

Resources, Concurrency and Local Reasoning

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Part I

An Illustrative Example

```

semaphore free:= 1; busy:= 0
⋮
⋮

P(free);                P(busy);

[10]:= m;           ||           n:= [10];

V(busy);              V(free);

⋮
⋮

```

Recall: $V(s)$ bumps s up by 1; $P(s)$ decrements by 1 if > 0 ,
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{emp}
:

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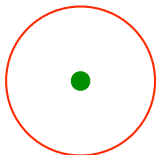
semaphore $free := 1; busy := 0$

\vdots		\vdots
$\{emp\}$		$\{emp\}$
$P(free);$		$P(busy);$
$\{10 \mapsto -\}$		$\{10 \mapsto -\}$
$[10] := m;$	\parallel	$n := [10];$
$\{10 \mapsto -\}$		$\{10 \mapsto -\}$
$V(busy);$		$V(free);$
$\{emp\}$		$\{emp\}$
\vdots		\vdots

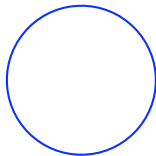
$$RI_{free} \stackrel{\text{def}}{=} (free = 0 \wedge emp) \vee (free = 1 \wedge 10 \mapsto -)$$

$$RI_{busy} \stackrel{\text{def}}{=} (busy = 0 \wedge emp) \vee (busy = 1 \wedge 10 \mapsto -)$$

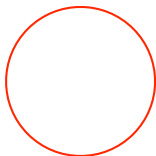
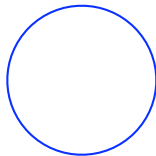
free semaphore



left process

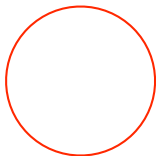


right process

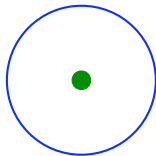


busy semaphore

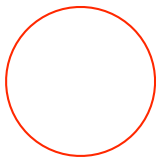
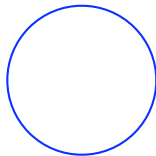
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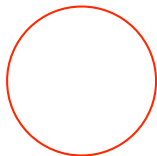


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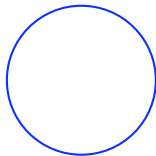


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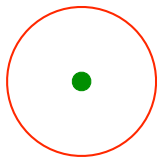
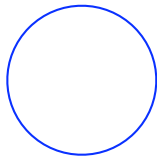
free semaphore



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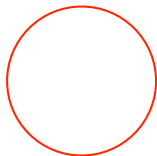


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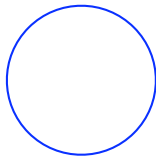


busy semaphore

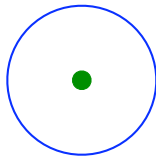
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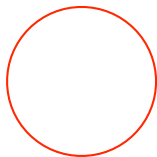
left process



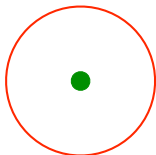
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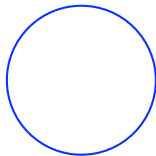
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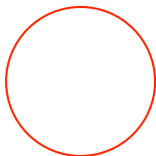
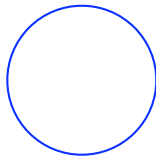
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busy semaphore

$\{10 \mapsto -\}$ semaphore $free := 1, busy := 0;$ $\{Rl_{free} * Rl_{busy} * emp * emp\}$ $\{emp\}$ while true do	$\{emp\}$ while true do
$produce\ m$	$P(busy);$
$P(free);$	$n := [10];$
$[10] := m;$	$V(free);$
$V(busy);$	$consume\ n$
$\{false\}$ $\{false\}$	$\{false\}$

$Rl_{free} \stackrel{\text{def}}{=} (free = 0 \wedge emp) \vee (free = 1 \wedge 10 \mapsto -)$

$Rl_{busy} \stackrel{\text{def}}{=} (busy = 0 \wedge emp) \vee (busy = 1 \wedge 10 \mapsto -)$

