Co-histogram and Image Degradation Evaluation*

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Abstract. The tool for image degradation evaluation addressed in this paper is called co-histogram, which is a statistic graph generated by counting the corresponding pixel pairs of two images. The graph is a two-dimensional joint probability distribution of the two images. A co-histogram shows how the pixels are distributed among combinations of two image pixel values. By means of co-histogram, we can have a visual understanding of PSNR, and the symmetry of a co-histogram is also significant for objective evaluation of image degradation. Our experiments with image degradation models of image compression, convolution blurring and geometric distortion perform the importance of the co-histogram.

1 Introduction

Images may be corrupted by some degradation sources, which may arise during image capture or processing, such as blurring, geometric distortion, or compression. A lot of image degradation models have been proposed in literature and a great deal of effort has been made to assess image quality objectively but close to subjective evaluation [1-3]. However, image degradation evaluation is so difficult that only limited success has been achieved [7].

The subjective evaluation methods are human vision oriented, considering human visual system (HVS) characteristics [3]. Mean opinion score (MOS) has been used for a long time, but it is inconvenient, slow and expensive. The subjective metrics also suffer from the physiological, psychological and environmental impact on the viewers.

So far, the widely used objective metrics for compressed image quality assessment are peak signal-to-noise ratio (PSNR) and mean square error (MSE). If the peak value of an image is 255, the mathematical representations are

$$PSNR = 10\log_{10}\frac{255^2}{MSE} \qquad MSE = \frac{1}{MN}\sum_{x=1}^{M}\sum_{y=1}^{N}\big(f(x,y)-g(x,y)\big)^2 \tag{1}$$

where $f(x,y)$ and $g(x,y)$ are two images of size MxN.

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However, they are considered not so satisfying. In many instances, it provides an inaccurate representation of image quality.

An ideal objective metric should be reliable, easily-computable and directly-intercomparable. There are two ways to overcome the shortcomings of PSNR: (i) to find another better objective metric to replace PSNR and to assess image degradation more effectively, and (ii) to find another objective metric as a complement to reinforce PSNR and to jointly assess image degradation more comprehensively. This paper is mainly based on the latter idea.

The tool we applied in this paper is called co-histogram [8]. A co-histogram is a statistic graph generated by counting the corresponding pixel pairs of two images. A co-histogram is also a two-dimensional joint probability distribution of the two images. A co-histogram shows how the pixels are distributed among combinations of two image pixel values, and visually it also gives an intuitive interpretation of PSNR.

For image degradation evaluation, a co-histogram can be easily obtained and then the metrics, the corresponding PSNR and its symmetry, are easily computable. Our experiments with the standard test images and using some degradation models, such as DCT-based JPEG and wavelet-based JPEG2000 compression methods, Gaussian blurring, and StirMark-like geometric distortion, perform the reliability of the co-histogram, its width and symmetry for image degradation evaluation.

2 Co-histogram and Its Properties

A histogram of an image is a statistical distribution of the image pixel values. It has found many applications such as image enhancement, thresholding, retrieval, classification and recognition.

The probability that pixel value p occurs in a digital image $f(x,y)$ of size M-by-N, $H_f(p)$, is counted as

$$H_f(p) = \frac{1}{MN}\sum_{y=1}^{M}\sum_{x=1}^{N}\delta(f(x,y),p) \tag{2}$$

where $\delta(f,p)$ is the Kronecker function: $\delta(f,p)=1$ (if $f = p$) and 0 (if $f \neq p$).

For all possible p, $H_f(p)$ gives the histogram of image $f(x,y)$.

For two images of the same size MxN, $f(x,y)$ and $g(x,y)$, the joint probability that pixel value pair (p,q) occurs is

$$H(p,q) = \frac{1}{MN}\sum_{y=1}^{M}\sum_{x=1}^{N}\delta(f(x,y),p)\cdot\delta(g(x,y),q) \tag{3}$$

For all possible pixel value pair (p,q), $H(p,q)$ makes the co-histogram of image pair $f(x,y)$ and $g(x,y)$.

