Cut-elimination for Multi-type DC

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Outline

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Main Quest

Proof-theory for Dynamic Logics.

Often, the hurdles are due to some of their **defining features** (e.g. lack of closure under uniform substitution).

Typically, these logics come in ***large families***:

- 'uniform' proof-theoretic approaches are in high demand.

Ongoing project with S. Frittella, A. Kurz, A. Palmigiano, V. Sikimić:

- case studies: **DEL**, **PDL**.

'Good' Proof Systems: Desiderata

- An **independent** account of dynamic logics:
  - Proof-theoretic semantic approach
- Intuitive, **user-friendly** rules.
- **Good performances**:
  - soundness & completeness,
  - cut-elimination & sub-formula property,
  - decidability.
- A **modular** account of dynamic logics:
  - charting the space of DLs by adding/subtracting rules,
  - transfer of results with minimal changes.

WE1: Dinamic Epistemic Logic

Interaction axioms: classical case

$$\langle\alpha\rangle p \leftrightarrow Pre(\alpha) \wedge p$$
$$\langle\alpha\rangle(A \vee B) \leftrightarrow \langle\alpha\rangle A \vee \langle\alpha\rangle B$$
$$\langle\alpha\rangle\neg A \leftrightarrow Pre(\alpha) \wedge \neg\langle\alpha\rangle A$$

$$\langle\alpha\rangle\langle\mathsf{a}\rangle A \leftrightarrow Pre(\alpha) \wedge \bigvee\{\langle\mathsf{a}\rangle\langle\beta\rangle A \mid \alpha\mathsf{a}\beta\}$$

Intuitionistic case: more axioms, e.g.

$$\langle\mathsf{a}\rangle(A \rightarrow B) \rightarrow ([\mathsf{a}]A \rightarrow \langle\mathsf{a}\rangle B)$$
$$[\alpha]\langle\mathsf{a}\rangle A \leftrightarrow Pre(\alpha) \rightarrow \bigvee\{\langle\mathsf{a}\rangle\langle\beta\rangle A \mid \alpha\mathsf{a}\beta\}$$

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$$[\alpha]\langle \mathsf{a}\rangle A \leftrightarrow Pre(\alpha) \rightarrow \bigvee\{\langle \mathsf{a}\rangle\langle\beta\rangle A \mid \alpha \mathsf{a} \beta\}$$

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WE2: Propositional Dynamic Logic

Box axioms

$$[\alpha]\,(A \to B) \to ([\alpha]\,A \to [\alpha]\,B)$$

$$[\alpha \cup \beta]\,A \leftrightarrow [\alpha]\,A \wedge [\beta]\,A$$

$$[\alpha\,;\beta]\,A \leftrightarrow [\alpha][\beta]\,A$$

$$[?A]\,B \leftrightarrow (A \to B)$$

$$[\alpha]\,(A \wedge B) \leftrightarrow [\alpha]\,A \wedge [\alpha]\,B$$

$$[\alpha^*]\,A \leftrightarrow A \wedge [\alpha]\,[\alpha^*]\,A$$

$$A \wedge [\alpha^*]\,(A \to [\alpha]\,A) \to [\alpha^*]\,A$$

WE2: Propositional Dynamic Logic

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$$[\alpha\,;\,\beta]\,A \leftrightarrow [\alpha][\beta]\,A$$

$$[?A]\,B \leftrightarrow (A \to B)$$

$$[\alpha]\,(A \wedge B) \leftrightarrow [\alpha]\,A \wedge [\alpha]\,B$$

$$[\alpha^*]\,A \leftrightarrow A \wedge [\alpha]\,[\alpha^*]\,A$$

$$A \wedge [\alpha^*]\,(A \to [\alpha]\,A) \to [\alpha^*]\,A$$

Diagnosis & cure

$$\textit{swap-out}_L \; \frac{\left(\color{red}{Pre(\alpha)} \,;\, \{\mathtt{a}\}\{\beta\}\, X \vdash Y \mid \alpha\mathtt{a}\beta \right)}{\color{red}{Pre(\alpha)} \,;\, \{\alpha\}\{\mathtt{a}\} X \vdash \mathbf{;}\left(Y \mid \color{red}{\alpha\mathtt{a}\beta} \right)}$$