<pre> {10 ↦ -} semaphore free:= 1, busy:= 0; {Rl_{free}*Rl_{busy}*emp*emp} {emp} while true do {emp ∧ true} produce m {emp} P(free); {10 ↦ -} [10]:= m; {10 ↦ -} V(busy); {emp} {false} {false} </pre>		<pre> {emp} while true do {emp ∧ true} P(busy); {emp} n:= [10]; {10 ↦ -} V(free); {10 ↦ -} consume n {emp} {false} </pre>
--	--	---

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- ▶ Each assertion within a process talks about only its own state, not the state of the other process or even the semaphores.
- ▶ We don't maintain $0 \leq \textit{free} + \textit{busy} \leq 1$ as a global invariant.
- ▶ Semaphores are “logically attached” to resource. P and V are ownership transformers.

Part II

Context

Resource Logics

- ▶ Linear Logic (Girard, 1987)
- ▶ BI (O'Hearn, Pym, 1999)
- ▶ Separation Logic (Reynolds, O'Hearn,... 2000...)

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- ▶ BI (O'Hearn, Pym, 1999)
- ▶ Separation Logic (Reynolds, O'Hearn,... 2000...)

- ▶ Resource is a central concern in concurrent programming
- ▶ A natural question: Could resource logic be used?

Local Reasoning

- ▶ It should be possible for reasoning and specification to be confined to the resources that a program actually accesses (it's *resource footprint*).¹

¹O'Hearn, Reynolds, Yang. Local Reasoning about Programs that Alter Date Structures. CSL'01

Local Reasoning

- ▶ It should be possible for reasoning and specification to be confined to the resources that a program actually accesses (it's *resource footprint*).¹
- ▶ Separation Logic's *frame rule*

$$\frac{\{P\}C\{Q\}}{\{P*R\}C\{Q*R\}} \text{Modifies}(C) \cap \text{Free}(R) = \emptyset$$

¹O'Hearn, Reynolds, Yang. Local Reasoning about Programs that Alter Date Structures. CSL'01

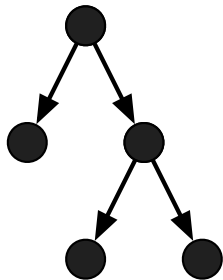
From

$\{ \text{tree}(z) \}$

$\text{disposetree}(z)$

$\{ \text{emp} \}$

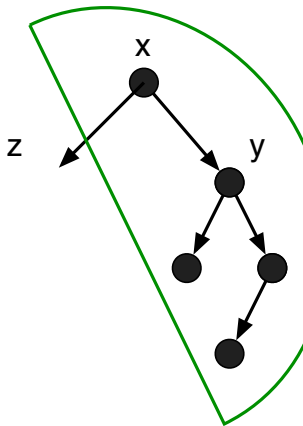
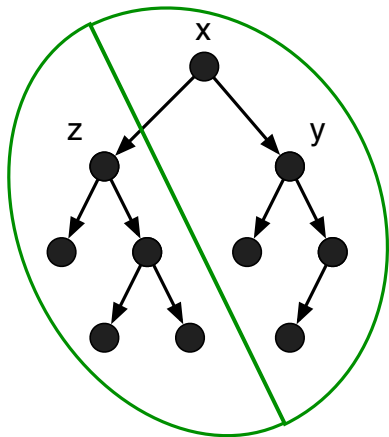
z



z

We Can Infer

$\{ \text{tree}(z) * \text{tree}(y) * x|y,z \}$ $\text{dispo} \text{setree}(z)$ $\{ \text{tree}(y) * x|y,z \}$



Parallel Disoposetree

- ▶ Disjoint Concurrency

$$\frac{\{P_1\}C_1\{Q_1\} \quad \{P_2\}C_2\{Q_2\}}{\{P_1*P_2\}C_1 \parallel C_2\{Q_1*Q_2\}}$$

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- ▶ Main Part of Proof

$$\frac{\{tree(y)*tree(z)\}}{\frac{\{tree(y)\} \quad \{tree(z)\}}{\text{disopsetree}(y) \parallel \text{disopsetree}(z)} \quad \{emp\} \quad \{emp\}} \quad \frac{\{emp*emp\}}{\{emp\}}$$

- ▶ Specification: $\{tree(x)\}\text{disopsetree}(x)\{emp\}$

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Reasoning about (shared-variable) Concurrency

- ▶ Owicki-Gries, Rely-Guarantee, Temporal Logic...

