Admissibility & Unification

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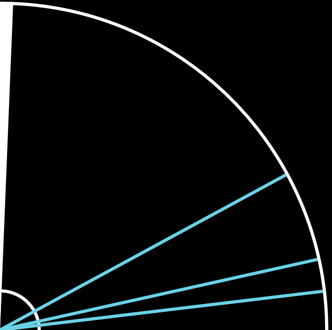
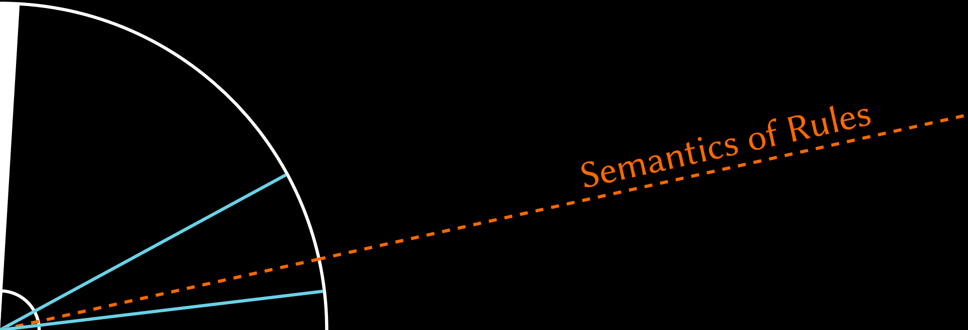
Utrecht University
ALCOP, May 15th 2014

Overview

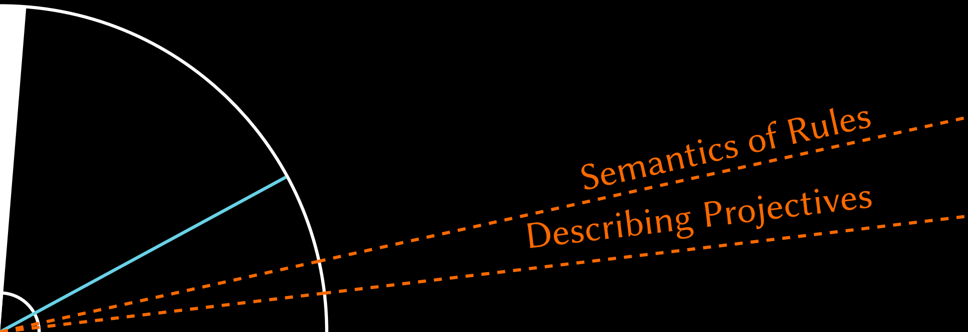
Overview



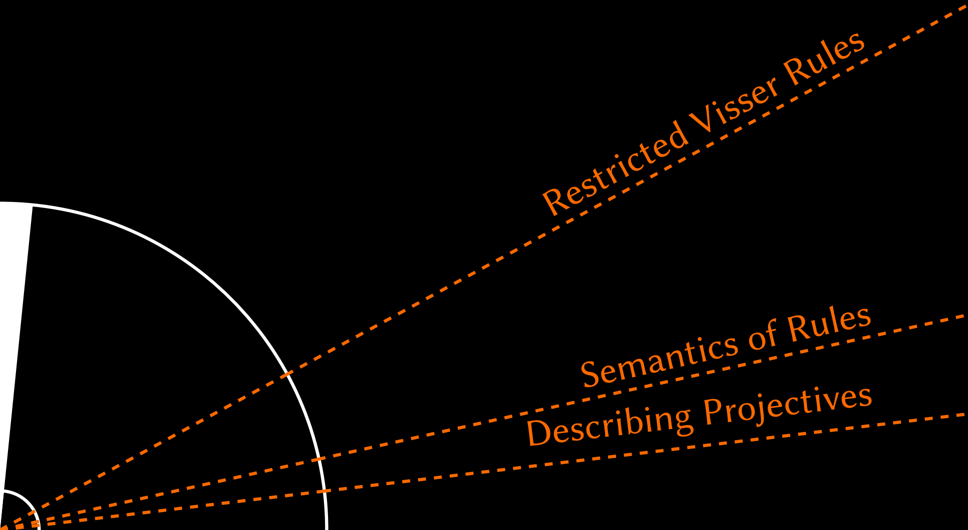
Overview



Overview



Overview


Restricted Visser Rules
Semantics of Rules
Describing Projectives

A / $\triangle$ admissible

σA is derivable

$A \;/\; \Delta$ admissible

σC is derivable for some $C \in \Delta$

σA is derivable

$A \mathrel{|\!\sim} \Delta$ admissible

σC is derivable for some $C \in \Delta$

Disjunction Property

$$\frac{A \lor B \text{ derivable}}{A \text{ derivable or } B \text{ derivable}}$$

Disjunction Property

$$\frac{p \vee q}{\{\ p, q\ \}}$$

$$\frac{\vdash A \vee B}{\vdash A \text{ or } \vdash B}$$

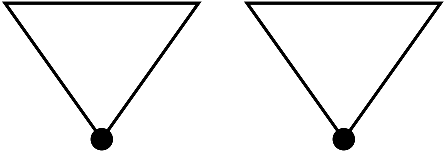
syntax

$$\frac{\vdash A \vee B}{\vdash A \text{ or } \vdash B}$$

semantics

syntax

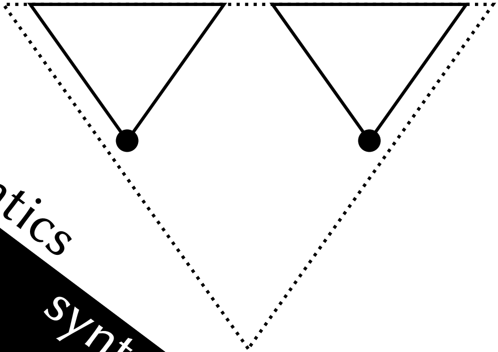
$$\frac{\vdash A \vee B}{\vdash A \text{ or } \vdash B}$$



