Comments on "The Multisynapse Neural Network and its Application to Fuzzy Clustering"

Jian Yu and Pengwei Hao

***Abstract*—In the above-mentioned paper, Wei and Fahn proposed a neural architecture, the multisynapse neural network, to solve constrained optimization problems including high-order, logarithmic, and sinusoidal forms, etc. As one of its main applications, a fuzzy bidirectional associative clustering network (FBACN) was proposed for fuzzy-partition clustering according to the objective-functional method. The connection between the objective-functional-based fuzzy c-partition algorithms and FBACN is the Lagrange multiplier approach. Unfortunately, the Lagrange multiplier approach was incorrectly applied so that FBACN does not equivalently minimize its corresponding constrained objective-function. Additionally, Wei and Fahn adopted traditional definition of fuzzy c-partition, which is not satisfied by FBACN. Therefore, FBACN can not solve constrained optimization problems, either.**

I. Introduction

In the above-mentioned paper, most existing partitional clustering algorithms need closed-form solutions [1], [2]. To avoid such a problem, Wei and Fahn proposed a "new" neural architecture, the multisynapse neural network [3]. They claimed that it could "open a new door for all the high-order optimization problems with constraints." As one of its main applications, a fuzzy bidirectional associative clustering network (FBACN) was proposed for fuzzy-partition clustering according to the objective-functional method in [3].

In the paper, they demonstrated the usefulness of FBACN by applying it to four fuzzy clustering examples. The connection between the objective-functional-based fuzzy c-partition algorithms and FBACN is the Lagrange multiplier approach. Unfortunately, the Lagrange multiplier approach was incorrectly applied so that FBACN does not equivalently minimize its corresponding constrained objective-function. In order to show their mistakes, we list the original objective functions with constraints, the corresponding Wei and Fahn's unconstrained Lagrangian forms, and the correct unconstrained Lagrangian forms as in Table I.

In brief, we show in the following the inconsistence between Wei and Fahn's original goal and FBACN only by minimizing the objective function of the FCM algorithm.

As for the objective function of the FCM algorithm, they designed a computational energy function corresponding to FBACN as follows [3, (23)]:

$$\mathrm{E}=\sum_{k=1}^{n}\sum_{i=1}^{c}u_{ik}^{m}\|x_k-v_i\|^2+\lambda\sum_{k=1}^{n}\left(\sum_{i=1}^{c}u_{ik}\right)^2-2\lambda\sum_{k=1}^{n}\left(\sum_{i=1}^{c}u_{ik}\right). \quad (1)$$

The corresponding Wei and Fahn's unconstrained Lagrangian form is

$$\sum_{k=1}^{n}\left[\sum_{i=1}^{c}(u_{ik})^m\|x_k-v_i\|^2+\lambda\left(\sum_{i=1}^{c}u_{ik}-1\right)^2\right]. \quad (2)$$

Manuscript received April 6, 2004; revised October 11, 2004. This work was supported in part by the Key Scientific Research Project 02031 of the Ministry of Education, P.R. China, in part by the National Natural Science Foundation of China under Grant 60303014, and in part by the Foundation for the Authors of National Excellent Doctoral Dissertation of China under Grant 200038.

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Digital Object Identifier 10.1109/TNN.2005.844852

As noted in [1], minimization of (1) and (2) are equivalent when taking λ as a constant. However, a Lagrange multiplier λ need be optimized as a parameter in (2). For given $v_1, v_2, \ldots, v_c$, the Lagrange multiplier approach tells us that the necessary conditions for minimizing (2) are

$$mu_{ik}^{m-1}\|x_k-v_i\|^2+2\lambda\left(\sum_{i=1}^{c}u_{ik}-1\right)=0$$
$$1\le i\le c,\ 1\le k\le n \quad (3a)$$

$$\sum_{k=1}^{n}\left(\sum_{i=1}^{c}u_{ik}-1\right)^2=0. \quad (3b)$$

The necessary conditions (3a) and (3b) can be easily simplified as

$$mu_{ik}^{m-1}\|x_k-v_i\|^2=0,\quad 1\le i\le c,\ 1\le k\le n \quad (4a)$$

$$\sum_{i=1}^{c}u_{ik}=1,\quad 1\le k\le n. \quad (4b)$$

In general cases, the conditions $\forall i,\ \forall k,\ \|x_k-v_i\|\neq 0$ hold, so it is easy to see that (4a) and (4b) are inconsistent. Therefore, Wei and Fahn's unconstrained Lagrangian form is not correct with regard to the objective function of the FCM algorithm.