Reasoning about (shared-variable) Concurrency

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- ▶ Hoare, 1972: Towards a Theory of Parallel Programming
 - ▶ revolves around “spatial separation”
 - ▶ extremely modular
 - ▶ limited, but...

Cautious and Daring Concurrency

- ▶ A program is *cautious* if, whenever concurrent processes access the same piece of state, they do so only within the same grouping of mutual exclusion. Otherwise, the program is *daring*.

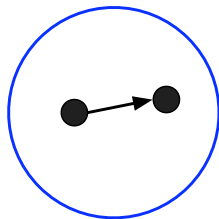
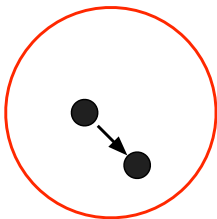
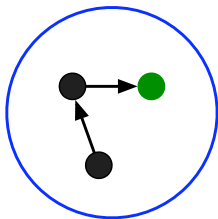
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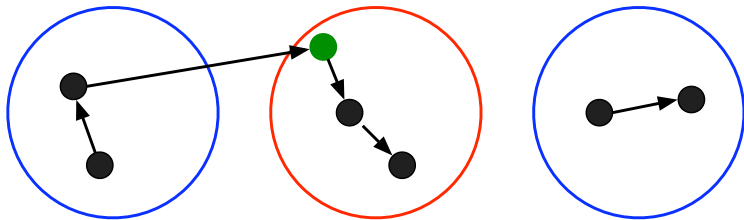
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- ▶ Daring programs are many
 - ▶ Memory Managers, Thread Pools, Connection Pools
 - ▶ Efficient Message Passing (copy avoiding)
 - ▶ Double-buffered I/O
 - ▶ Microkernel OS designs
 - ▶ Essentially all semaphore programs

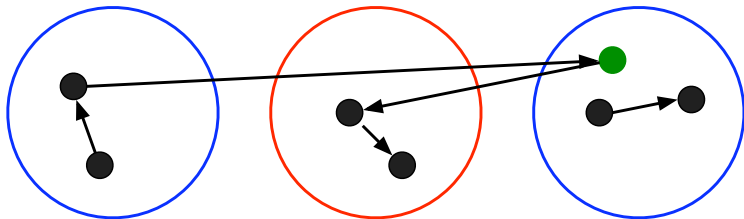
Dynamic Partitioning and Ownership Transfer



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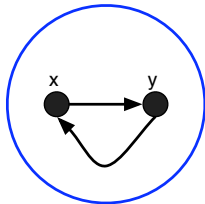
Part III

Separation Logic²

²Separation Logic: A logic for shared mutable data structure.

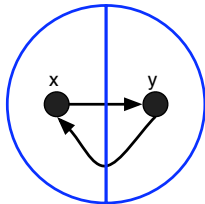
Separation Logic

$x \mapsto y * y \mapsto x$



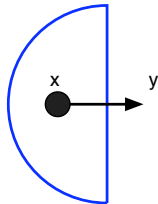
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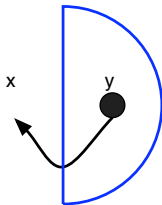
Separation Logic

$x \vdash y$



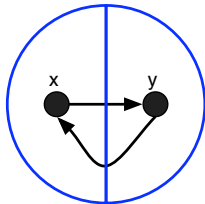
Separation Logic

$|y| \rightarrow x$



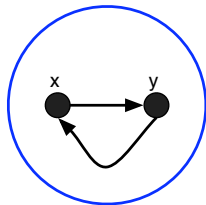
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Separation Logic

