAYOUT "^E" ALL
    ("^E(R)"«20,21/A,B»)
    (hyp«19/A»)
  (cut«9,5/B,C»)
  (LAYOUT "^E" ALL
    ("^E(R)"«19,9/A,B»)
    (hyp«18/A»)
  (cut«10,5/B,C»)
  ("rewrite="«15,16,12,17/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("symmetric="«12,15/A,B»)
    (hyp«14/A»)
  (hyp«9/A»)
  (cut«8,5/B,C»)
  (LAYOUT "R¬∩S@T≐R¬∩S^R¬∩T" ALL
    ("rewrite="«8,13,10,13/A,xx,B,P»)
    (LAYOUT HIDEROOT
      ("symmetric≐"«10,8/A,B»)
      (LAYOUT HIDEROOT
        ("R¬∩S@T≐R¬∩S^R¬∩T"«11,11,12/R,S,T»)))
        (hyp«10/A»)
  (cut«6,5/B,C»)
  (LAYOUT "^E" ALL
    ("^E(L)"«9,6/B,A»)
    (hyp«8/A»)
  (cut«9,5/B,C»)
  (LAYOUT "^E" ALL
    ("^E(R)"«6,9/A,B»)
    (hyp«8/A»)
  (cut«3,5/B,C»)
  (LAYOUT "(A)¬∩(B)≐A≠B" ALL
    ("rewrite≐"«3,7,6,7/A,xx,B,P»)
    (LAYOUT HIDEROOT
      ("symmetric≐"«6,3/A,B»)
      (LAYOUT HIDEROOT
        ("(A)¬∩(B)≐A≠B"«1,1/A,B»)))
        (hyp«6/A»)
  (cut«0,5/B,C»)
  (LAYOUT "¬(A=B)≐A≠B" ALL
    ("rewrite="«3,0,4,4/B,A,xx,P»)
    (LAYOUT HIDEROOT
      ("¬(A=B)≐A≠B"«1,1/A,B»)
      (hyp«3/A»)

```

```

("¬-E"«2/B»)
(LAYOUT HIDEROOT
 ("reflexive="«1/A»)
 (hyp«0/A»)

```

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DERIVED RULE IS FROM  $A \neq B$  AND  $\text{list}(A \Rightarrow E \Rightarrow B)$  INFER  $\text{list}(A.E \Rightarrow E \Rightarrow B)$

1: $\text{list}(A \Rightarrow E \Rightarrow B)$	$\text{list}(A \Rightarrow E \Rightarrow B)$
2: $A \neq B$	$A \neq B$
3: $\text{list}((A)@A.E \Rightarrow E \Rightarrow B)$	FROM $A \neq B$ INFER $A \Rightarrow E \Rightarrow B = (A)@A.E \Rightarrow E \Rightarrow B$ 2,1
4: $\text{list}(A) \wedge \text{list}(A.E \Rightarrow E \Rightarrow B) \wedge (A) \neg \neg A.E \Rightarrow E \Rightarrow B$	$\text{list}(R@S) \neq \text{list } R \wedge \text{list } S \wedge R \neg \neg S$ 3
5: $\text{list}(A)$	$\wedge$ -E 4
6: $\text{list}(A.E \Rightarrow E \Rightarrow B)$	$\wedge$ -E 4
7: $(A) \neg \neg A.E \Rightarrow E \Rightarrow B$	$\wedge$ -E 4
8: $\text{list}(A.E \Rightarrow E \Rightarrow B)$	hyp 6

Given:

```

A ≠ B
list(A ⇒ E ⇒ B)

```

FORMULAE 0  $\text{list}(A.E \Rightarrow E \Rightarrow B)$ , 1  $\text{list}(A) \wedge \text{list}(A.E \Rightarrow E \Rightarrow B) \wedge (A) \neg \neg A.E \Rightarrow E \Rightarrow B$ , 2  $\text{list}(A) \wedge \text{list}(A.E \Rightarrow E \Rightarrow B)$ , 3  $(A) \neg \neg A.E \Rightarrow E \Rightarrow B$ , 4  $\text{list}(A)$ , 5  $\text{list}((A)@A.E \Rightarrow E \Rightarrow B)$ , 6  $(A)$ , 7  $A.E \Rightarrow E \Rightarrow B$ , 8  $xx1$ , 9  $\text{list}(A \Rightarrow E \Rightarrow B)$ , 10  $A$ , 11  $B$ , 12  $E$ , 13  $A \Rightarrow E \Rightarrow B$ , 14  $(A)@A.E \Rightarrow E \Rightarrow B$ , 15  $xx$ , 16  $\text{list } xx$

SEQ

```

(cut«9,0/B,C»)
(GIVEN 1)
(cut«5,0/B,C»)
(LAYOUT "FROM A ≠ B INFER A ⇒ E ⇒ B = (A)@A.E ⇒ E ⇒ B" ALL
 ("rewrite="«14,15,13,16/A,xx,B,P»)
 (LAYOUT HIDEROOT
 ("symmetric="«13,14/A,B»)
 (LAYOUT HIDEROOT
 ("FROM A ≠ B INFER A ⇒ E ⇒ B = (A)@A.E ⇒ E ⇒ B"«10,11,12/A,B,E»)
 (GIVEN 0)))
 (hyp«9/A»)
(cut«1,0/B,C»)
(LAYOUT "list(R@S) ≠ list R ∧ list S ∧ R ¬ ¬ S" ALL
 ("rewrite≠"«1,8,5,8/A,xx,B,P»)
 (LAYOUT HIDEROOT
 ("symmetric≠"«5,1/A,B»)
 (LAYOUT HIDEROOT
 ("list(R@S) ≠ list R ∧ list S ∧ R ¬ ¬ S"«6,7/R,S»)))
 (hyp«5/A»)
(LAYOUT HIDE CUT
 (cut«2,0/B,C»)
 ("∧-E(L)"«3,2/B,A»)
 (hyp«1/A»)
 (cut«4,0/B,C»)
 (LAYOUT "∧-E" ALL
 ("∧-E(L)"«0,4/B,A»)
 (hyp«2/A»)
 (cut«0,0/B,C»)
 (LAYOUT "∧-E" ALL
 ("∧-E(R)"«4,0/A,B»)
 (hyp«2/A»)
 (cut«3,0/B,C»)
 (LAYOUT "∧-E" ALL
 ("∧-E(R)"«2,3/A,B»)

```

(hyp«1/A»)  
(hyp«0/A»)

-----  
DERIVED RULE IS FROM  $A \neq B$  AND  $\text{list}(A \Rightarrow E \Rightarrow B)$  INFER  $(A) \neg \neg A.E \Rightarrow E \Rightarrow B$

1: $\text{list}(A \Rightarrow E \Rightarrow B)$	$\text{list}(A \Rightarrow E \Rightarrow B)$
2: $A \neq B$	$A \neq B$
3: $\text{list}((A)@A.E \Rightarrow E \Rightarrow B)$	FROM $A \neq B$ INFER $A \Rightarrow E \Rightarrow B = (A)@A.E \Rightarrow E \Rightarrow B$ 2,1
4: $\text{list}(A) \wedge \text{list}(A.E \Rightarrow E \Rightarrow B) \wedge (A) \neg \neg A.E \Rightarrow E \Rightarrow B$	$\text{list}(R@S) \neq \text{list } R \wedge \text{list } S \wedge R \neg \neg S$ 3
5: $\text{list}(A)$	$\wedge$ -E 4
6: $\text{list}(A.E \Rightarrow E \Rightarrow B)$	$\wedge$ -E 4
7: $(A) \neg \neg A.E \Rightarrow E \Rightarrow B$	$\wedge$ -E 4

Given:

$A \neq B$   
 $\text{list}(A \Rightarrow E \Rightarrow B)$

FORMULAE 0  $(A) \neg \neg A.E \Rightarrow E \Rightarrow B$ , 1  $\text{list}(A) \wedge \text{list}(A.E \Rightarrow E \Rightarrow B) \wedge (A) \neg \neg A.E \Rightarrow E \Rightarrow B$ , 2  $\text{list}(A) \wedge \text{list}(A.E \Rightarrow E \Rightarrow B)$ , 3  $\text{list}(A)$ , 4  $\text{list}(A.E \Rightarrow E \Rightarrow B)$ , 5  $\text{list}((A)@A.E \Rightarrow E \Rightarrow B)$ , 6  $(A)$ , 7  $A.E \Rightarrow E \Rightarrow B$ , 8  $xx1$ , 9  $\text{list}(A \Rightarrow E \Rightarrow B)$ , 10  $A$ , 11  $B$ , 12  $E$ , 13  $A \Rightarrow E \Rightarrow B$ , 14  $(A)@A.E \Rightarrow E \Rightarrow B$ , 15  $xx$ , 16  $\text{list } xx$

SEQ

```
(cut«9,0/B,C»)
(GIVEN 1)
(cut«5,0/B,C»)
(LAYOUT "FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B" ALL
("rewrite="«14,15,13,16/A,xx,B,P»)
(LAYOUT HIDEROOT
("symmetric="«13,14/A,B»)
(LAYOUT HIDEROOT
("FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B"«10,11,12/A,B,E»)
(GIVEN 0)))
(hyp«9/A»)
(cut«1,0/B,C»)
(LAYOUT "list(R@S)≠list R∧list S∧R¬¬S" ALL
("rewrite="«1,8,5,8/A,xx,B,P»)
(LAYOUT HIDEROOT
("symmetric="«5,1/A,B»)
(LAYOUT HIDEROOT
("list(R@S)≠list R∧list S∧R¬¬S"«6,7/R,S»)))
(hyp«5/A»)
(LAYOUT HIDEDECUT
(cut«2,0/B,C»)
("∧-E(L)"«0,2/B,A»)
(hyp«1/A»)
(cut«3,0/B,C»)
(LAYOUT "∧-E" ALL
("∧-E(L)"«4,3/B,A»)
(hyp«2/A»)
(cut«4,0/B,C»)
(LAYOUT "∧-E" ALL
("∧-E(R)"«3,4/A,B»)
(hyp«2/A»)
(cut«0,0/B,C»)
(LAYOUT "∧-E" ALL
("∧-E(R)"«2,0/A,B»)
(hyp«1/A»)
(hyp«0/A»))
```

DERIVED RULE IS FROM  $A \neq A.E$  INFER  $A \Rightarrow E \Rightarrow A.E = (A)$

- 1:  $A \neq A.E$   $A \neq A.E$   
 2:  $(A)@() = (A)$   $S@() \triangleq S$   
 3:  $(A)@A.E \Rightarrow E \Rightarrow A.E = (A)$   $A \Rightarrow E \Rightarrow A = ()$  2  
 4:  $A \Rightarrow E \Rightarrow A.E = (A)$  FROM  $A \neq B$  INFER  $A \Rightarrow E \Rightarrow B = (A)@A.E \Rightarrow E \Rightarrow B$  1,3

Given:  
 $A \neq A.E$

FORMULAE 0 (A), 1 (A)@(), 2 xx2, 3 xx2=(A), 4 A.E, 5 E, 6 A.E⇒E⇒A.E, 7 xx1, 8 (), 9 (A)@xx1=(A), 10 A, 11 A⇒E⇒A.E, 12 xx, 13 (A)@A.E⇒E⇒A.E, 14 xx=(A)

LAYOUT "FROM  $A \neq B$  INFER  $A \Rightarrow E \Rightarrow B = (A)@A.E \Rightarrow E \Rightarrow B$ " ALL  
 ("rewrite="«11,12,13,14/A,xx,B,P») (LAYOUT HIDEROOT  
 ("FROM  $A \neq B$  INFER  $A \Rightarrow E \Rightarrow B = (A)@A.E \Rightarrow E \Rightarrow B$ "«10,4,5/A,B,E») (GIVEN 0))  
 (LAYOUT " $A \Rightarrow E \Rightarrow A = ()$ " ALL ("rewrite="«6,7,8,9/A,xx,B,P») (LAYOUT HIDEROOT  
 (" $A \Rightarrow E \Rightarrow A = ()$ "«4,5/A,E»)) (LAYOUT " $S@() \triangleq S$ " ALL ("rewrite="«1,2,0,3/A,xx,B,P») (LAYOUT HIDEROOT  
 (" $S@() \triangleq S$ "«0/S»)) (LAYOUT HIDEROOT ("reflexive="«0/A»))))

DERIVED RULE IS FROM  $A \neq A.E$  AND  $A \neq B$  INFER  $A \Rightarrow E \Rightarrow A.E = A \Rightarrow (E \oplus A \mapsto B) \Rightarrow B$

- 1:  $A \neq A.E$   $A \neq A.E$   
 2:  $A \neq B$   $A \neq B$   
 3:  $(A) = (A)@()$   $S@() \triangleq S$   
 4:  $(A) = (A)@B \Rightarrow (E \oplus A \mapsto B) \Rightarrow B$   $A \Rightarrow E \Rightarrow A = ()$  3  
 5:  $(A) = A \Rightarrow (E \oplus A \mapsto B) \Rightarrow B$  FROM  $A \neq B$  INFER  $A \Rightarrow E \Rightarrow B = (A)@A.E \Rightarrow E \Rightarrow B$  2,4  
 6:  $A \Rightarrow E \Rightarrow A.E = A \Rightarrow (E \oplus A \mapsto B) \Rightarrow B$  FROM  $A \neq A.E$  INFER  $A \Rightarrow E \Rightarrow A.E = (A)$  1,5

Given:  
 $A \neq A.E$   
 $A \neq B$

FORMULAE 0 (A), 1 (A)@(), 2 xx4, 3 (A)=xx4, 4 B, 5  $E \oplus A \mapsto B$ , 6  $B \Rightarrow (E \oplus A \mapsto B) \Rightarrow B$ , 7 xx3, 8 (), 9 (A)=(A)@xx3, 10 A, 11 E, 12 A.( $E \oplus A \mapsto B$ ), 13 xx2, 14 (A)=(A)@xx2⇒( $E \oplus A \mapsto B$ )⇒B, 15  $A \Rightarrow (E \oplus A \mapsto B) \Rightarrow B$ , 16 xx1, 17 (A)@A.( $E \oplus A \mapsto B$ )⇒( $E \oplus A \mapsto B$ )⇒B, 18 (A)=xx1, 19  $A \Rightarrow E \Rightarrow A.E$ , 20 xx, 21  $xx = A \Rightarrow (E \oplus A \mapsto B) \Rightarrow B$

LAYOUT "FROM  $A \neq A.E$  INFER  $A \Rightarrow E \Rightarrow A.E = (A)$ " ALL  
 ("rewrite="«19,20,0,21/A,xx,B,P») (LAYOUT HIDEROOT  
 ("FROM  $A \neq A.E$  INFER  $A \Rightarrow E \Rightarrow A.E = (A)$ "«10,11/A,E») (GIVEN 0))  
 (LAYOUT "FROM  $A \neq B$  INFER  $A \Rightarrow E \Rightarrow B = (A)@A.E \Rightarrow E \Rightarrow B$ " ALL ("rewrite="«15,16,17,18/A,xx,B,P») (LAYOUT HIDEROOT  
 ("FROM  $A \neq B$  INFER  $A \Rightarrow E \Rightarrow B = (A)@A.E \Rightarrow E \Rightarrow B$ "«10,4,5/A,B,E») (GIVEN 1)) (LAYOUT HIDEROOT  
 (LAYOUT "iterate" (1) ("rewrite="«12,13,4,14/A,xx,B,P»))

