A resource aware picalculus

(work in progress)

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TCD, Ireland

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Outline

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Introduction

The language Pi_{exc}

Examples

Amortised weighted bisimulations

Conclusions

Resource aware processes

- processes: P, Q, etc provide and use resources
- environments: Γ, Δ, etc: records resource costs/funding/usage
- reductions: $\Gamma \triangleright P \longrightarrow \Delta \triangleright Q$

Behavioural theory

$\Gamma \triangleright P \sqsubseteq_{cost} \Delta \triangleright Q$ means:

- $\Gamma \triangleright P$ and $\Delta \triangleright Q$ exhibit the same behaviour
- $\Delta \triangleright Q$ is at least as efficient as $\Gamma \triangleright P$ hopefully more

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- channels interpreted as resources
- resources require funding
- funding provided by owners
  - it costs to use resources
  - it costs to provide resources
  - provider may profit
- code runs under (financial) responsibility of owners
- interaction with resources paid for by code's owner

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M, N ::=

$[T]_{\circ}$	Owned code
$M \mid N$	Composition
$(\text{new } r : R)M$	Scoped resource
0	Identity

T, U ::=

$u?(x).T$	Provide resource u
$u!\langle v\rangle.T$	Use resource u
if $u = v$ then T else U	Matching
$(\text{new } r : \text{R})T$	Resource creation
$T \mid U$	Concurrency
rec x. T	Repetition
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fixed finite set of owners: Own

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Cost environment Γ

Keeps control of accounts

- funds available to each owner: $\Gamma^o : \text{Own} \longrightarrow K$
- cost of resources:
  - usage: $\Gamma^u : \text{Chan} \rightharpoonup K$
  - provision: $\Gamma^p : \text{Chan} \rightharpoonup K$
- record of expenditure: $\Gamma^{\text{rec}} \in K$

Resource charging: $\Gamma \xrightarrow{(\text{u},a,\text{p})} \Delta$

whenever

- owner u can afford to use a
- owner p can afford to provide a
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Reduction semantics

Novel rule:

$$\text{(R-COMM)}\ \frac{\Gamma \xrightarrow{(u,a,p)} \Delta}{\Gamma \triangleright [a!\langle v\rangle.Q]_u \mid [a?(x).P]_p \longrightarrow \Delta \triangleright [Q \mid P\{\!\!\{^v/_x\}\!\!\}]_p}$$

Standard rules:

- $$\text{(R-STRUCT)}\ \frac{M \equiv M',\ \Gamma \triangleright M \longrightarrow \Delta \triangleright N,\ N \equiv N'}{\Gamma \triangleright M' \longrightarrow \Delta \triangleright N'}$$
- $$\text{(R-SPLIT)}\ \Gamma \triangleright [M \mid N]_o \longrightarrow \Gamma \triangleright [M]_o \mid [N]_o$$
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Outline

Example: library

Two owners: pub and lib

$$\text{Sys} \Leftarrow (\ [\text{Reader}]_{\text{pub}}\ \mid\ [\text{Library} \mid \text{Store}]_{\text{lib}}\ )$$

$$\text{Reader} \Leftarrow \text{rec } R.\ \text{goLib?(name)}\ .(\text{new } r)\ \text{reqR!}\langle r, \text{name}\rangle. \\ r?(b)\ .\text{goHome!}\langle b\rangle.R$$

$$\text{Library} \Leftarrow \text{rec } L.\ \text{reqR?}(y, z)\ .\quad y!\langle \text{book}(z)\rangle.L \\ \oplus\ (\text{new } r)\ \text{reqS!}\langle r, z\rangle.r?(b)\ .y!\langle b\rangle.L$$

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Example: library

- Two owners: pub and lib
- Four resources: goLib, reqR, reqS, goHome

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Costing library facilities

Local v central provision:

Compare behaviour of

- $\Gamma_{\text{local}} \triangleright \text{Sys}$
- $\Gamma_{\text{central}} \triangleright \text{Sys}$

Costings:

- usage free $\Gamma^{u}_{\text{local}}(-) = \Gamma^{u}_{\text{central}}(-) = 0$
- provision: $\Gamma^{p}_{\text{local}}(-) = \Gamma^{p}_{\text{central}}(-)$ given by:

	local	central
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Publishing

Three owners:

- publisher p
- news provider n
- advertising agency a

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where

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Publishing with kickbacks

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Type K determines size of kickback

Comparision:

- $\Gamma \triangleright \mathrm{PA}$
- $\Gamma \triangleright \mathrm{PA}_K$

Γ controls costs involved records profit/loss for p

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Weighted labelled transition systems