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$x=10$

10

42

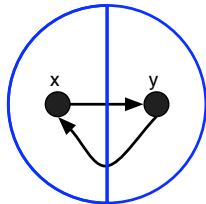
42

10

$y=42$

Separation Logic

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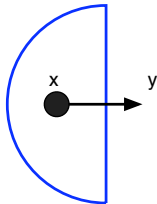
42

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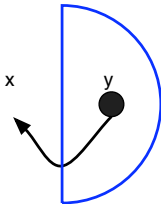
10

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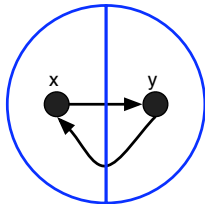
42

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Separation Logic

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The Assertion Language and Model

Assertions

$$P, Q, R ::= \text{emp} \mid E \mapsto F \mid P * Q \mid P \rightarrow Q \mid \forall x. P \mid \dots$$

Semantics: $s, h \models P$, where

$$s \in S = \text{Variables} \rightarrow \text{Val} \quad h \in H = \text{Addresses} \rightarrow \text{Val}$$

RAM Model: $\text{Addresses} = \text{Naturals}$, $\text{Val} = \text{Integers}$.

* splits h component, unioning heaps with disjoint domains. H has a partial commutative monoid structure.

The powerset of H is a boolean algebra, *and* a residuated monoid.
(A Boolean BI Algebra.)³

³Possible Worlds and Resources, the Semantics of BI.

Well specified programs don't go wrong

$$\{(x \mapsto -)*P\} [x]:=7 \{(x \mapsto 7)*P\}$$

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$\{\text{true}\} [x]:=7 \{\text{???\}$

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$\{P*(x \mapsto -)\} \text{dispose}(x) \{P\}$

Well specified programs don't go wrong

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$\{\text{true}\} \text{dispose}(x) \{\text{???\}$

$\{P\} x = \text{cons}(a, b) \{P*(x \mapsto a, b)\} \quad (x \notin \text{free}(P))$

Part IV

Concurrency Proof Rules⁴

⁴Following Hoare, and Owicki-Gries

Process Interaction: Conditional Critical Regions (Hoare, 72)

init;
resource r_1 (variable list), \dots , r_m (variable list)
 $C_1 \parallel \dots \parallel C_n$

with r when B do C endwith.

- if a variable belongs to a resource, it cannot appear in a parallel process except in a critical section for that resource
- if a variable is changed in one process, it cannot appear in another unless it belongs to a resource.

$$\frac{\{P\} \text{init}\{RI_{r_1} * \dots * RI_{r_m} * P'\} \quad \{P'\} C_1 \parallel \dots \parallel C_n \{Q\}}{\{P\}}$$

$\{P\}$

init;

resource r_1 (variable list), ..., r_m (variable list)

$C_1 \parallel \dots \parallel C_n$

$\{RI_{r_1} * \dots * RI_{r_m} * Q\}$

RI * ... * RI



P'



$$\frac{\{P\} \text{init} \{RI_{r_1} * \dots * RI_{r_m} * P'\} \quad \{P'\} C_1 \parallel \dots \parallel C_n \{Q\}}{\{P\}}$$

$\{P\}$

init;

resource r_1 (variable list), ..., r_m (variable list)

$C_1 \parallel \dots \parallel C_n$

$\{RI_{r_1} * \dots * RI_{r_m} * Q\}$

RI * ... * RI



P1 * ... * Pn



$$\frac{\{P_1\} C_1 \{Q_1\} \cdots \{P_n\} C_n \{Q_n\}}{\{P_1 * \cdots * P_n\} C_1 \parallel \cdots \parallel C_n \{Q_1 * \cdots * Q_n\}}$$

