Unifying Functional Interpretations

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Oberwolfach, 21 March 2005





Outline

- Functional interpretations
- Parametrised Formula Translation
- Parametrised Proof Translation





Functional Interpretation

A formula mapping

$$A \mapsto |A|_{V}^{X}$$

- x marks the witness required by A (i.e. $\forall y |A|_y^t$)
- y marks the refutation of a given witness for Å.
- A proof mapping

$$\vdash A \mapsto \vdash B$$
,

for some B such that $B \vdash \exists x \forall y |A|_y^x$.



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for some B such that $B \vdash \exists x \forall y |A|_y^x$.

E.g. $B \equiv \forall y |A|_y^t$, for some term t.





- 1958. Gödel's Dialectica interpretation
 - Relative consistency of PA
- 1959. Kreisel's modified realizability
 - Independence results, unwinding proofs
- 1974. Diller-Nahm variant of Dialectica interpretation
 - Solve contraction problem
- 1978. Stein's family of functional interpretations
 - Relate modified realizability and Diller-Nahm's
- 1992. Kohlenbach's "monotone" interpretations
 - Proof mining





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Goal

Relation between Dialectica interpretation and modified realizability.



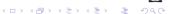


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Relation between Dialectica interpretation and modified realizability.

Common framework for all functional interpretations via a **parametrised functional interpretation**.





Logical Framework

$A \wedge B$	conjunction	
$A \vee B$	disjunction	
<i>A ∀ B</i>	classical disjunction	
$\neg A$	negation	
<i>¬A</i> ∃ <i>x A</i>	negation existential quantifier	





Logical Framework

$A \wedge B$	conjunction	
$A \vee B$	disjunction	
A⊽B	classical disjunction	
$\neg A$	negation	
∃ <i>x A</i>	existential quantifier	
∀ <i>x A</i>	universal quantifier	

$$A \rightarrow B :\equiv \neg A \triangledown B$$





$$A \vdash A$$
 (Axiom)

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$$\frac{\Gamma \vdash A(z), \Delta}{\Gamma \vdash \forall z A(z), \Delta} \forall I \qquad \frac{\Gamma \vdash \forall z A(z), \Delta}{\Gamma \vdash A(s), \Delta} \forall E$$

$$\frac{\Gamma \vdash A(s), \Delta}{\Gamma \vdash \exists z A(z), \Delta} \exists I \qquad \frac{\Gamma, A(z) \vdash \Delta}{\Gamma, \exists z A(z) \vdash \Delta} \exists E$$

$$\frac{\Gamma_0 \vdash A \quad \Gamma_1, A \vdash \Delta}{\Gamma_0, \Gamma_1 \vdash \Delta} \text{ (cut)} \qquad \frac{\Gamma \vdash \Delta}{\Gamma' \vdash \Delta'} \text{ (per)}$$

$$\frac{\Gamma, A, A \vdash B}{\Gamma, A \vdash B} \text{ (con)} \qquad \frac{\Gamma \vdash \Delta}{\Gamma, \Gamma_0 \vdash \Delta, \Delta_0} \text{ (wkn)}$$

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Logical Framework

- ullet Language of finite types ${\mathcal T}$
 - $\mathbb{N} \in \mathcal{T}$
 - $\rho, \sigma \in \mathcal{T} \quad \Rightarrow \quad \rho \to \sigma \in \mathcal{T}$
- Variable and quantifiers for each finite type $\rho \in \mathcal{T}$
- Combinatorial completeness
- ullet Equality for $\mathbb N$





Heyting Arithmetic HA^{ω}

- Universal axioms for 0 and S
- Gödel's primitive recursion
- Induction rule

$$\frac{\vdash A(0) \qquad A(n) \vdash A(n+1)}{\vdash A(n)} \text{ (IND)}$$





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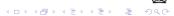
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Assume we have already defined $|A|_{v}^{x}$ and $|B|_{w}^{v}$, we define

$$|A \wedge B|_{y,w}^{x,v}$$
 := $|A|_y^x \wedge |B|_w^v$





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Notation:
$$|A|_{V}^{X} \vee_{n} |B|_{W}^{V} := (n = 0 \wedge |A|_{V}^{X}) \vee (n \neq 0 \wedge |B|_{W}^{V})$$