semantics

syntax

$$\frac{\vdash A \vee B}{\vdash A \text{ or } \vdash B}$$

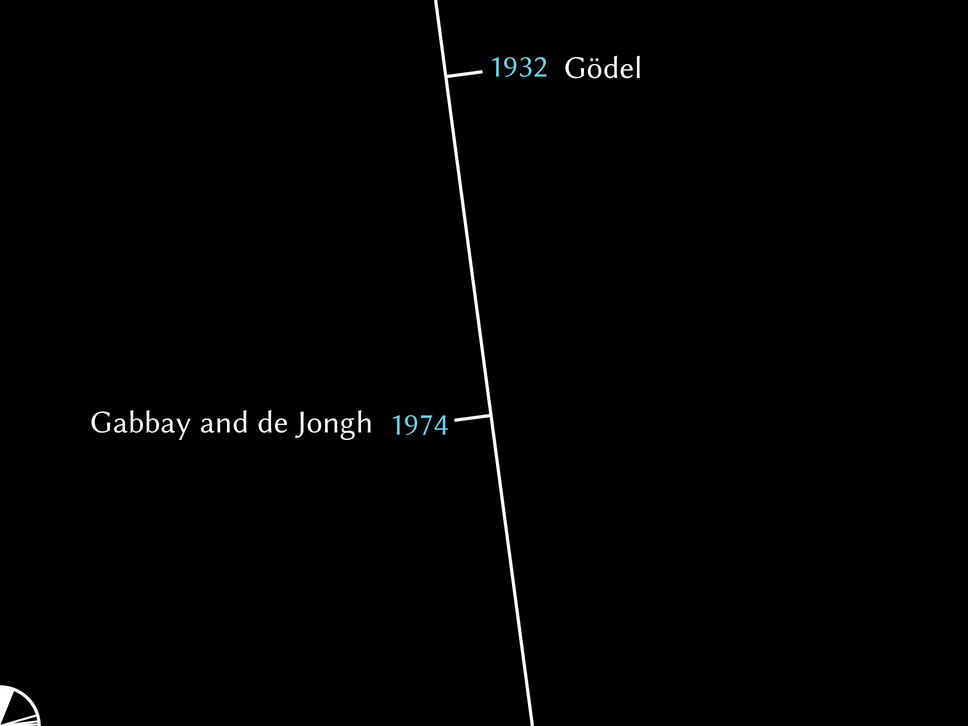


semantics

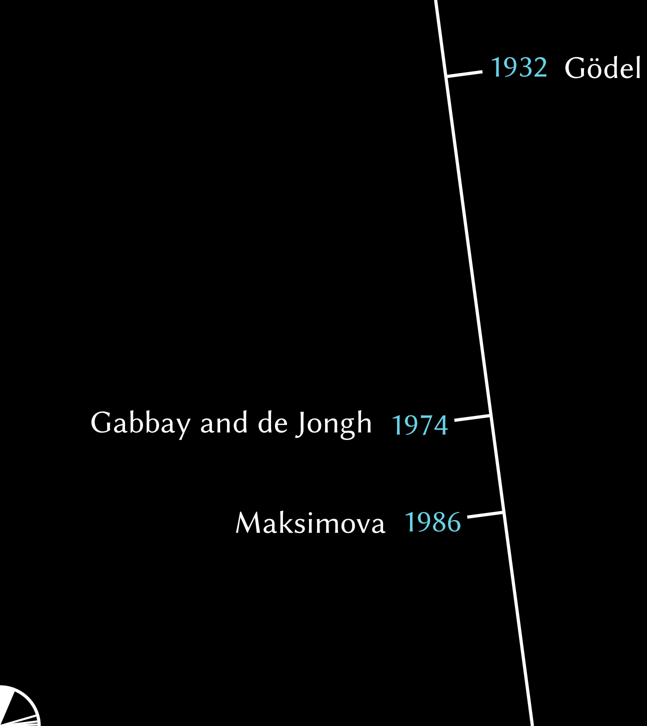
syntax

$$\frac{\vdash A \vee B}{\vdash A \text{ or } \vdash B}$$

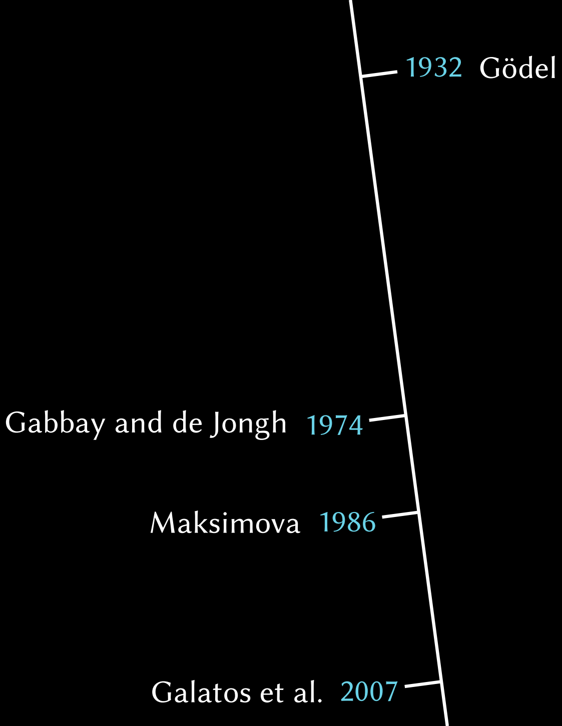
1932 Gödel



1932 Gödel
Gabbay and de Jongh 1974



1932 Gödel
Gabbay and de Jongh 1974
Maksimova 1986



1932 Gödel
Gabbay and de Jongh 1974
Maksimova 1986
Galatos et al. 2007


1932 Gödel
1952 Łukasiewicz
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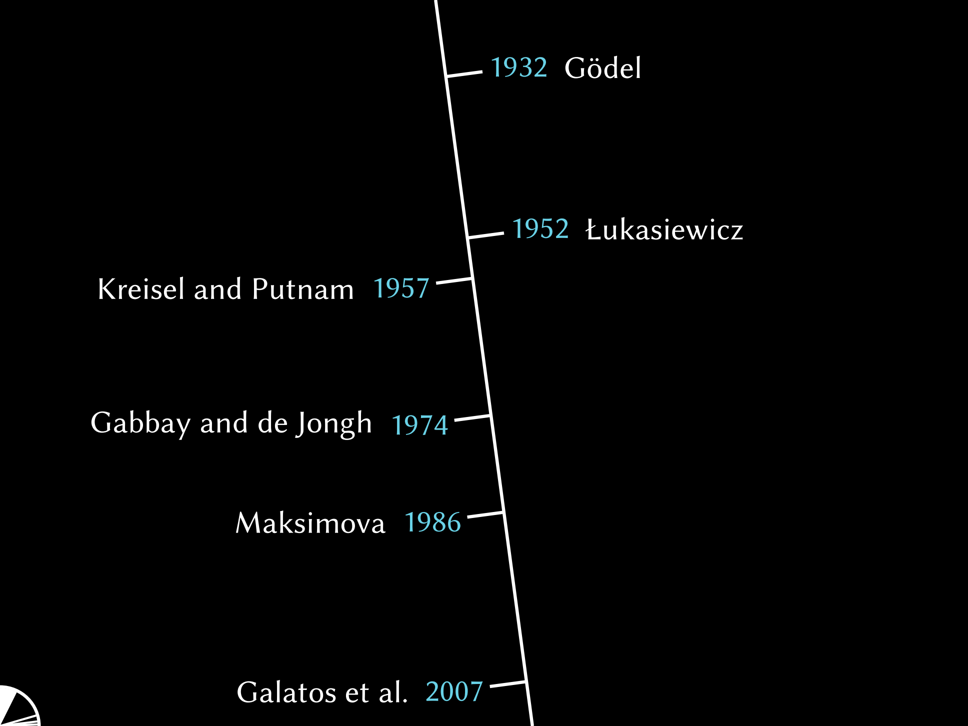
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Kreisel and Putnam 1957
Gabbay and de Jongh 1974
Maksimova 1986
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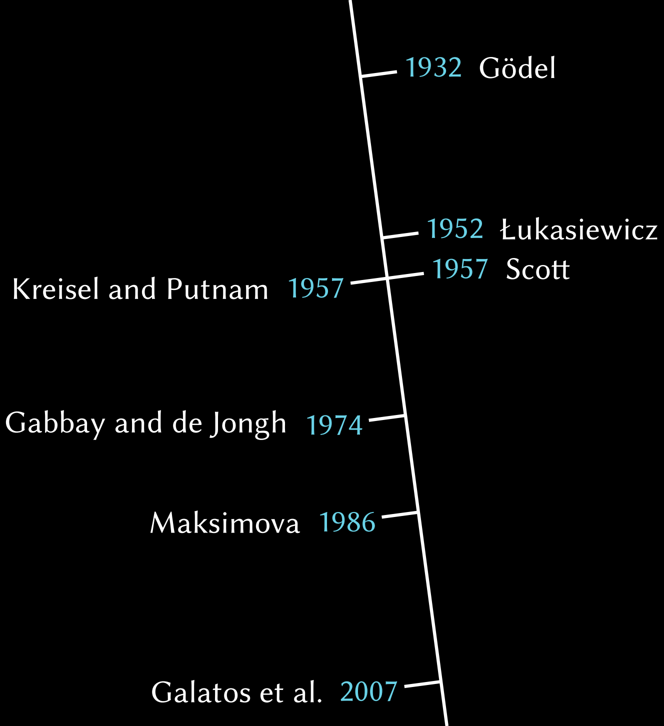