$RI * \cdots * RI$



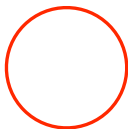
$P1 * \cdots * Pn$



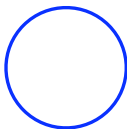
- no variable free in P_i or Q_i is changed in C_j when $j \neq i$.
- any modified variable free in RI_r must occur only within a critical region for r .

$$\frac{\{(P * RI_r) \wedge B\} C \{Q * RI_r\}}{\{P\} \text{ with } r \text{ when } B \text{ do } C \text{ endwith } \{Q\}}$$

RI



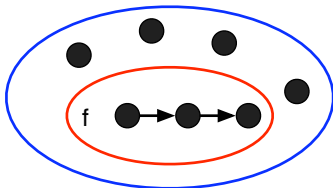
P



- No other process modifies variables in P or Q

Brookes's Semantic Analysis

- ▶ **Soundness Theorem.** The proof rules are sound, as long as resource invariants are *precise*.
- ▶ Reynolds counterexample shows unsoundness for imprecise invariants.
- ▶ A predicate is precise if, for every state, there is at most one substate that satisfies it.



Brookes's Semantic Analysis, II

- ▶ **Safety Theorem.** If $\{P\}C\{Q\}$ holds then there will be *no race condition* and *no memory fault* in any execution starting in a state satisfying P .
- ▶ **Slogan.** *Well specified processes mind their own business.*

Part V

Toy Resource Manager

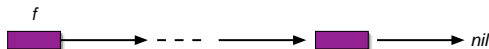
```
resource mm(f).
```

```
alloc(x, a, b)  $\stackrel{\text{def}}{=} \text{with mm when true do}$   
    if f = nil then x := cons(a, b)  
    else x := f; f := x.2; z.1 := a; x.2 := b  
endwith
```

```
dealloc(y)  $\stackrel{\text{def}}{=} \text{with mm when true do}$   
    y.2 := f;  
    f := y;  
endwith
```

Resource Invariant: *list*(*f*)

The manager owns the free list.



```
list x  $\stackrel{\text{def}}{\text{iff}}$  (x = nil  $\wedge$  emp)  $\vee$  ( $\exists y. x \mapsto -, y * \text{list } y$ )
```


- ▶ PROOF ESTABLISHING $\{y \mapsto -, -\} \text{dealloc}(y) \{\text{emp}\}$:

$\{(y \mapsto -, -) * \text{list } f\}$

$y.2 := f;$

$f := y;$

$\{\text{list } f\}$

$\{\text{emp} * \text{list } f\}$



$\{(P * RI_r) \wedge B\} C \{Q * RI_r\}$

- ▶ $\frac{\{(P * RI_r) \wedge B\} C \{Q * RI_r\}}{\{P\} \text{ with } r \text{ when } B \text{ do } C \text{ endwith } \{Q\}}$

- ▶ Similarly, $\{\text{emp}\} \text{alloc}(x, a, b) \{x \mapsto a, b\}$.

A Safe Example

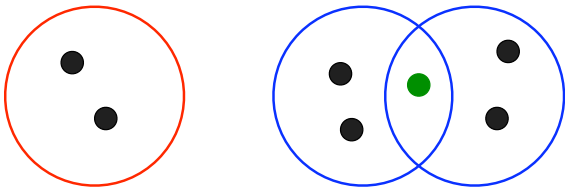
```
⋮
{x ↦ -, -}
x.1 := 42;
{x ↦ -, -}
dealloc(x);
{emp}
⋮

||

⋮
{emp}
alloc(y, a, b);
{y ↦ a, b}
y.1 := 7;
{y ↦ 7, b}
⋮
```

An Unsafe Example

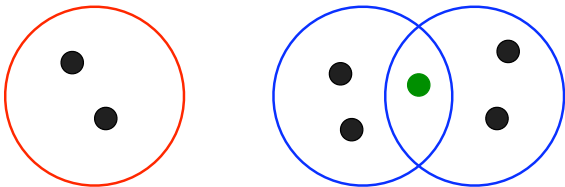
```
⋮  
{x ↦ -, -}  
x.1 := 42;    ||  
{x ↦ -, -}  
dealloc(x);  
{emp}  
  
⋮  
{emp}  
alloc(y, a, b);  
{y ↦ a, b}  
y.2 := 17;  
{y ↦ a, y}  
  
⋮  
⋮
```