A co-histogram has following properties.

(1) Axial projection

$$\sum_q H(p,q) = \frac{1}{MN}\sum_q\sum_{y=1}^{M}\sum_{x=1}^{N}\delta(f(x,y),p)\cdot\delta(g(x,y),q) = H_f(p) \tag{4}$$

(Histogram of image $f(x,y)$)

$$\sum_p H(p,q) = \frac{1}{MN} \sum_p \sum_{y=1}^{M} \sum_{x=1}^{N} \delta(f(x,y),p) \cdot \delta(g(x,y),q) = H_g(q) \tag{5}$$

(Histogram of image $g(x,y)$)

(2) Mean pixel value

$$I_f = \sum_p pH_f(p) = \sum_{p,q} pH(p,q) \tag{6}$$

$$I_g = \sum_q qH_g(q) = \sum_{p,q} qH(p,q) \tag{7}$$

If $I_f = I_g$, we have $\sum_{p,q}(p-q)H(p,q) = 0.$

(3) Variance of pixel values

$$Var(f) = \sum_p (p - I_f)^2 H_f(p) = \sum_{p,q} p^2 H(p,q) - I_f^{\,2} \tag{8}$$

$$Var(g) = \sum_q (q - I_g)^2 H_g(q) = \sum_{p,q} q^2 H(p,q) - I_g^{\,2} \tag{9}$$

If $I_f = I_g$, we have $Var(f) - Var(g) = \sum_{p,q}(p^2 - q^2)H(p,q).$

(4) Diagonal projection

To project in the direction of $p=q$, or its parametric representation, $p(t) = t + r$ and $q(t) = t$, we have

$$\begin{aligned}\sum_t H(p(t),q(t)) &= \sum_t H(t+r,t) = \sum_{x,y}\sum_t \delta(f,t+r)\cdot\delta(g,t) \\ &= \sum_{x,y} \delta(f(x,y) - g(x,y), r) = H_{f-g}(r)\end{aligned} \tag{10}$$

In fact, it is the histogram of the difference image $d(x,y) = f(x,y) - g(x,y)$.

(5) Mean of the difference image

$$I_d = \sum_r rH_d(r) = \sum_r r\sum_t H(t+r,t) = \sum_{p,q}(p-q)H(p,q) = I_f - I_g \tag{11}$$

(6) Variance of the difference image

$$\begin{aligned}Var(d) &= \sum_r (r - I_d)^2 H_d(r) = \sum_r r^2 \sum_t H(t+r,t) - I_d^2 \\ &= \sum_{p,q}(p-q)^2 H(p,q) - (I_f - I_g)^2 = MSE - (I_f - I_g)^2\end{aligned} \tag{12}$$

where MSE is the mean square error between two images.

3 PSNR and Symmetry of Co-histogram

For image degradation evaluation, we take the original image and the degraded image as above two images $f(x,y)$ and $g(x,y)$. Thus, for most image degradations, we generally have $I_f \approx I_g$. Accordingly, the degradation evaluation metrics PSNR and MSE correspond to the variance of the difference image, and then to the variance of the diagonal projection of the co-histogram, which is visually a "width" measure of the co-histogram. Therefore, PSNR is a measure of the co-histogram width along the diagonal.

If $Var(d) = 0$, or the whole co-histogram is strictly on the diagonal, the two images must be identical, and no degradation occurs. If $Var(d) \neq 0$, or the off-diagonal distribution is not entirely zero, the image must have been degraded. Thus, the PSNR can be calculated directly from the co-histogram:

$$PSNR = 10\log_{10}\frac{255^2}{MSE} \approx 48.1308 - 10\log_{10}\left(Var(d)\right) \tag{13}$$

The co-histogram symmetry (CHS) is defined as:

$$CHS = \frac{\alpha\sum_{p} H^2(p,p) + \sum_{p,q}(p-q)^2 H(p,q)H(q,p)}{\alpha\sum_{p} H^2(p,p) + \sum_{p,q}(p-q)^2 H^2(p,q)} \tag{14}$$

where α is a positive constant and less than 1, which plays a role of the weight of the unchanged pixel possibilities (distributed on the diagonal in the co-histogram). We use 1/4 in our experiments.