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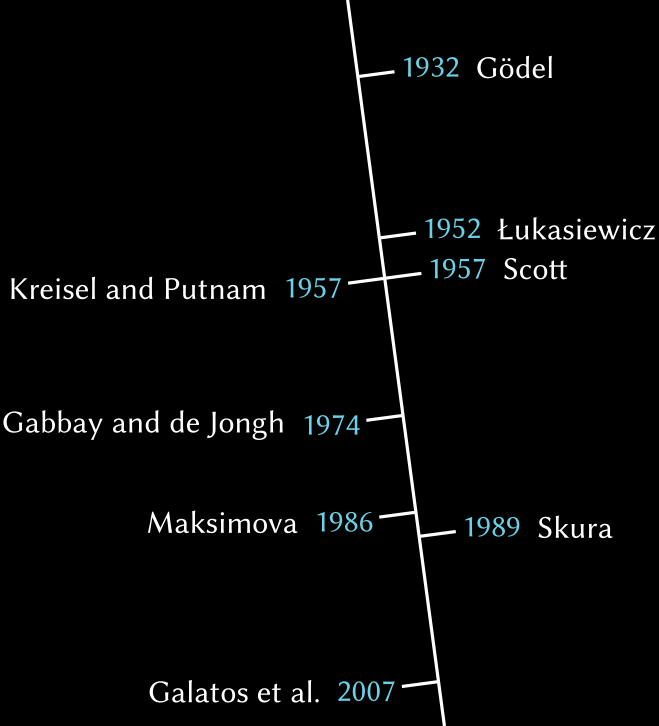
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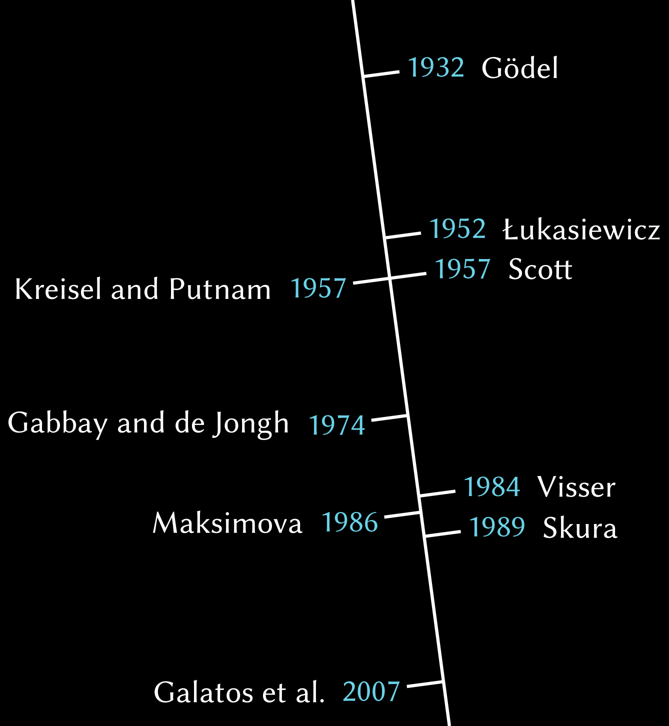
1989 Skura
Galatos et al. 2007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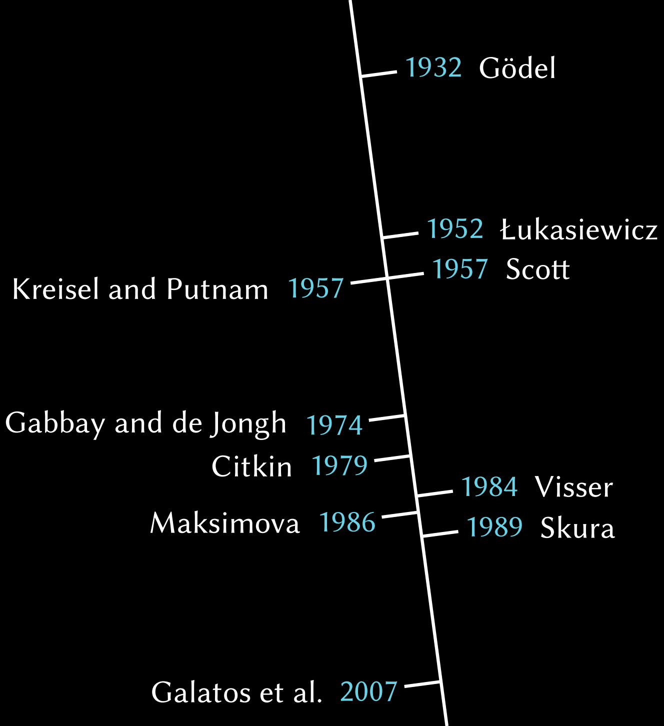
1984 Visser
Maksimova 1986
1989 Skura
Galatos et al. 2007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Citkin 1979
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Maksimova 1986
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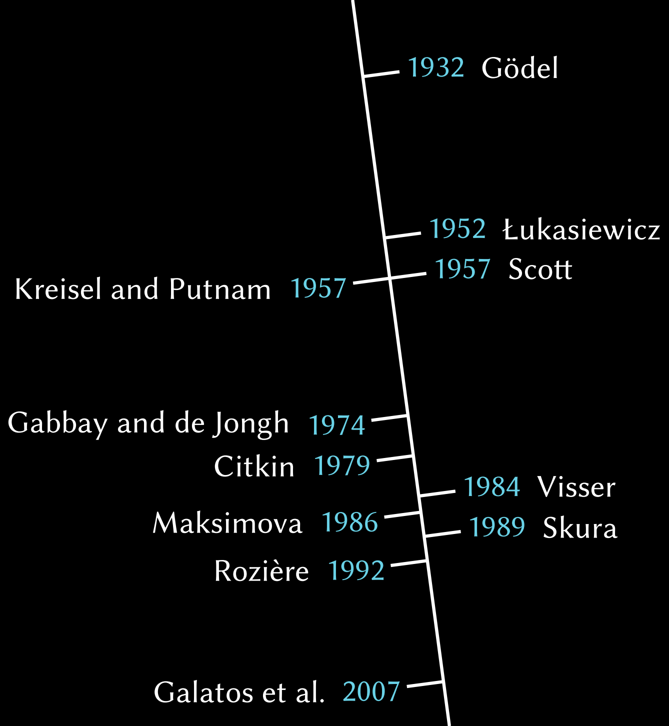
Galatos et al. 2007


1932 Gödel
1952 Łukasiewicz
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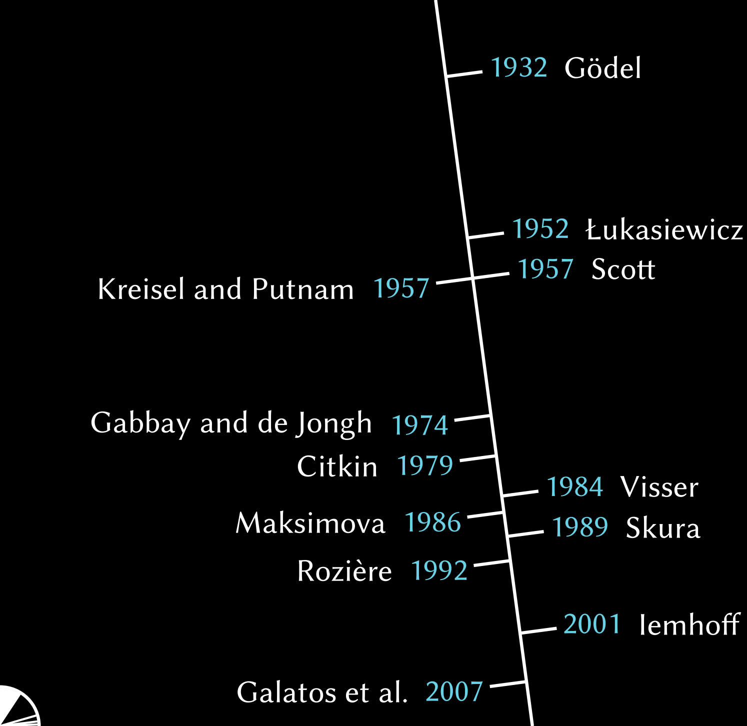
1984 Visser
Maksimova 1986
1989 Skura
Rozière 1992
Galatos et al. 2007