× Diagnosis:

- lack of expressivity
- lack of modularity

√ Cure:

- add structural connectives
- add types

Display Calculi

- Natural generalization of sequent calculi;
- sequents $X \vdash Y$, where X and Y are STRUCTURES:
  - built by **structural connectives** (generalising the role of the comma)
  - **binary trees** (not sequences)
- DISPLAY PROPERTY: **adjunction** at the structural level
- Canonical proof of cut elimination

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DC: Two levels of language

Structural connectives are ***contextual*** (as the Gentzen's comma) :

I		$;$		$>$	
$\top$	$\bot$	$\wedge$	$\vee$	$\succ\!\!-$	$\rightarrow$

$\{\mathtt{a}\}$		$\overbrace{\underbrace{\mathtt{a}}}$		$\{\alpha\}$		$\overbrace{\underbrace{\alpha}}$	
$\langle\mathtt{a}\rangle$	$[\mathtt{a}]$	$\widehat{\widecheck{\mathtt{a}}}$	$\overline{\underline{\mathtt{a}}}$	$\langle\alpha\rangle$	$[\alpha]$	$\widehat{\widecheck{\alpha}}$	$\overline{\underline{\alpha}}$

DC: Three groups of rules, 1/3

(1) Display Postulates

$$\frac{X\,;Y \vdash Z}{Y \vdash X > Z} \qquad \frac{Z \vdash Y\,;X}{Y > Z \vdash X}$$

Theorem (Display Property)

Each substructure in a display-sequent is 'displayable' in precedent or, exclusively, succedent position.

$$\cfrac{\cfrac{\cfrac{Y \vdash {\color{blue}X} > Z}{X\,;Y \vdash Z}}{Y\,;X \vdash Z}}{{\color{blue}X} \vdash Y > Z}$$

DC: Three groups of rules, 2/3

(2) Operational Rules

$$\frac{X \vdash A \qquad B \vdash Y}{A \rightarrow B \vdash X > Y} \qquad \frac{Z \vdash A > B}{Z \vdash A \rightarrow B}$$

$$\frac{\{\alpha\} A \vdash X}{\langle\alpha\rangle A \vdash X} \qquad \frac{X \vdash A}{\{\alpha\} X \vdash \langle\alpha\rangle A}$$

$$\frac{A \vdash X}{[\alpha] A \vdash \{\alpha\} X} \qquad \frac{X \vdash \{\alpha\} A}{X \vdash [\alpha] A}$$

DC: Three groups of rules, 3/3

(3) Structural Rules

$$Gri_L \; \begin{array}{c} X > (Y; Z) \vdash W \\ \hline\hline (X > Y); Z \vdash W \end{array} \qquad \begin{array}{c} W \vdash X > (Y; Z) \\ \hline\hline W \vdash (X > Y); Z \end{array} \; Gri_R$$

$$FS^1_L \; \frac{\{a\}X > \{a\}Y \vdash Z}{\{a\}(X > Y) \vdash Z} \qquad \frac{Z \vdash \overbrace{\underbrace{\mathtt{a}}}\, Y > \overbrace{\underbrace{\mathtt{a}}}\, X}{Z \vdash \overbrace{\underbrace{\mathtt{a}}}\,(Y > X)} \; FS^2_R$$

