

Bayesianism: Objections and Rebuttals

Norman Fenton and David Lagnado

n.fenton@qmul.ac.uk, d.lagnado@ucl.ac.uk

1 Introduction

While the laws of probability are rarely disputed, the question of how we should interpret probability judgments is less straightforward. Broadly, there are two ways to conceive of probability – either as an objective feature of the world, or as a subjective measure of our uncertainty¹. Both notions have their place in science, but it is the latter subjective notion (the Bayesian approach) that is crucial in legal reasoning. This is because we are usually concerned with events that must have either already happened or not happened, and which (from an objective viewpoint) have redundant probabilities of either one or zero. OJ Simpson either *did* or *did not* murder his ex-wife but, with the possible exception of OJ Simpson himself, nobody knows for certain which; thus, we aim to gather evidence to reduce our uncertainty about what actually happened. Probability theory is just as able to capture our uncertainty about whether an event did or did not happen in the past as it can capture uncertainty about an event that may or may not happen in the future. Unfortunately, a failure to understand this point about the nature of uncertainty lies at the heart of one of the most persistent objections among some members of the legal profession to the use of probability theory (especially Bayesian probability) –namely that “there is no such thing as probability”. This is normally expressed informally such as in the following (these are based on actual words we have heard used on many occasions by legal professionals):

“Look the guy either committed the crime or he didn’t. If he did it then the probability is one and if he didn’t then the probability is 0. There is nothing in between so, there is no such thing as probability other than 0 or 1.”

Indeed, it was essentially this argument used in a 2013 UK Appeal Court case ruling² (discussed in³) to reject the use of Bayes (in a civil dispute about the cause of a fire). Specifically, Point 37 of the ruling asserted (about the use of Bayes and probabilities):

I would reject that approach. It is not only over-formulaic but it is intrinsically unsound. The chances of something happening in the future may be expressed in terms of percentage. Epidemiological evidence may enable doctors to say that on average smokers increase their risk of lung cancer by X%. But you cannot properly say that there is a 25 per cent chance that something has happened ... Either it has or it has not.

The ‘no such thing as probability’ objection is also closely tied to the general objection to the notion of subjective probability on the grounds that it should not be used in legal contexts, because it depends on the vagaries of someone’s personal opinion. But probabilities are always

¹ Gillies 2000, 807.

² *Nulty & Ors v Milton Keynes Borough Council EWCA Civ 15 (24 January 2013)*.

³ Spiegelhalter 2013.

‘personal’ to some extent because different people will generally have different relevant information (i.e. evidence) available to them about an event (whether it already happened or may happen in the future). Your view of the probability that OJ Simpson murdered his ex-wife will be based on incomplete information about what happened and so will certainly be different to that of OJ Simpson himself who has the most information, or the lawyers and jurors on the case (who all have different levels of relevant information). But, crucially, subjective and personal does not mean arbitrary; the same rules of probability apply to both objective and subjective notions. For example, if you believe that the probability that someone committed a crime is p , then on pain of inconsistency you should believe that the probability that they did *not* commit the crime is $1-p$. Moreover, subjective probabilities should be updated in a rational way via Bayes’ rule. So, whatever your starting point (your prior beliefs), the Bayesian framework tells you how you should update these in light of new evidence (to give your posterior beliefs). It ensures that your posterior beliefs are rationally derived from your prior beliefs, in the same way that formal logic tells you what conclusions are deductively implied by your premises.

Both the ‘no such thing as probability’ objection and the objection to subjective probability have long been proven to be irrational because with either objection you will be open to a Dutch book (see for example ⁴): this means you can be made to lose money irrespective of the outcomes of the events bet upon; also, obeying the laws of probability minimizes your overall inaccuracy.

A number of other commonly repeated objections to probability and Bayes in the law were raised in the highly influential paper ⁵, which was written as a criticism of the prosecutor’s presentation in ⁶. Tribe’s objections especially pertinent for Bayes ⁷ were:

- That an accurate and/or non-overpowering prior cannot be devised.
- That in using statistical evidence to formulate priors jurors might use it twice in reaching a posterior
- That not all evidence can be considered or valued in probabilistic terms.
- That no probability value can ever be reconciled with “Beyond A Reasonable Doubt”
- That due to the complexity of cases and non-sequential nature of evidence presentation, any application of Bayes would be too cumbersome for a jury to use effectively and efficiently.
- That probabilistic reasoning is not compatible with the law, for policy reasons. In particular, that jurors are asked to formulate an opinion of the defendant’s guilt during the prosecutor’s case, which violates the obligation to keep an open mind until all evidence is in.

⁴ Fenton & Neil 2018; Pettigrew 2020.

⁵ Tribe 1971.

⁶ *People v Collins*, 438 P 2d 33 (68 Cal 2d 319 1968) .