1932 Gödel
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Gabbay and de Jongh 1974
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1984 Visser
Maksimova 1986
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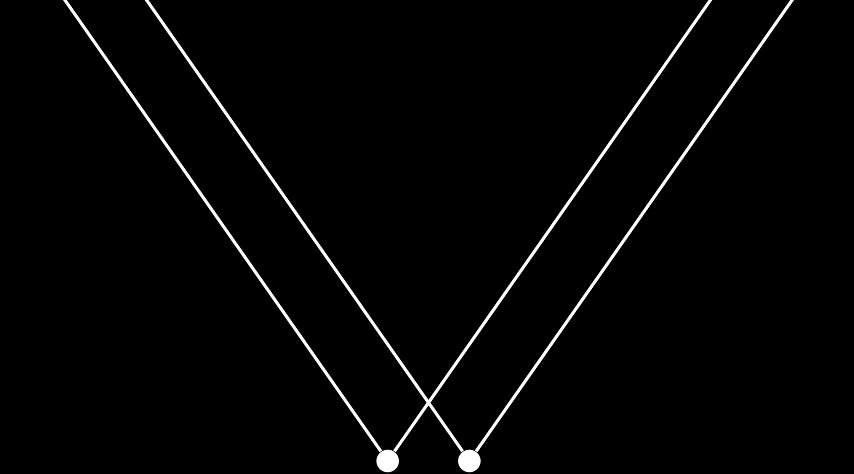
Rozière 1992
2001 Iemhoff
Galatos et al. 2007

Analogous

Analogous



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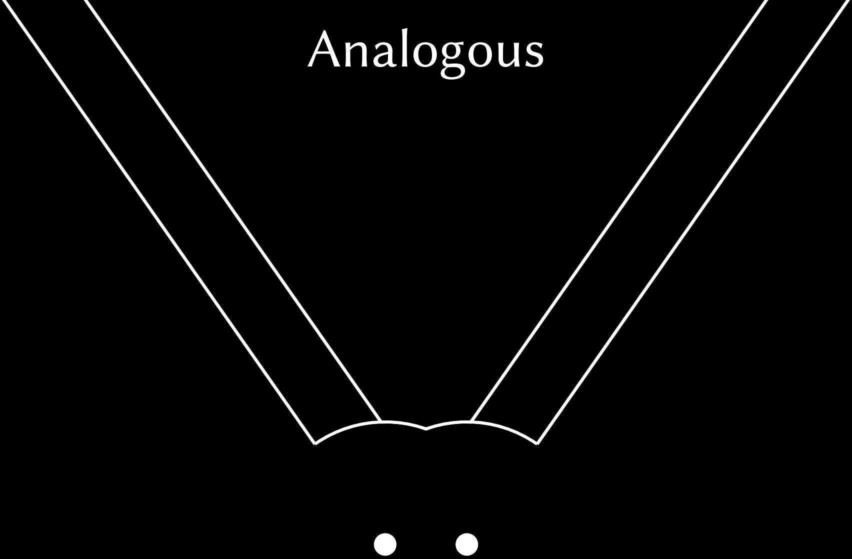

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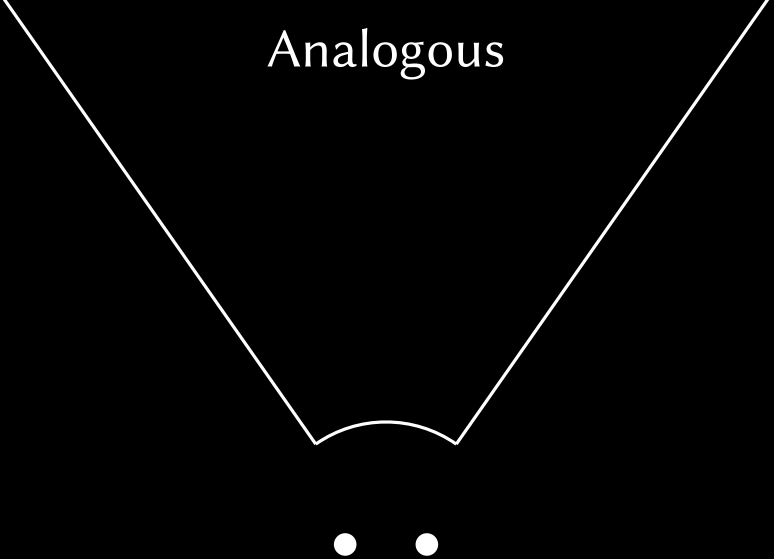

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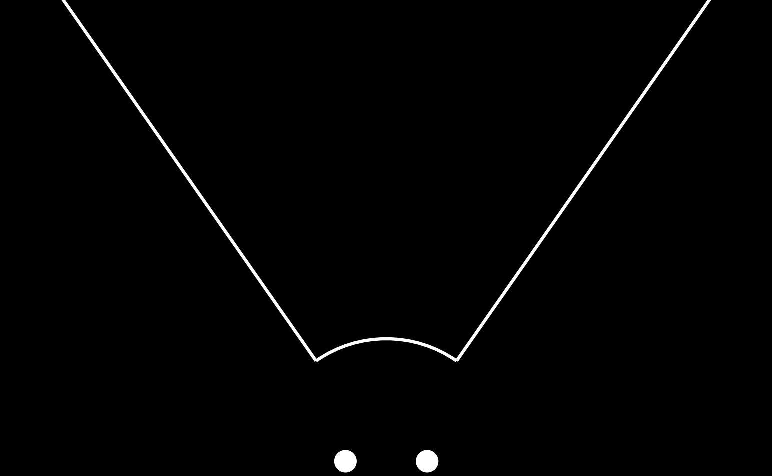
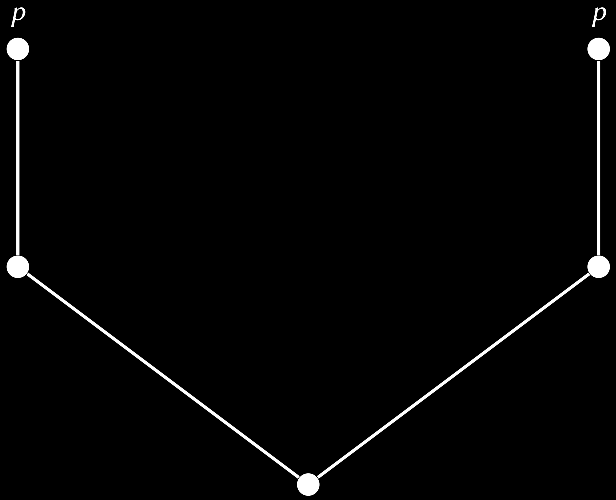
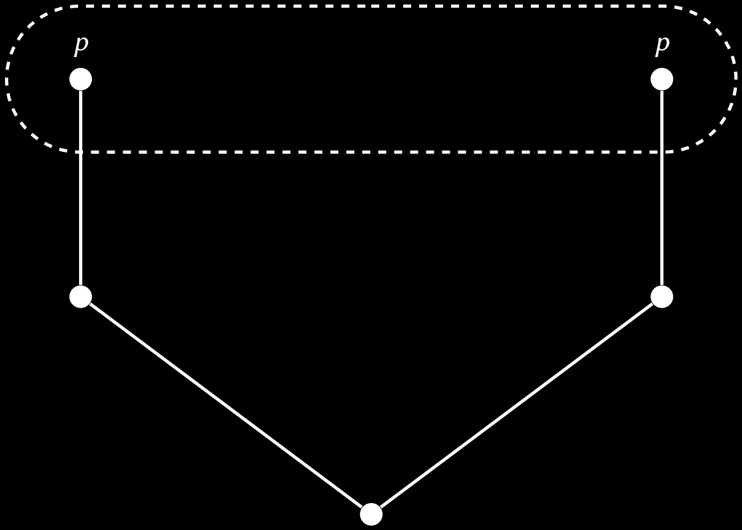