```

("A.(B⊗A⇒E)=E"«10,11,4/A,B,E»)
(LAYOUT "A⇒E⇒A=()" ALL
  ("rewrite="«6,7,8,9/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("A⇒E⇒A=()"«4,5/A,E»))
  (LAYOUT "S@()≠S" ALL
    ("rewrite≠"«1,2,0,3/A,xx,B,P»)
    (LAYOUT HIDEROOT
      ("S@()≠S"«0/S»))
    (LAYOUT HIDEROOT
      ("reflexive="«0/A»))))))

```

DERIVED RULE IS FROM  $F \neg \neg A \Rightarrow E \Rightarrow B$  AND  $A \neq B$  INFER  $F \neg \neg (A) \wedge F \neg \neg A.E \Rightarrow E \Rightarrow B$

- 1:  $F \neg \neg A \Rightarrow E \Rightarrow B$                        $F \neg \neg A \Rightarrow E \Rightarrow B$
- 2:  $A \neq B$                                                $A \neq B$
- 3:  $F \neg \neg (A) @ A.E \Rightarrow E \Rightarrow B$       FROM  $A \neq B$  INFER  $A \Rightarrow E \Rightarrow B = (A) @ A.E \Rightarrow E \Rightarrow B$  2,1
- 4:  $F \neg \neg (A) \wedge F \neg \neg A.E \Rightarrow E \Rightarrow B$      $R \neg \neg S @ T \neq R \neg \neg S \wedge R \neg \neg T$  3

Given:

```

F ¬ ¬ A ⇒ E ⇒ B
A ≠ B

```

FORMULAE 0  $F \neg \neg (A) \wedge F \neg \neg A.E \Rightarrow E \Rightarrow B$ , 1  $F \neg \neg (A) @ A.E \Rightarrow E \Rightarrow B$ , 2 F, 3 (A), 4  $A.E \Rightarrow E \Rightarrow B$ , 5  $xx1$ , 6  $F \neg \neg A \Rightarrow E \Rightarrow B$ , 7 A, 8 B, 9 E, 10  $A \Rightarrow E \Rightarrow B$ , 11  $(A) @ A.E \Rightarrow E \Rightarrow B$ , 12  $xx$ , 13  $F \neg \neg xx$

SEQ

```

(cut«6,0/B,C»)
(GIVEN 0)
(cut«1,0/B,C»)
(LAYOUT "FROM A ≠ B INFER A ⇒ E ⇒ B = (A) @ A.E ⇒ E ⇒ B" ALL
  ("rewrite="«11,12,10,13/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("symmetric="«10,11/A,B»)
    (LAYOUT HIDEROOT
      ("FROM A ≠ B INFER A ⇒ E ⇒ B = (A) @ A.E ⇒ E ⇒ B"«7,8,9/A,B,E»)
      (GIVEN 1)))
    (hyp«6/A»)
  (cut«0,0/B,C»)
  (LAYOUT "R ¬ ¬ S @ T ≠ R ¬ ¬ S ∧ R ¬ ¬ T" ALL
    ("rewrite≠"«0,5,1,5/A,xx,B,P»)
    (LAYOUT HIDEROOT
      ("symmetric≠"«1,0/A,B»)
      (LAYOUT HIDEROOT
        ("R ¬ ¬ S @ T ≠ R ¬ ¬ S ∧ R ¬ ¬ T"«2,3,4/R,S,T»)))
    (hyp«1/A»)
  (hyp«0/A»)

```

DERIVED RULE "list induction"(A, OBJECT cA, OBJECT xA) WHERE FRESH cA IS  
 FROM list(A⇒E⇒B) AND P«B/xA» AND cA≠B, P«cA.E/xA» ⊢ P«cA/xA»  
 INFER P«A/xA»

1: list(A⇒E⇒B)	list(A⇒E⇒B)
2: finitesequence(A⇒E⇒B)	FROM list S INFER finitesequence S 1
3: cc⇒E⇒B=()	assumption
4: cc=B	Derived Rule FROM A⇒E⇒B=() INFER A=B 3
5: P«B/xA»	P«B/xA»
6: P«cc/xA»	rewrite= 4,5
7: cc⇒E⇒B=()→P«cc/xA»	→I 3-6
8: ∀yA:(yA⇒E⇒B=()→P«yA/xA»)	∀I 7
9: ∀yA:(yA⇒E⇒B=ys→P«yA/xA»)	assumption
10: cA⇒E⇒B=(y)@ys	assumption
11: cA≠B ∧ cA=y ∧ cA.E⇒E⇒B=ys	Derived Rule FROM A⇒E⇒B=(C)@S INFER A≠B ∧ A=C ∧ A.E⇒E⇒B=S 10
12: cA≠B	∧-E 11
13: cA=y	∧-E 11
14: cA.E⇒E⇒B=ys	∧-E 11
15: cA.E⇒E⇒B=ys→P«cA.E/xA»	∀-E 9
16: P«cA.E/xA»	→-E 14,15
17: P«cA/xA»	cA≠B, P«cA.E/xA» ⊢ P«cA/xA» 12,16
18: cA⇒E⇒B=(y)@ys→P«cA/xA»	→I 10-17
19: ∀yA:(yA⇒E⇒B=(y)@ys→P«yA/xA»)	∀I 18
20: ∀xt:(finitesequence xt→∀yA:(yA⇒E⇒B=xt→P«yA/xA»))	finite sequence induction (L) 8,9-19
21: finitesequence(A⇒E⇒B)→∀yA:(yA⇒E⇒B=A⇒E⇒B→P«yA/xA»)	∀-E 20
22: ∀yA:(yA⇒E⇒B=A⇒E⇒B→P«yA/xA»)	→-E 2,21
23: A⇒E⇒B=A⇒E⇒B→P«A/xA»	∀-E 22
24: P«A/xA»	→-E 23

Given:  
 list(A⇒E⇒B)  
 P«B/xA»  
 cA≠B, P«cA.E/xA» ⊢ P«cA/xA»  
 Provided:  
 FRESH cA

FORMULAE 0 cA.E⇒E⇒B=ys→P«cA.E/xA», 1 cA.E⇒E⇒B=ys, 2 P«cA.E/xA», 3 P«cA/xA»,  
 4 ∀yA:(yA⇒E⇒B=ys→P«yA/xA»), 5 cA.E, 6 yA⇒E⇒B=ys→P«yA/xA», 7 yA, 8 cA≠B ∧ cA=y ∧ cA.E⇒E⇒B=ys,  
 9 cA≠B ∧ cA=y, 10 cA≠B, 11 cA=y, 12 cA⇒E⇒B=(y)@ys, 13 cA, 14 B, 15 y, 16 E, 17 ys,  
 18 yA⇒E⇒B=(y)@ys→P«yA/xA», 19 cc=B, 20 P«x/xA», 21 cc, 22 x, 23 cc⇒E⇒B=(), 24 P«cc/xA»,  
 25 yA⇒E⇒B=()→P«yA/xA», 26 ∀yA:(yA⇒E⇒B=xt→P«yA/xA»), 27 xt, 28 A⇒E⇒B,  
 29 finitesequence xt→∀yA:(yA⇒E⇒B=xt→P«yA/xA»), 30 (A⇒E⇒B), 31 finitesequence(A⇒E⇒B),  
 32 ∀yA:(yA⇒E⇒B=A⇒E⇒B→P«yA/xA»), 33 A, 34 yA⇒E⇒B=A⇒E⇒B→P«yA/xA», 35 A⇒E⇒B=A⇒E⇒B,  
 36 P«A/xA»

SEQ

```

("→-E"«35,36/A,B»)
(LAYOUT HIDEROOT
  ("reflexive="«28/A»)
  ("∀-E"«33,34,7/B,A,x»)
  ("→-E"«31,32/A,B»)
  ("FROM list S INFER finitesequence S"«30/S»)
  (GIVEN 0)
  ("∀-E"«28,29,27/B,A,x»)
  ("finite sequence induction (L)"«15,17,26,27/y,ys,P,xs»)
  ("∀-I"«21,25,7/cc,A,x»)
  ("→-I"«23,24/A,B»)
  (cut«19,24/B,C»)
  ("FROM A⇒E⇒B=() INFER A=B"«21,14,16/A,B,E»)
  (hyp«23/A»)
  ("rewrite="«20,21,22,14/P,A,xx,B»)
  (hyp«19/A»)
  (GIVEN 1)
  ("∀-I"«13,18,7/cc,A,x»)
  ("→-I"«12,3/A,B»)

```

```

(cut«8,3/B,C»)
("FROM A⇒E⇒B=(C)@S INFER A≠B∧A=C∧A.E⇒E⇒B=S"«13,14,15,16,17/A,B,C,E,S»)
(hyp«12/A»)
(LAYOUT HIDE CUT
  (cut«9,3/B,C»)
  ("∧-E(L)"«1,9/B,A»)
  (hyp«8/A»)
  (cut«10,3/B,C»)
  (LAYOUT "∧-E" ALL
    ("∧-E(L)"«11,10/B,A»)
    (hyp«9/A»))
  (cut«11,3/B,C»)
  (LAYOUT "∧-E" ALL
    ("∧-E(R)"«10,11/A,B»)
    (hyp«9/A»))
  (cut«1,3/B,C»)
  (LAYOUT "∧-E" ALL
    ("∧-E(R)"«9,1/A,B»)
    (hyp«8/A»))
  (cut«0,3/B,C»)
  ("∀-E"«5,6,7/B,A,x»)
  (hyp«4/A»)
  (cut«2,3/B,C»)
  ("→-E"«1,2/A,B»)
  (hyp«1/A»)
  (hyp«0/A»)
  (GIVEN 2))

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DERIVED RULE IS FROM  $(C)\neg\eta A \Rightarrow E \Rightarrow B$  AND  $\text{list}(A \Rightarrow E \Rightarrow B)$  INFER  $A \Rightarrow E \Rightarrow B = A \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$

1: $(C)\neg\eta A \Rightarrow E \Rightarrow B$	$(C)\neg\eta A \Rightarrow E \Rightarrow B$
2: $\text{list}(A \Rightarrow E \Rightarrow B)$	$\text{list}(A \Rightarrow E \Rightarrow B)$
3: $(C)\neg\eta B \Rightarrow E \Rightarrow B$	assumption
4: $B \Rightarrow E \Rightarrow B = ()$	Theorem $A \Rightarrow E \Rightarrow A = ()$
5: $B \Rightarrow E \Rightarrow B = B \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$	$A \Rightarrow E \Rightarrow A = ()$ 4
6: $(C)\neg\eta B \Rightarrow E \Rightarrow B \rightarrow B \Rightarrow E \Rightarrow B = B \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$	$\rightarrow$ -I 3-5
7: $C \neq cA, (C)\neg\eta cA.E \Rightarrow E \Rightarrow B \rightarrow cA.E \Rightarrow E \Rightarrow B = cA.E \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$	assumptions
8: $(C)\neg\eta cA \Rightarrow E \Rightarrow B$	assumption
9: $(C)\neg\eta(cA) @ cA.E \Rightarrow E \Rightarrow B$	FROM $A \neq B$ INFER $A \Rightarrow E \Rightarrow B = (A) @ A.E \Rightarrow E \Rightarrow B$ 7.1,8
10: $(C)\neg\eta(cA) \wedge (C)\neg\eta cA.E \Rightarrow E \Rightarrow B$	$R\neg\eta S @ T = R\neg\eta S \wedge R\neg\eta T$ 9
11: $(C)\neg\eta(cA)$	$\wedge$ -E 10
12: $(C)\neg\eta cA.E \Rightarrow E \Rightarrow B$	$\wedge$ -E 10
13: $cA.E \Rightarrow E \Rightarrow B = cA.E \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$	$\rightarrow$ -E 12,7.2
14: $C \neq cA$	$(A)\neg\eta(B) = A \neq B$ 11
15: $(cA) @ cA.E \Rightarrow (E \oplus C \mapsto F) \Rightarrow B = (cA) @ cA.(E \oplus C \mapsto F) \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$	FROM $A \neq C$ INFER $A.(B \oplus C \mapsto E) = A.B$ 14
16: $cA \Rightarrow E \Rightarrow B$	
17: $= (cA) @ cA.E \Rightarrow E \Rightarrow B$	FROM $A \neq B$ INFER $A \Rightarrow E \Rightarrow B = (A) @ A.E \Rightarrow E \Rightarrow B$ 7.1
18: $= (cA) @ cA.E \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$	rewrite= 13
19: $= (cA) @ cA.(E \oplus C \mapsto F) \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$	transitive= 15
20: $= cA \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$	FROM $A \neq B$ INFER $A \Rightarrow E \Rightarrow B = (A) @ A.E \Rightarrow E \Rightarrow B$ 7.1
21: $(C)\neg\eta cA \Rightarrow E \Rightarrow B \rightarrow cA \Rightarrow E \Rightarrow B = cA \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$	$\rightarrow$ -I 8-20
22: $(C)\neg\eta A \Rightarrow E \Rightarrow B \rightarrow A \Rightarrow E \Rightarrow B = A \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$	Derived Rule list induction 2,6,7-21
23: $A \Rightarrow E \Rightarrow B = A \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$	$\rightarrow$ -E 1,22