CHS reflects statistic information of pixel pairs and some similarity between the individual histograms of the two images. It is 1 if a co-histogram is exactly symmetric.

CHS is mathematically independent of the PSNR, and therefore is a valid complement to PSNR. Symmetry together with PSNR makes the depiction of co-histogram more comprehensive.

CHS also gives a good estimation of accuracy in image classification and other applications [8]. Actually, the CHS given in (14) is a weighted version of a co-histogram symmetry, which enhances the difference between two images.

4 Evaluation Methodology

For evaluation purposes, we employed some image degradation models, including pixel value degradation and geometric distortion.

Geometric distortion was usually ignored in image degradation evaluation. We use StirMark-like bending [4] and image rotation, cropping and scaling for comparison.

Pixel value variation methods include image blurring, and image compression.

The blurring of an image can be caused by many factors [2], such as out-of-focus, motion, and atmospheric turbulence. Blurs are generally modeled as convolution filters. In this paper, we use Gaussian low-pass filter to generate the blurred images.

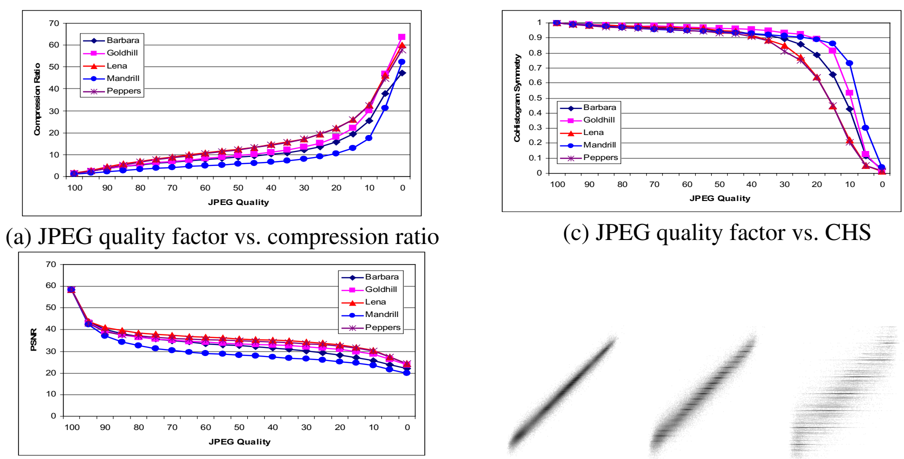
The image compression methods we considered are two international standardized image-coding frameworks, DCT-based JPEG and wavelet-based JPEG2000. Due to the block artifacts of former JPEG and many other shortcomings, new standard named JPEG2000 was published in 2000, which provides many more flexibilities and functions besides better compression quality.

Test images we used are 5 widely-used standard test images, `Barbara`, `Goldhill`, `Lena`, `Mandrill` and `Peppers`. All of them are 512x512 8-bit gray-level images. We take each degraded version and its original image as a pair of images, and make the co-histogram by statistics. Then, we use the co-histogram to find its "width" measure PSNR and its symmetry. Having collected all the PSNRs and the corresponding symmetries, we try to find out if there is any relationship between PSNR and CHS, and how much information of image degradation they manifest.

5 Experiments and Analysis

5.1 Image Compression

JPEG. JPEG compression is configured by image quality options [6]. All the 5 test images are compressed at image quality factor of 100 through 0 decreased by 5. The relation between the quality factors and the compression ratios are shown in Figure 1(a). JPEG quality factor vs. PSNR and co-histogram symmetry are in Figure 1 (b) and (c). Three co-histograms with Lena are shown in Figure 1(d).