$k \equiv l$ when $v(k) = v(l)$ and $k \leq u$ iff $l \leq u$ for all $u \neq k, l$


p
p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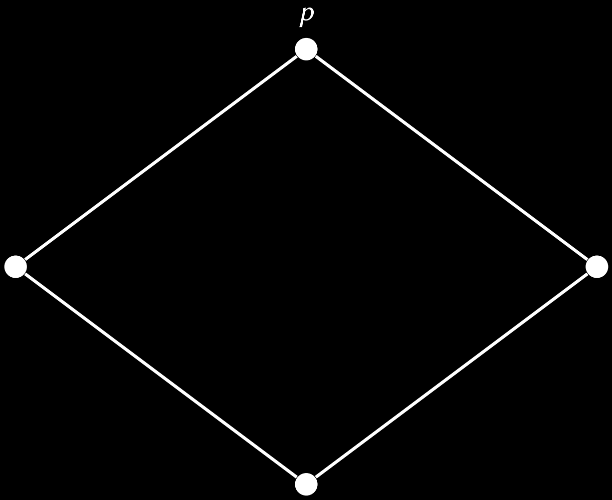
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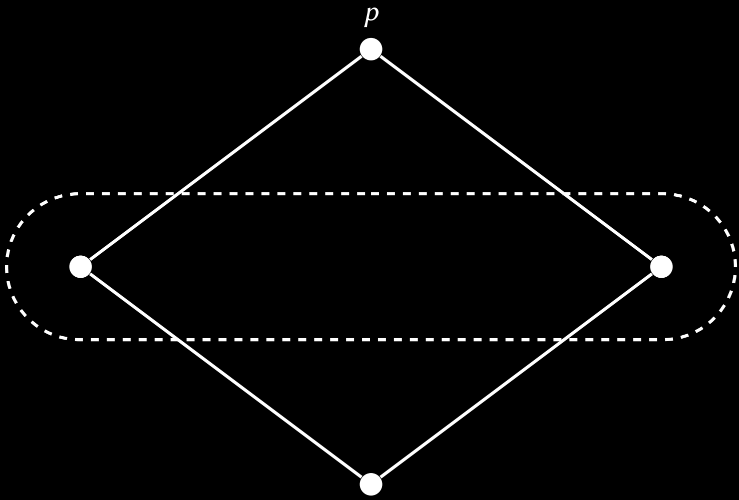
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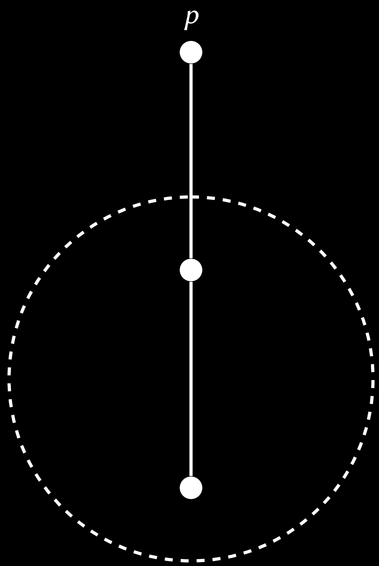
p


p

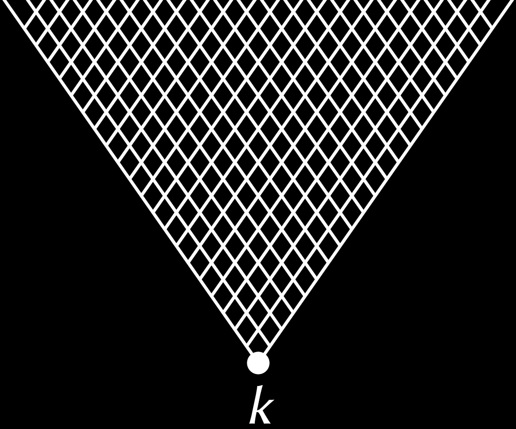

p

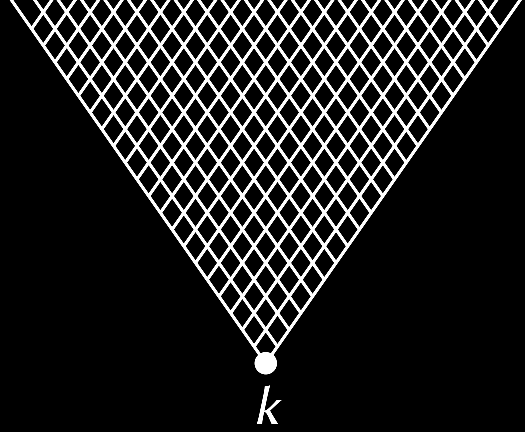
Jankov–de Jongh formulae

In suitable models have

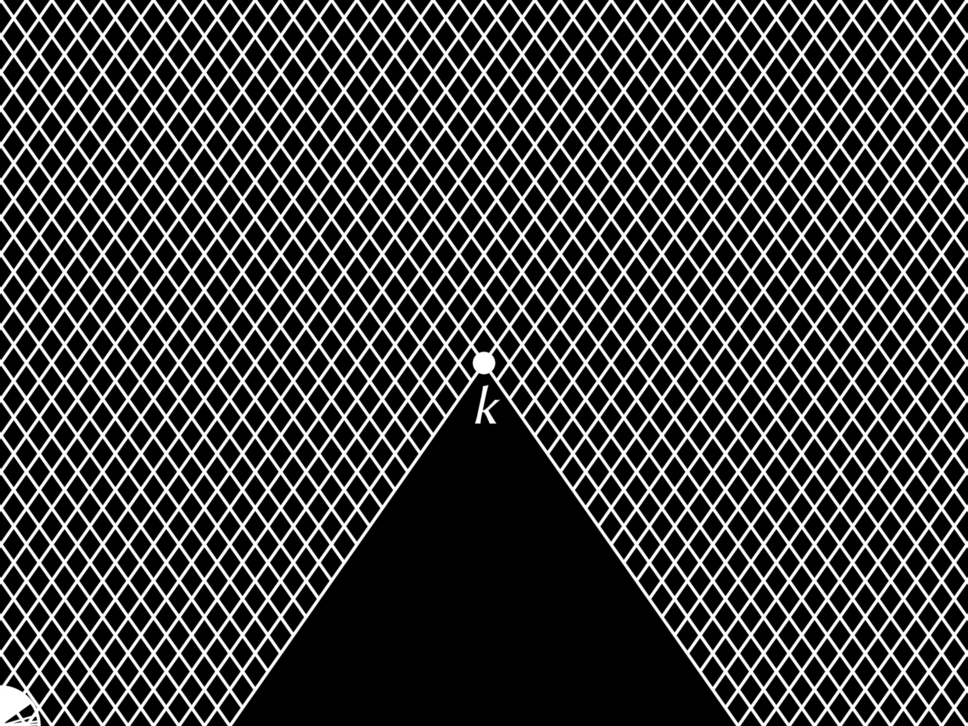
$$l \Vdash \mathrm{up}\, k \quad \text{iff} \quad k \leq l$$
$$l \Vdash \mathrm{nd}\, k \quad \text{iff} \quad l \not\leq k$$