⁷ Berger 2014; Fienberg & Finkelstein 1996.

However, most of these concerns have long been systematically demolished in ⁸ and more recently in ⁹. Although we revisit some of these objections in this chapter, our focus is on those objections that are most cited even once the Bayesian framework is accepted as a rational procedure for updating our subjective probabilities. Specifically, we will deal with three common objections, all of which can at least in part be addressed by using Bayesian networks ¹⁰: (1) that failing to constrain personal priors means no reasonable consensus posterior can ever be reached. (2) that Bayes - as encapsulated by the likelihood ratio – leads to multiple problems (including legal paradoxes); (3) that Bayes is too complex to be used in court or in legal arguments.

The chapter is structured as follows: we first review (section 2) the historical perspective for objections to Bayes. In Section 3 we provide necessary definitions of Bayes and the likelihood ratio. In Section 4 we address the above objections to Bayes, and explain why, despite the rebuttals to the objections, the use of Bayes has been extremely limited. Section 5 points the way forward.

2 Historical perspective

The reluctance to accept Bayes in the law is just the latest manifestation of a long-time historical reticence to accept any statistical analysis as valid evidence. Sadly, there is good reason for this reticence. When, in 1894, a statistical analysis was used in the Dreyfus case it turned out to be fundamentally flawed ¹¹. Not until 1968 was there another well-documented case, ¹² in which statistical analysis played a key role. In that case another flawed statistical argument further set back the cause of statistics in court. The Collins case was characterised by two errors:

- 1) It underestimated the probability that some evidence would be observed if the defendants were innocent by failing to consider dependence between components of the evidence; and
- 2) It implied that the low probability from the calculation in 1) was synonymous with innocence (the so-called ‘prosecutors’ fallacy).

Since then the same errors (either in combination or individually) have occurred in well reported cases such as R v Sally Clark ¹³, R v Barry George ¹⁴, Lucia de Berk ¹⁵. Although original ‘bad statistics’ used in each case (presented by forensic or medical expert witnesses without statistical training) was exposed through ‘good statistics’ on appeal, it is the ‘bad statistics’ which leaves an indelible stain. Yet, the role of legal professionals (who allow expert witnesses to commit the same well-known statistical errors repeatedly) is rarely questioned.

⁸ Edwards 1991; Koehler 1992.

⁹ Berger 2014; Tillers & Gottfried 2007.

¹⁰ Fenton & Neil 2018.

¹¹ Kaye 2007.

¹² *People v. Collins*, 438 P. 2d 33 (68 Cal. 2d 319 1968) (n 6).

¹³ Forrest 2003; Hill 2005.

¹⁴ Fenton et al 2014.

¹⁵ Meester et al 2007.

Hence, although the last 40 years has seen considerable growth in the use of statistics in legal proceedings, its use in the courtroom has been mostly restricted to a small class of cases where classical statistical methods of hypothesis testing using p-values and confidence intervals are used for probabilistic inference. Yet, even this type of statistical reasoning has severe limitations as discussed extensively in ¹⁶, including specifically in the context of legal and forensic evidence ¹⁷. In particular:

- The use of p-values can also lead to the prosecutor’s fallacy since a p-value (which says something about the probability of observing the evidence given a hypothesis) is often wrongly interpreted as being the same as the probability of the hypothesis given the evidence ¹⁸.
- Confidence intervals are almost invariably misinterpreted since their proper definition is both complex and counter-intuitive (indeed it is not properly understood even by many trained statisticians) ¹⁹.

The poor experience – and difficulties in interpretation - with classical statistics means that there is also strong resistance to any alternative approaches. This resistance extends to the Bayesian approach, despite the fact that it is especially well suited for a broad range of legal reasoning ²⁰.

Although the natural resistance within the legal profession to a new statistical approach is one reason why Bayes has, to date, made only minimal impact, it is certainly not the only reason. Many previous papers have discussed the social, legal and logical impediments to the use of Bayes in legal proceedings ²¹ and in more general policy decision making ²².

3 Basics of Bayes for legal reasoning

The following terminology and assumptions will be used:

- A ***hypothesis*** is a statement (typically Boolean) whose truth value we seek to determine, but is generally unknown - and which may never be known with certainty. Examples include:
 - “Defendant is innocent of the crime charged” (this is an example of an ***offense level hypothesis*** also called the ***ultimate hypothesis***, since in many criminal cases it is ultimately the only hypothesis we are really interested in)
 - “Defendant was the source of DNA found at the crime scene” (this is an example of what is often referred to as a ***source level hypothesis*** ²³)
- The ***alternative hypothesis*** is a statement which is the negation of a hypothesis.
- A piece of ***evidence*** is a statement that, if true, lends support to one or more hypotheses.