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Notation:
$$|A|_{V}^{X} \vee_{n} |B|_{W}^{V} :\equiv (n = 0 \wedge |A|_{V}^{X}) \vee (n \neq 0 \wedge |B|_{W}^{V})$$





Interpretation of negation

Assume A has interpretation $|A|_{\nu}^{x}$

Gödel's Dialectica interpretation
 Functionals producing counter-examples for A, i.e.

$$|\neg A|_X^f :\equiv \neg |A|_{f_X}^X$$

Modified Realizability

 $\neg A$ does not ask for witnesses, i.e.

$$|\neg A|_x :\equiv \neg \forall y |A|_y^x$$





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In General

Functionals producing "bound" on counter-examples

$$|\neg A|_X^f :\equiv \neg \forall y \sqsubset f_X |A|_Y^X \equiv \neg \forall y (y \sqsubset f_X \to |A|_Y^X)$$





 $|A \wedge B|_{y,w}^{x,v} :\equiv |A|_{y}^{x} \wedge |B|_{w}^{v}$ $|A \vee B|_{y,w}^{x,v,n} :\equiv |A|_{y}^{x} \vee_{n} |B|_{w}^{v}$ $|A \nabla B|_{y,w}^{f,g} :\equiv |A|_{y}^{gw} \nabla |B|_{w}^{fy}$ $|\forall z A(z)|_{y,z}^{f} :\equiv |A(z)|_{y}^{fz}$ $|\exists z A(z)|_{y}^{x,z} :\equiv |A(z)|_{y}^{x}$ $|\neg A|_{x}^{f} :\equiv \neg \forall y \sqsubseteq fx |A|_{y}^{x}$

$$|A \wedge B|_{y,w}^{x,v} :\equiv |A|_y^x \wedge |B|_w^v$$

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$$|A \nabla B|_{y,w}^{f,g} :\equiv |A|_y^{gw} \nabla |B|_w^{fy}$$

$$|\forall z A(z)|_{y,z}^f :\equiv |A(z)|_y^{fz}$$

$$|\exists z A(z)|_y^{x,z} :\equiv |A(z)|_y^x$$

$$|\neg A|_x^f :\equiv \neg \forall y \sqsubseteq fx |A|_y^x$$

$$|y < x| \equiv (y < x)$$

$$|\forall y(y < x)|_{y} \equiv (y < x)$$

$$|\exists x \forall y(y < x)|_{y}^{x} \equiv (y < x)$$

$$|\neg \exists x \forall y(y < x)|_{y}^{f} \equiv \neg \forall y \sqsubseteq fx(y < x)$$

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$$|\neg \exists x \forall y(y < x)|_{x}^{f} \equiv \neg \forall y \sqsubset fx(y < x) \begin{cases} \neg (fx < x) \\ \neg \forall y(y < x) \end{cases}$$

Choice for abbreviation	Provable in HA^ω
Modified realizability $\forall x \sqsubset^{r} aA(x) :\equiv \forall xA(x)$	$\exists x \forall y A _y^x \leftrightarrow \exists x (x \text{ mr } A)$
Dialectica interpretation $\forall x \sqsubset^g aA(x) :\equiv A(a)$	$\exists x \forall y A _y^x \leftrightarrow \exists x \forall y A_D(x,y)$
Diller-Nahm variant $\forall x \sqsubset^{\wedge} aA(x) :\equiv \forall x \in aA(x)$	$\exists x \forall y A _y^x \leftrightarrow \exists x \forall y A_{\wedge}(x,y)$
Stein's family of interp. $\forall x^{\tau} \sqsubset^{n} aA(x) :\equiv \begin{cases} \forall x \in aA(x) \\ \forall xA(x) \end{cases}$	$\exists x \forall y A _y^x \leftrightarrow \exists x \forall y A_n(x,y)$