Given:

$(C)\neg\eta A \Rightarrow E \Rightarrow B$   
 $\text{list}(A \Rightarrow E \Rightarrow B)$

FORMULAE 0  $(cA) @ cA.(E \oplus C \mapsto F) \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$ , 1  $C \neq cA$ , 2  $cA$ , 3  $B$ , 4  $E \oplus C \mapsto F$ , 5  $cA \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$ , 6  $xx4$ , 7  $(cA) @ cA.(E \oplus C \mapsto F) \Rightarrow (E \oplus C \mapsto F) \Rightarrow B = xx4$ , 8  $(cA) @ cA.E \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$ , 9  $C \neq cA$ , 10  $C$ , 11  $E$ , 12  $F$ , 13  $cA.(E \oplus C \mapsto F)$ , 14  $xx5$ , 15  $cA.E$ , 16  $(cA) @ cA.E \Rightarrow (E \oplus C \mapsto F) \Rightarrow B = (cA) @ xx5 \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$ , 17  $cA.E \Rightarrow E \Rightarrow B = cA.E \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$ , 18  $(cA) @ x = (cA) @ cA.E \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$ , 19  $cA.E \Rightarrow E \Rightarrow B$ , 20  $x$ , 21  $cA.E \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$ , 22  $(cA) @ cA.E \Rightarrow E \Rightarrow B$ , 23  $cA \Rightarrow E \Rightarrow B$ , 24  $xx3$ , 25  $xx3 = (cA) @ cA.E \Rightarrow E \Rightarrow B$ , 26  $(C)\neg\eta(cA)$ , 27  $xx6$ , 28  $cA \Rightarrow E \Rightarrow B = cA \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$ , 29  $(C)\neg\eta cA.E \Rightarrow E \Rightarrow B \rightarrow cA.E \Rightarrow E \Rightarrow B = cA.E \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$ , 30  $(C)\neg\eta cA.E \Rightarrow E \Rightarrow B$ , 31  $(C)\neg\eta(cA) \wedge (C)\neg\eta cA.E \Rightarrow E \Rightarrow B$ , 32  $(C)\neg\eta(cA) @ cA.E \Rightarrow E \Rightarrow B$ , 33  $(C)$ , 34  $(cA)$ , 35  $xx2$ , 36  $(C)\neg\eta cA \Rightarrow E \Rightarrow B$ , 37  $xx1$ , 38  $(C)\neg\eta xx1$ , 39  $B \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$ , 40  $xx$ , 41  $()$ , 42  $B \Rightarrow E \Rightarrow B = xx$ , 43  $(C)\neg\eta B \Rightarrow E \Rightarrow B$ , 44  $B \Rightarrow E \Rightarrow B = B \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$ , 45  $A$ , 46  $xA$ , 47  $(C)\neg\eta xA \Rightarrow E \Rightarrow B \rightarrow xA \Rightarrow E \Rightarrow B = xA \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$ , 48  $(C)\neg\eta A \Rightarrow E \Rightarrow B$ , 49  $A \Rightarrow E \Rightarrow B = A \Rightarrow (E \oplus C \mapsto F) \Rightarrow B$

SEQ

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("→-E"«48,49/A,B»)
(GIVEN 0)
("list induction"«45,2,46,3,11,47/A,cA,xA,B,E,P»)
(GIVEN 1)
("→-I"«43,44/A,B»)
(LAYOUT "A⇒E⇒A=()" ALL
  ("rewrite="«39,40,41,42/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("A⇒E⇒A=()"«3,4/A,E»)
    ("A⇒E⇒A=()"«3,11/A,E»)
  )
("→-I"«36,28/A,B»)
(cut«32,28/B,C»)
(LAYOUT "FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B" ALL
  ("rewrite="«22,37,23,38/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("symmetric="«23,22/A,B»)
  )
(LAYOUT HIDEROOT

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    ("FROM  $A \neq B$  INFER  $A \Rightarrow B = (A) @ A.E \Rightarrow B$ " «2,3,11/A,B,E»)
    (hyp«1/A»))
  (hyp«36/A»))
(cut«31,28/B,C»)
(LAYOUT "R¬S@T≠R¬S∧R¬T" ALL
  ("rewrite≠"«31,35,32,35/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("symmetric≠"«32,31/A,B»)
    (LAYOUT HIDEROOT
      ("R¬S@T≠R¬S∧R¬T"«33,34,19/R,S,T»)))
  (hyp«32/A»))
(cut«26,28/B,C»)
(LAYOUT "∧-E" ALL
  ("∧-E(L)"«30,26/B,A»)
  (hyp«31/A»))
(cut«30,28/B,C»)
(LAYOUT "∧-E" ALL
  ("∧-E(R)"«26,30/A,B»)
  (hyp«31/A»))
(cut«17,28/B,C»)
("→-E"«30,17/A,B»)
(hyp«30/A»)
(hyp«29/A»)
(cut«9,28/B,C»)
(LAYOUT "(A)¬(B)≠A≠B" ALL
  ("rewrite≠"«9,27,26,27/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("symmetric≠"«26,9/A,B»)
    (LAYOUT HIDEROOT
      ("(A)¬(B)≠A≠B"«10,2/A,B»)))
  (hyp«26/A»))
("transitive="«23,22,5/A,B,C»)
(LAYOUT "FROM  $A \neq B$  INFER  $A \Rightarrow B = (A) @ A.E \Rightarrow B$ " ALL
  ("rewrite="«23,24,22,25/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("FROM  $A \neq B$  INFER  $A \Rightarrow B = (A) @ A.E \Rightarrow B$ "«2,3,11/A,B,E»)
    (hyp«1/A»))
  (LAYOUT HIDEROOT
    ("reflexive="«22/A»)))
("transitive="«22,8,5/A,B,C»)
("rewrite="«18,19,20,21/P,A,xx,B»)
(hyp«17/A»)
(LAYOUT HIDEROOT
  ("reflexive="«8/A»)
  ("transitive="«8,0,5/A,B,C»)
  ("transitive="«8,8,0/A,B,C»)
  (LAYOUT HIDEROOT
    ("reflexive="«8/A»)
    (LAYOUT "FROM  $A \neq C$  INFER  $A.(B \oplus C \rightarrow E) = A.B$ " ALL
      ("rewrite="«13,14,15,16/A,xx,B,P»)
      (LAYOUT HIDEROOT
        ("FROM  $A \neq C$  INFER  $A.(B \oplus C \rightarrow E) = A.B$ "«2,11,10,12/A,B,C,E»)
        (LAYOUT HIDEROOT
          ("symmetric≠"«10,2/A,B»)
          (hyp«9/A»)))
        (LAYOUT HIDEROOT
          ("reflexive="«8/A»)))
      (LAYOUT "FROM  $A \neq B$  INFER  $A \Rightarrow B = (A) @ A.E \Rightarrow B$ " ALL
        ("rewrite="«5,6,0,7/A,xx,B,P»)
        (LAYOUT HIDEROOT
          ("FROM  $A \neq B$  INFER  $A \Rightarrow B = (A) @ A.E \Rightarrow B$ "«2,3,4/A,B,E»)
          (hyp«1/A»))
        (LAYOUT HIDEROOT
          ("reflexive="«0/A»)))
    )
  )
)

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DERIVED RULE IS FROM  $(C)\neg\eta A \Rightarrow E \Rightarrow B$  AND  $\text{list}(A \Rightarrow E \Rightarrow B)$  INFER  $A \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C = A \Rightarrow E \Rightarrow C$

1: $(C)\neg\eta A \Rightarrow E \Rightarrow B$	$(C)\neg\eta A \Rightarrow E \Rightarrow B$
2: $\text{list}(A \Rightarrow E \Rightarrow B)$	$\text{list}(A \Rightarrow E \Rightarrow B)$
3: $(C)\neg\eta B \Rightarrow E \Rightarrow B$	assumption
4: $()@B \Rightarrow E \Rightarrow C = B \Rightarrow E \Rightarrow C$	$()@S \doteq S$
5: $B \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C$	
6: $= ()@B \Rightarrow E \Rightarrow C$	$A \Rightarrow E \Rightarrow A = ()$
7: $= B \Rightarrow E \Rightarrow C$	transitive= 4
8: $(C)\neg\eta B \Rightarrow E \Rightarrow B \rightarrow B \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C = B \Rightarrow E \Rightarrow C$	$\rightarrow\text{-I}$ 3-7
9: $cA \neq B, (C)\neg\eta cA.E \Rightarrow E \Rightarrow B \rightarrow cA.E \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C = cA.E \Rightarrow E \Rightarrow C$	assumptions
10: $(C)\neg\eta cA \Rightarrow E \Rightarrow B$	assumption
11: $(C)\neg\eta(cA)@cA.E \Rightarrow E \Rightarrow B$	FROM $A \neq B$ INFER $A \Rightarrow E \Rightarrow B = (A)@A.E \Rightarrow E \Rightarrow B$ 9.1,10
12: $(C)\neg\eta(cA) \wedge (C)\neg\eta cA.E \Rightarrow E \Rightarrow B$	$R\neg\eta S @ T \doteq R\neg\eta S \wedge R\neg\eta T$ 11
13: $(C)\neg\eta(cA)$	$\wedge\text{-E}$ 12
14: $(C)\neg\eta cA.E \Rightarrow E \Rightarrow B$	$\wedge\text{-E}$ 12
15: $cA.E \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C = cA.E \Rightarrow E \Rightarrow C$	$\rightarrow\text{-E}$ 14,9.2
16: $C \neq cA$	$(A)\neg\eta(B) \doteq A \neq B$ 13
17: $(cA)@cA.E \Rightarrow E \Rightarrow C = cA \Rightarrow E \Rightarrow C$	FROM $A \neq B$ INFER $A \Rightarrow E \Rightarrow B = (A)@A.E \Rightarrow E \Rightarrow B$ 16
18: $cA \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C$	
19: $= (cA)@cA.E \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C$	FROM $A \neq B$ INFER $A \Rightarrow E \Rightarrow B = (A)@A.E \Rightarrow E \Rightarrow B$ 9.1
20: $= (cA)@(cA.E \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C)$	$(R@S)@T \doteq R@(S@T)$
21: $= (cA)@cA.E \Rightarrow E \Rightarrow C$	rewrite= 15
22: $= cA \Rightarrow E \Rightarrow C$	transitive= 17
23: $(C)\neg\eta cA \Rightarrow E \Rightarrow B \rightarrow cA \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C = cA \Rightarrow E \Rightarrow C$	$\rightarrow\text{-I}$ 10-22
24: $(C)\neg\eta A \Rightarrow E \Rightarrow B \rightarrow A \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C = A \Rightarrow E \Rightarrow C$	Derived Rule list induction 2,8,9-23
25: $A \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C = A \Rightarrow E \Rightarrow C$	$\rightarrow\text{-E}$ 1,24

Given:

$(C)\neg\eta A \Rightarrow E \Rightarrow B$   
 $\text{list}(A \Rightarrow E \Rightarrow B)$

FORMULAE 0  $cA \Rightarrow E \Rightarrow C$ , 1  $C \neq cA$ , 2  $C$ , 3  $cA$ , 4  $E$ , 5  $(cA)@cA.E \Rightarrow E \Rightarrow C$ , 6  $xx6$ , 7  $xx6 = cA \Rightarrow E \Rightarrow C$ , 8  $cA.E \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C = cA.E \Rightarrow E \Rightarrow C$ , 9  $(cA)@x = (cA)@cA.E \Rightarrow E \Rightarrow C$ , 10  $cA.E \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C$ , 11  $x$ , 12  $cA.E \Rightarrow E \Rightarrow C$ , 13  $(cA)@(cA.E \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C)$ , 14  $(cA)$ , 15  $cA.E \Rightarrow E \Rightarrow B$ , 16  $B \Rightarrow E \Rightarrow C$ , 17  $(cA)@cA.E \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C$ , 18  $xx5$ , 19  $xx5 = (cA)@(cA.E \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C)$ , 20  $cA \neq B$ , 21  $B$ , 22  $cA \Rightarrow E \Rightarrow B$ , 23  $xx4$ , 24  $(cA)@cA.E \Rightarrow E \Rightarrow B$ , 25  $xx4 @ B \Rightarrow E \Rightarrow C = (cA)@cA.E \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C$ , 26  $cA \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C$ , 27  $(C)\neg\eta(cA)$ , 28  $xx7$ , 29  $cA \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C = cA \Rightarrow E \Rightarrow C$ , 30  $(C)\neg\eta cA.E \Rightarrow E \Rightarrow B \rightarrow cA.E \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C = cA.E \Rightarrow E \Rightarrow C$ , 31  $(C)\neg\eta cA.E \Rightarrow E \Rightarrow B$ , 32  $(C)\neg\eta(cA) \wedge (C)\neg\eta cA.E \Rightarrow E \Rightarrow B$ , 33  $(C)\neg\eta(cA)@cA.E \Rightarrow E \Rightarrow B$ , 34  $(C)$ , 35  $xx3$ , 36  $(C)\neg\eta cA \Rightarrow E \Rightarrow B$ , 37  $xx2$ , 38  $(C)\neg\eta xx2$ , 39  $()@B \Rightarrow E \Rightarrow C$ , 40  $xx1$ , 41  $xx1 = B \Rightarrow E \Rightarrow C$ , 42  $B \Rightarrow E \Rightarrow B$ , 43  $xx$ , 44  $()$ , 45  $xx @ B \Rightarrow E \Rightarrow C = ()@B \Rightarrow E \Rightarrow C$ , 46  $B \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C$ , 47  $(C)\neg\eta B \Rightarrow E \Rightarrow B$ , 48  $B \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C = B \Rightarrow E \Rightarrow C$ , 49  $A$ , 50  $xA$ , 51  $(C)\neg\eta xA \Rightarrow E \Rightarrow B \rightarrow xA \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C = xA \Rightarrow E \Rightarrow C$ , 52  $(C)\neg\eta A \Rightarrow E \Rightarrow B$ , 53  $A \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C = A \Rightarrow E \Rightarrow C$

