

# The Frequency Structure Matrix: A Representation of Color Filter Arrays

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## Abstract

This letter introduces the frequency structure matrix as a new representation of color filter arrays (CFAs). The matrix records the frequency components of CFA filtered images and their positions in the spectrum. The matrix can be conveniently obtained by applying the symbolic DFT to the CFA pattern. With this new representation, it is easy to analyze the characteristics of CFAs and to formulate the CFA design as an optimization problem.

**Key Words:** color filter array (CFA), discrete Fourier transform (DFT), sampling, multiplexing, demosaicking

## 1 Introduction

To reduce cost, size and complexity, color filter arrays (CFAs) are commonly used in consumer digital cameras. A CFA is a mosaic of optically selective filters, each filtering the incident light projected to each pixel. Therefore, the sensed image has only one color at each pixel. The missing two colors of each pixel have to be estimated by methods called demosaicking. The most widely used CFA was the Bayer

CFA [2] (Fig. 1(a)), whose sampling rates for green, red and blue (G, R and B) are  $1/2$ ,  $1/4$  and  $1/4$ , respectively.

The representation of CFA is crucial. Different representations lead to different demosaicking algorithms. For the CFA design, an appropriate representation is also necessary as it requires proper mathematical modeling for optimization. The spatial representation could not provide enough insight into why a demosaicking algorithm can work well, hence is inadequate for the CFA design. While the spectral representation can reveal the structure and composition of frequencies, it is more suitable for theoretical analysis on the performance of CFAs and the corresponding demosaicking algorithms.

There has been some prior work on analyzing the spectral components of CFA filtered images [1, 5, 7, 8, 6]. In [1, 5], it was shown that in the frequency domain, a Bayer CFA filtered image has one luminance component at the baseband and several chrominance ones modulated at higher frequencies (Fig. 1(b)). In [7, 8], an image sampled with any CFA was represented with a green component and those that correspond to differences between colors. To design a CFA, one can first specify the modulation points of frequency components and then select a set of parameters satisfying some constraints. In [6], a CFA filtered image was represented as a sum of different combinations of the original components, which correspond to components modulated at different frequencies. This combination was represented using several matrices, which record parameters of the components in the frequency domain. Based on this representation, a demosaicking method by demultiplexing of the fre-

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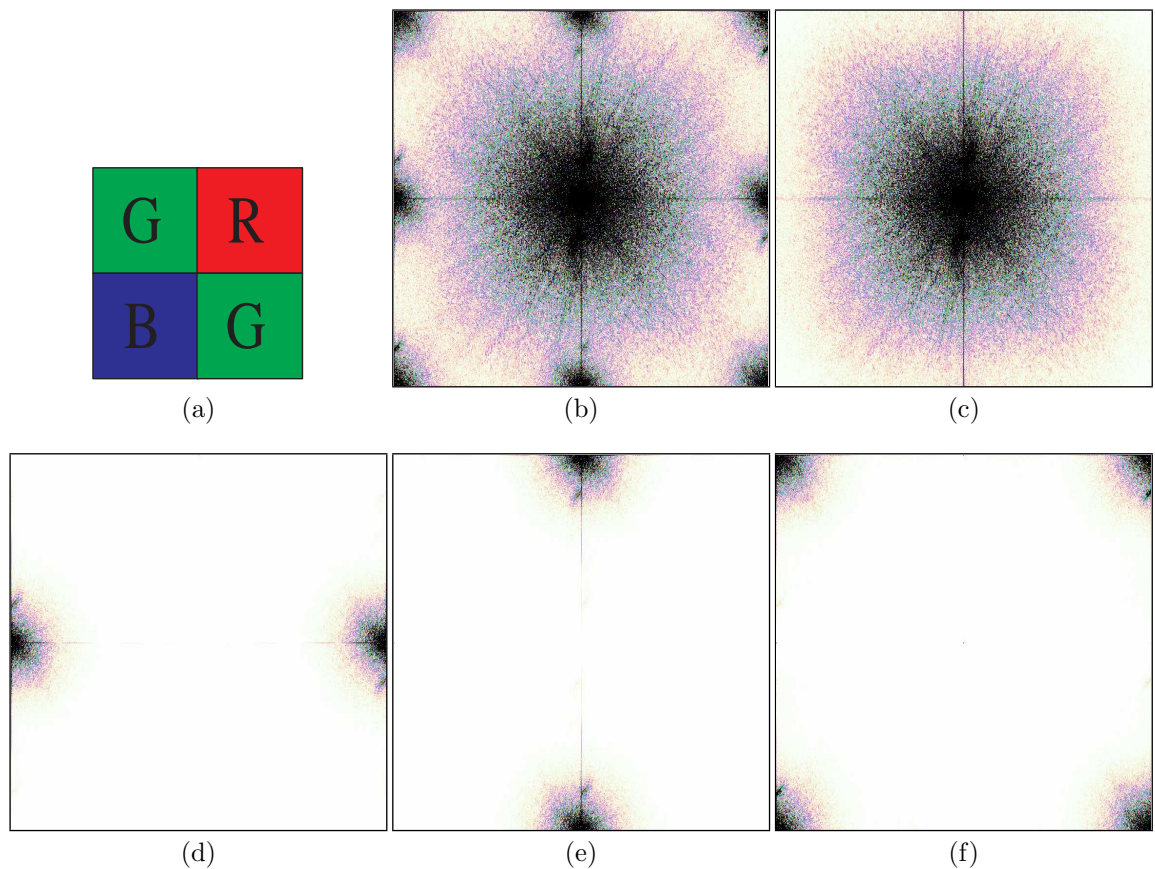