Conditions on $\forall x \sqsubset aA(x)$

For all formulas A there are a_1, a_2, a_3 such that

(A1)
$$\forall y' \sqsubset a_1 y |A|_{y'}^x \vdash |A|_y^x$$

(A2)
$$\forall y' \sqsubset a_2 y_0 y_1 |A|_{y'}^x \vdash \forall y' \sqsubset y_0 |A|_{y'}^x \land \forall y' \sqsubset y_1 |A|_{y'}^x$$

(A3)
$$\forall y' \sqsubset a_3 h z |A|_{y'}^x \vdash \forall y \sqsubset z \forall y' \sqsubset h y |A|_{y'}^x$$





Theorem (Over HA^{ω})

If conditions (A1), (A2), (A3) hold and

 $\vdash A$

then there is a sequence of terms t such that

$$\vdash \forall y |A|_y^t$$



(Standard) Proof Translation

Theorem (Over HA^{ω})

If conditions (A1), (A2), (A3) hold and

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Theorem (Over HA^{ω})

If conditions (A1), (A2), (A3) hold and

 $\Gamma \vdash A$

then there are sequences of terms t and q such that

$$\forall w \sqsubset q[v,y]|\Gamma|_w^v \vdash |A|_v^{t[v]}$$





Condition (A1) - Logical axioms

Any counter-example has a bound

$$\forall y' \sqsubset \mathsf{a_1} \, y \, |A|_{y'}^x \vdash |A|_y^x$$

$$A \vdash A$$
 (axiom)





Condition (A1) - Logical axioms

Any counter-example has a bound
 ∀y' □ a₁ y |A|^x_{v'} ⊢ |A|^x_v

$$A \vdash A$$
 (axiom)

$$\forall y' \sqsubset a_1 \ y \ |A|_{y'}^x \vdash |A|_y^x$$





Condition (A2) - Contraction

Joining two sets of counter-examples into one

$$\forall y' \sqsubset a_2 \, y_0 \, y_1 \, |A|_{y'}^x \vdash \forall y' \sqsubset y_0 \, |A|_{y'}^x \wedge \forall y' \sqsubset y_1 \, |A|_{y'}^x$$

$$\frac{A, A \vdash B}{A \vdash B}$$
 (con)





Condition (A2) - Contraction

Joining two sets of counter-examples into one

$$\forall y' \sqsubset a_2 y_0 y_1 |A|_{y'}^x \vdash \forall y' \sqsubset y_0 |A|_{y'}^x \land \forall y' \sqsubset y_1 |A|_{y'}^x$$

$$\frac{A, A \vdash B}{A \vdash B} \text{ (con)}$$

$$\downarrow \qquad \qquad \qquad \downarrow$$

$$\frac{\forall y' \sqsubseteq q_0 |A|_{y'}^{x_0}, \forall y' \sqsubseteq q_1 |A|_{y'}^{x_1} \vdash |B|_w^t}{\forall y' \sqsubseteq a_2 \ q_0 \ q_1 |A|_{y'}^{x} \vdash |B|_w^t} \text{ (A2)}$$





Condition (A3) - Cut

Bounded family of sets into a single set

$$\forall y' \sqsubset \mathsf{a}_3 \, \mathit{h} \, z \, |A|_{y'}^\mathsf{x} \vdash \forall y \sqsubset z \forall y' \sqsubset \mathit{h} y \, |A|_{y'}^\mathsf{x}$$

$$\frac{\Gamma \vdash A \quad A \vdash B}{\Gamma \vdash B}$$
(cut)