SEQ

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("→-E"«52,53/A,B»)
(GIVEN 0)
("list induction"«49,3,50,21,4,51/A,cA,xA,B,E,P»)
(GIVEN 1)
("→-I"«47,48/A,B»)
("transitive="«46,39,16/A,B,C»)
(LAYOUT "A ⇒ E ⇒ A = ()" ALL
("rewrite="«42,43,44,45/A,xx,B,P»)
(LAYOUT HIDEROOT
("A ⇒ E ⇒ A = ()"«21,4/A,E»)
(LAYOUT HIDEROOT
("reflexive="«39/A»)))
    
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("transitive="«39,16,16/A,B,C»)
(LAYOUT "{}@S=S" ALL
  ("rewrite="«39,40,16,41/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("{}@S=S"«16/S»))
  (LAYOUT HIDEROOT
    ("reflexive="«16/A»)))
(LAYOUT HIDEROOT
  ("reflexive="«16/A»))
("→-I"«36,29/A,B»)
(cut«33,29/B,C»)
(LAYOUT "FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B" ALL
  ("rewrite="«24,37,22,38/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("symmetric="«22,24/A,B»)
    (LAYOUT HIDEROOT
      ("FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B"«3,21,4/A,B,E»)
      (hyp«20/A»)))
    (hyp«36/A»))
(cut«32,29/B,C»)
(LAYOUT "R¬∩S@T≠R¬∩S∧R¬∩T" ALL
  ("rewrite="«32,35,33,35/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("symmetric="«33,32/A,B»)
    (LAYOUT HIDEROOT
      ("R¬∩S@T≠R¬∩S∧R¬∩T"«34,14,15/R,S,T»)))
    (hyp«33/A»))
(cut«27,29/B,C»)
(LAYOUT "∧-E" ALL
  ("∧-E(L)"«31,27/B,A»)
  (hyp«32/A»))
(cut«31,29/B,C»)
(LAYOUT "∧-E" ALL
  ("∧-E(R)"«27,31/A,B»)
  (hyp«32/A»))
(cut«8,29/B,C»)
("→-E"«31,8/A,B»)
(hyp«31/A»)
(hyp«30/A»)
(cut«1,29/B,C»)
(LAYOUT "(A)¬∩(B)≠A≠B" ALL
  ("rewrite="«1,28,27,28/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("symmetric="«27,1/A,B»)
    (LAYOUT HIDEROOT
      ("(A)¬∩(B)≠A≠B"«2,3/A,B»)))
    (hyp«27/A»))
("transitive="«26,17,0/A,B,C»)
(LAYOUT "FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B" ALL
  ("rewrite="«22,23,24,25/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B"«3,21,4/A,B,E»)
    (hyp«20/A»))
  (LAYOUT HIDEROOT
    ("reflexive="«17/A»)))
("transitive="«17,13,0/A,B,C»)
(LAYOUT "(R@S)@T≠R@(S@T)" ALL
  ("rewrite="«17,18,13,19/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("(R@S)@T≠R@(S@T)"«14,15,16/R,S,T»))
  (LAYOUT HIDEROOT
    ("reflexive="«13/A»)))
("transitive="«13,5,0/A,B,C»)
("rewrite="«9,10,11,12/P,A,xx,B»)
(hyp«8/A»)
(LAYOUT HIDEROOT
  ("reflexive="«5/A»))

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("transitive="«5,0,0/A,B,C»)
(LAYOUT "FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B" ALL
  ("rewrite="«5,6,0,7/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("symmetric="«0,5/A,B»)
    (LAYOUT HIDEROOT
      ("FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B"«3,2,4/A,B,E»)
      (LAYOUT HIDEROOT
        ("symmetric="«2,3/A,B»)
        (hyp«1/A»)))
    (LAYOUT HIDEROOT
      ("reflexive="«0/A»)))
  (LAYOUT HIDEROOT
    ("reflexive="«0/A»))

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DERIVED RULE IS FROM  $\text{list}(A \Rightarrow E \Rightarrow B)$  INFER  $(B) \neg \neg A \Rightarrow E \Rightarrow B$

1: $\text{list}(A \Rightarrow E \Rightarrow B)$	$\text{list}(A \Rightarrow E \Rightarrow B)$
2: $(B) \neg \neg ()$	$S \neg \neg ()$
3: $(B) \neg \neg B \Rightarrow E \Rightarrow B$	$A \Rightarrow E \Rightarrow A = ()$ 2
4: $cA \neq B, (B) \neg \neg cA.E \Rightarrow E \Rightarrow B$	assumptions
5: $(B) \neg \neg (cA)$	$(A) \neg \neg (B) \neq A \neq B$ 4.1
6: $cA \Rightarrow E \Rightarrow B = (cA) @ cA.E \Rightarrow E \Rightarrow B$	Derived Rule FROM $A \neq B$ INFER $A \Rightarrow E \Rightarrow B = (A) @ A.E \Rightarrow E \Rightarrow B$ 4.1
7: $(B) \neg \neg (cA) @ cA.E \Rightarrow E \Rightarrow B \neq (B) \neg \neg (cA) \wedge (B) \neg \neg cA.E \Rightarrow E \Rightarrow B$	$R \neg \neg S @ T \neq R \neg \neg S \wedge R \neg \neg T$
8: $(B) \neg \neg (cA) \wedge (B) \neg \neg cA.E \Rightarrow E \Rightarrow B$	$\wedge I$ 5,4,2
9: $(B) \neg \neg (cA) @ cA.E \Rightarrow E \Rightarrow B$	$R \neg \neg S @ T \neq R \neg \neg S \wedge R \neg \neg T$ 7,8
10: $(B) \neg \neg cA \Rightarrow E \Rightarrow B$	FROM $A \neq B$ INFER $A \Rightarrow E \Rightarrow B = (A) @ A.E \Rightarrow E \Rightarrow B$ 6,9
11: $(B) \neg \neg A \Rightarrow E \Rightarrow B$	Derived Rule list induction 1,3,4-10

Given:  
 $\text{list}(A \Rightarrow E \Rightarrow B)$

FORMULAE 0  $(B) \neg \neg cA.E \Rightarrow E \Rightarrow B$ , 1  $(B) \neg \neg (cA)$ , 2  $(B)$ , 3  $(cA)$ , 4  $cA.E \Rightarrow E \Rightarrow B$ , 5  $(B) \neg \neg (cA) @ cA.E \Rightarrow E \Rightarrow B$ , 6  $xx2$ , 7  $(B) \neg \neg (cA) \wedge (B) \neg \neg cA.E \Rightarrow E \Rightarrow B$ , 8  $cA \neq B$ , 9  $cA$ , 10  $B$ , 11  $E$ , 12  $cA \Rightarrow E \Rightarrow B$ , 13  $xx1$ , 14  $(cA) @ cA.E \Rightarrow E \Rightarrow B$ , 15  $(B) \neg \neg xx1$ , 16  $(B) \neg \neg cA \Rightarrow E \Rightarrow B$ , 17  $xx3$ , 18  $B \neq cA$ , 19  $B \Rightarrow E \Rightarrow B$ , 20  $xx$ , 21  $()$ , 22  $(B) \neg \neg xx$ , 23  $A$ , 24  $xA$ , 25  $(B) \neg \neg xA \Rightarrow E \Rightarrow B$

SEQ

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("list induction"«23,9,24,10,11,25/A,cA,xA,B,E,P»)
(GIVEN 0)
(LAYOUT "A⇒E⇒A=()" ALL
  ("rewrite="«19,20,21,22/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("A⇒E⇒A=()"«10,11/A,E»)
    ("S¬¬()"«2/S»))
  (cut«1,16/B,C»)
  (LAYOUT "(A)¬¬(B)≠A≠B" ALL
    ("rewrite="«1,17,18,17/A,xx,B,P»)
    (LAYOUT HIDEROOT
      ("(A)¬¬(B)≠A≠B"«10,9/A,B»)
      (LAYOUT HIDEROOT
        ("symmetric="«9,10/A,B»)
        (hyp«8/A»)))
    (cut«0,16/B,C»)
    (hyp«0/A»)
  (LAYOUT "FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B" ALL
    ("rewrite="«12,13,14,15/A,xx,B,P»)
    (LAYOUT HIDEROOT
      ("FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B"«9,10,11/A,B,E»)
      (hyp«8/A»)
    (LAYOUT "R¬¬S@T≠R¬¬S∧R¬¬T" ALL
      ("rewrite="«5,6,7,6/A,xx,B,P»)

```

```
(LAYOUT HIDEROOT
 ("R¬∩S@T≐R¬∩S∧R¬∩T"«2,3,4/R,S,T»))
(LAYOUT COMPRESS "∧-!" ALL
 ("∧-!"«1,0/A,B»)
 (hyp«1/A»)
 (hyp«0/A»))))
```

-----

THEOREM IS  $A \Rightarrow E \Rightarrow +B \neq \{\}$

- 1:  $A \Rightarrow E \Rightarrow B @ (B) @ \{\} \neq \{\}$   $R @ (A) @ T \neq \{\}$
- 2:  $A \Rightarrow E \Rightarrow B @ (B) \neq \{\}$   $S @ \{\} \neq S \ 1$
- 3:  $A \Rightarrow E \Rightarrow +B \neq \{\}$   $A \Rightarrow E \Rightarrow +B \neq A \Rightarrow E \Rightarrow B @ (B) \ 2$

FORMULAE 0 B, 1  $A \Rightarrow E \Rightarrow B$ , 2  $\{\}$ , 3  $A \Rightarrow E \Rightarrow B @ (B)$ , 4  $A \Rightarrow E \Rightarrow B @ (B) @ \{\}$ , 5  $xx1$ , 6  $xx1 \neq \{\}$ , 7 A, 8 E, 9  $A \Rightarrow E \Rightarrow +B$ , 10  $xx$ , 11  $xx \neq \{\}$

```
LAYOUT "A⇒E⇒+B≐A⇒E⇒B@(B)" ALL
 ("rewrite≐"«9,10,3,11/A,xx,B,P»)
(LAYOUT HIDEROOT
 ("A⇒E⇒+B≐A⇒E⇒B@(B)"«7,0,8/A,B,E»))
(LAYOUT "S@{}≐S" ALL
 ("rewrite≐"«3,5,4,6/A,xx,B,P»)
(LAYOUT HIDEROOT
 ("symmetric≐"«4,3/A,B»)
(LAYOUT HIDEROOT
 ("S@{}≐S"«3/S»)))
("R@(A)@T≐{}"«0,1,2/A,R,T»))
```

-----

THEOREM IS  $A \Rightarrow E \Rightarrow +A = \{A\}$

- 1:  $\{\} @ (A) = \{A\}$   $\{\} @ S \neq S$
- 2:  $A \Rightarrow E \Rightarrow A @ (A) = \{A\}$   $A \Rightarrow E \Rightarrow A = \{\} \ 1$
- 3:  $A \Rightarrow E \Rightarrow +A = \{A\}$   $A \Rightarrow E \Rightarrow +B \neq A \Rightarrow E \Rightarrow B @ (B) \ 2$

FORMULAE 0  $\{A\}$ , 1  $\{\} @ (A)$ , 2  $xx2$ , 3  $xx2 = \{A\}$ , 4 A, 5 E, 6  $A \Rightarrow E \Rightarrow A$ , 7  $xx1$ , 8  $\{\}$ , 9  $xx1 @ (A) = \{A\}$ , 10  $A \Rightarrow E \Rightarrow +A$ , 11  $xx$ , 12  $A \Rightarrow E \Rightarrow A @ (A)$ , 13  $xx = \{A\}$

```
LAYOUT "A⇒E⇒+B≐A⇒E⇒B@(B)" ALL
 ("rewrite≐"«10,11,12,13/A,xx,B,P»)
(LAYOUT HIDEROOT
 ("A⇒E⇒+B≐A⇒E⇒B@(B)"«4,4,5/A,B,E»))
(LAYOUT "A⇒E⇒A=()" ALL
 ("rewrite="«6,7,8,9/A,xx,B,P»)
(LAYOUT HIDEROOT
 ("A⇒E⇒A=()"«4,5/A,E»))
(LAYOUT "{}@S≐S" ALL
 ("rewrite="«1,2,0,3/A,xx,B,P»)
(LAYOUT HIDEROOT
 ("{}@S≐S"«0/S»))
(LAYOUT HIDEROOT
 ("reflexive="«0/A»))))
```

-----

DERIVED RULE IS FROM  $A=B$  INFER  $A \Rightarrow E \Rightarrow +B = (A)$

- 1:  $A=B$   $A=B$
- 2:  $(B)=(A)$  rewrite= 1
- 3:  $()@(B)=(A)$   $()@S \triangleq S$  2
- 4:  $A \Rightarrow E \Rightarrow B@(B)=(A)$  FROM  $A=B$  INFER  $A \Rightarrow E \Rightarrow B=()$  1,3
- 5:  $A \Rightarrow E \Rightarrow +B=(A)$   $A \Rightarrow E \Rightarrow +B \triangleq A \Rightarrow E \Rightarrow B@(B)$  4

Given:  
 $A=B$

FORMULAE 0 (B), 1  $A=B$ , 2  $(B)=(x)$ , 3 A, 4 x, 5 B, 6  $()@(B)$ , 7  $xx2$ , 8  $xx2=(A)$ , 9 E, 10  $A \Rightarrow E \Rightarrow B$ , 11  $xx1$ , 12  $()$ , 13  $xx1@(B)=(A)$ , 14  $A \Rightarrow E \Rightarrow +B$ , 15  $xx$ , 16  $A \Rightarrow E \Rightarrow B@(B)$ , 17  $xx=(A)$ , 18  $A \Rightarrow E \Rightarrow +B=(A)$

SEQ