(a) JPEG quality factor vs. compression ratio

(c) JPEG quality factor vs. CHS

(b) JPEG quality factor vs. PSNR (dB)

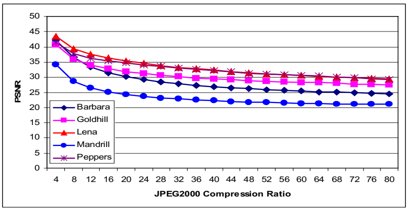
(d) Co-histograms at quality factors 30, 10, 0

Fig. 1. JPEG Compression

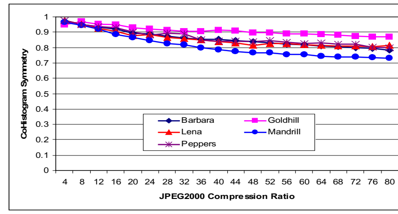
For JPEG compression, PSNRs are around 30dB or less for compression quality factor 80 or less, and CHS' are very low (even close to 0) when the JPEG quality is very low, and the compression ratio is between 30 and about 60.

JPEG2000. JPEG2000 compression can be controlled directly by bit rates or compression ratios [5]. We use the compression ratios from 4 to 80, increased by 4. The results are in Figure 2 (a) and (b). Three co-histograms of compression at ratio 32, 64 and 80 with Lena are shown in Figure 2 (c).

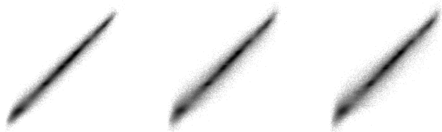
JPEG2000 compression gives higher PSNRs than JPEG at similar compression ratios, and the co-histogram symmetries are all larger than 0.7. It leads to a conclusion that JPEG2000 scheme is better than the old JPEG.



(a) JPEG2000 compression ratio vs. PSNR

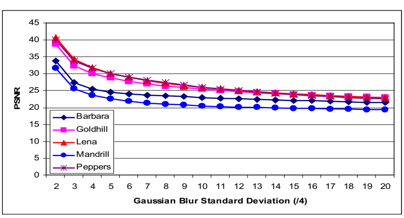


(b) JPEG2000 compression ratio vs. CHS

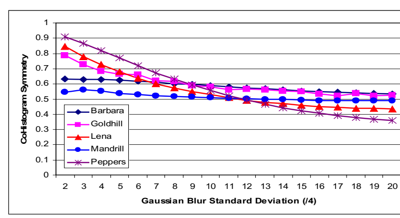


(c) Co-histograms at compression ratios 32, 64 and 80 (Lena)

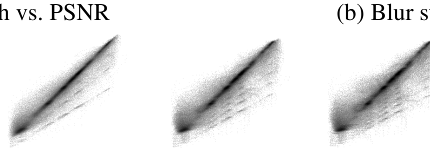
Fig. 2. JPEG2000 Compression



(a) Blur strength vs. PSNR



(b) Blur strength vs. CHS



(c) Co-histograms at standard deviation 8, 15 and 20 (Lena)

Fig. 3. Gaussian Image Blurring

5.2 Blurring

There are many models for image blurring [2]. In this paper, we use a widely-used typical model, convolution Gaussian blur. The model is defined as a 2D Gaussian filter:

$$h(x,y) = \frac{h_g(x,y)}{\sum_{x,y} h_g(x,y)} \qquad \text{where} \qquad h_g(x,y) = \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) \tag{15}$$

The filter is sized in 21 pixels and the standard deviation σ is tested from 1/4 to 20/4, increased by 1/4 as shown in Figure 3 (a) and (b). Three co-histograms at Gaussian blue standard deviation of 8, 15 and 20 with Lena are shown in Figure 3 (c).

PSNRs are almost under 20, and the co-histogram symmetries are mostly low, below 0.6.

Some other experiments with motion blur are very similar to above with Gaussian blur.