Condition (A3) - Cut

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 (cut)

$$\Downarrow$$

$$\frac{\forall w \sqsubset q[y]|\Gamma|_{w}^{v} \vdash |A|_{y}^{t}}{\forall y \sqsubseteq r \forall w \sqsubseteq q[y]|\Gamma|_{w}^{v} \vdash \forall y \sqsubseteq r|A|_{y}^{t}} \frac{\forall y \sqsubseteq r|A|_{y}^{x} \vdash |B|_{z}^{s}}{\forall y \sqsubseteq r|A|_{y}^{t} \vdash |B|_{z}^{s}}}{\forall y \sqsubseteq r \forall w \sqsubseteq q[y]|\Gamma|_{w}^{v} \vdash |B|_{z}^{s}}$$
(cut)
$$\frac{\forall y \sqsubseteq r \forall w \sqsubseteq q[y]|\Gamma|_{w}^{v} \vdash |B|_{z}^{s}}{\forall y \sqsubseteq a_{3} \ q \ r \vdash |B|_{z}^{s}}$$
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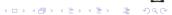


Parametrised Proof Translation

$$\vdash A \Rightarrow \vdash B$$

for some *B* such that $B \vdash \exists x \forall y | A|_y^x$





Parametrised Proof Translation

$$\vdash A \Rightarrow \vdash B$$

for some *B* such that $B \vdash \exists x \forall y |A|_{v}^{x}$

- $B \equiv \forall y |A|_{V}^{t}$, for some term t
- $B \equiv \exists x \leq^* t \forall y |A|_v^t$ (\leq^* is Howard/Bezem majorizability)
- $B \equiv \exists x \forall y |A|_v^x$





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- $B \equiv \exists x \leq^* t \forall y |A|_y^t \ (\leq^* \text{ is Howard/Bezem majorizability})$
- $\bullet \ B \equiv \exists x \forall y |A|_y^x$
- $\bullet \ B \equiv \exists x \prec t \forall y |A|_y^x$





Conditions on $\forall x \sqsubset aA(x)$ and $\exists x \prec aA(x)$

For all formulas A there are terms a_1^*, a_2^*, a_3^* such that

$$(\mathbf{A1})^* \vdash \exists \nu \prec \mathbf{a}_1^* \forall x, y (|\neg A|_x^{\nu y} \triangledown |A|_y^x)$$

$$(\mathbf{A2})^* \vdash \exists \chi \prec \mathbf{a}_2^* \forall x, y_0, y_1(|\neg A|_x^{\chi y_0 y_1} \triangledown \bigwedge_{i=0}^1 \forall y' \sqsubset y_i |A|_{y'}^x)$$

$$(\textbf{A3})^* \vdash \exists \xi \prec a_3^* \forall x, h, z (|\neg A|_x^{\xi hz} \triangledown \forall y \sqsubseteq z \forall y' \sqsubseteq hy |A|_{y'}^x)$$





Condition on $\exists x \prec aA(x)$

(E) For each formula A, closed term s and term t[z], if $\vdash \exists z \prec s \forall a, y | A|_y^{t[z]a}$ then there exists a closed term t^* such that $\vdash \exists F \prec t^* \forall a, y | A|_y^{Fa}$. We call t^* a \prec -majorizing term for t.



Parametrised Proof Translation

Theorem (Over HA^{ω})

If conditions (A)* and (E) hold, and

$$\Gamma \vdash A$$
,

then there are sequences of closed terms t, r such that

$$\vdash \exists f \prec t \exists g \prec r \forall v, y | \neg \Gamma \triangledown A|_{v,v}^{g,f}.$$



Summary

$$\forall x \sqsubset aA(x) := \begin{cases} A(a) \\ \forall x \in aA(x) \\ \forall x \in aA(x) \text{ or } \forall x A(x) \text{ (type of } x) \\ \forall x A(x) \end{cases}$$

$$\exists x \prec aA(x) :\equiv \begin{cases} A(a) \\ \exists x \leq^* aA(x) \\ \exists x A(x) \end{cases}$$





Work in Progress...

- Study relation between interpretations
 - different principles interpreted
 - concrete case studies
- Extend to classical context
- Interpretations not covered
 - Bounded functional interpretation [Ferreira, O. '04]
 - Bounded modified realizability [Ferreira, Nunes '05





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