An Unsafe Example

```
⋮  
{x ↦ -, -}  
x.1 := 42;      ||  
{x ↦ -, -}  
dealloc(x);  
{emp}  
x.2 := x  
{???
```

```
⋮  
{emp}  
alloc(y, a, b);  
{y ↦ a, b}  
y.2 := 17;  
{y ↦ a, y}  
⋮  
⋮
```



Part VI

Modularity

Pointer-transferring Buffer

full := false;

resource *buf*(*c*, *full*)

$\text{put}(x) \stackrel{\text{def}}{=} \text{with } \textit{buf} \text{ when } \neg \textit{full} \text{ do}$
 c := *x*; *full* := true
 endwith

$\text{get}(y) \stackrel{\text{def}}{=} \text{with } \textit{buf} \text{ when } \textit{full} \text{ do}$
 y := *c*; *full* := false
 $\text{endwith};$

Pointer-transferring Buffer

$full := false;$

resource $buf(c, full)$

$put(x) \stackrel{\text{def}}{=} \text{with } buf \text{ when } \neg full \text{ do}$
 $c := x; full := true$
 endwith

$get(y) \stackrel{\text{def}}{=} \text{with } buf \text{ when } full \text{ do}$
 $y := c; full := false$
 $\text{endwith};$

$\{x \mapsto -, -\} put(x) \{emp\} \quad \{emp\} get(y) \{y \mapsto -, -\}$

$RI_{buf} \stackrel{\text{def}}{=} (full \wedge c \mapsto -, -) \vee (\neg full \wedge emp)$

Code Snippet

```

      {emp*emp}
:
{emp}
x:= cons(a, b)   ||
{x ↦ -, -}
put(x);
{emp}
      {emp*emp}
      {emp}
:
{emp}
get(y);
{y ↦ -, -}
use(y);
{y ↦ -, -}
dispose(y);
{emp}

```

$$RI_{buf} \stackrel{\text{def}}{=} (full \wedge c \mapsto -, -) \vee (\neg full \wedge emp)$$

Code Snippet

{emp*emp}

:	:
{emp}	{emp}
$x := \text{cons}(a, b)$	\parallel $\text{get}(y);$
$\{x \mapsto -, -\}$	$\{y \mapsto -, -\}$
$\text{put}(x);$	$\text{use}(y);$
{emp}	$\{y \mapsto -, -\}$
$x.l := 44$	$\text{dispose}(y);$
{???	{emp}
{emp*emp}	
{emp}	

$RI_{buf} \stackrel{\text{def}}{=} (\text{full} \wedge c \mapsto -, -) \vee (\neg \text{full} \wedge \text{emp})$

Code Snippet

```

      {emp*emp}
:
{emp}
x:= cons(a, b)   ||
{x ↦ -, -}
put(x);
{emp}
      {emp*emp}
      {emp}
:
{emp}
get(y);
{y ↦ -, -}
use(y);
{y ↦ -, -}
dispose(y);
{emp}

```

$$RI_{buf} \stackrel{\text{def}}{=} (full \wedge c \mapsto -, -) \vee (\neg full \wedge emp)$$

Combining *buf* and *mm*

```

      {emp*emp}
:
{emp}
alloc(x, a, b); ||
{x ↦ -, -}
put(x);
{emp}

      {emp*emp}
:
{emp}
get(y);
{y ↦ -, -}
use(y);
{y ↦ -, -}
dealloc(y);
{emp}

      {emp*emp}
      {emp}

```

$RI_{buf} \stackrel{\text{def}}{=} (full \wedge c \mapsto -, -) \vee (\neg full \wedge emp)$