Figure 1: Bayer CFA pattern [2] and the spectrum of the ‘window’ image filtered with it. (c)-(f) show each component in the spectrum (b). (c)  $(F^{(R)}+2F^{(G)}+F^{(B)})/4$  at frequency point  $(0,0)$ , (d)  $(-F^{(R)}+F^{(B)})/4$  modulated at  $(0.5,0)$ , (e)  $(F^{(R)}-F^{(B)})/4$  at  $(0,0.5)$  and (f)  $(-F^{(R)}+2F^{(G)}-F^{(B)})/4$  at  $(0.5,0.5)$ . The ‘window’ is a commonly used test image from the Kodak dataset [1].

quency components was also proposed in that paper.

However, the existing representations are not good enough. For example, they are not very intuitive, not revealing enough of the relationship between a CFA and its spectral representation and a bit too complex to calculate. So when applied to the CFA design, these representations often cause some difficulties. For example, one cannot easily express a CFA and its spectral representation with the same set of parameters. Thus it is difficult to design CFAs using a unified framework, by expressing all the constraints and the objective function mathematically.

In this letter, we propose a new representation, the frequency structure matrix, which records the frequency components at all the modulation points. It is more intuitive, informative and directly related to the CFA. The matrix can be easily obtained by computing the symbolic DFT of the CFA pattern. With this representation, the CFA design can be formulated as an optimization problem [10].

## 2 The Frequency Structure Matrix

Fig. 1(b) shows the spectrum of the image ‘window’ filtered with the Bayer CFA. As shown in [1, 5], it contains several frequency components:  $(F^{(R)} + 2F^{(G)} + F^{(B)})/4$  at frequency point  $(0, 0)$ ,  $(-F^{(R)} + F^{(B)})/4$  modulated at  $(0.5, 0)$ ,  $(F^{(R)} - F^{(B)})/4$  at  $(0, 0.5)$  and  $(-F^{(R)} + 2F^{(G)} - F^{(B)})/4$  at  $(0.5, 0.5)$ , where  $F^{(R)}$ ,  $F^{(G)}$  and  $F^{(B)}$  denote the DFT spectra of the original image of primary colors R, G and B, respectively, and one period in the frequency domain is a square  $[0, 1]^2$ . It is also shown that images filtered with any CFA have similar spectra, consisting of several components modulated at different frequency points [7, 8, 6].