The excluded middle is derivable using *Grishin's rules* :

$$
\begin{array}{cl}
A \vdash A & \\ \hline
A ; \mathrm{I} \vdash A & \\ \hline
A ; \mathrm{I} \vdash \bot ; A & \\ \hline
\mathrm{I} \vdash A > (\bot ; A) & \\ \hline
\mathrm{I} \vdash (A > \bot) ; A & \mathit{Gri} \\ \hline
\mathrm{I} \vdash A ; (A > \bot) & \\ \hline
A > \mathrm{I} \vdash A > \bot & \\ \hline
A > \mathrm{I} \vdash A \rightarrow \bot & \\ \hline
A > \mathrm{I} \vdash \neg A & \\ \hline
\mathrm{I} \vdash A ; \neg A & \\ \hline
\mathrm{I} \vdash A \vee \neg A &
\end{array}
$$

The distinctive axioms for intuitionistic modal logic are derivable using *Fischer Servi's rules* :

$$
\begin{prooftree}
\AxiomC{$A \vdash A$}
\UnaryInfC{$[\mathtt{a}]A \vdash \{\mathtt{a}\}A$}
\UnaryInfC{$\overbrace{\underbrace{\mathtt{a}}}[\mathtt{a}]A \vdash A$}
\AxiomC{$B \vdash B$}
\UnaryInfC{$\{\mathtt{a}\}B \vdash \langle\mathtt{a}\rangle B$}
\UnaryInfC{$B \vdash \overbrace{\underbrace{\mathtt{a}}}\langle\mathtt{a}\rangle B$}
\BinaryInfC{$A \rightarrow B \vdash \overbrace{\underbrace{\mathtt{a}}}[\mathtt{a}]A > \overbrace{\underbrace{\mathtt{a}}}\langle\mathtt{a}\rangle B$}
\RightLabel{FS}
\UnaryInfC{$A \rightarrow B \vdash \overbrace{\underbrace{\mathtt{a}}}([\mathtt{a}]A > \langle\mathtt{a}\rangle B)$}
\UnaryInfC{$\{\mathtt{a}\}(A \rightarrow B) \vdash [\mathtt{a}]A > \langle\mathtt{a}\rangle B$}
\UnaryInfC{$\{\mathtt{a}\}(A \rightarrow B) \vdash [\mathtt{a}]A \rightarrow \langle\mathtt{a}\rangle B$}
\UnaryInfC{$\langle\mathtt{a}\rangle(A \rightarrow B) \vdash [\mathtt{a}]A \rightarrow \langle\mathtt{a}\rangle B$}
\UnaryInfC{$\langle\mathtt{a}\rangle(A \rightarrow B) \,;\mathrm{I} \vdash [\mathtt{a}]A \rightarrow \langle\mathtt{a}\rangle B$}
\UnaryInfC{$\mathrm{I} \vdash \langle\mathtt{a}\rangle(A \rightarrow B) > ([\mathtt{a}]A \rightarrow \langle\mathtt{a}\rangle B)$}
\UnaryInfC{$\mathrm{I} \vdash \langle\mathtt{a}\rangle(A \rightarrow B) \rightarrow ([\mathtt{a}]A \rightarrow \langle\mathtt{a}\rangle B)$}
\end{prooftree}
$$

But ...

Rules such as the following are problematic:

$$\text{swap-out}_L \; \frac{\left(Pre(\alpha) \,;\, \{\mathtt{a}\}\{\beta\}\, X \vdash Y \mid \alpha\mathtt{a}\beta \right)}{Pre(\alpha) \,;\, \{\alpha\}\{\mathtt{a}\}X \vdash \mathbf{;}\left(Y \mid \alpha\mathtt{a}\beta \right)}$$

$$\frac{\left(Y \vdash Pre(\alpha) > \{\mathtt{a}\}\{\beta\}\, X \mid \alpha\mathtt{a}\beta \right)}{\mathbf{;}\left(Y \mid \alpha\mathtt{a}\beta \right) \vdash Pre(\alpha) > \{\alpha\}\{\mathtt{a}\}X} \; \text{swap-out}_R$$