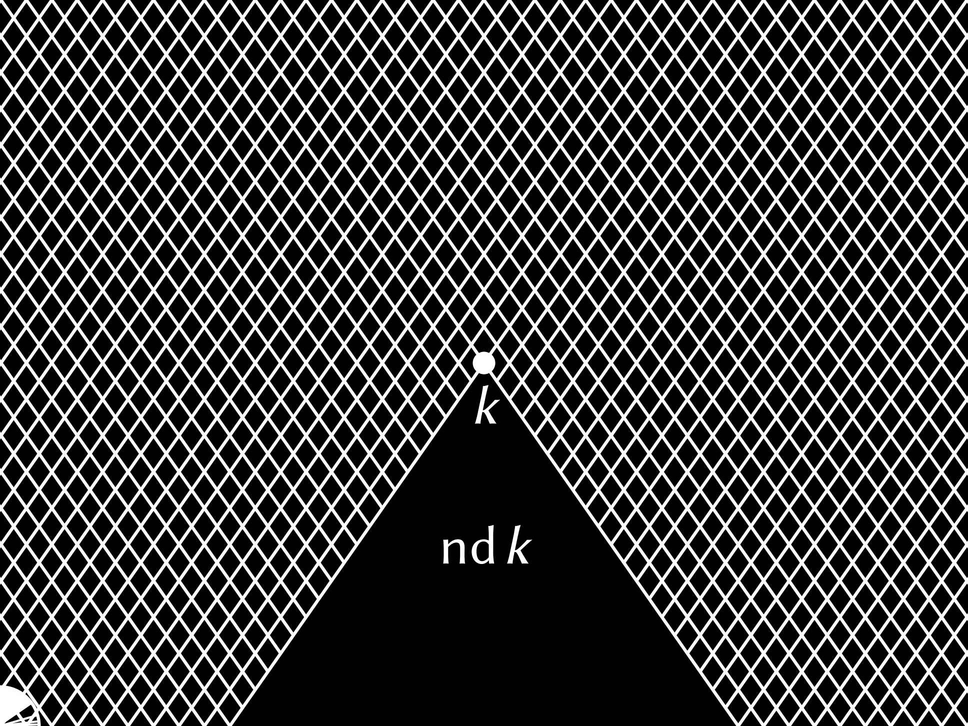

k


k

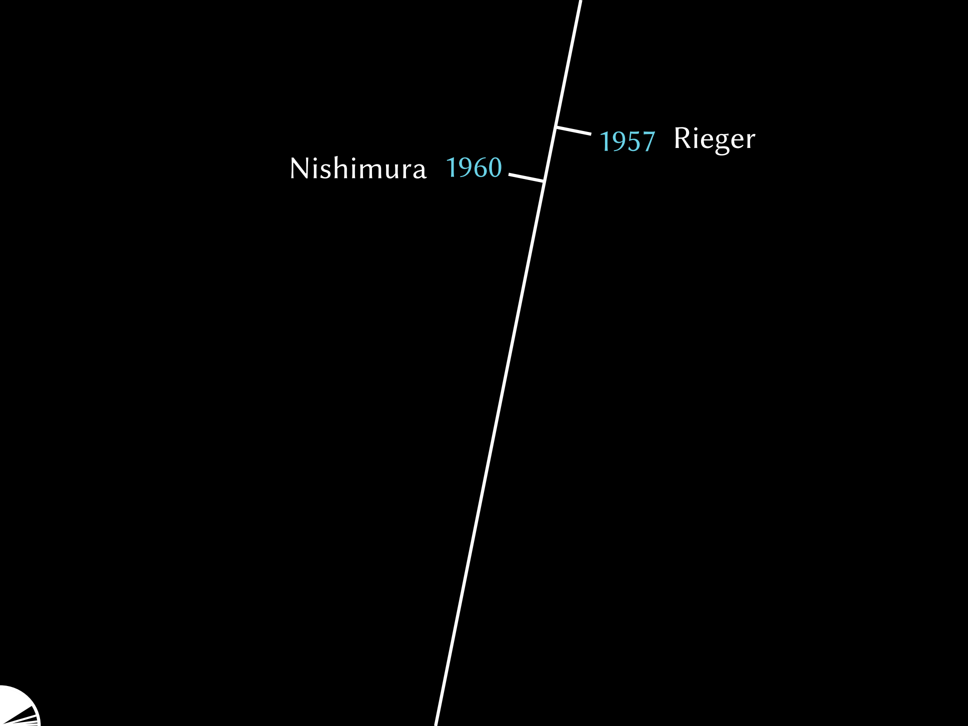


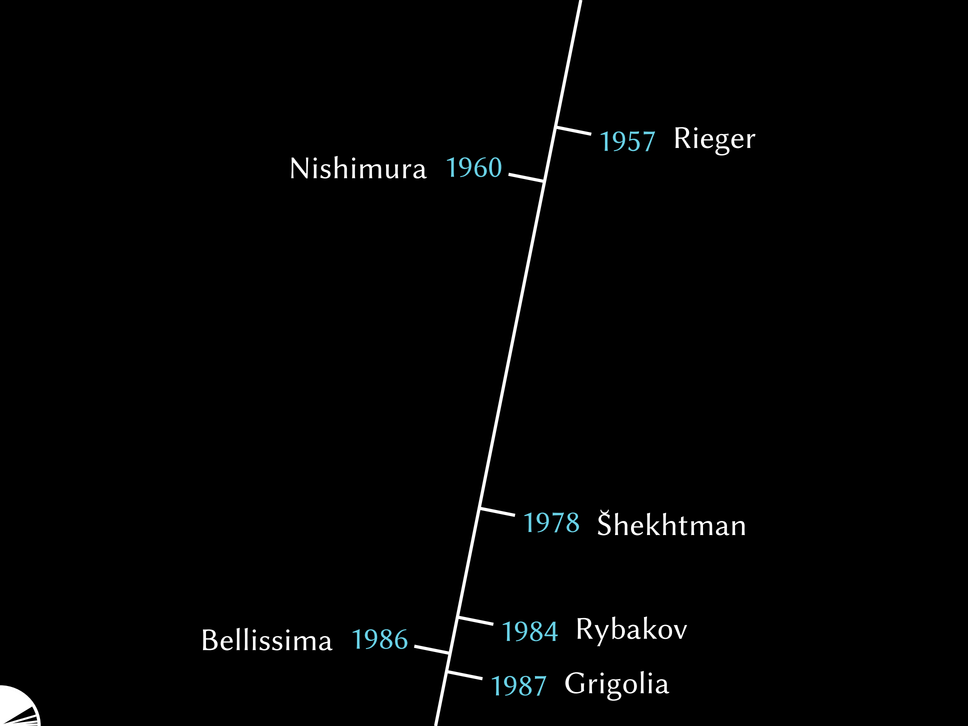
up k


k

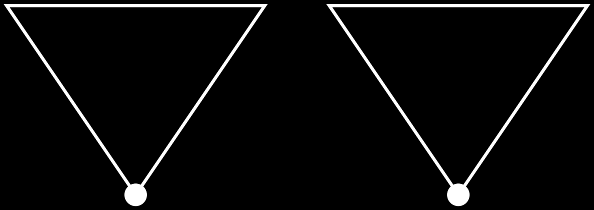

k
nd k

There exists a suitable model containing all rooted finite models.