Inspired by the observed patterns of the spectra of CFA filtered images, we propose to represent the spectra by faithfully recording the frequency components: their frequency details and their positions. Such information can naturally be arranged in a matrix form. So we call it the frequency structure matrix. For example, we may represent the spectrum of an image filtered with the Bayer CFA as:

$$S_{Bayer} = \frac{1}{4} \begin{bmatrix} F^{(R)} + 2F^{(G)} + F^{(B)} & -F^{(R)} + F^{(B)} \\ F^{(R)} - F^{(B)} & -F^{(R)} + 2F^{(G)} - F^{(B)} \end{bmatrix}.$$

(Note that by convention the spectra of images are displayed with  $(0, 0)$  being at the center, while in the frequency structure matrix,  $(0, 0)$  is at the top-left corner of the matrix. The readers should circularly shift either of them in order to match them.) One can see that all the information of the spectra that is of interest to him/her can be found in  $S_{Bayer}$ .

In the following, we present the formal definition of the frequency structure matrix. We also prove that the frequency structure matrix can be easily calculated by applying the symbolic DFT to the CFA pattern. For brevity, sometimes we may write ‘‘frequency structure’’ instead of ‘‘frequency structure matrix’’.

## 3 From Spectra to Frequency Structure

To obtain the frequency-domain representation, a CFA filtered image is first represented in the spatial domain by decomposing the image and the CFA pattern into three channels, corresponding to three primary colors, respectively. Then the DFT of the CFA filtered image is computed using the convolution theory. Once the expression of the spectrum is available, we can identify its frequency components and their corresponding positions in the frequency domain. Then we can arrange these components into a matrix form, according to their positions, and obtain the frequency structure matrix.

### 3.1 Spectra of CFA filtered images

A CFA  $h_{CFA}(x, y)$  is usually a periodic tiling of a much smaller array, called the *CFA pattern*  $h_p(x, y)$ . Using the well-established tri-primary color theory, a CFA pattern can be decomposed into three *primary CFA patterns*  $h_p^{(C)}(x, y)$ , each accounting for one primary color  $\mathcal{C}$  [1, 6, 8]. Then symbolically we can write:

$$h_p = \sum_{\mathcal{C}} h_p^{(C)} \cdot \mathcal{C}. \quad (1)$$

To ensure the same dynamic range of the sensed image at all pixels, the sum of all primary CFA patterns should be an all-one matrix

$$\sum_{\mathcal{C}} h_p^{(C)}(x, y) = 1, \quad \forall x, y, \quad (2)$$

For example, for the Bean CFA pattern  $h_{Bean} = [C \ M; B \ Y]$  (Fig. 2(a)), the primary CFA patterns of colors R, G and B are respectively:

$$h_{Bean}^{(R)} = \begin{bmatrix} 0 & \frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix}, h_{Bean}^{(G)} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}, h_{Bean}^{(B)} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix}, \quad (3)$$

as  $M = (R + B)/2$ ,  $Y = (R + G)/2$  and  $C = (G + B)/2$ .