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The multi-type approach

- Ag Act Fnc Fm;
  - no ancillary symbols; all types are **first-class citizens**;
- Additional expressivity:
  - operational connectives **merging different types** (à la Abramsky, Vickers):

$$
\begin{array}{lclr}
\vartriangle_1, \blacktriangle_1 & : & \mathsf{Act} \times \mathsf{Fm} \rightarrow \mathsf{Fm} & \langle \alpha \rangle A \rightsquigarrow \alpha \vartriangle_1 A \\
\vartriangle_2, \blacktriangle_2 & : & \mathsf{Ag} \times \mathsf{Fm} \rightarrow \mathsf{Fm} & \langle \mathsf{a} \rangle A \rightsquigarrow \mathsf{a} \vartriangle_2 A \\
\vartriangle_3, \blacktriangle_3 & : & \mathsf{Ag} \times \mathsf{Fnc} \rightarrow \mathsf{Act} & \mathsf{a} \vartriangle_3 \alpha
\end{array}
$$

- Modularity: by adding or subtracting types (games, strategies, coalitions) the whole space of dynamic logics can be charted.

  For $1 \leq i \leq 3$,

$\vartriangle_i$		$\blacktriangle_i$		$\vartriangleright_i$		$\blacktriangleright_i$	
$\vartriangle_i$		$\blacktriangle_i$			$-\!\!\vartriangleright_i$		$-\!\!\blacktriangleright_i$

A glimpse at rules for DEL

Single-type, first version: rules with side conditions & labels;

$$\text{swap-out}_L \frac{\left(Pre(\alpha)\,;\{\mathtt{a}\}\{\beta\}\,X \vdash Y \mid \alpha\mathtt{a}\beta\right)}{Pre(\alpha)\,;\{\alpha\}\{\mathtt{a}\}X \vdash \boldsymbol{;}\left(Y \mid \alpha\mathtt{a}\beta\right)}$$

Single-type, emended: purely structural, but labels still there;

$$\text{swap-out}'_L \frac{\left(\Phi_\alpha\,;\{\mathtt{a}\}\{\beta\}\,X \vdash Y \mid \alpha\mathtt{a}\beta\right)}{\Phi_\alpha\,;\{\alpha\}\{\mathtt{a}\}X \vdash \boldsymbol{;}\left(Y \mid \alpha\mathtt{a}\beta\right)}$$

Multi-type: no side conditions and no labels.

+ (4) Interaction Rules

$$\text{swap-out}_L \frac{(\mathrm{a} \blacktriangle F) \blacktriangle (\mathrm{a} \blacktriangle X) \vdash Y}{\mathrm{a} \blacktriangle (F \blacktriangle X) \vdash Y}$$

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Let $\{\beta | \alpha \mathtt{a} \beta\} = \{\beta_1, \ldots, \beta_n\}$,

$$\dfrac{\dfrac{\dfrac{\dfrac{A \vdash A}{\{\beta_1\}A \vdash \langle\beta_1\rangle A}}{\{\mathtt{a}\}\{\beta_1\}A \vdash \langle\mathtt{a}\rangle\langle\beta_1\rangle A}}{Pre(\alpha)\,;\{\mathtt{a}\}\{\beta_1\}A \vdash \langle\mathtt{a}\rangle\langle\beta_1\rangle A} \qquad \begin{array}{c}\cdots\\ \cdots\\ \cdots\\ \cdots\end{array} \qquad \dfrac{\dfrac{\dfrac{A \vdash A}{\{\beta_n\}A \vdash \langle\beta_n\rangle A}}{\{\mathtt{a}\}\{\beta_n\}A \vdash \langle\mathtt{a}\rangle\langle\beta_n\rangle A}}{Pre(\alpha)\,;\{\mathtt{a}\}\{\beta_n\}A \vdash \langle\mathtt{a}\rangle\langle\beta_n\rangle A}}{Pre(\alpha)\,;\{\alpha\}\{\mathtt{a}\}A \vdash \boldsymbol{;}\left(\langle\mathtt{a}\rangle\langle\beta_i\rangle A\right)}$$