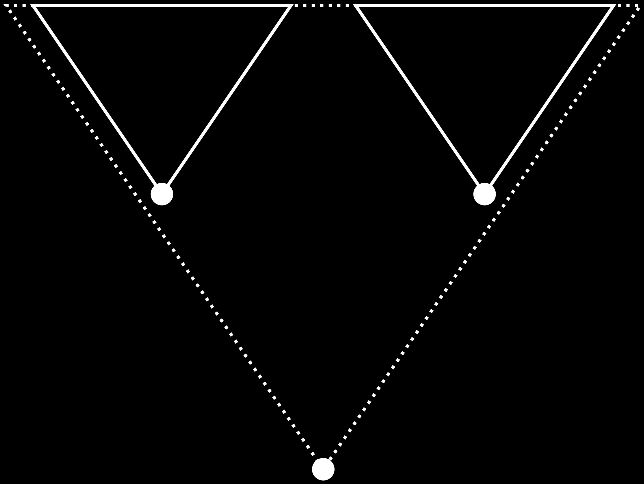

1957 Rieger
Nishimura 1960


1957 Rieger
Nishimura 1960
1978 Šhekhtman
1984 Rybakov
Bellissima 1986
1987 Grigolia

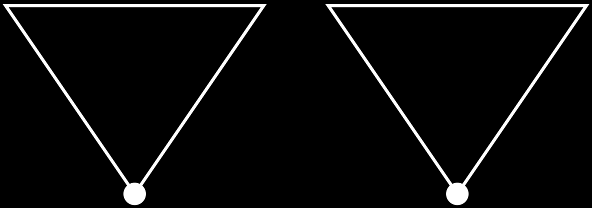
Disjunction Property



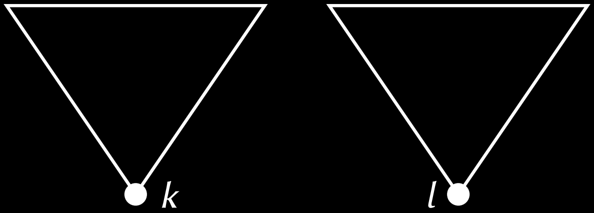
Disjunction Property



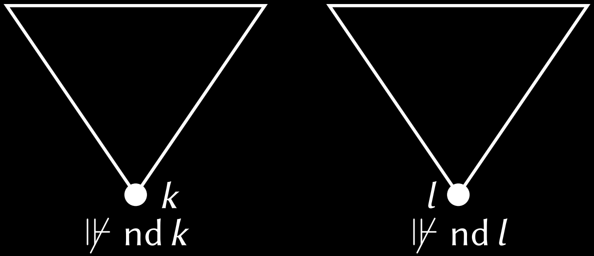
Disjunction Property



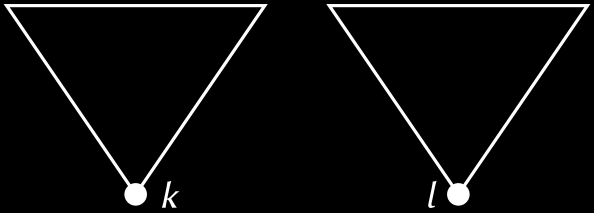
Disjunction Property



Disjunction Property

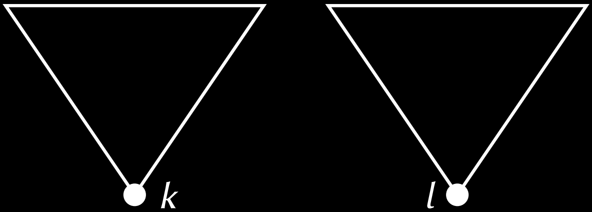


Disjunction Property



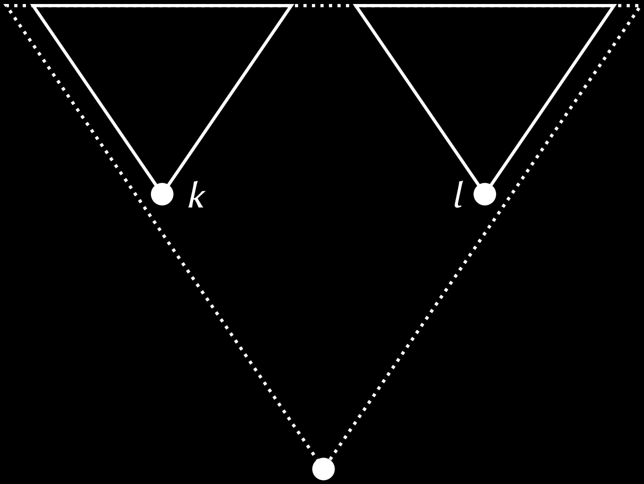
$$\nvdash \mathrm{nd}\, k \vee \mathrm{nd}\, l$$

Disjunction Property



$\nVdash \mathrm{nd}\, k \vee \mathrm{nd}\, l$

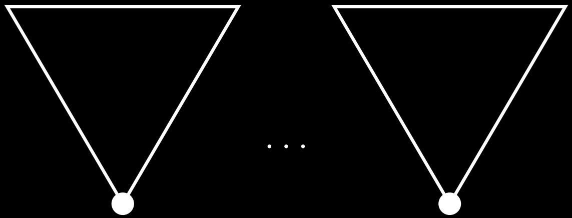
Disjunction Property



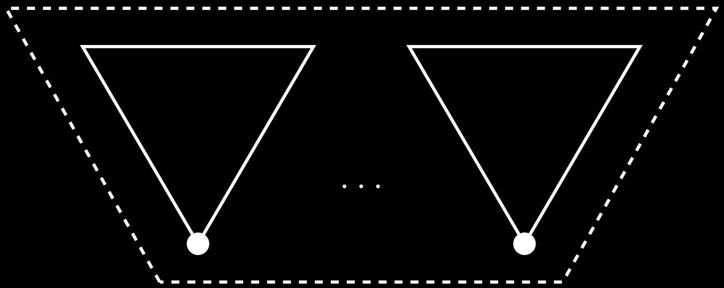
$$\frac{(\bigvee \Delta \to A) \to \bigvee \Delta}{\left\{ (\bigvee \Delta \to A) \to C \;\middle|\; C \in \Delta \right\}}$$