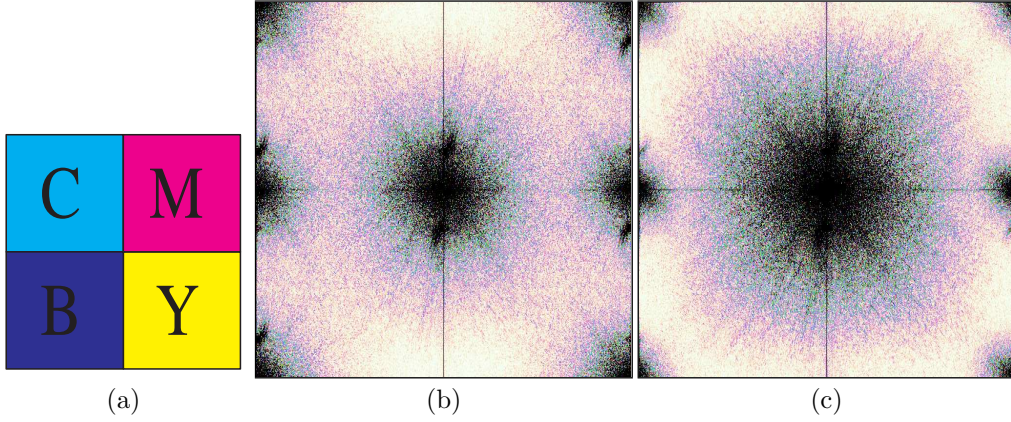


Figure 2: The Bean CFA pattern [3] and the spectra of the ‘window’ image filtered with it. (a) The Bean CFA pattern, (b) the spectrum  $F_{Bean}^{(B)}$  of the blue channel of the Bean CFA filtered image, represented by the frequency structure (8), (c) the spectrum  $F_{Bean}$  of the Bean CFA filtered image, represented by the frequency structure (11) or (13).

Let  $f(x, y)$  be the full color image of size  $(N_x, N_y)$  and the CFA pattern  $h_p(x, y)$  be of size  $(n_x, n_y)$ . Then the CFA filtered image is:

$$f_{CFA}(x, y) = \sum_C f^{(C)}(x, y) \cdot h_{CFA}^{(C)}(x, y), \quad (4)$$

where  $f^{(C)}(x, y)$  is the color  $C$  component of  $f$  and  $h_{CFA}^{(C)}(x, y)$  is the corresponding CFA of color  $C$  defined as the periodic replica of the primary CFA pattern  $h_p^{(C)}(x, y)$ :

$$h_{CFA}^{(C)}(x, y) = h_p^{(C)}(x \bmod n_x, y \bmod n_y).$$

Without loss of generality, we assume that  $N_x$  and  $N_y$  are multiples of  $n_x$  and  $n_y$ , respectively. We first compute the spectrum of  $h_{CFA}^{(C)}(x, y)$  [9]:

$$\begin{aligned} H_{CFA}^{(C)}(\omega_x, \omega_y) &= DFT[h_{CFA}^{(C)}(x, y)] \\ &= \frac{1}{N_x N_y} \sum_{x=0}^{N_x-1} \sum_{y=0}^{N_y-1} h_{CFA}^{(C)}(x, y) e^{-i2\pi(x\omega_x + y\omega_y)} \\ &= \begin{cases} H_p^{(C)}(\omega_x, \omega_y), & \text{if } n_x \omega_x \in Z \text{ and } n_y \omega_y \in Z, \\ 0, & \text{otherwise,} \end{cases} \end{aligned} \quad (5)$$

where  $H_p^{(C)}(\omega_x, \omega_y)$  is the DFT of the primary CFA pattern  $h_p^{(C)}(x, y)$ . Note that here  $(\omega_x, \omega_y)$  takes discrete values in the square  $[0, 1)^2$  (at a stepsize of  $(1/N_x, 1/N_y)$  for  $H_{CFA}^{(C)}$  and  $(1/n_x, 1/n_y)$  for  $H_p^{(C)}$ , respectively) instead of discrete indices of the signals because we have found that it is more convenient to normalize the frequencies. The above equality shows that the spectrum of a CFA is a *sampling* of the spectrum of its CFA pattern at frequencies  $(k_x/n_x, k_y/n_y)$ , where  $(k_x, k_y) \in Z^2$ .