$$\vdots$$

$$\dfrac{\dfrac{\dfrac{Pre(\alpha)\,;\{\alpha\}\langle\mathtt{a}\rangle A \vdash \bigvee\left(\langle\mathtt{a}\rangle\langle\beta_i\rangle A\right)}{Pre(\alpha)\,;[\alpha]\langle\mathtt{a}\rangle A \vdash \bigvee\left(\langle\mathtt{a}\rangle\langle\beta_i\rangle A\right)}\,rev}{[\alpha]\langle\mathtt{a}\rangle A \vdash Pre(\alpha) > \bigvee\left(\langle\mathtt{a}\rangle\langle\beta_i\rangle A\right)}}{[\alpha]\langle\mathtt{a}\rangle A \vdash Pre(\alpha) \rightarrow \bigvee\left(\langle\mathtt{a}\rangle\langle\beta_i\rangle A\right)}$$

$$
\dfrac{\dfrac{\dfrac{A \vdash A}{\{\beta_1\}A \vdash \langle\beta_1\rangle A}}{\{\mathsf{a}\}\{\beta_1\}A \vdash \langle\mathsf{a}\rangle\langle\beta_1\rangle A} \quad \cdots \quad \dfrac{\dfrac{A \vdash A}{\{\beta_n\}A \vdash \langle\beta_n\rangle A}}{\{\mathsf{a}\}\{\beta_n\}A \vdash \langle\mathsf{a}\rangle\langle\beta_n\rangle A}}{\{\alpha\}\{\mathsf{a}\}A \vdash \mathbf{;}\left(\langle\mathsf{a}\rangle\langle\beta_i\rangle A\right)}\ \textit{s-out}
$$

$$
\vdots
$$

$$
\dfrac{[\alpha]\langle\mathsf{a}\rangle A \vdash \{\alpha\}\, \alpha \,\bigvee \left(\langle\mathsf{a}\rangle\langle\beta_i\rangle A\right)}{[\alpha]\langle\mathsf{a}\rangle A \vdash \Phi_\alpha > \bigvee \left(\langle\mathsf{a}\rangle\langle\beta_i\rangle A\right)}\ \textit{com}
$$

$$
\vdots
$$

$$
[\alpha]\langle\mathsf{a}\rangle A \vdash 1_\alpha \rightarrow \bigvee \left(\langle\mathsf{a}\rangle\langle\beta_i\rangle A\right)
$$