```
(cut«1,18/B,C»)
(GIVEN 0)
(LAYOUT "A⇒E⇒+B△A⇒E⇒B@(B)" ALL
("rewrite△"«14,15,16,17/A,xx,B,P»)
(LAYOUT HIDEROOT
("A⇒E⇒+B△A⇒E⇒B@(B)"«3,5,9/A,B,E»))
(LAYOUT "FROM A=B INFER A⇒E⇒B=()" ALL
("rewrite="«10,11,12,13/A,xx,B,P»)
(LAYOUT HIDEROOT
("FROM A=B INFER A⇒E⇒B=()"«3,5,9/A,B,E»)
(hyp«1/A»)
(LAYOUT "()@S△S" ALL
("rewrite△"«6,7,0,8/A,xx,B,P»)
(LAYOUT HIDEROOT
("()@S△S"«0/S»)
("rewrite="«2,3,4,5/P,A,xx,B»)
(hyp«1/A»)
(LAYOUT HIDEROOT
("reflexive="«0/A»))))
```

DERIVED RULE IS FROM  $A \neq B$  INFER  $A \Rightarrow E \Rightarrow +B = (A)@A.E \Rightarrow E \Rightarrow +B$

- 1:  $A \neq B$   $A \neq B$
- 2:  $(A)@(A.E \Rightarrow E \Rightarrow B@(B))=(A)@A.E \Rightarrow E \Rightarrow +B$   $A \Rightarrow E \Rightarrow +B \triangleq A \Rightarrow E \Rightarrow B@(B)$
- 3:  $(A)@A.E \Rightarrow E \Rightarrow B@(B)=(A)@A.E \Rightarrow E \Rightarrow +B$   $(R@S)@T \triangleq R@(S@T)$  2
- 4:  $A \Rightarrow E \Rightarrow B@(B)=(A)@A.E \Rightarrow E \Rightarrow +B$  FROM  $A \neq B$  INFER  $A \Rightarrow E \Rightarrow B=(A)@A.E \Rightarrow E \Rightarrow B$  1,3
- 5:  $A \Rightarrow E \Rightarrow +B=(A)@A.E \Rightarrow E \Rightarrow +B$   $A \Rightarrow E \Rightarrow +B \triangleq A \Rightarrow E \Rightarrow B@(B)$  4

Given:  
 $A \neq B$

FORMULAE 0  $(A)@A.E \Rightarrow E \Rightarrow +B$ , 1 A.E, 2 B, 3 E, 4  $A.E \Rightarrow E \Rightarrow +B$ , 5  $A.E \Rightarrow E \Rightarrow B@(B)$ , 6  $xx3$ , 7  $(A)@xx3=(A)@A.E \Rightarrow E \Rightarrow +B$ , 8 (A), 9  $A.E \Rightarrow E \Rightarrow B$ , 10 (B), 11  $(A)@A.E \Rightarrow E \Rightarrow B@(B)$ , 12  $xx2$ , 13  $(A)@(A.E \Rightarrow E \Rightarrow B@(B))$ , 14  $xx2=(A)@A.E \Rightarrow E \Rightarrow +B$ , 15 A, 16  $A \Rightarrow E \Rightarrow B$ , 17  $xx1$ , 18  $(A)@A.E \Rightarrow E \Rightarrow B$ , 19  $xx1@(B)=(A)@A.E \Rightarrow E \Rightarrow +B$ , 20  $A \Rightarrow E \Rightarrow +B$ , 21  $xx$ , 22  $A \Rightarrow E \Rightarrow B@(B)$ , 23  $xx=(A)@A.E \Rightarrow E \Rightarrow +B$

```
LAYOUT "A⇒E⇒+B△A⇒E⇒B@(B)" ALL
("rewrite△"«20,21,22,23/A,xx,B,P»)
(LAYOUT HIDEROOT
("A⇒E⇒+B△A⇒E⇒B@(B)"«15,2,3/A,B,E»))
(LAYOUT "FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B" ALL
("rewrite="«16,17,18,19/A,xx,B,P»)
(LAYOUT HIDEROOT
("FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B"«15,2,3/A,B,E»))
```

```

(GIVEN 0)
(LAYOUT "(R@S)@T≐R@(S@T)" ALL
("rewrite≐"«11,12,13,14/A,xx,B,P»)
(LAYOUT HIDEROOT
("R@S)@T≐R@(S@T)"«8,9,10/R,S,T»)
(LAYOUT "A⇒E⇒+B≐A⇒E⇒B@(B)" ALL
("rewrite≐"«5,6,4,7/A,xx,B,P»)
(LAYOUT HIDEROOT
("symmetric≐"«4,5/A,B»)
(LAYOUT HIDEROOT
("A⇒E⇒+B≐A⇒E⇒B@(B)"«1,2,3/A,B,E»)))
(LAYOUT HIDEROOT
("reflexive≐"«0/A»))))

```

-----

DERIVED RULE IS FROM  $A \Rightarrow E \Rightarrow +B = (A)$  INFER  $A = B$

- |                                                      |                                                                             |
|------------------------------------------------------|-----------------------------------------------------------------------------|
| 1: $A \Rightarrow E \Rightarrow +B = (A)$            | $A \Rightarrow E \Rightarrow +B = (A)$                                      |
| 2: $A \Rightarrow E \Rightarrow B@(B) = (A)$         | $A \Rightarrow E \Rightarrow +B \doteq A \Rightarrow E \Rightarrow B@(B)$ 1 |
| 3: $A \Rightarrow E \Rightarrow B@(B) = ()@(A)$      | $()@S \doteq S$ 2                                                           |
| 4: $A \Rightarrow E \Rightarrow B = () \wedge B = A$ | $R@(A) = S@(B) \doteq R = S \wedge A = B$ 3                                 |
| 5: $A \Rightarrow E \Rightarrow B = ()$              | $\wedge$ -E 4                                                               |
| 6: $B = A$                                           | $\wedge$ -E 4                                                               |
| 7: $A = B$                                           | symmetric= 6                                                                |

Given:

$A \Rightarrow E \Rightarrow +B = (A)$

FORMULAE 0  $A = B$ , 1  $B = A$ , 2  $B$ , 3  $A$ , 4  $A \Rightarrow E \Rightarrow B = () \wedge B = A$ , 5  $A \Rightarrow E \Rightarrow B = ()$ , 6  $A \Rightarrow E \Rightarrow B@(B) = ()@(A)$ , 7  $A \Rightarrow E \Rightarrow B$ , 8  $()$ , 9  $xx2$ , 10  $A \Rightarrow E \Rightarrow B@(B) = (A)$ , 11  $(A)$ , 12  $()@(A)$ , 13  $xx1$ , 14  $A \Rightarrow E \Rightarrow B@(B) = xx1$ , 15  $A \Rightarrow E \Rightarrow +B = (A)$ , 16  $E$ , 17  $A \Rightarrow E \Rightarrow +B$ , 18  $A \Rightarrow E \Rightarrow B@(B)$ , 19  $xx$ , 20  $xx = (A)$

SEQ

```

(cut«0,0/B,C»)
(cut«15,0/B,C»)
(GIVEN 0)
(cut«10,0/B,C»)
(LAYOUT "A⇒E⇒+B≐A⇒E⇒B@(B)" ALL
("rewrite≐"«18,19,17,20/A,xx,B,P»)
(LAYOUT HIDEROOT
("symmetric≐"«17,18/A,B»)
(LAYOUT HIDEROOT
("A⇒E⇒+B≐A⇒E⇒B@(B)"«3,2,16/A,B,E»)))
(hyp«15/A»)
(cut«6,0/B,C»)
(LAYOUT "()@S≐S" ALL
("rewrite≐"«12,13,11,14/A,xx,B,P»)
(LAYOUT HIDEROOT
("()@S≐S"«11/S»)
(hyp«10/A»)
(cut«4,0/B,C»)
(LAYOUT "R@(A)=S@(B)≐R=S∧A=B" ALL
("rewrite≐"«4,9,6,9/A,xx,B,P»)
(LAYOUT HIDEROOT
("symmetric≐"«6,4/A,B»)
(LAYOUT HIDEROOT
("R@(A)=S@(B)≐R=S∧A=B"«2,3,7,8/A,B,R,S»)))
(hyp«6/A»)
(cut«5,0/B,C»)
(LAYOUT "∧-E" ALL
("∧-E(L)"«1,5/B,A»)
(hyp«4/A»)

```

```
(cut«1,0/B,C»)
(LAYOUT "∧-E" ALL
 ("∧-E(R)"«5,1/A,B»)
 (hyp«4/A»)
(LAYOUT HIDEROOT
 ("symmetric="«2,3/A,B»)
 (hyp«1/A»)
 (hyp«0/A»)
```

-----

DERIVED RULE IS FROM  $A \neq B$  AND  $\text{list}(A \Rightarrow E \Rightarrow +B)$  INFER  $A \neq A.E$

1: $\text{list}(A \Rightarrow E \Rightarrow +B)$	$\text{list}(A \Rightarrow E \Rightarrow +B)$
2: $\text{list}(A \Rightarrow E \Rightarrow B @ (B))$	$A \Rightarrow E \Rightarrow +B \hat{=} A \Rightarrow E \Rightarrow B @ (B)$ 1
3: $\text{list}(A \Rightarrow E \Rightarrow B) \wedge \text{list}(B) \wedge A \Rightarrow E \Rightarrow B \neg \cap (B)$	$\text{list}(R @ S) \hat{=} \text{list } R \wedge \text{list } S \wedge R \neg \cap S$ 2
4: $\text{list}(A \Rightarrow E \Rightarrow B)$	$\wedge$ -E 3
5: $\text{list}(B)$	$\wedge$ -E 3
6: $A \Rightarrow E \Rightarrow B \neg \cap (B)$	$\wedge$ -E 3
7: $A \neq B$	$A \neq B$
8: $A \neq A.E$	Derived Rule FROM $A \neq B$ AND $\text{list}(A \Rightarrow E \Rightarrow B)$ INFER $A \neq A.E$ 7,4

Given:

```
A ≠ B
list(A ⇒ E ⇒ +B)
```

FORMULAE 0  $\text{list}(A \Rightarrow E \Rightarrow B)$ , 1 A, 2 B, 3 E, 4  $\text{list}(A \Rightarrow E \Rightarrow B) \wedge \text{list}(B) \wedge A \Rightarrow E \Rightarrow B \neg \cap (B)$ , 5  $\text{list}(A \Rightarrow E \Rightarrow B) \wedge \text{list}(B)$ , 6  $A \Rightarrow E \Rightarrow B \neg \cap (B)$ , 7  $A \neq A.E$ , 8  $\text{list}(B)$ , 9  $\text{list}(A \Rightarrow E \Rightarrow B @ (B))$ , 10  $A \Rightarrow E \Rightarrow B$ , 11  $(B)$ , 12  $xx1$ , 13  $\text{list}(A \Rightarrow E \Rightarrow +B)$ , 14  $A \Rightarrow E \Rightarrow +B$ , 15  $A \Rightarrow E \Rightarrow B @ (B)$ , 16  $xx$ , 17  $\text{list}(xx)$

SEQ

```
(cut«13,7/B,C»)
(GIVEN 1)
(cut«9,7/B,C»)
(LAYOUT "A ⇒ E ⇒ +B ≐ A ⇒ E ⇒ B @ (B)" ALL
 ("rewrite≐"«15,16,14,17/A,xx,B,P»)
(LAYOUT HIDEROOT
 ("symmetric≐"«14,15/A,B»)
(LAYOUT HIDEROOT
 ("A ⇒ E ⇒ +B ≐ A ⇒ E ⇒ B @ (B)"«1,2,3/A,B,E»)))
 (hyp«13/A»)
(cut«4,7/B,C»)
(LAYOUT "list(R @ S) ≐ list R ∧ list S ∧ R ¬ ∩ S" ALL
 ("rewrite≐"«4,12,9,12/A,xx,B,P»)
(LAYOUT HIDEROOT
 ("symmetric≐"«9,4/A,B»)
(LAYOUT HIDEROOT
 ("list(R @ S) ≐ list R ∧ list S ∧ R ¬ ∩ S"«10,11/R,S»)))
 (hyp«9/A»)
(LAYOUT HIDE CUT
 (cut«5,7/B,C»)
 ("∧-E(L)"«6,5/B,A»)
 (hyp«4/A»)
 (cut«0,7/B,C»)
 (LAYOUT "∧-E" ALL
 ("∧-E(L)"«8,0/B,A»)
 (hyp«5/A»)
 (cut«8,7/B,C»)
 (LAYOUT "∧-E" ALL
 ("∧-E(R)"«0,8/A,B»)
 (hyp«5/A»)
 (cut«6,7/B,C»)
```