As multiplication in the spatial domain corresponds to the *circular convolution* in the frequency domain, the DFT of  $f_{CFA}^{(C)}$  can be found to be [9]:

$$\begin{aligned} F_{CFA}^{(C)}(\omega_x, \omega_y) &= DFT[f^{(C)}(x, y) \cdot h_{CFA}^{(C)}(x, y)] \\ &= \sum_{k_x=0}^{n_x-1} \sum_{k_y=0}^{n_y-1} H_p^{(C)}\left(\frac{k_x}{n_x}, \frac{k_y}{n_y}\right) \cdot F^{(C)}\left(\omega_x - \frac{k_x}{n_x}, \omega_y - \frac{k_y}{n_y}\right). \end{aligned} \quad (6)$$

where  $F^{(C)}\left(\omega_x - \frac{k_x}{n_x}, \omega_y - \frac{k_y}{n_y}\right)$  has been *circularly shifted*. This implies that in the frequency domain the spectrum  $F_{CFA}^{(C)}$  is a multiplexing of  $n_x n_y$  frequency components centered at  $(k_x/n_x, k_y/n_y)$ ,  $k_x = 0, 1, \dots, n_x - 1$ ;  $k_y = 0, 1, \dots, n_y - 1$ , and each com-

ponent is the original spectrum  $F^{(C)}$  weighted by  $H_p^{(C)}(k_x/n_x, k_y/n_y)$ , the spectral value of the CFA pattern at the corresponding frequency. Fig. 2(b) shows an example of the spectrum.

### 3.2 The frequency structure

As exemplified in Section 2 and shown in [9], it will be more intuitive and useful to arrange the identified frequency components in a matrix form. So we use a matrix  $S_{CFA}^{(C)}$  to represent the spectrum  $F_{CFA}^{(C)}$  in Eq. (6):

$$S_{CFA}^{(C)} = \left[ H_p^{(C)} \left( \frac{k_x}{n_x}, \frac{k_y}{n_y} \right) \cdot F^{(C)}(\omega) \right]_{\substack{k_x = 0, 1, \dots, n_x - 1, \\ k_y = 0, 1, \dots, n_y - 1.}} \quad (7)$$

It records all the information about the frequency components of  $F_{CFA}^{(C)}$ : the  $(k_x, k_y)$ -th entry  $S_{CFA}^{(C)}(k_x, k_y)$  is the frequency component centered at  $(k_x/n_x, k_y/n_y)$ . We call this matrix the *frequency structure* of the primary CFA  $h_{CFA}^{(C)}$  of color  $\mathcal{C}$ .

For example, for the primary Bean CFA pattern of color blue (Eq. (3)), its DFT is  $DFT[h_{Bean}^{(B)}] = [1/2 \ 1/4; 0 \ -1/4]$  and thus its frequency structure is

$$S_{Bean}^{(B)} = \begin{bmatrix} \frac{1}{2}F^{(B)} & \frac{1}{4}F^{(B)} \\ 0 & -\frac{1}{4}F^{(B)} \end{bmatrix}, \quad (8)$$

where  $F^{(B)}$  denotes the spectrum of the blue channel of  $f$ .  $S_{Bean}^{(B)}$  shows that  $F_{Bean}^{(B)}$  has three nonzero spectra: one is  $F^{(B)}/2$ , at the baseband, one is  $F^{(B)}/4$ , modulated at frequency  $(1/2, 0)$  and another is  $-F^{(B)}/4$ , at  $(1/2, 1/2)$ . Fig. 2(b) shows the spectrum  $F_{Bean}^{(B)}$ .

Now we are equipped to define the frequency structure matrix of a CFA pattern. According to (4), we may define the *frequency structure* of a CFA pattern as the following matrix:

$$S_{CFA} = \sum_{\mathcal{C}} S_{CFA}^{(C)}. \quad (9)$$

The entries of the matrix  $S_{CFA}$  are actually:

$$S_{CFA}(k_x, k_y) = \sum_{\mathcal{C}} H_p^{(C)} \left( \frac{k_x}{n_x}, \frac{k_y}{n_y} \right) \cdot F^{(C)}(\omega_x, \omega_y), \quad (10)$$