¹⁶ Royal Statistical Society 2015; Ziliak & McCloskey 2008.

¹⁷ Finkelstein 2009; Vosk & Emery 2014.

¹⁸ Gastwirth 2000.

¹⁹ Fenton & Neil 2018.

²⁰ Fienberg & Finkelstein 1996.

²¹ Faigman & Baglioni 1988; Fienberg 2011; Tillers & Green 1988; Tribe 1971.

²² Fienberg & Finkelstein 1996.

²³ Cook et al 1998.

The relationship between a hypothesis H and a piece of evidence E can be represented graphically as in the example in Figure 1 where we assume that:

- The evidence E is a DNA trace found at the scene of the crime (for simplicity we assume the crime was committed on an island with 10,000 people who therefore represent the entire set of possible suspects)
- The defendant was arrested and some of his DNA was sampled and analysed

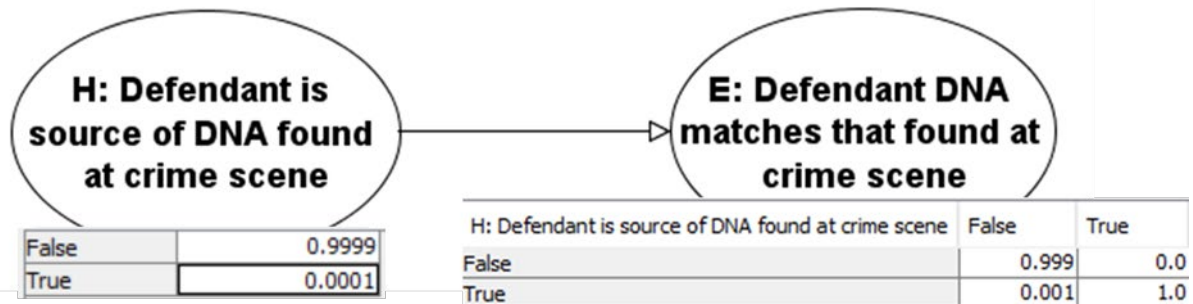


Figure 1 Causal view of evidence, with prior probabilities shown in tables. This is a very simple example of a Bayesian Network (BN)

The direction of the causal structure makes sense here because H being true (resp. false) can **cause** E to be true (resp. false), while E cannot ‘cause’ H . However, inference can go in **both** directions. If we observe E to be true (resp. false) then our belief in H being true (resp. false) increases. It is this latter type of inference that is central to all legal reasoning since, informally, lawyers and jurors normally use the following widely accepted procedure for reasoning about evidence:

- Start with some (unconditional) prior assumption about the ultimate hypothesis H (for example, the ‘innocent until proven guilty’ assumption equates to a belief that “the defendant is no more likely to be guilty than any other member of the relevant population”).
- Update our prior belief about H once we observe evidence E . This updating takes account of the *likelihood* of the evidence.

This informal reasoning is a perfect match for Bayesian inference where the prior assumption about H and the likelihood of the evidence E are captured formally by the probability tables shown in Figure 1. Specifically, these are the tables for the **prior probability** of H , written $P(H)$, and the conditional probability of E given H , which we write as $P(E | H)$. Bayes’ theorem provides the formula for updating our prior belief about H in the light of observing E to arrive at a **posterior belief about H** which we write as $P(H | E)$. In other words Bayes calculates $P(H | E)$ in terms of $P(H)$ and $P(E | H)$. Specifically:

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)} = \frac{P(E|H)P(H)}{P(E|H)P(H) + P(E|not H)P(not H)}$$

The first table (the probability table for H) captures our knowledge that the defendant is one of 10,000 people who could have been the source of the DNA. The second table (the probability table for $E | H$) captures the assumptions that:

- The probability of correctly matching a DNA trace is one (so there is no chance of a false negative DNA match). This probability $P(E | H)$ is called the **prosecution likelihood** for the evidence E .
- The probability of a match in a person who did not leave their DNA at the scene (the ‘random DNA match probability’) is 1 in 1,000. This probability $P(E | \text{not } H)$ is called the **defence likelihood** for the evidence E .

With these assumptions, it follows from Bayes’ theorem that, in our example, the posterior belief in H after observing the evidence E being true is about 9%, i.e. our belief in the defendant being the source of the DNA at the crime scene moves from a prior of 1 in a 10,000 to a posterior of 9%. Alternatively, our belief in the defendant not being the source of the DNA moves from a prior of 99.99% to a posterior of 91%.