```
(LAYOUT "∧-E" ALL
  ("∧-E(R)"«5,6/A,B»)
  (hyp«4/A»))
("FROM A≠B AND list(A⇒E⇒B) INFER A≠A.E"«1,2,3/A,B,E»)
(GIVEN 0)
(hyp«0/A»))
```

DERIVED RULE IS FROM  $A \Rightarrow E \Rightarrow +B = (C) @ S$  AND  $S \neq ()$  INFER  $A \neq B \wedge A = C \wedge A.E \Rightarrow E \Rightarrow +B = S$

1: $A \Rightarrow E \Rightarrow +B = (C) @ S$	$A \Rightarrow E \Rightarrow +B = (C) @ S$
2: $A = B$	assumption
3: $\langle A \rangle = (C) @ S$	FROM $A = B$ INFER $A \Rightarrow E \Rightarrow +B = (A)$ 2,1
4: $\langle A \rangle @ () = (C) @ S$	$S @ () \neq S$ 3
5: $A = C \wedge () = S$	$\langle A \rangle @ R = (B) @ S \neq A = B \wedge R = S$ 4
6: $A = C$	$\wedge$ -E 5
7: $() = S$	$\wedge$ -E 5
8: $S \neq ()$	$S \neq ()$
9: $\neg ( () = S )$	$\neg (A = B) \neq A \neq B$ 8
10: $\perp$	$\neg$ -E 7,9
11: $\neg (A = B)$	$\neg$ -I 2-10
12: $A \neq B$	$\neg (A = B) \neq A \neq B$ 11
13: $\langle A \rangle @ A.E \Rightarrow E \Rightarrow +B = (C) @ S$	FROM $A \neq B$ INFER $A \Rightarrow E \Rightarrow +B = \langle A \rangle @ A.E \Rightarrow E \Rightarrow +B$ 12,1
14: $A = C \wedge A.E \Rightarrow E \Rightarrow +B = S$	$\langle A \rangle @ R = (B) @ S \neq A = B \wedge R = S$ 13
15: $A = C$	$\wedge$ -E 14
16: $A.E \Rightarrow E \Rightarrow +B = S$	$\wedge$ -E 14
17: $A \neq B \wedge A = C$	$\wedge$ -I 12,15
18: $A \neq B \wedge A = C \wedge A.E \Rightarrow E \Rightarrow +B = S$	$\wedge$ -I 17,16

Given:

```
A ⇒ E ⇒ +B = (C) @ S
S ≠ ()
```

FORMULAE 0  $A.E \Rightarrow E \Rightarrow +B = S$ , 1  $A = C$ , 2  $A \neq B$ , 3  $A \neq B \wedge A = C$ , 4  $A \neq B \wedge A = C \wedge A.E \Rightarrow E \Rightarrow +B = S$ , 5  $A = C \wedge A.E \Rightarrow E \Rightarrow +B = S$ , 6  $\langle A \rangle @ A.E \Rightarrow E \Rightarrow +B = (C) @ S$ , 7  $A$ , 8  $C$ , 9  $A.E \Rightarrow E \Rightarrow +B$ , 10  $S$ , 11  $xx6$ , 12  $A \Rightarrow E \Rightarrow +B = (C) @ S$ , 13  $B$ , 14  $E$ , 15  $A \Rightarrow E \Rightarrow +B$ , 16  $\langle A \rangle @ A.E \Rightarrow E \Rightarrow +B$ , 17  $xx5$ , 18  $xx5 = (C) @ S$ , 19  $()$ , 20  $\neg ( () = S )$ , 21  $xx4$ , 22  $() \neq S$ , 23  $() = S$ , 24  $A = C \wedge () = S$ , 25  $\perp$ , 26  $\langle A \rangle @ () = (C) @ S$ , 27  $xx3$ , 28  $\langle A \rangle = (C) @ S$ , 29  $\langle A \rangle$ , 30  $\langle A \rangle @ ()$ , 31  $xx2$ , 32  $xx2 = (C) @ S$ , 33  $A = B$ , 34  $xx1$ , 35  $xx1 = (C) @ S$ , 36  $\neg (A = B)$ , 37  $xx$

SEQ

```
(cut«12,4/B,C»)
(GIVEN 0)
(cut«2,4/B,C»)
(LAYOUT "¬(A=B)≠A≠B" ALL
  ("rewrite≠"«2,37,36,37/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("symmetric≠"«2,36/B,A»)
    (LAYOUT HIDEROOT
      ("¬(A=B)≠A≠B"«7,13/A,B»)))
  ("¬-I"«33/A»)
  (cut«28,25/B,C»)
  (LAYOUT "FROM A=B INFER A⇒E⇒+B=(A)" ALL
    ("rewrite="«29,34,15,35/A,xx,B,P»)
    (LAYOUT HIDEROOT
```

```

("symmetric="«15,29/A,B»)
(LAYOUT HIDEROOT
 ("FROM A=B INFER A⇒E⇒+B=(A)"«7,13,14/A,B,E»)
 (hyp«33/A»))
(hyp«12/A»)
(cut«26,25/B,C»)
(LAYOUT "S@()≠S" ALL
 ("rewrite="«30,31,29,32/A,xx,B,P»)
 (LAYOUT HIDEROOT
 ("S@()≠S"«29/S»)
 (hyp«28/A»)
 (cut«24,25/B,C»)
 (LAYOUT "(A)@R=(B)@S≠A=B∧R=S" ALL
 ("rewrite="«24,27,26,27/A,xx,B,P»)
 (LAYOUT HIDEROOT
 ("symmetric="«26,24/A,B»)
 (LAYOUT HIDEROOT
 ("(A)@R=(B)@S≠A=B∧R=S"«7,8,19,10/A,B,R,S»)))
 (hyp«26/A»)
 (cut«1,25/B,C»)
 (LAYOUT "^-E" ALL
 ("^-E(L)"«23,1/B,A»)
 (hyp«24/A»)
 (cut«23,25/B,C»)
 (LAYOUT "^-E" ALL
 ("^-E(R)"«1,23/A,B»)
 (hyp«24/A»)
 ("^-E"«23/B»)
 (hyp«23/A»)
 (LAYOUT "~(A=B)≠A≠B" ALL
 ("rewrite="«20,21,22,21/A,xx,B,P»)
 (LAYOUT HIDEROOT
 ("~(A=B)≠A≠B"«19,10/A,B»)
 (LAYOUT HIDEROOT
 ("symmetric="«10,19/A,B»)
 (GIVEN 1))))
 (cut«6,4/B,C»)
 (LAYOUT "FROM A≠B INFER A⇒E⇒+B=(A)@A.E⇒E⇒+B" ALL
 ("rewrite="«16,17,15,18/A,xx,B,P»)
 (LAYOUT HIDEROOT
 ("symmetric="«15,16/A,B»)
 (LAYOUT HIDEROOT
 ("FROM A≠B INFER A⇒E⇒+B=(A)@A.E⇒E⇒+B"«7,13,14/A,B,E»)
 (hyp«2/A»)
 (hyp«12/A»)
 (cut«5,4/B,C»)
 (LAYOUT "(A)@R=(B)@S≠A=B∧R=S" ALL
 ("rewrite="«5,11,6,11/A,xx,B,P»)
 (LAYOUT HIDEROOT
 ("symmetric="«6,5/A,B»)
 (LAYOUT HIDEROOT
 ("(A)@R=(B)@S≠A=B∧R=S"«7,8,9,10/A,B,R,S»)))
 (hyp«6/A»)
 (cut«1,4/B,C»)
 (LAYOUT "^-E" ALL
 ("^-E(L)"«0,1/B,A»)
 (hyp«5/A»)
 (cut«0,4/B,C»)
 (LAYOUT "^-E" ALL
 ("^-E(R)"«1,0/A,B»)
 (hyp«5/A»)
 (cut«1,4/B,C»)
 (hyp«1/A»)
 (cut«0,4/B,C»)
 (hyp«0/A»)
 (LAYOUT COMPRESS "^-I" ALL
 ("^-I"«3,0/A,B»)

```

```
(LAYOUT COMPRESS "^-!" ALL
 ("^-!"«2,1/A,B»)
 (hyp«2/A»)
 (hyp«1/A»)
 (hyp«0/A»))
```

DERIVED RULE IS FROM  $\text{list}(A \Rightarrow E \Rightarrow +B)$  INFER  $\text{list}(A \Rightarrow E \Rightarrow B)$

1: $\text{list}(A \Rightarrow E \Rightarrow +B)$	$\text{list}(A \Rightarrow E \Rightarrow +B)$
2: $\text{list}(A \Rightarrow E \Rightarrow B @ (B))$	$A \Rightarrow E \Rightarrow +B \triangle A \Rightarrow E \Rightarrow B @ (B)$ 1
3: $\text{list}(A \Rightarrow E \Rightarrow B) \wedge \text{list}(B) \wedge A \Rightarrow E \Rightarrow B \neg \cap (B)$	$\text{list}(R @ S) \triangle \text{list } R \wedge \text{list } S \wedge R \neg \cap S$ 2
4: $\text{list}(A \Rightarrow E \Rightarrow B)$	$\wedge\text{-E}$ 3
5: $\text{list}(B)$	$\wedge\text{-E}$ 3
6: $A \Rightarrow E \Rightarrow B \neg \cap (B)$	$\wedge\text{-E}$ 3
7: $\text{list}(A \Rightarrow E \Rightarrow B)$	hyp 4

Given:

$\text{list}(A \Rightarrow E \Rightarrow +B)$

FORMULAE 0  $\text{list}(A \Rightarrow E \Rightarrow B)$ , 1  $\text{list}(A \Rightarrow E \Rightarrow B) \wedge \text{list}(B) \wedge A \Rightarrow E \Rightarrow B \neg \cap (B)$ , 2  $\text{list}(A \Rightarrow E \Rightarrow B) \wedge \text{list}(B)$ , 3  $A \Rightarrow E \Rightarrow B \neg \cap (B)$ , 4  $\text{list}(B)$ , 5  $\text{list}(A \Rightarrow E \Rightarrow B @ (B))$ , 6  $A \Rightarrow E \Rightarrow B$ , 7  $(B)$ , 8  $xx1$ , 9  $\text{list}(A \Rightarrow E \Rightarrow +B)$ , 10  $A$ , 11  $B$ , 12  $E$ , 13  $A \Rightarrow E \Rightarrow +B$ , 14  $A \Rightarrow E \Rightarrow B @ (B)$ , 15  $xx$ , 16  $\text{list}(xx)$

SEQ

```
(cut«9,0/B,C»)
(GIVEN 0)
(cut«5,0/B,C»)
(LAYOUT "A⇒E⇒+B△A⇒E⇒B@(B)" ALL
 ("rewrite△"«14,15,13,16/A,xx,B,P»)
 (LAYOUT HIDEROOT
 ("symmetric△"«13,14/A,B»)
 (LAYOUT HIDEROOT
 ("A⇒E⇒+B△A⇒E⇒B@(B)"«10,11,12/A,B,E»)))
 (hyp«9/A»)
(cut«1,0/B,C»)
(LAYOUT "list(R@S)△list R∧list S∧R¬∩S" ALL
 ("rewrite△"«1,8,5,8/A,xx,B,P»)
 (LAYOUT HIDEROOT
 ("symmetric△"«5,1/A,B»)
 (LAYOUT HIDEROOT
 ("list(R@S)△list R∧list S∧R¬∩S"«6,7/R,S»)))
 (hyp«5/A»)
(LAYOUT HIDE CUT
 (cut«2,0/B,C»)
 ("∧-E(L)"«3,2/B,A»)
 (hyp«1/A»)
 (cut«0,0/B,C»)
 (LAYOUT "∧-E" ALL
 ("∧-E(L)"«4,0/B,A»)
 (hyp«2/A»)
 (cut«4,0/B,C»)
 (LAYOUT "∧-E" ALL
 ("∧-E(R)"«0,4/A,B»)
 (hyp«2/A»)
 (cut«3,0/B,C»)
 (LAYOUT "∧-E" ALL
 ("∧-E(R)"«2,3/A,B»)
 (hyp«1/A»)
 (hyp«0/A»))
```

DERIVED RULE IS FROM  $\text{list}(A \Rightarrow E \Rightarrow B)$  INFER  $\text{list}(A \Rightarrow E \Rightarrow +B)$

1: $\text{list}(A \Rightarrow E \Rightarrow B)$	$\text{list}(A \Rightarrow E \Rightarrow B)$
2: $(B) \neg \neg A \Rightarrow E \Rightarrow B$	Derived Rule FROM $\text{list}(A \Rightarrow E \Rightarrow B)$ INFER $(B) \neg \neg A \Rightarrow E \Rightarrow B$ 1
3: $A \Rightarrow E \Rightarrow B \neg \neg (B) \triangleq (B) \neg \neg A \Rightarrow E \Rightarrow B$	$R \neg \neg S \triangleq S \neg \neg R$
4: $A \Rightarrow E \Rightarrow B \neg \neg (B)$	$R \neg \neg S \triangleq S \neg \neg R$ 3,2
5: $A \Rightarrow E \Rightarrow +B \triangleq A \Rightarrow E \Rightarrow B @ (B)$	$A \Rightarrow E \Rightarrow +B \triangleq A \Rightarrow E \Rightarrow B @ (B)$
6: $\text{list}(A \Rightarrow E \Rightarrow B @ (B)) \triangleq \text{list}(A \Rightarrow E \Rightarrow B) \wedge \text{list}(B) \wedge A \Rightarrow E \Rightarrow B \neg \neg (B)$	$\text{list}(R @ S) \triangleq \text{list } R \wedge \text{list } S \wedge R \neg \neg S$
7: $\text{list}(A \Rightarrow E \Rightarrow B) \wedge \text{list}(B)$	$\wedge -I$ 1
8: $\text{list}(A \Rightarrow E \Rightarrow B) \wedge \text{list}(B) \wedge A \Rightarrow E \Rightarrow B \neg \neg (B)$	$\wedge -I$ 7,4
9: $\text{list}(A \Rightarrow E \Rightarrow B @ (B))$	$\text{list}(R @ S) \triangleq \text{list } R \wedge \text{list } S \wedge R \neg \neg S$ 6,8
10: $\text{list}(A \Rightarrow E \Rightarrow +B)$	$A \Rightarrow E \Rightarrow +B \triangleq A \Rightarrow E \Rightarrow B @ (B)$ 5,9

Given:

$\text{list}(A \Rightarrow E \Rightarrow B)$

FORMULAE 0  $A \Rightarrow E \Rightarrow B \neg \neg (B)$ , 1  $\text{list}(B)$ , 2  $\text{list}(A \Rightarrow E \Rightarrow B)$ , 3  $\text{list}(A \Rightarrow E \Rightarrow B) \wedge \text{list}(B)$ , 4  $A \Rightarrow E \Rightarrow B$ , 5  $(B)$ , 6  $\text{list}(A \Rightarrow E \Rightarrow B @ (B))$ , 7  $xx1$ , 8  $\text{list}(A \Rightarrow E \Rightarrow B) \wedge \text{list}(B) \wedge A \Rightarrow E \Rightarrow B \neg \neg (B)$ , 9  $A$ , 10  $B$ , 11  $E$ , 12  $A \Rightarrow E \Rightarrow +B$ , 13  $xx$ , 14  $A \Rightarrow E \Rightarrow B @ (B)$ , 15  $\text{list } xx$ , 16  $(B) \neg \neg A \Rightarrow E \Rightarrow B$ , 17  $xx2$ , 18  $\text{list}(A \Rightarrow E \Rightarrow +B)$

SEQ