$k_x = 0, 1, \dots, n_x - 1; k_y = 0, 1, \dots, n_y - 1$ . The entry  $S_{CFA}(k_x, k_y)$  denotes the frequency component of the spectrum  $F_{CFA}$  centered (or modulated) at frequency  $(k_x/n_x, k_y/n_y)$ . Thus the spectrum of  $F_{CFA}$  is a multiplexing of  $n_x \cdot n_y$  components  $S_{CFA}(k_x, k_y)$  centered at grid points  $(k_x/n_x, k_y/n_y)$  ( $k_x = 0, 1, \dots, n_x - 1; k_y = 0, 1, \dots, n_y - 1$ ). For this reason, we refer to the entries (10) in  $S_{CFA}$  as the *multiplex components*, which are sums of the spectra  $F^{(C)}$  weighted by  $H_p^{(C)}$ .

For example, the frequency structure of the Bean CFA pattern, as the sum of  $S_{Bean}^{(C)}$ ,  $\mathcal{C} = R, G, B$ , is:

$$\begin{aligned} S_{Bean} &= \sum_{\mathcal{C}=R,G,B} S_{Bean}^{(C)} = \begin{bmatrix} \frac{1}{4}F^{(R)} & -\frac{1}{4}F^{(R)} \\ 0 & 0 \end{bmatrix} + \\ &\begin{bmatrix} \frac{1}{4}F^{(G)} & 0 \\ 0 & \frac{1}{4}F^{(G)} \end{bmatrix} + \begin{bmatrix} \frac{1}{2}F^{(B)} & \frac{1}{4}F^{(B)} \\ 0 & -\frac{1}{4}F^{(B)} \end{bmatrix} \\ &= \frac{1}{4} \begin{bmatrix} F^{(R)} + F^{(G)} + 2F^{(B)} & -F^{(R)} + F^{(B)} \\ 0 & F^{(G)} - F^{(B)} \end{bmatrix} \end{aligned} \quad (11)$$

This shows that the spectrum  $F_{CFA}$  of the Bean CFA filtered image has three nonzero multiplex components:  $(F^{(R)} + F^{(G)} + 2F^{(B)})/4$  at the baseband,  $(-F^{(R)} + F^{(B)})/4$  at  $(1/2, 0)$  and  $(F^{(G)} - F^{(B)})/4$  at  $(1/2, 1/2)$ . Fig. 2(c) shows the spectrum of the ‘window’ image filtered by Bean CFA.

By applying DFT to both sides of Eq. (2), we have that:

$$\sum_{\mathcal{C}} H_p^{(C)} \left( \frac{k_x}{n_x}, \frac{k_y}{n_y} \right) = \delta(k_x)\delta(k_y), \quad (12)$$

which means that the sums of the coefficients for all multiplex components (10) are zero, except the one at the baseband (frequency  $(0, 0)$ ), which is 1. As in [1] and [5], we shall call the multiplex component at the baseband the luminance component (luma) and the others the chrominance components (chromas).

## 4 Symbolic DFT to Compute the Frequency Structure

By the definition in Eq. (10), it seems a little tedious to compute the frequency structure as we may have

to compute the DFT of all the primary CFA patterns. However, we have found that there is a simple way to compute the frequency structure of a CFA. To proceed, we introduce the *symbolic* DFT of a sequence of symbols. For a string  $s = s_0 s_1 \cdots s_{N-1}$ , its symbolic DFT is a sequence of order-1 polynomials  $S = S_0 S_1 \cdots S_{N-1}$ , where

$$S_k = \frac{1}{N} \sum_{l=0}^{N-1} s_l e^{-2\pi i k l / N}.$$

For the 2D case, the symbolic DFT can be defined in an analogous way. With this definition, we can claim that:

**Theorem 1.** *If we rewrite “ $F^{(\mathcal{C})}(\omega)$ ” as “ $\mathcal{C}$ ”, then the frequency structure  $S_{CFA}$  is the symbolic DFT of the CFA pattern  $h_p$ .*