$RI_{mm} \stackrel{\text{def}}{=} list(f)$

$$\frac{\{P_1\} C_1 \{Q_1\} \cdots \{P_n\} C_n \{Q_n\}}{\{P_1 * \cdots * P_n\} C_1 \parallel \cdots \parallel C_n \{Q_1 * \cdots * Q_n\}}$$

$$\frac{\{(P * RI_r) \wedge B\} C \{Q * RI_r\}}{\{P\} \text{ with } r \text{ when } B \text{ do } C \text{ endwith } \{Q\}}$$

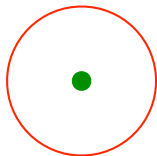
RI * ... * RI



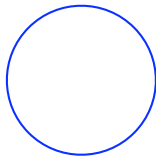
P1 * ... * Pn



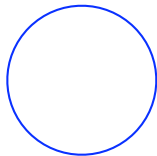
memory manager



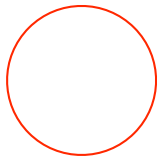
left process



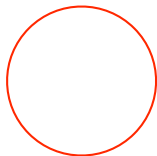
right process



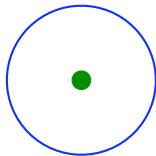
buffer



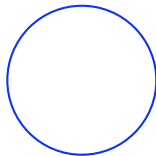
memory manager



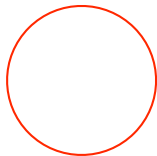
left process



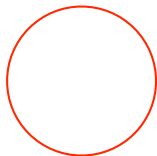
right process



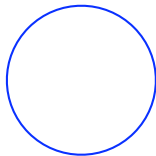
buffer



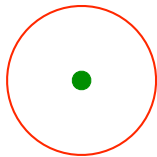
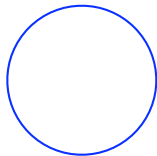
memory manager



left process

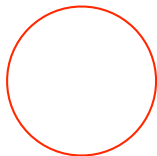


right process

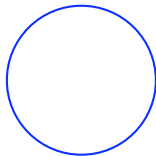


buffer

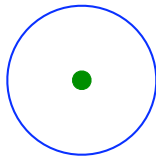
memory manager



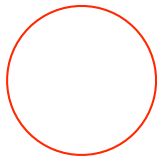
left process



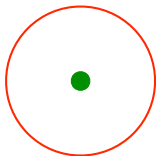
right process



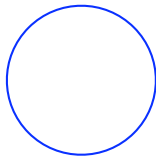
buffer



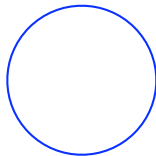
memory manager



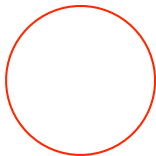
left process



right process



buffer



Part VII

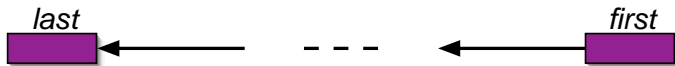
Counting Semaphore Example⁵

⁵From: Cooperating Sequential Processes, Dijkstra, 1968

```
begin integer number of queuing portions;
      number of queuing portions := 0;
      parbegin
        producer:begin
          again 1: produce the next portion;
                  add portion to buffer;
                  V(number of queuing positions);
                  goto again 1;
        end;
        consumer: begin
          again 2: P(number of queuing positions);
                  take portion from buffer;
                  process portion taken;
                  goto again 2;
        end;
      parend;
end;
```

Add and Take: No Mutexes Needed

We use a non-empty linked list.



The *last* element is a dummy node.

Producer puts message in dummy, then creates new dummy.

Consumer takes *first* element of queue.

Concurrent access is possible.

Relevant Parts of Code

```
semaphore number:= 0;

:
produce(m)
last.1:= m;
last.2:= cons(-, -);    ||
last:= last.2
V(number);
:

:
P(number);
n:= first.1; t:= first
first:= first.2
dispose(t);
consume(n);
:
```

Auxiliary Variables

- ▶ To describe ownership we need auxiliary variables l and f .
- ▶ The semaphore will own a linked list of size `number` from f to l (but not including l).
- ▶ We insert auxiliary manipulations into CCRs simulating the semaphores.