```
(cut«2,18/B,C»)
(GIVEN 0)
(cut«16,18/B,C»)
("FROM list(A⇒E⇒B) INFER (B)¬¬A⇒E⇒B"«9,10,11/A,B,E»)
(hyp«2/A»)
(cut«2,18/B,C»)
(hyp«2/A»)
(cut«1,18/B,C»)
(LAYOUT HIDEROOT
  ("list(A)"«10/A»))
(cut«0,18/B,C»)
(LAYOUT "R¬¬S≐S¬¬R" ALL
  ("rewrite≐"«0,17,16,17/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("R¬¬S≐S¬¬R"«4,5/R,S»)
    (hyp«16/A»))
(LAYOUT "A⇒E⇒+B≐A⇒E⇒B@(B)" ALL
  ("rewrite≐"«12,13,14,15/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("A⇒E⇒+B≐A⇒E⇒B@(B)"«9,10,11/A,B,E»))
(LAYOUT "list(R@S)≐list R∧list S∧R¬¬S" ALL
  ("rewrite≐"«6,7,8,7/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("list(R@S)≐list R∧list S∧R¬¬S"«4,5/R,S»)
  (LAYOUT COMPRESS "∧-I" ALL
    ("∧-I"«3,0/A,B»)
    (LAYOUT COMPRESS "∧-I" ALL
      ("∧-I"«2,1/A,B»)
      (hyp«2/A»)
      (hyp«1/A»)
      (hyp«0/A»))))
```

DERIVED RULE IS FROM  $F \neg \neg A \Rightarrow E \Rightarrow +B$  INFER  $F \neg \neg A \Rightarrow E \Rightarrow B \wedge F \neg \neg(B)$

- 1:  $F \neg \neg A \Rightarrow E \Rightarrow +B$   $F \neg \neg A \Rightarrow E \Rightarrow +B$
- 2:  $F \neg \neg A \Rightarrow E \Rightarrow B @ (B)$   $A \Rightarrow E \Rightarrow +B \triangleq A \Rightarrow E \Rightarrow B @ (B)$  1
- 3:  $F \neg \neg A \Rightarrow E \Rightarrow B \wedge F \neg \neg(B)$   $R \neg \neg S @ T \triangleq R \neg \neg S \wedge R \neg \neg T$  2

Given:

$F \neg \neg A \Rightarrow E \Rightarrow +B$

FORMULAE 0  $F \neg \neg A \Rightarrow E \Rightarrow B \wedge F \neg \neg(B)$ , 1  $F \neg \neg A \Rightarrow E \Rightarrow B @ (B)$ , 2 F, 3  $A \Rightarrow E \Rightarrow B$ , 4 (B), 5 xx1, 6  $F \neg \neg A \Rightarrow E \Rightarrow +B$ , 7 A, 8 B, 9 E, 10  $A \Rightarrow E \Rightarrow +B$ , 11  $A \Rightarrow E \Rightarrow B @ (B)$ , 12 xx, 13  $F \neg \neg xx$

SEQ

```
(cut«6,0/B,C»)
(GIVEN 0)
(cut«1,0/B,C»)
(LAYOUT "A⇒E⇒+B△A⇒E⇒B@(B)" ALL
("rewrite△"«11,12,10,13/A,xx,B,P»)
(LAYOUT HIDEROOT
("symmetric△"«10,11/A,B»)
(LAYOUT HIDEROOT
("A⇒E⇒+B△A⇒E⇒B@(B)"«7,8,9/A,B,E»)))
(hyp«6/A»)
(cut«0,0/B,C»)
(LAYOUT "R¬¬S@T△R¬¬S∧R¬¬T" ALL
("rewrite△"«0,5,1,5/A,xx,B,P»)
(LAYOUT HIDEROOT
("symmetric△"«1,0/A,B»)
(LAYOUT HIDEROOT
("R¬¬S@T△R¬¬S∧R¬¬T"«2,3,4/R,S,T»)))
(hyp«1/A»)
(hyp«0/A»)
```

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DERIVED RULE IS FROM  $(C) \neg \neg A \Rightarrow E \Rightarrow B$  AND  $\text{list}(A \Rightarrow E \Rightarrow +B)$  INFER  $A \Rightarrow E \Rightarrow +B = A \Rightarrow (E \oplus C \rightarrow F) \Rightarrow +B$

- 1:  $(C) \neg \neg A \Rightarrow E \Rightarrow B$   $(C) \neg \neg A \Rightarrow E \Rightarrow B$
- 2:  $\text{list}(A \Rightarrow E \Rightarrow +B)$   $\text{list}(A \Rightarrow E \Rightarrow +B)$
- 3:  $\text{list}(A \Rightarrow E \Rightarrow B)$  Derived Rule FROM  $\text{list}(A \Rightarrow E \Rightarrow +B)$  INFER  $\text{list}(A \Rightarrow E \Rightarrow B)$  2
- 4:  $A \Rightarrow E \Rightarrow B = A \Rightarrow (E \oplus C \rightarrow F) \Rightarrow B$  Derived Rule FROM  $(C) \neg \neg A \Rightarrow E \Rightarrow B$  AND  $\text{list}(A \Rightarrow E \Rightarrow B)$  INFER  $A \Rightarrow E \Rightarrow B = A \Rightarrow (E \oplus C \rightarrow F) \Rightarrow B$  1,3
- 5:  $A \Rightarrow E \Rightarrow B = A \Rightarrow (E \oplus C \rightarrow F) \Rightarrow B \wedge B = B$   $\wedge \neg$  4
- 6:  $A \Rightarrow E \Rightarrow +B$
- 7: =  $A \Rightarrow E \Rightarrow B @ (B)$   $A \Rightarrow E \Rightarrow +B \triangleq A \Rightarrow E \Rightarrow B @ (B)$
- 8: =  $A \Rightarrow (E \oplus C \rightarrow F) \Rightarrow B @ (B)$   $R @ (A) = S @ (B) \triangleq R = S \wedge A = B$  5
- 9: =  $A \Rightarrow (E \oplus C \rightarrow F) \Rightarrow +B$   $A \Rightarrow E \Rightarrow +B \triangleq A \Rightarrow E \Rightarrow B @ (B)$

Given:

$(C) \neg \neg A \Rightarrow E \Rightarrow B$   
 $\text{list}(A \Rightarrow E \Rightarrow +B)$

FORMULAE 0  $A \Rightarrow (E \oplus C \rightarrow F) \Rightarrow B @ (B)$ , 1 A, 2 B, 3  $E \oplus C \rightarrow F$ , 4  $A \Rightarrow (E \oplus C \rightarrow F) \Rightarrow +B$ , 5 xx1, 6  $A \Rightarrow (E \oplus C \rightarrow F) \Rightarrow B @ (B) = xx1$ , 7  $B = B$ , 8  $A \Rightarrow E \Rightarrow B = A \Rightarrow (E \oplus C \rightarrow F) \Rightarrow B$ , 9  $A \Rightarrow E \Rightarrow B$ , 10  $A \Rightarrow (E \oplus C \rightarrow F) \Rightarrow B$ , 11  $A \Rightarrow E \Rightarrow B @ (B) = A \Rightarrow (E \oplus C \rightarrow F) \Rightarrow B @ (B)$ , 12 xx2, 13  $A \Rightarrow E \Rightarrow B = A \Rightarrow (E \oplus C \rightarrow F) \Rightarrow B \wedge B = B$ , 14  $A \Rightarrow E \Rightarrow B @ (B)$ , 15 E, 16  $A \Rightarrow E \Rightarrow +B$ , 17 xx, 18  $xx = A \Rightarrow E \Rightarrow B @ (B)$ , 19  $A \Rightarrow E \Rightarrow +B = A \Rightarrow (E \oplus C \rightarrow F) \Rightarrow +B$ , 20 C, 21 F

SEQ

```
(cut«8,19/B,C»)
("FROM (C)¬¬A⇒E⇒B AND list(A⇒E⇒B) INFER A⇒E⇒B=A⇒(E⊕C→F)⇒B"«1,2,20,15,21/A,B,C,E,F»)
(GIVEN 0)
("FROM list(A⇒E⇒+B) INFER list(A⇒E⇒B)"«1,2,15/A,B,E»)
(GIVEN 1)
(cut«7,19/B,C»)