However, Wei and Fahn took λ as a predefined parameter in [3]. Consequently, the necessary conditions for minimizing (1) or (2) should be

$$mu_{ik}^{m-1}\|x_k-v_i\|^2+2\lambda\left(\sum_{i=1}^{c}u_{ik}-1\right)=0,\quad 1\le i\le c,\ 1\le k\le n. \quad (5)$$

Let $s_k=\sum_{i=1}^{c}u_{ik}$, $u_{ik}=(2\lambda(1-s_k)/m\|x_k-v_i\|^2)^{1/(m-1)}$, we have $s_k=[2\lambda(1-s_k)]^{1/(m-1)}(\sum_{i=1}^{c}(m\|x_k-v_i\|^2)^{1/(1-m)})$. It implies that $s_k<1$.

In particular, if $m=2$, $s_k=\lambda(1-s_k)(\sum_{i=1}^{c}(\|x_k-v_i\|^2)^{-1})$, we have $s_k=(\lambda\sum_{i=1}^{c}(\|x_k-v_i\|^2)^{-1})/(1+\lambda\sum_{i=1}^{c}(\|x_k-v_i\|^2)^{-1})$, and $u_{ik}=1/(\lambda^{-1}\|x_k-v_i\|^2+\|x_k-v_i\|^2\sum_{i=1}^{c}(\|x_k-v_i\|^2)^{-1})$, then we know that $s_k<1$. In theory, any correct iterative sequence of minimizing (1) or (2) should converge to $u_{ik}=1/(\lambda^{-1}\|x_k-v_i\|^2+\|x_k-v_i\|^2\sum_{i=1}^{c}(\|x_k-v_i\|^2)^{-1})$, therefore $\sum_{i=1}^{c}u_{ik}<1$, which means that the output of FBACN does not satisfy the traditional definition of fuzzy c-partition. It is well known that when $\mathrm{m}=2$, minimizing the objective function of the FCM algorithm results in

$$u_{ik}=\frac{1}{\|x_k-v_i\|^2\sum_{i=1}^{c}(\|x_k-v_i\|^2)^{-1}}.$$

Therefore, minimizing (1) or (2) is not equivalent to minimizing the objective function of the FCM algorithm. Notice that the original idea of FBACN is not to minimize (1) or (2) but to minimize the objective function of the FCM algorithm, it is obvious to draw a conclusion that FBACN does not achieve its original goal.

As for FBACN, the most important part is layer 2, its architecture and convergence both depend on one uniform parameter λ for n constraints. In other words, the corresponding Wei and Fahn's unconstrained Lagrangian forms at most have a parameter more than its original objective functions with constraints. In order to do this, Wei and Fahn ignored the difference between the Lagrange multiplier λ and the original parameter λ in the objective functions [3, (58)] and [3, (59)], and took such two parameters as the same. But correct unconstrained Lagrangian forms have no such "merit". Therefore, there does not exist a neural network like FBACN to minimize the correct unconstrained Lagrangian forms.

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TABLE I
ORIGINAL OBJECTIVE FUNCTIONS WITH CONSTRAINS, CORRESPONDING WEI AND FAHN'S UNCONSTRAINED LAGRANGIAN FORMS, AND CORRECT UNCONSTRAINED LAGRANGIAN FORMS