$$
\begin{prooftree}
\AxiomC{$\mathsf{a} \vdash \mathsf{a}$}
\AxiomC{$\mathsf{a} \vdash \mathsf{a}$}
\AxiomC{$\alpha \vdash \alpha$}
\BinaryInfC{$\mathsf{a} \blacktriangle \alpha \vdash \mathsf{a} \blacktriangle \alpha$}
\AxiomC{$A \vdash A$}
\BinaryInfC{$(\mathsf{a} \blacktriangle \alpha) \triangle A \vdash (\mathsf{a} \blacktriangle \alpha) \vartriangle A$}
\BinaryInfC{$\mathsf{a} \triangle ((\mathsf{a} \blacktriangle \alpha) \triangle A) \vdash \mathsf{a} \vartriangle ((\mathsf{a} \blacktriangle \alpha) \vartriangle A)$}
\UnaryInfC{$(\mathsf{a} \blacktriangle \alpha) \triangle A \vdash \mathsf{a} \blacktriangleright (\mathsf{a} \vartriangle ((\mathsf{a} \blacktriangle \alpha) \vartriangle A))$}
\UnaryInfC{$\color{blue} A \vdash (\mathsf{a} \blacktriangle \alpha) \blacktriangleright (\mathsf{a} \blacktriangleright (\mathsf{a} \vartriangle ((\mathsf{a} \blacktriangle \alpha) \vartriangle A)))$}
\LeftLabel{\color{blue}\textit{s-out}}
\UnaryInfC{$\color{blue} A \vdash \mathsf{a} \blacktriangleright (\alpha \blacktriangleright (\mathsf{a} \vartriangle ((\mathsf{a} \blacktriangle \alpha) \vartriangle A)))$}
\UnaryInfC{$\vdots$}
\noLine
\UnaryInfC{$\alpha \triangle (\alpha \blacktriangle (\alpha \mathrel{-\!\!\triangleright} (\mathsf{a} \vartriangle A)) \,;\, \mathrm{I}) \vdash \mathsf{a} \vartriangle ((\mathsf{a} \blacktriangle \alpha) \vartriangle A)$}
\RightLabel{\color{blue}\textit{conj}}
\UnaryInfC{$(\alpha \mathrel{-\!\!\triangleright} (\mathsf{a} \vartriangle A)) \,;\, (\alpha \triangle \mathrm{I}) \vdash \mathsf{a} \vartriangle ((\mathsf{a} \blacktriangle \alpha) \vartriangle A)$}
\UnaryInfC{$\vdots$}
\noLine
\UnaryInfC{$\color{blue} \alpha \mathrel{-\!\!\triangleright} (\mathsf{a} \vartriangle A) \vdash \alpha \vartriangle \top \to \mathsf{a} \vartriangle ((\mathsf{a} \blacktriangle \alpha) \vartriangle A)$}
\UnaryInfC{$\color{blue} [\alpha]\langle \mathsf{a} \rangle A \vdash \mathit{Pre}(\alpha) \to \bigvee \left(\langle \mathsf{a} \rangle \langle \beta_i \rangle A \right)$}
\end{prooftree}
$$

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Cut rules in Gentzen's Calculi

$$\frac{\Gamma \vdash C, \Delta \qquad \Gamma', C \vdash \Delta'}{\Gamma', \Gamma \vdash \Delta', \Delta} \qquad \frac{\Gamma \vdash C, \Delta \qquad \Gamma, C \vdash \Delta}{\Gamma \vdash \Delta} \qquad \frac{\Gamma \vdash C \qquad \Gamma, C \vdash \Delta}{\Gamma \vdash \Delta}$$

$$\frac{\Gamma \vdash C \qquad \Gamma', C \vdash \Delta}{\Gamma', \Gamma \vdash \Delta} \qquad \frac{\Gamma \vdash C, \Delta \qquad C \vdash \Delta'}{\Gamma \vdash \Delta', \Delta} \qquad \frac{\Gamma \vdash C \qquad C \vdash \Delta}{\Gamma \vdash \Delta}$$

Theorem (Cut-elimination)

If $\Gamma \vdash \Delta$ is derivable, then it is derivable without Cut.

- ✓ A Cut is an intermediate step in a deduction. 'Eliminating the cut' generates a ***new and lemma-free proof***, which employs ***syntactic material coming from the end-sequent***.
- × Typically, syntactic proofs of Cut-elimination are ***non-modular***, i.e. if a new rule is added, it must be proved from scratch.

Canonical cut elimination, 1/4

Definition

A sequent $x \vdash y$ is ***type-uniform*** if x and y are of the same type.

A (cut) rule is ***strongly type-uniform*** if its premises and conclusion are of the same type.

Theorem (Canonical cut elimination)

If a calculus satisfies the properties below, then it enjoys cut elimination.

Canonical cut elimination, 2/4

1. structures can disappear, formulas are **forever**;
2. **tree-traceable** formula-occurrences, via suitably defined congruence:
   - same shape, same position, same type, non-proliferation;
3. **principal** = **displayed** (Exception: principal fma's in axioms)
   - Generaliz.: axioms are **closed** under display rules (when applicable);
4. rules are closed under **uniform substitution** of congruent parameters within each type;
5. **reduction strategy** exists when cut formulas are both principal.