5.3 Geometric Distortion

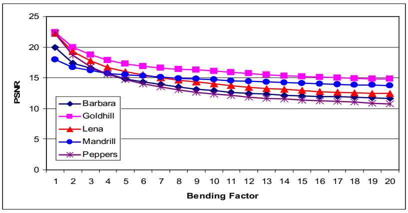
Punch-pinch bending. All the pixels are moved using a smooth sine function. Pixels at the corners are not displaced while the pixel in the center of the image is moved the most. This is similar to a "punch-pinch" effect. The degradation parameter is defined by the number of pixel displacement allowed for the center of the image. The displacement is defined as in StirMark software [4]:

$$\begin{cases} x' = x + b \cdot \sin x\pi/W \\ y' = y + b \cdot \sin y\pi/W \end{cases} \tag{16}$$

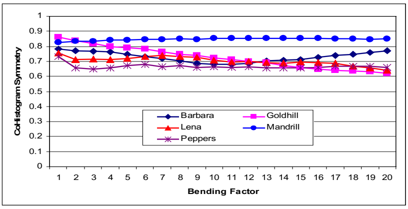
where the bending factor b varies from 1 to 20.

In our experiments, we use quadratic interpolation method and point sampling after geometric bending. Results and some co-histograms are shown in Figure 4.

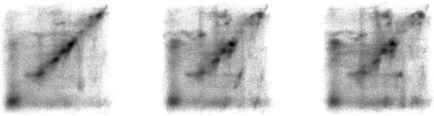
In these experiments, PSNRs are very low, while the co-histogram symmetries stay high.



(a) Bending factor vs. PSNR (dB)



(b) Bending factor vs. CHS

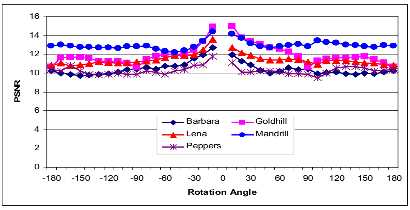


(c) Co-histograms at bending factors 8, 15 and 20 (Lena)

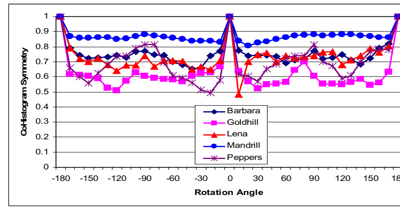
Fig. 4. Geometric Bending

Rotation. Rotation by an angle followed by centered cropping and rescaling to keep the original size of the image. Rotation, cropping and scaling are commonly used methods of geometric distortion, and the random distortion of an image can also be modeled as such distortion combinations for individual image blocks. We test the rotation angles from -180 to 180 degrees, increased by 5 degrees.

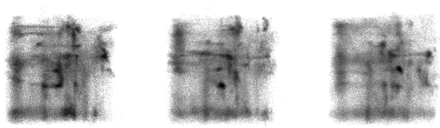
As shown in Figure 5, the results are similar to geometric bending. PSNRs are very low, while the co-histogram symmetries are high.



(a) Rotation angle vs. PSNR



(b) Rotation angle vs. CHS



(c) Co-histograms at rotation angles 30, 60 and 90 (Lena)

Fig. 5. Rotation, Cropping and Scaling

6 Conclusions

(1) Co-histogram is the joint probability distribution of two images. It gives the perfect statistic depiction of the pixel difference between two images.
(2) Co-histogram provides a visual perception of PSNR, which reflects the width of co-histogram along the diagonal.
(3) Co-histogram symmetry is independent of and a valid complement to PSNR, and it is also an objective statistic metric. Symmetry together with PSNR makes the depiction of a co-histogram more comprehensive. Its calculation is direct and as easy as that of PSNR.
(4) Co-histogram is significant in evaluation of image degradation.
(5) Some by-products of our experiments with PSNR and co-histogram symmetry: JPEG 2000 performs better than JPEG; geometric distortion changes images less than intensity distortion.

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