

Permission-based separation logic for message-passing concurrency

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[
joint work with
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Obvious starting point

O'Hearn and Reynolds on “Resources, Concurrency and Local Reasoning”

Promotes the idea of transfer of ‘ownership’ of resources between threads.
Logic rules tailored to this notion of sharing. Some very neat ideas here.

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$$\frac{\{(P * R) \wedge B\} C \{Q * R\}}{\{P\} \text{ with } r \text{ when } B \text{ do } C \text{ endwith } \{Q\}}$$

R is the “resource invariant” - all shared state expressed here.

Some example invariants

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In many cases, the intended concurrent control flow is clear and inferrable from the code without the need for such codification. e.g. Producer/Consumer problem.

This is partly a problem of the concurrency model used - no signalling!

Message-passing concurrency

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Idea : leverage such inference to isolate the reasoning about (transfer of) ownership of resources from reasoning about data flow.

Step One : Analyse the control flow

Step Two : Prove data manipulation is correct

Our target setting (for starters):

Simple value-passing calculus:

asynchronous messages

no shared state

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- functional relationship between inputs and outputs

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Driving example:

- parallel mergesort

Step 1: Control Flow

Determinism via resource access control

The main (only) source of non-determinism in message-passing systems is the race for synchronisation on a channel:

$$a?x.P \parallel a?y.Q \parallel a!v$$

Here there is a competition to receive a value on a given channel.

Similarly for

$$a?x.P \parallel a!v \parallel a!w$$

So we could easily delimit deterministic processes by using linear channels.

This would rule out a large class of interesting processes - e.g. Producer/Consumers, or parallel mergesort

An analysis of determinism

We check to see whether processes can be assigned linear permissions.

Permissions can be transferred implicitly through communication.

Exactly which permissions will be transferred during each communication is a parameter to the analysis :

Γ is a map from channels to sets of permissions - $c \mapsto \{a! , b? \}$

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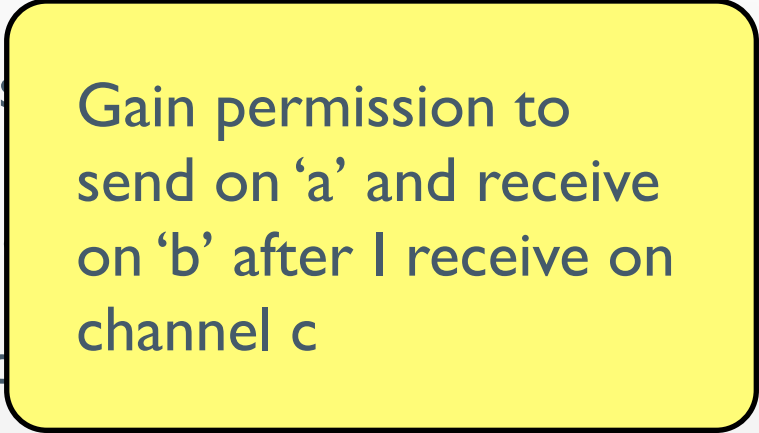
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Gain permission to
send on 'a' and receive
on 'b' after I receive on
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We think of Γ as representing the intended control flow of the process. It acts as an assertion regarding control which must be checked. This is in accord with O'Hearn's tenet that

“Ownership is in the eye of the asserter”

Our simple calculus

Grammar:

$$e ::= v \mid x \mid e + e \mid \dots \qquad v ::= 0 \mid 1 \mid 2$$

$$\begin{aligned} P, Q & ::= a!e \mid a?x.P \\ & \mid \text{if } e \leq e' \text{ then } P \text{ else } Q \\ & \mid \text{rec } X.P \mid X \\ & \mid \text{nil} \mid P \parallel Q \mid (\text{new } a)P \end{aligned}$$

With obvious reduction rules including:

$$\frac{e \Downarrow v}{a!e \parallel a?x.P \rightarrow P[v/x]}$$

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Rule for par

$$\frac{\Gamma \vdash P : \epsilon \quad \Gamma \vdash Q : \epsilon'}{\Gamma \vdash P \parallel Q : \epsilon \oplus \epsilon'}$$

Example

$$Pr = \text{rec } X.(p?().s?().(c!\langle \rangle \parallel X))$$

$$Con = \text{rec } X.(c?().s?().(p!\langle \rangle \parallel X))$$

$$\Gamma =$$

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Properties of effect system

Proposition (subject reduction) :

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Proposition (confluence) :

$\Gamma \vdash P : \epsilon$ and $P \rightarrow P_1$ and $P \rightarrow P_2$ implies
 $P_1 \equiv P_2$ or $P_1 \rightarrow P_3$ and $P_2 \rightarrow P_3$ for some P_3

Step 2: Proof System

Logic

$$\phi, \varphi ::= \text{emp} \mid a\langle v \rangle \mid \phi \triangleright \phi \mid \phi * \phi$$

The $a\langle v \rangle$ formulas represent outputs on the wire -
analogous to memory locations

Satisfaction is defined by:

$\Gamma \triangleright P \models \text{emp}$	iff	<i>always</i>
$\Gamma \triangleright P \models a\langle v \rangle$	iff	$P \equiv a!e \parallel P'$ where $e \Downarrow v$
$\Gamma \triangleright P \models \phi \triangleright \psi$	iff	$\forall Q. \Gamma \triangleright Q \models \phi, \Gamma \vdash Q \perp P$ implies $\Gamma \triangleright P \parallel Q \models \psi$
$\Gamma \triangleright P \models \phi_1 * \phi_2$	iff	$\exists P_1, P_2. P \equiv P_1 \parallel P_2$ and $\Gamma \triangleright P_1 \models \phi_1, \Gamma \triangleright P_2 \models \phi_2$
$\Gamma \triangleright P \Vdash \phi$	iff	$\exists Q. P \rightarrow^* Q, \Gamma \triangleright Q \models \phi$

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Separate permissions needed

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Sequents are interpreted as follows:

$$\Gamma \models \{\phi\}P\{\psi\} \stackrel{def}{=} \Gamma \vdash Q \perp P, \quad \Gamma \triangleright Q \models \phi$$

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The actual form of sequents is $\Gamma, b \vdash \{\phi\} P \{\phi\}$ with b boolean

Soundness

Theorem:

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This interpretation is not conventional and fairly weak - but recall that we are working with a class of confluent processes. One path is much the same as any other.