The original objective function with constraints	Wei and Fahn's unconstrained Lagrangian form	Correct unconstrained Lagrangian form
$\sum_{k=1}^{n}\sum_{i=1}^{c}(u_{ik})^m\|x_k-v_i\|^2$ where $u_{ik}\ge 0,\ \sum_{i=1}^{c}u_{ik}=1, 0<\sum_{k=1}^{n}u_{ik}<n$ [1]	$\sum_{k=1}^{n}\left[\sum_{i=1}^{c}(u_{ik})^m\|x_k-v_i\|^2+\lambda\left(\sum_{i=1}^{c}u_{ik}-1\right)^2\right]$ [3,(23)]	$\sum_{k=1}^{n}\left[\sum_{i=1}^{c}(u_{ik})^m\|x_k-v_i\|^2+\lambda_k\left(\sum_{i=1}^{c}u_{ik}-1\right)\right]$
$\sum_{k=1}^{n}\sum_{i=1}^{c}u_{ik}\|x_k-v_i\|^2+\lambda^{-1}\sum_{k=1}^{n}\sum_{i=1}^{c}u_{ik}\log u_{ik}$ where $u_{ik}\ge 0,\ \sum_{i=1}^{c}u_{ik}=1, 0<\sum_{k=1}^{n}u_{ik}<n$ [2]	$\sum_{k=1}^{n}\left[\sum_{i=1}^{c}u_{ik}\|x_k-v_i\|^2+\lambda\left(\sum_{i=1}^{c}u_{ik}-1\right)^2\right]$ $+\lambda^{-1}\sum_{k=1}^{n}\sum_{i=1}^{c}u_{ik}\log u_{ik}$ [3,(58)]*	$\sum_{k=1}^{n}\left[\sum_{i=1}^{c}u_{ik}\|x_k-v_i\|^2+\lambda_k\left(\sum_{i=1}^{c}u_{ik}-1\right)\right]$ $+\lambda^{-1}\sum_{k=1}^{n}\sum_{i=1}^{c}u_{ik}\log u_{ik}$
$\sum_{k=1}^{n}\sum_{i=1}^{c}(u_{ik})^m\|x_k-v_i\|^2+\lambda^{-1}\sum_{k=1}^{n}\sum_{i=1}^{c}u_{ik}\log u_{ik}$ where $u_{ik}\ge 0,\ \sum_{i=1}^{c}u_{ik}=1, 0<\sum_{k=1}^{n}u_{ik}<n$ [3]	$\sum_{k=1}^{n}\left[\sum_{i=1}^{c}(u_{ik})^m\|x_k-v_i\|^2+\lambda\left(\sum_{i=1}^{c}u_{ik}-1\right)^2\right]$ $+\lambda^{-1}\sum_{k=1}^{n}\sum_{i=1}^{c}u_{ik}\log u_{ik}$ [3,(62)]	$\sum_{k=1}^{n}\left[\sum_{i=1}^{c}(u_{ik})^m\|x_k-v_i\|^2+\lambda_k\left(\sum_{i=1}^{c}u_{ik}-1\right)\right]$ $+\lambda^{-1}\sum_{k=1}^{n}\sum_{i=1}^{c}u_{ik}\log u_{ik}$

*To minimizing [3, (58)], Wei & Fahn thought that the Lagrangian term was $\lambda\left(\sum_{i=1}^{c}u_{ik}-1\right)$, but the Lagrangian term used by Wie & Fahn for implementation is still $\lambda\left(\sum_{i=1}^{c}u_{ik}-1\right)^2$ according to [3, (62)] when minimizing [3, (58)] or [3, (59)].

II. CONCLUSION

In this correspondence, we point out that the connection between the objective-functional-based fuzzy c-partition algorithms and FBACN is the Lagrange multiplier approach. Unfortunately, the Lagrange multiplier approach was incorrectly used so that FBACN is an illusion for objective-functional-based fuzzy clustering with constraints. Additionally, the FBACN does not satisfy the traditional definition of fuzzy c-partition, either.

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Comments on "A Generalized LMI-Based Approach to the Global Asymptotic Stability of Delayed Cellular Neural Networks"

Hongtao Lu

***Abstract*—In this letter, we point out that the linear matrix inequality (LMI)-based criterion obtained in the above paper for the global exponential stability of the delayed neural networks can be simplified to a simpler but equivalent form and, thus, show that it is not necessary to have such complex form of condition in the above paper. As a result, we also answer the question raised by the author of the above paper.**

***Index Terms*—Delayed cellular neural networks (DCNNs), global exponential stability, linear matrix inequality (LMI).**

In the above paper, the author obtained a linear matrix inequality (LMI) based condition for the global exponential stability of the delayed cellular neural networks (DCNNs) described by the following:

$$\dot{x}_i(t) = -x_i(t) + \sum_{j=1}^{n} a_{ij} y_j(x_j(t)) + \sum_{j=1}^{n} a_{ij}^{\tau} y_j(x_j(t-\tau)) + u_i, \quad i = 1, 2, \ldots, n \qquad (1)$$

or in vector-matrix form

$$\dot{\mathbf{x}}(t) = -\mathbf{x}(t) + \mathbf{A}\mathbf{y}(\mathbf{x}(t)) + \mathbf{A}^{\boldsymbol{\tau}}\mathbf{y}(\mathbf{x}(t-\tau)) + \mathbf{u} \qquad (2)$$

where $\mathbf{x}(\cdot) = [x_1(\cdot), \ldots, x_n(\cdot)]^T, \mathbf{y}(\mathbf{x}(\cdot)) = [y_1(x_1(\cdot)), \ldots, y_n(x_n(\cdot))]^T, \mathbf{A} = \{a_{ij}\}_{n\times n}, \mathbf{A}^{\boldsymbol{\tau}} = \{a_{ij}^{\tau}\}_{n\times n}, \mathbf{u} = [u_1, \ldots, u_n]^T$, and $y_i(x_i) = 0.5(|x_i+1| - |x_i-1|)$.

The LMI-based criterion established in the above paper is restated as follows.

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Digital Object Identifier 10.1109/TNN.2005.844094

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