Extension Property

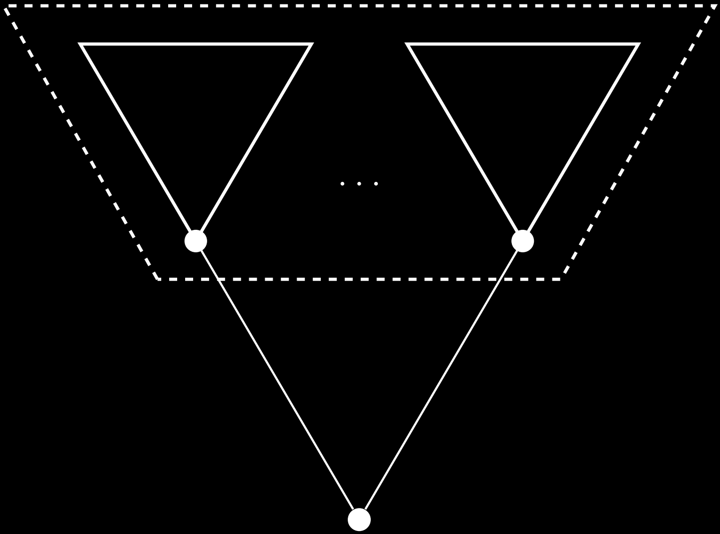
Extension Property



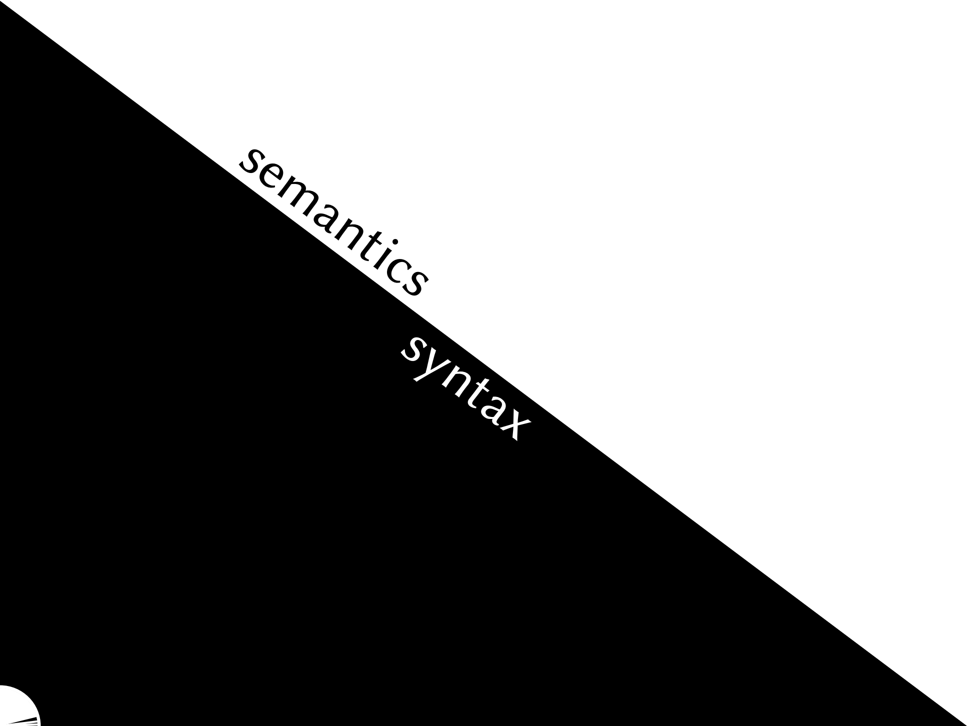
Extension Property

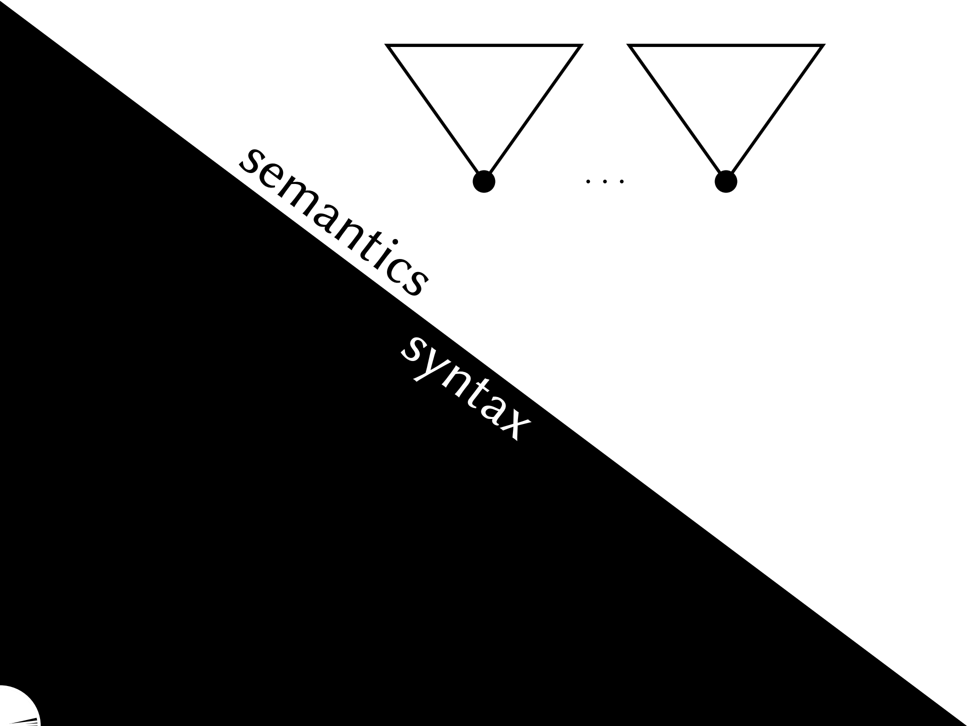


Extension Property



semantics


semantics
syntax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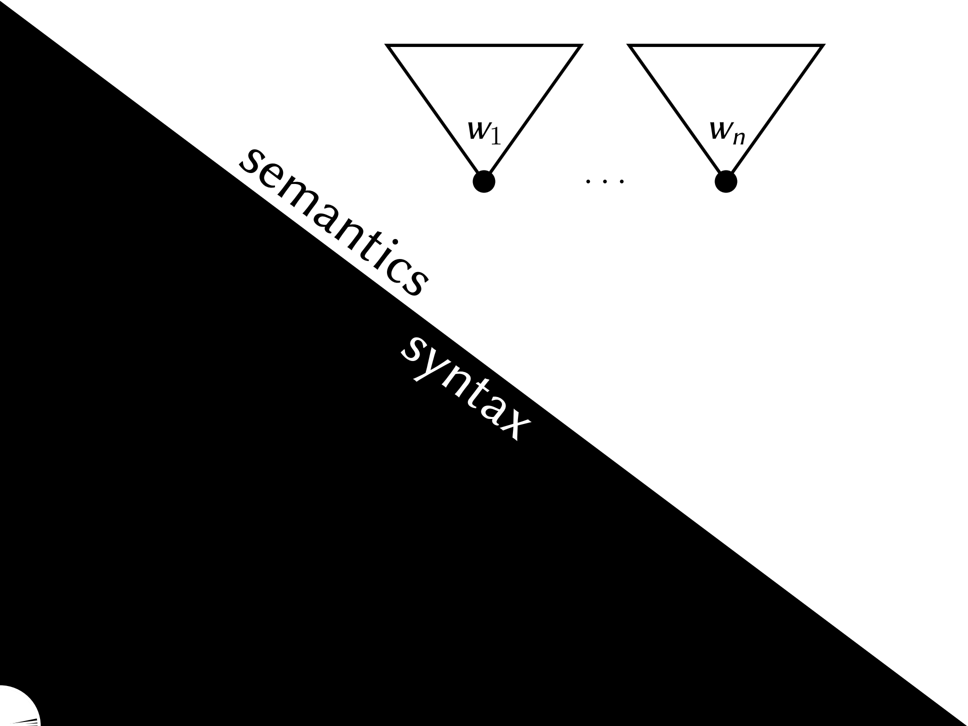
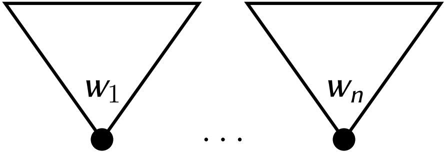
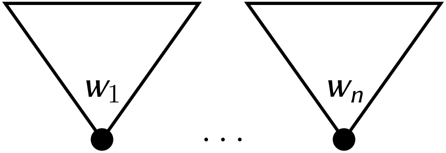

w_1
. . .
w_n
semantics
syntax



semantics

syntax

$$\left\{ \left(\bigvee_{i=1}^{n} \mathsf{nd}\, w_i \to \bigvee_{i=1}^{n} \mathsf{up}\, w_i \right) \to \mathsf{nd}\, w_j \right\}_{j=1}^{n}$$



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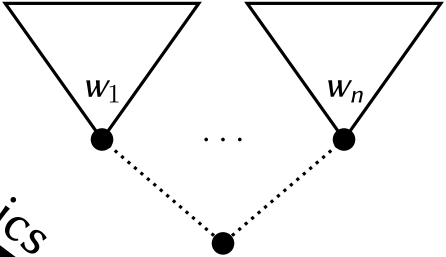
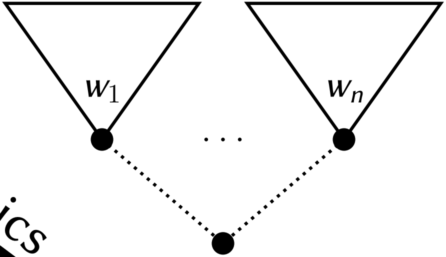
$$\frac{\left(\bigvee_{i=1}^{n} \mathsf{nd}\, w_i \to \bigvee_{i=1}^{n} \mathsf{up}\, w_i\right) \to \bigvee_{i=1}^{n} \mathsf{nd}\, w_i}{\left\{ \left(\bigvee_{i=1}^{n} \mathsf{nd}\, w_i \to \bigvee_{i=1}^{n} \mathsf{up}\, w_i\right) \to \mathsf{nd}\, w_j \right\}_{j=1}^{n}}$$



semantics

syntax

$$\frac{\left(\bigvee_{i=1}^{n} \mathsf{nd}\, w_i \to \bigvee_{i=1}^{n} \mathsf{up}\, w_i\right) \to \bigvee_{i=1}^{n} \mathsf{nd}\, w_i}{\left\{ \left(\bigvee_{i=1}^{n} \mathsf{nd}\, w_i \to \bigvee_{i=1}^{n} \mathsf{up}\, w_i\right) \to \mathsf{nd}\, w_j \right\}_{j=1}^{n}}$$



semantics

syntax

$$\frac{\left(\bigvee \Delta \to A\right) \to \bigvee \Delta}{\left\{ \left(\bigvee \Delta \to A\right) \to C \right\}_{C \in \Delta}}$$

A is projective when
$\vdash \sigma A$ and $A \vdash \sigma B \equiv B$
for some σ.

A is admissibly saturated when

$A \mathrel{|\!\!\sim} \Delta$ implies $A \vdash C$

for some $C \in \Delta$.

Ghilardi (1999) and Ghilardi (2004)

A formula is IPC-projective
precisely if it has
the extension property.

A formula B is IPC-projective iff

$$B \vdash (\bigvee \Delta \to A) \to \bigvee \Delta \text{ entails}$$

$$B \vdash (\bigvee \Delta \to A) \to C$$

for some $C \in \Delta$.

Similar characterisations exists for BD_2, T_n and $\mathsf{BD}_2 + \mathsf{T}_n$.

References I

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