We make use of this fact to lift soundness to an 'all paths' interpretation.

Satisfaction preservation

We'd like to prove

Proposition:

$$\Gamma \triangleright P \models \phi, P \rightarrow^* Q \text{ implies } \Gamma \triangleright Q \models \phi$$

But unfortunately, this is not true for all formulas:

$$\phi = a\langle v \rangle * (a\langle v \rangle \triangleright b\langle v \rangle)$$

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However, if we rule out adjoints and check that P does not contain the 'co-permissions' specified by Φ , then the above proposition holds.

Example - parallel mergesort

$$\mathbf{srt}(X)_{i,j} \triangleq \begin{cases} X! & i = j \\ (\text{new } c_1, c_2) & i < j, \\ \left(\mathbf{srt}(c_1)_{i,m} \parallel \mathbf{srt}(c_2)_{m+1,j} \right) & m = \frac{j-i}{2} \\ \left(\parallel c_1?.c_2?.\mathbf{mrg}(X)_{i,m+1,j} \right) & \end{cases} \quad \mathbf{shft}(X)_{i,j} \triangleq \begin{cases} X! & i = j \\ a_{j-1}?x. & \\ \left(\mathbf{shft}(X)_{i,j-1} \right) & i < j \\ \left(\parallel a_j!x \right) & \end{cases}$$

$$\mathbf{mrg}(X)_{i,m,j} \triangleq \begin{cases} X! & i = m \vee m = j + 1 \\ a_i?x.a_m?y.\text{if } x \leq y \text{ then } (a_i!x \parallel a_m!y \parallel \mathbf{mrg}(X)_{i+1,m,j}) & \\ \text{else } \left(a_i!y. \parallel (\text{new } b) \left(\mathbf{shft}(b)_{i+1,m} \parallel \right. \right. & i < m \leq j \\ \left. \left. b?. \left(\begin{array}{l} a_{i+1}!x \\ \parallel \mathbf{mrg}(X)_{i+1,m+1,j} \end{array} \right) \right) \right) & \end{cases}$$

The arrays are represented using a collection of outputs:

$$a_1!v_1 \parallel a_2!v_2 \parallel \cdots \parallel a_n!v_n$$

Example specified

$$\begin{aligned}
 A_i^j \langle \vec{v}_i^j \rangle &\triangleq \begin{cases} \mathbf{emp} & i > j \\ a_i \langle v_i \rangle * A_{i+1}^j \langle \vec{v}_{i+1}^j \rangle & i \leq j \end{cases} & \vec{u}_i^j - v &\stackrel{\text{def}}{=} \begin{cases} \vec{u}_i^{m-1} \vec{u}_{m+1}^j & \text{if } \vec{u}_i^j = \vec{u}_i^{m-1} v \vec{u}_{m+1}^j \\ \text{undefined} & \text{otherwise} \end{cases} \\
 v \leq \vec{u}_i^j &\stackrel{\text{def}}{=} \forall i \leq k \leq j. v \leq u_k & v \in \vec{u}_i^j &\stackrel{\text{def}}{=} \exists i \leq m \leq j. \vec{u}_i^j = \vec{u}_i^{m-1} v \vec{u}_{m+1}^j \\
 \vec{v}_i^j \doteq \vec{u}_i^j &\stackrel{\text{def}}{=} v_i \in \vec{u}_i^j \wedge \vec{v}_{i+1}^j \doteq (\vec{u}_i^j - v) & \mathbf{srt}(\vec{v}_i^j) &\stackrel{\text{def}}{=} v_i \leq \vec{v}_{i+1}^j \wedge \mathbf{srt}(\vec{v}_{i+1}^j)
 \end{aligned}$$

We can derive the sequent by building a proof tree by induction over the indices:

$$\{A_i^j \langle \vec{v}_i^j \rangle\} \quad \mathbf{Srt}(b)_{i,j} \quad \{b \langle \rangle * A_i^j \langle \vec{u}_i^j \rangle\}$$

where

$$\underline{(\vec{v}_i^j \doteq \vec{w}_i^j) \wedge (\vec{w}_i^j \doteq \vec{u}_i^j) \wedge \mathbf{srt}(\vec{w}_i^{m-1}) \wedge \mathbf{srt}(\vec{w}_m^j) \wedge \mathbf{srt}(\vec{u}_i^j)}$$

Other related work

Hennessy Milner Logic for CCS (Stirling)

Hennessy Milner Logic for the Pi-calculus (Dam)

Modal logics for Typed Pi-calculus (Berger+)

Rely-Guarantee Separation Logic (Feng+, Vafeiadis+)

Permission accounting in Separation Logic (Bornat+)

Compositional proof systems - path blow-up

Need to study this further

Could be very useful model for extensions

Conclusions

We proposed a two-step analysis for local reasoning about message-passing concurrency:

- Effect analysis of deterministic flow

- Local Hoare logic for reasoning about data

Plenty more work to do to strengthen the class of programs we address

- controlled introduction of interference and racy-programs

- name-passing (a la pi-calculus)

- lots more examples

Thank You