**Proof.** The symbolic DFT of the CFA pattern  $h_p$  is  $H_p = DFT[h_p]$ , where

$$\begin{aligned} & H_p(k_x, k_y) \\ &= \frac{1}{n_x n_y} \sum_{x=0}^{n_x-1} \sum_{y=0}^{n_y-1} h_p(x, y) e^{-2\pi i (x k_x / n_x + y k_y / n_y)} \\ &= \frac{1}{n_x n_y} \sum_{\mathcal{C}} \sum_{x=0}^{n_x-1} \sum_{y=0}^{n_y-1} h_p^{(\mathcal{C})}(x, y) e^{-2\pi i (x k_x / n_x + y k_y / n_y)} \\ &= \sum_{\mathcal{C}} H_p^{(\mathcal{C})}(k_x / n_x, k_y / n_y) \cdot \mathcal{C}. \end{aligned}$$

where  $\sum_{\mathcal{C}}$  denotes summation among all primary colors  $\mathcal{C}$ . Hence, the claim is true by comparing the above with (10). ■

From this theorem, in the sequel, we also use  $\mathcal{C}$  to represent the spectrum of the color channel  $\mathcal{C}$  of the original image.

Similarly, for the frequency structure of the primary CFA patterns, we also have  $S_{CFA}^{(\mathcal{C})} = DFT[h_p^{(\mathcal{C})}] \cdot \mathcal{C}$ .

## 5 Examples of Frequency Structures

Thanks to Theorem 1, the frequency structures of any CFAs can be easily computed. For example, the

frequency structure of the Bean CFA [3] can be found to be:

$$\begin{aligned} S_{Bean} &= DFT \begin{bmatrix} \frac{G+B}{2} & \frac{R+B}{2} \\ B & \frac{R+G}{2} \end{bmatrix} \\ &= \frac{1}{4} \begin{bmatrix} R+G+2B & -R+B \\ 0 & G-B \end{bmatrix}. \end{aligned} \quad (13)$$

As proven by Theorem 1, this representation is the same as that of Eq. (11), if “ $F^{(\mathcal{C})}(\omega)$ ” is rewritten as “ $\mathcal{C}$ ”.

Now we show more examples. The frequency structures of the CFA [12], the Diagonal CFA [11] and the Dillon CFA [4] are respectively as follows:

$$\begin{aligned} S_{Yam} &= DFT \begin{bmatrix} G & R & G & B \\ G & B & G & R \end{bmatrix} \\ &= \begin{bmatrix} F_L & 0 & F_{C1} & 0 \\ 0 & F_{C2} & 0 & -F_{C2} \end{bmatrix}, \end{aligned}$$

where  $F_L = (R+2G+B)/4$ ,  $F_{C1} = (-R+2G-B)/4$  and  $F_{C2} = (-R+B)/4$ ;

$$S_{Diag} = DFT \begin{bmatrix} R & B & G \\ G & R & B \\ B & G & R \end{bmatrix} = \begin{bmatrix} F_L & 0 & 0 \\ 0 & 0 & F_{C1} \\ 0 & F_{C2} & 0 \end{bmatrix},$$

where  $F_L = (R+G+B)/3$ ,  $F_{C1} = (2R - (1+i\sqrt{3})G - (1-i\sqrt{3})B)/6$  and  $F_{C2} = (2R - (1-i\sqrt{3})G - (1+i\sqrt{3})B)/6$ ; and

$$\begin{aligned} S_{Dillon} &= DFT \begin{bmatrix} W & R & W & B \\ B & W & R & W \\ W & B & W & R \\ R & W & B & W \end{bmatrix} \\ &= \begin{bmatrix} F_L & 0 & 0 & 0 \\ 0 & 0 & 0 & F_{C2} \\ 0 & 0 & F_{C1} & 0 \\ 0 & F_{C2} & 0 & 0 \end{bmatrix}, \end{aligned}$$

where  $W = (R+G+B)/3$ ,  $F_L = (5R+2G+5B)/12$ ,  $F_{C1} = (-R+2G-B)/12$  and  $F_{C2} = -i(R-B)/4$ . To illustrate, the spectra of the ‘window’ image filtered by the Yamanaka CFA, the Diagonal CFA and the Dillon CFA are shown in Fig. 3.