$P'(\text{number}) \stackrel{\text{def}}{=} \text{with number when number} > 0 \text{ do } f := \text{first.2}; \text{number} --$

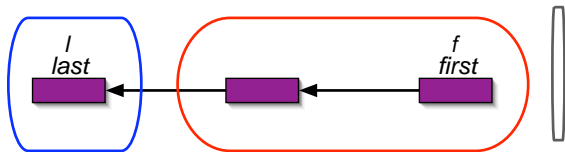
$V'(\text{number}) \stackrel{\text{def}}{=} \text{with number when true do } l := \text{last}; \text{number} ++$

Relevant Parts of Code

```
semaphore number:= 0;

:
produce(m)
last.1:= m;
last.2:= cons(-, -);    ||
last:= last.2
V'(number);
:

:
P'(number);
n:= first.1; t:= first
first:= first.2
dispose(t);
consume(n);
:
```



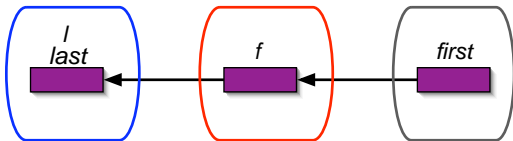
produce(m) || P(number)

producer

semaphore

consumer

produce(m) || P(number)



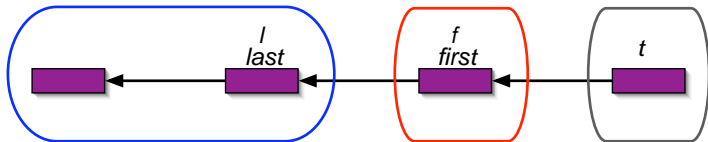
last.1:=m
last.2:=cons(-,-) || *n:=first.1; t:=first*
first:=first.2

producer

semaphore

consumer

$last.1 := m$
 $last.2 := cons(-, -) \parallel$
 $n := first.1; t := first$
 $first := first.2$



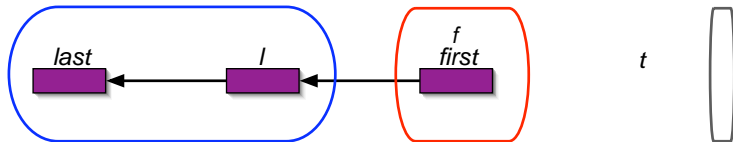
$last := last.2 \parallel$ $dispose(t)$

producer

semaphore

consumer

$last := last.2 \quad || \quad dispose(t)$



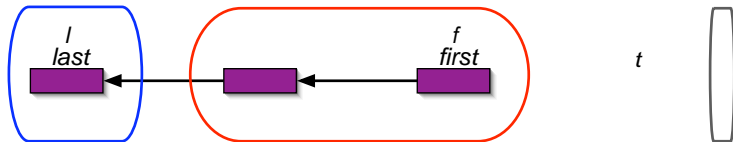
$V'(number) \quad || \quad consume(m)$

producer

semaphore

consumer

$V(\text{number}) \parallel \text{consume}(m)$



producer

semaphore

consumer

No Race

{emp}

semaphore number:= 0;

while true do

produce(*m*)

last.1:= *m*;

last.2:= cons(-, -); ||

last := *last*.2

V(number);

while true do

P(number);

n:= *first*.1; *t*:= *first*

first:= *first*.2

dispose(*t*);

consume(*n*);

{false}

Future Directions

- ▶ True concurrency model (for proven programs only!)
- ▶ Passivity (shared read access)
- ▶ Rely/guarantee formalism
- ▶ Temporal formalism
- ▶ Process calculi (CSP, pi-calculus)
- ▶ Mechanized proof
- ▶ Model checking, abstract interpretation.