Specific to multi-type setting:

6. **type-uniformity** of derivable sequents;
7. **strongly uniform cuts** in each/some type(s).

Canonical cut elimination, 3/4

Two main cases + subcases.

(a) **Both cut formulas are principal.** by 5. (cut is either eliminated or “broken down” into cuts of lower rank).

(b) **At least one cut formula is parametric.**

- Subcase (b1): a_u principal in axiom. Then,

$$
\dfrac{\begin{array}{c} \vdots\, \pi_1 \\ x \vdash a \end{array} \qquad \begin{array}{c} (x' \vdash y')[a_u^{pre}, a_{suc}] \\ \vdots\, \pi_2 \\ a \vdash y \end{array}}{x \vdash y}
\quad \rightsquigarrow \quad
\begin{array}{c}
\dfrac{\begin{array}{c} \vdots\, \pi_1 \\ x \vdash a \end{array} \qquad a_u \vdash y''[a_{suc}]}{x \vdash y''[a_{suc}]} \\
\vdots\, \pi'' \\
(x' \vdash y')[x^{pre}, a_{suc}] \\
\vdots\, \pi_2[x/a_u] \\
x \vdash y
\end{array}
$$

Canonical cut elimination, 4/4

- Subcase (b2): a_u principal in other rule. Then, a_u is in display, and hence:

$$
\dfrac{\begin{array}{c}\vdots\ \pi_1\\ x \vdash a\end{array} \qquad \begin{array}{c}\vdots\ \pi_2'\\ a_u \vdash y'\\ \vdots\ \pi_2\\ a \vdash y\end{array}}{x \vdash y}
\quad \rightsquigarrow \quad
\begin{array}{c}\dfrac{\begin{array}{c}\vdots\ \pi_1\\ x \vdash a\end{array} \qquad \begin{array}{c}\vdots\ \pi_2'\\ a_u \vdash y'\end{array}}{x \vdash y'}\\ \vdots\ \pi_2[x/a]\\ x \vdash y\end{array}
$$

Canonical cut elimination, 4/4

- Subcase (b2): a_u principal in other rule. Then, a_u is in display, and hence:

$$
\dfrac{\dfrac{\vdots\ \pi_1}{x \vdash a} \qquad \dfrac{\dfrac{\vdots\ \pi'_2}{a_u \vdash y'}}{\dfrac{\vdots\ \pi_2}{a \vdash y}}}{x \vdash y}
\quad \rightsquigarrow \quad
\dfrac{\dfrac{\dfrac{\vdots\ \pi_1}{x \vdash a} \qquad \dfrac{\vdots\ \pi'_2}{a_u \vdash y'}}{x \vdash y'}}{\dfrac{\vdots\ \pi_2[x/a]}{x \vdash y}}
$$

- Subcase (b3): a_u parametric. Then:

$$
\dfrac{\dfrac{\vdots\ \pi_1}{x \vdash a} \qquad \dfrac{\dfrac{\vdots\ \pi'_2}{(x' \vdash y')[a_u]^{pre}}}{\dfrac{\vdots\ \pi_2}{a \vdash y}}}{x \vdash y}
\quad \rightsquigarrow \quad
\dfrac{\dfrac{\vdots\ \pi'_2}{(x' \vdash y')[x/a_u^{pre}]}}{\dfrac{\vdots\ \pi_2[x/a_u^{pre}]}{x \vdash y}}
$$

Conclusions & Future works

To summarise

✓ Display Calculi $\rightsquigarrow$ Multi-type DC $\rightsquigarrow$ Display-type Calculi

Working papers

- Linear Logic: avoiding *closed-enough rules*
- PDL: avoiding *omega-rule*
- Game Logic: avoiding *non-contextual rules*
- Predicative Logic: *quantification*?
- Display-type Sequent Calculus for Monotonic Modal Logic

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