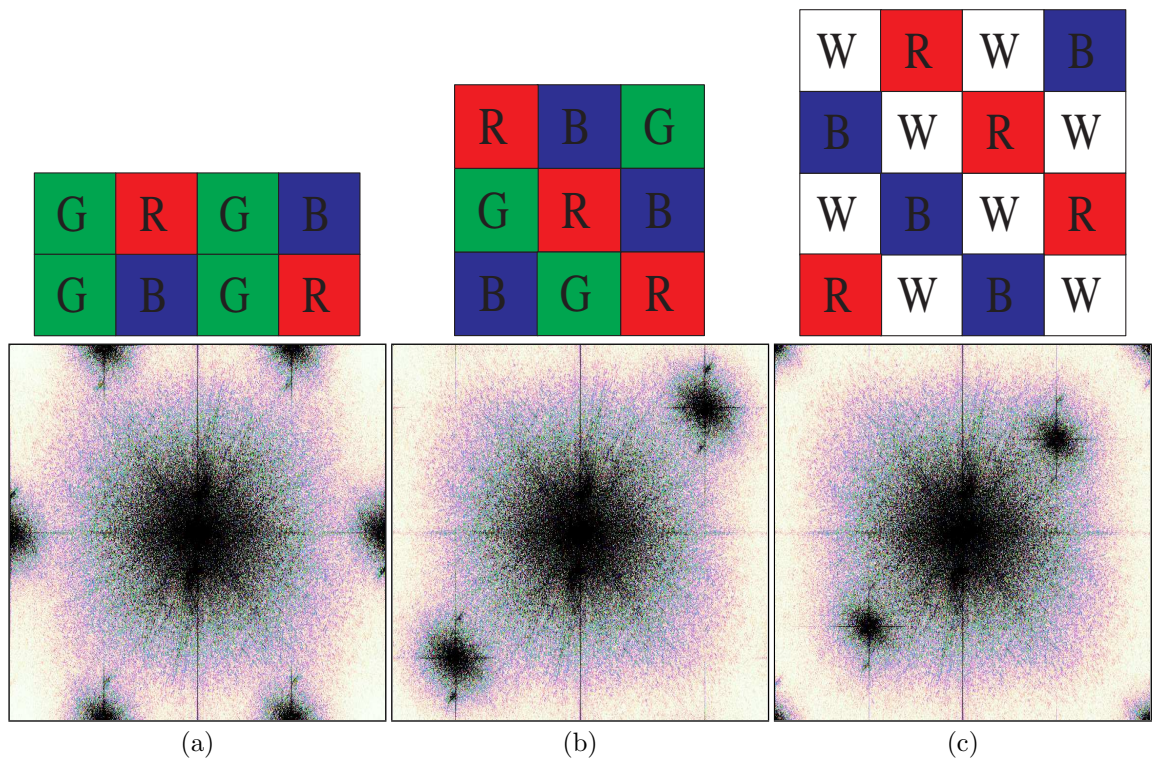


Figure 3: Some existing CFA patterns. The second row are the spectra of image ‘window’ sampled with the corresponding CFAs in the first row. (a) Yamanaka [12]. (b) Diagonal Stripe [11]. (c) Dillon [4].

## 6 Conclusions

A matrix, named the frequency structure, is introduced to represent a CFA filtered image in the frequency domain. The frequency structure not only records the frequency components of the CFA filtered image, but also their arrangement in the frequency domain. It is also proven that the frequency structure is just the symbolic DFT of the CFA pattern. With this simple relationship between the frequency structure and the CFA pattern, one can easily formulate the CFA design as an optimization problem [10], satisfying some constraints in both the spatial and the frequency domains. One may refer to [10] for more details to see the effectiveness of this new representation.

Although in this letter we only consider CFAs replicated on rectangular lattices, the above results can be

easily extended to non-rectangular (e.g., hexagonal) lattices.

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