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(LAYOUT HIDEROOT
  ("reflexive="«2/A»)
  ("transitive="«16,14,4/A,B,C»)
(LAYOUT "A⇒E⇒+B⇒A⇒E⇒B@{B}" ALL
  ("rewrite="«16,17,14,18/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("A⇒E⇒+B⇒A⇒E⇒B@{B}"«1,2,15/A,B,E»))
  (LAYOUT HIDEROOT
    ("reflexive="«14/A»)))
  ("transitive="«14,0,4/A,B,C»)
(LAYOUT "R@{A}=S@{B}⇒R=S^A=B" ALL
  ("rewrite="«11,12,13,12/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("R@{A}=S@{B}⇒R=S^A=B"«2,2,9,10/A,B,R,S»))
  (LAYOUT COMPRESS "^-I" ALL
    ("^-I"«8,7/A,B»)
    (hyp«8/A»)
    (hyp«7/A»)))
(LAYOUT "A⇒E⇒+B⇒A⇒E⇒B@{B}" ALL
  ("rewrite="«4,5,0,6/A,xx,B,P»)
  (LAYOUT HIDEROOT
    ("A⇒E⇒+B⇒A⇒E⇒B@{B}"«1,2,3/A,B,E»))
  (LAYOUT HIDEROOT
    ("reflexive="«0/A»)))

```

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DERIVED RULE IS FROM  $(C)\neg\eta A \Rightarrow E \Rightarrow +B$  AND  $\text{list}(A \Rightarrow E \Rightarrow +B)$   
 INFER  $A \Rightarrow E \Rightarrow +B @ B.E \Rightarrow E \Rightarrow +C = A \Rightarrow E \Rightarrow +C$

1: $(C)\neg\eta A \Rightarrow E \Rightarrow +B$	$(C)\neg\eta A \Rightarrow E \Rightarrow +B$
2: $(C)\neg\eta A \Rightarrow E \Rightarrow B \wedge (C)\neg\eta(B)$	Derived Rule FROM $F\neg\eta A \Rightarrow E \Rightarrow +B$ INFER $F\neg\eta A \Rightarrow E \Rightarrow B \wedge F\neg\eta(B)$ 1
3: $(C)\neg\eta A \Rightarrow E \Rightarrow B$	$\wedge$ -E 2
4: $(C)\neg\eta(B)$	$\wedge$ -E 2
5: $C \neq B$	$(A)\neg\eta(B) \neq A \neq B$ 4
6: $\text{list}(A \Rightarrow E \Rightarrow +B)$	$\text{list}(A \Rightarrow E \Rightarrow +B)$
7: $\text{list}(A \Rightarrow E \Rightarrow B)$	Derived Rule FROM $\text{list}(A \Rightarrow E \Rightarrow +B)$ INFER $\text{list}(A \Rightarrow E \Rightarrow B)$ 6
8: $A \Rightarrow E \Rightarrow +B \neq A \Rightarrow E \Rightarrow B @ (B)$	$A \Rightarrow E \Rightarrow +B \neq A \Rightarrow E \Rightarrow B @ (B)$
9: $A \Rightarrow E \Rightarrow B @ (B) @ B.E \Rightarrow E \Rightarrow +C = A \Rightarrow E \Rightarrow B @ (B) @ B.E \Rightarrow E \Rightarrow +C$	reflexive=
10: $A \Rightarrow E \Rightarrow B @ (B) @ B.E \Rightarrow E \Rightarrow +C \neq A \Rightarrow E \Rightarrow B @ ((B) @ B.E \Rightarrow E \Rightarrow +C)$	$(R @ S) @ T \neq R @ (S @ T)$
11: $A \Rightarrow E \Rightarrow B @ ((B) @ B.E \Rightarrow E \Rightarrow +C) = A \Rightarrow E \Rightarrow B @ ((B) @ B.E \Rightarrow E \Rightarrow +C)$	reflexive=
12: $B \Rightarrow E \Rightarrow +C = (B) @ B.E \Rightarrow E \Rightarrow +C$	Derived Rule FROM $A \neq B$ INFER $A \Rightarrow E \Rightarrow +B = (A) @ A.E \Rightarrow E \Rightarrow +B$ 5
13: $(B) @ B.E \Rightarrow E \Rightarrow +C = B \Rightarrow E \Rightarrow +C$	symmetric= 12
14: $A \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow +C = A \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow +C$	reflexive=
15: $B \Rightarrow E \Rightarrow +C \neq B \Rightarrow E \Rightarrow C @ (C)$	$A \Rightarrow E \Rightarrow +B \neq A \Rightarrow E \Rightarrow B @ (B)$
16: $A \Rightarrow E \Rightarrow B @ (B \Rightarrow E \Rightarrow C @ (C)) = A \Rightarrow E \Rightarrow B @ (B \Rightarrow E \Rightarrow C @ (C))$	reflexive=
17: $A \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C @ (C) \neq A \Rightarrow E \Rightarrow B @ (B \Rightarrow E \Rightarrow C @ (C))$	$(R @ S) @ T \neq R @ (S @ T)$
18: $A \Rightarrow E \Rightarrow B @ (B \Rightarrow E \Rightarrow C @ (C)) \neq A \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C @ (C)$	symmetric= 17
19: $A \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C @ (C) = A \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C @ (C)$	reflexive=
20: $A \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C = A \Rightarrow E \Rightarrow C$	Derived Rule FROM $(C)\neg\eta A \Rightarrow E \Rightarrow B$ AND $\text{list}(A \Rightarrow E \Rightarrow B)$ INFER $A \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C = A \Rightarrow E \Rightarrow C$ 3,7
21: $A \Rightarrow E \Rightarrow C @ (C) = A \Rightarrow E \Rightarrow C @ (C)$	reflexive=
22: $A \Rightarrow E \Rightarrow +C \neq A \Rightarrow E \Rightarrow C @ (C)$	$A \Rightarrow E \Rightarrow +B \neq A \Rightarrow E \Rightarrow B @ (B)$
23: $A \Rightarrow E \Rightarrow C @ (C) \neq A \Rightarrow E \Rightarrow +C$	symmetric= 22
24: $A \Rightarrow E \Rightarrow +C = A \Rightarrow E \Rightarrow +C$	reflexive=
25: $A \Rightarrow E \Rightarrow +B @ B.E \Rightarrow E \Rightarrow +C$	
26: = $A \Rightarrow E \Rightarrow B @ (B) @ B.E \Rightarrow E \Rightarrow +C$	$A \Rightarrow E \Rightarrow +B \neq A \Rightarrow E \Rightarrow B @ (B)$ 8,9
27: = $A \Rightarrow E \Rightarrow B @ ((B) @ B.E \Rightarrow E \Rightarrow +C)$	$(R @ S) @ T \neq R @ (S @ T)$ 10,11
28: = $A \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow +C$	FROM $A \neq B$ INFER $A \Rightarrow E \Rightarrow +B = (A) @ A.E \Rightarrow E \Rightarrow +B$ 13,14
29: = $A \Rightarrow E \Rightarrow B @ (B \Rightarrow E \Rightarrow C @ (C))$	$A \Rightarrow E \Rightarrow +B \neq A \Rightarrow E \Rightarrow B @ (B)$ 15,16
30: = $A \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C @ (C)$	$(R @ S) @ T \neq R @ (S @ T)$ 18,19
31: = $A \Rightarrow E \Rightarrow C @ (C)$	FROM $(C)\neg\eta A \Rightarrow E \Rightarrow B$ AND $\text{list}(A \Rightarrow E \Rightarrow B)$ INFER $A \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C = A \Rightarrow E \Rightarrow C$ 20,21
32: = $A \Rightarrow E \Rightarrow +C$	$A \Rightarrow E \Rightarrow +B \neq A \Rightarrow E \Rightarrow B @ (B)$ 23,24

Given:

$(C)\neg\eta A \Rightarrow E \Rightarrow +B$   
 $\text{list}(A \Rightarrow E \Rightarrow +B)$

FORMULAE 0  $A \Rightarrow E \Rightarrow +C$ , 1 A, 2 C, 3 E, 4  $A \Rightarrow E \Rightarrow C @ (C)$ , 5  $\text{xx}6$ , 6  $\text{xx}6 = A \Rightarrow E \Rightarrow +C$ , 7  $\text{list}(A \Rightarrow E \Rightarrow B)$ ,  
 8  $(C)\neg\eta A \Rightarrow E \Rightarrow B$ , 9 B, 10  $A \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C$ , 11  $\text{xx}5$ , 12  $A \Rightarrow E \Rightarrow C$ , 13  $\text{xx}5 @ (C) = A \Rightarrow E \Rightarrow C @ (C)$ ,  
 14  $A \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C @ (C)$ , 15  $A \Rightarrow E \Rightarrow B$ , 16  $B \Rightarrow E \Rightarrow C$ , 17 (C), 18  $A \Rightarrow E \Rightarrow B @ (B \Rightarrow E \Rightarrow C @ (C))$ , 19  $\text{xx}4$ ,  
 20  $\text{xx}4 = A \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow C @ (C)$ , 21  $B \Rightarrow E \Rightarrow +C$ , 22  $\text{xx}3$ , 23  $B \Rightarrow E \Rightarrow C @ (C)$ ,  
 24  $A \Rightarrow E \Rightarrow B @ \text{xx}3 = A \Rightarrow E \Rightarrow B @ (B \Rightarrow E \Rightarrow C @ (C))$ , 25  $A \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow +C$ , 26  $B \neq C$ , 27  $(B) @ B.E \Rightarrow E \Rightarrow +C$ , 28  $\text{xx}2$ ,  
 29  $A \Rightarrow E \Rightarrow B @ \text{xx}2 = A \Rightarrow E \Rightarrow B @ B \Rightarrow E \Rightarrow +C$ , 30  $A \Rightarrow E \Rightarrow B @ ((B) @ B.E \Rightarrow E \Rightarrow +C)$ , 31 (B), 32  $B.E \Rightarrow E \Rightarrow +C$ ,  
 33  $A \Rightarrow E \Rightarrow B @ (B) @ B.E \Rightarrow E \Rightarrow +C$ , 34  $\text{xx}1$ , 35  $\text{xx}1 = A \Rightarrow E \Rightarrow B @ ((B) @ B.E \Rightarrow E \Rightarrow +C)$ , 36  $A \Rightarrow E \Rightarrow +B$ , 37  $\text{xx}$ ,  
 38  $A \Rightarrow E \Rightarrow B @ (B)$ , 39  $\text{xx} @ B.E \Rightarrow E \Rightarrow +C = A \Rightarrow E \Rightarrow B @ (B) @ B.E \Rightarrow E \Rightarrow +C$ , 40  $A \Rightarrow E \Rightarrow +B @ B.E \Rightarrow E \Rightarrow +C$ ,  
 41  $A \Rightarrow E \Rightarrow +B @ B.E \Rightarrow E \Rightarrow +C = A \Rightarrow E \Rightarrow +C$ , 42  $C \neq B$ , 43  $(C)\neg\eta(B)$ , 44  $\text{xx}7$ , 45  $(C)\neg\eta A \Rightarrow E \Rightarrow B \wedge (C)\neg\eta(B)$

SEQ

(cut«45,41/B,C»)  
 ("FROM  $F\neg\eta A \Rightarrow E \Rightarrow +B$  INFER  $F\neg\eta A \Rightarrow E \Rightarrow B \wedge F\neg\eta(B)$ "«1,9,3,17/A,B,E,F»)  
 (GIVEN 0)  
 (cut«8,41/B,C»)  
 (LAYOUT " $\wedge$ -E" ALL  
 (" $\wedge$ -E(L)"«43,8/B,A»)  
 (hyp«45/A»)  
 (cut«43,41/B,C»)  
 (LAYOUT " $\wedge$ -E" ALL  
 (" $\wedge$ -E(R)"«8,43/A,B»)  
 (hyp«45/A»)  
 (cut«42,41/B,C»)  
 (LAYOUT " $(A)\neg\eta(B) \neq A \neq B$ " ALL  
 ("rewrite $\neq$ "«42,44,43,44/A,xx,B,P»)

```

(LAYOUT HIDEROOT
  ("symmetric="«43,42/A,B»)
  (LAYOUT HIDEROOT
    ("(A)¬∩(B)≠A≠B"«2,9/A,B»)))
(hyp«43/A»)
(cut«26,41/B,C»)
(LAYOUT HIDEROOT
  ("symmetric="«2,9/A,B»)
  (hyp«42/A»)
  (cut«8,41/B,C»)
  (hyp«8/A»)
  (cut«7,41/B,C»)
  ("FROM list(A⇒E⇒+B) INFER list(A⇒E⇒B)"«1,9,3/A,B,E»)
  (GIVEN 1)
  ("transitive="«40,33,0/A,B,C»)
  (LAYOUT "A⇒E⇒+B≠A⇒E⇒B@(B)" ALL
    ("rewrite="«36,37,38,39/A,xx,B,P»)
    (LAYOUT HIDEROOT
      ("A⇒E⇒+B≠A⇒E⇒B@(B)"«1,9,3/A,B,E»)
      (LAYOUT HIDEROOT
        ("reflexive="«33/A»)))
    ("transitive="«33,30,0/A,B,C»)
    (LAYOUT "(R@S)@T≠R@(S@T)" ALL
      ("rewrite="«33,34,30,35/A,xx,B,P»)
      (LAYOUT HIDEROOT
        ("(R@S)@T≠R@(S@T)"«15,31,32/R,S,T»)
        (LAYOUT HIDEROOT
          ("reflexive="«30/A»)))
      ("transitive="«30,25,0/A,B,C»)
      (LAYOUT "FROM A≠B INFER A⇒E⇒+B=(A)@A.E⇒E⇒+B" ALL
        ("rewrite="«27,28,21,29/A,xx,B,P»)
        (LAYOUT HIDEROOT
          ("symmetric="«21,27/A,B»)
          (LAYOUT HIDEROOT
            ("FROM A≠B INFER A⇒E⇒+B=(A)@A.E⇒E⇒+B"«9,2,3/A,B,E»)
            (hyp«26/A»)))
          (LAYOUT HIDEROOT
            ("reflexive="«25/A»)))
        ("transitive="«25,18,0/A,B,C»)
        (LAYOUT "A⇒E⇒+B≠A⇒E⇒B@(B)" ALL
          ("rewrite="«21,22,23,24/A,xx,B,P»)
          (LAYOUT HIDEROOT
            ("A⇒E⇒+B≠A⇒E⇒B@(B)"«9,2,3/A,B,E»)
            (LAYOUT HIDEROOT
              ("reflexive="«18/A»)))
          ("transitive="«18,14,0/A,B,C»)
          (LAYOUT "(R@S)@T≠R@(S@T)" ALL
            ("rewrite="«18,19,14,20/A,xx,B,P»)
            (LAYOUT HIDEROOT
              ("symmetric="«14,18/A,B»)
              (LAYOUT HIDEROOT
                ("(R@S)@T≠R@(S@T)"«15,16,17/R,S,T»)))
            (LAYOUT HIDEROOT
              ("reflexive="«14/A»)))
          ("transitive="«14,4,0/A,B,C»)
          (LAYOUT "FROM (C)¬∩A⇒E⇒B AND list(A⇒E⇒B) INFER A⇒E⇒B@B⇒E⇒C=A⇒E⇒C" ALL
            ("rewrite="«10,11,12,13/A,xx,B,P»)
            (LAYOUT HIDEROOT
              ("FROM (C)¬∩A⇒E⇒B AND list(A⇒E⇒B) INFER A⇒E⇒B@B⇒E⇒C=A⇒E⇒C"«1,9,2,3/A,B,C,E»)
              (hyp«8/A»)
              (hyp«7/A»)
              (LAYOUT HIDEROOT
                ("reflexive="«4/A»)))
            ("transitive="«4,0,0/A,B,C»)
            (LAYOUT "A⇒E⇒+B≠A⇒E⇒B@(B)" ALL
              ("rewrite="«4,5,0,6/A,xx,B,P»)
              (LAYOUT HIDEROOT

```

```

("symmetric="«0,4/A,B»)
(LAYOUT HIDEROOT
 ("A⇒E⇒+B⇒A⇒E⇒B@(B)"«1,2,3/A,B,E»))
(LAYOUT HIDEROOT
 ("reflexive="«0/A»))
(LAYOUT HIDEROOT
 ("reflexive="«0/A»))

```

-----

DERIVED RULE IS FROM  $A \neq B$  INFER  $A \Rightarrow (E \oplus A \mapsto B) \Rightarrow B = (A)$

```

A≠B 1: A≠B
      2: A⇒(E⊕A↦B)⇒B
(1) FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B 3: = (A)@A.(E⊕A↦B)⇒(E⊕A↦B)⇒B
      A.(B⊕A↦E)=E 4: = (A)@B⇒(E⊕A↦B)⇒B
      A⇒E⇒A=() 5: = (A)@()
      S@()≠S 6: = (A)

```

Given:  
A≠B

FORMULAE 0 (A), 1 (A)@(), 2 xx2, 3 xx2=(A), 4 B, 5 E⊕A↦B, 6 B⇒(E⊕A↦B)⇒B, 7 xx1, 8 (),  
9 (A)@xx1=(A)@(), 10 (A)@B⇒(E⊕A↦B)⇒B, 11 A, 12 E, 13 A.(E⊕A↦B), 14 xx,  
15 (A)@xx⇒(E⊕A↦B)⇒B=(A)@B⇒(E⊕A↦B)⇒B, 16 (A)@A.(E⊕A↦B)⇒(E⊕A↦B)⇒B, 17 A⇒(E⊕A↦B)⇒B

SEQ

```

("transitive="«17,16,0/A,B,C»)
("FROM A≠B INFER A⇒E⇒B=(A)@A.E⇒E⇒B"«11,4,5/A,B,E»)
(GIVEN 0)
("transitive="«16,10,0/A,B,C»)
(LAYOUT "A.(B⊕A↦E)=E" ALL
 ("rewrite="«13,14,4,15/A,xx,B,P»)
 (LAYOUT HIDEROOT
 ("A.(B⊕A↦E)=E"«11,12,4/A,B,E»)
 (LAYOUT HIDEROOT
 ("reflexive="«10/A»)))
("transitive="«10,1,0/A,B,C»)
(LAYOUT "A⇒E⇒A=()" ALL
 ("rewrite="«6,7,8,9/A,xx,B,P»)
 (LAYOUT HIDEROOT
 ("A⇒E⇒A=()"«4,5/A,E»)
 (LAYOUT HIDEROOT
 ("reflexive="«1/A»)))
(LAYOUT "S@()≠S" ALL
 ("rewrite="«1,2,0,3/A,xx,B,P»)
 (LAYOUT HIDEROOT
 ("S@()≠S"«0/S»)
 (LAYOUT HIDEROOT
 ("reflexive="«0/A»)))

```