$\langle S, \mathrm{Act}_\tau, W, \longrightarrow \rangle$

- S - states
- W - weights positive, negative integers
- $\longrightarrow \subseteq S \times \mathrm{Act}_\tau \times W \times S$

Weights:

- $s \xrightarrow{\mu}_w s'$: weight of action is w
- w: interpreted as cost
- w: or interpreted as profit/loss

Weights can vary:

- $s \xrightarrow{\tau}_{10} s_1$
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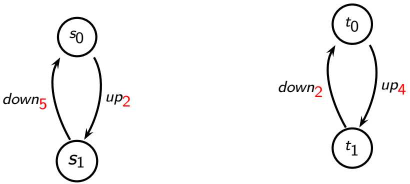
Amortisation Kumar and Kiehn



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- t_0 and s_o have same extensional behaviour
- t_0 is *better* than s_o t_0 is *lighter* than s_o
- extra weight of *up* more than compensated by the relative weight of *down*

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$s_o \sqsubseteq^{2}_{\mathrm{wgt}} t_0$ process t_0 given an initial boost of 2

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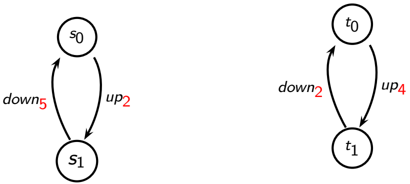

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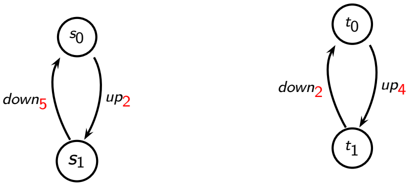

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- t_0 and s_o have same extensional behaviour
- t_0 is *better* than s_o t_0 is *lighter* than s_o
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Amortised weighted bisimulation preorder:

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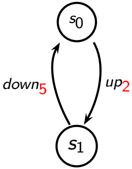

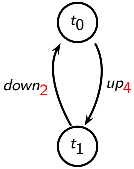

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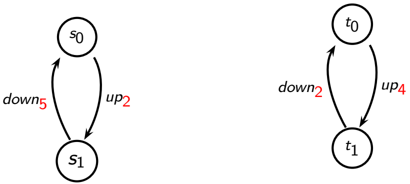

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Amortised weighted bisimulations

$s \sqsubseteq^k_{\mathrm{wgt}} t$, $k \in N$, is the *largest* family of relations satisfying:

- $s \sqsubseteq^k_{\mathrm{wgt}} t$, $s \overset{\mu}{\Longrightarrow}_w s'$ implies $t \overset{\mu}{\Longrightarrow}_v t'$ such that $s' \sqsubseteq^{(k+w-v)}_{\mathrm{wgt}} t'$
- $s \sqsubseteq^k_{\mathrm{wgt}} t$, $t \overset{\mu}{\Longrightarrow}_v t'$ implies . . . . . .

Properties

- $\sqsubseteq^k_{\mathrm{wgt}}$ is reflexive
- $s_1 \sqsubseteq^m_{\mathrm{wgt}} s_2$, $s_2 \sqsubseteq^n_{\mathrm{wgt}} s_3$ implies $s_1 \sqsubseteq^{(m+n)}_{\mathrm{wgt}} s_3$
- $s_1 \sqsubseteq^m_{\mathrm{wgt}} s_2$ implies $s_1 \sqsubseteq^n_{\mathrm{wgt}} s_2$ whenever $m \leq n$.

Application to Pi$_{exc}$

Step 1:

concrete LTS semantics

- $(\Gamma \triangleright M) \xrightarrow{\mu} (\Delta \triangleright N)$
- μ being τ, $(\mathrm{u}, \mathrm{input}, \mathrm{p})$ or $(\mathrm{u}, \mathrm{output}, \mathrm{p})$

Step 2:

abstract LTS semantics

- Add costs: $(\Gamma \triangleright M) \xrightarrow{\mu}_{w} (\Delta \triangleright N)$ where $w = (\Delta^{rec} - \Gamma^{rec})$
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Step 3:

More actions required

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Γ: owner o has lots
resource a costs 20
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Δ: owner o has only 10
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$$\Gamma \triangleright [a!]_o \sqsubseteq_{\mathrm{wgt}} \Delta \triangleright [a!]_o$$

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- Publishing:

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- owners who:
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  - may reap rewards
- a behavioural theory:
  - based on efficiency

Further work:

- Language design:
  - ownership types   localisation of owners . . .
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