Note that the posterior probability of the defendant not being the source of the DNA is very different from the random match probability of 1 in 1,000. The incorrect assumption that the two probabilities $P(\text{not } H | E)$ and $P(E | \text{not } H)$ are the same characterises what is known as the **prosecutor’s fallacy** (or error of transposed conditional). A prosecutor might state, for example, that “the probability the defendant was not the source of this evidence is one in a thousand”, when actually it is 91%. This fallacy of probabilistic reasoning has affected numerous cases²⁴, but can always be avoided by a basic understanding of Bayes’ Theorem. A closely related error of probabilistic reasoning is the **defendant’s fallacy**, whereby the defence argues that since $P(\text{not } H | E)$ is still low after taking into account the prior and the evidence, the evidence should be ignored.

Unfortunately, people without statistical training find Bayes’ theorem both difficult to understand and counter-intuitive²⁵. Legal professionals are also concerned that the use of Bayes requires us to assign prior probabilities. In fact, an equivalent formulation of Bayes (called the ‘odds’ version of Bayes) enables us to interpret the value of evidence E without having to consider the prior probability of H . Specifically, this version of Bayes’ tells us that the posterior odds of H are the prior odds of H times the **likelihood ratio**:

$$\frac{P(H|E)}{P(\text{not } H|E)} = \frac{P(H)}{P(\text{not } H)} \times \frac{P(E|H)}{P(E|\text{not } H)}$$

where the likelihood ratio (LR) is simply the prosecution likelihood of E divided by the defence likelihood of E , i.e.

$$\frac{P(E | H)}{P(E | \text{not } H)}$$

In the example in Figure 1 the prosecution likelihood for the DNA match evidence is 1, while the defence likelihood is 1/1,000. So the LR is 1,000. This means that, whatever the prior odds were in favour of the prosecution hypothesis, the posterior odds must increase by a factor of

²⁴ Balding & Donnelly 1994; Fenton & Neil 2011.

²⁵ Casscells & Graboys 1978; Cosmides & Tooby 1996.

1,000 as a result of seeing the evidence. In general, if the LR is bigger than 1 then the evidence results in an increased posterior probability of H (with higher values leading to the posterior probability getting closer to 1), while if it is less than 1 it results in a decreased posterior probability of H (and the closer it gets to zero the closer the posterior probability gets to zero). If the LR is equal to 1 then E offers no value since it leaves the posterior probability is unchanged.

The LR is therefore an important and meaningful measure of the probative value of evidence. In our example the fact that the DNA match evidence had a LR of 1000 meant the evidence was highly probative in favour of the prosecution. But as impressive as that sounds, whether or not it is sufficient to convince you of which hypothesis is true still depends entirely on the prior $P(H)$. If $P(H)$ is, say 0.5 (so the prior odds are evens 1:1), then a LR of 1000 results in posterior odds of 1000 to 1 in favour of H . That may be sufficient to convince a jury that H is true. But if $P(H)$ is very low - as in our example (9999 to 1 against) - then the same LR of 1000 results in posterior odds that still strongly favour the defence hypothesis by 10 to 1.

It is important to note that the properties of the LR (as a meaningful measure of probative value of evidence) depend both on Bayes' theorem and the assumption that the defence hypothesis is the negation of the prosecution hypothesis (i.e. the hypotheses must be mutually exclusive). Unfortunately, in practice there is much misunderstanding of LR lawyers, and even forensic experts and statisticians. An indication of the extent of the confusion can be found in one of the many responses by the latter community to the RvT judgement²⁶. Specifically, in the otherwise excellent position statement is the extraordinary point 9 that asserts:

“It is regrettable that the judgment confuses the Bayesian approach with the use of Bayes' Theorem. The Bayesian approach does not necessarily involve the use of Bayes' Theorem.”

By the “Bayesian approach” the authors are specifically referring to the use of the LR, thereby implying that the use of the LR is appropriate, while the use of Bayes' Theorem may not be.

Notwithstanding these misunderstandings (and other problems with the LR that we discuss in Section 4) the fact that it does determine the probative value of evidence and can be calculated without reference to the prior probability of H , has meant that it has become a potentially powerful application of Bayesian reasoning in the law. Indeed, its use is a core recommendation in Guidelines such as²⁷. Forcing expert witnesses to consider both the prosecution and defence likelihood of their evidence – instead of just one or the other – also avoids most common cases of the prosecutor's fallacy.

While Bayes' Theorem provides a natural match to intuitive legal reasoning in the case of a single hypothesis H and a single piece of evidence E , practical legal arguments normally involve multiple hypotheses and pieces of evidence with complex causal dependencies. For example, even the simplest case of DNA evidence strictly speaking involves three unknown hypotheses and two pieces of evidence with the causal links shown in Figure 2²⁸ once we take account of the possibility of different types of DNA collection and testing errors²⁹.

²⁶ Aitken and many other signatories, 2011 .

²⁷ ENFSI, ‘Guideline for Evaluative Reporting in Forensic Science’; Puch-Solis et al 2012.

²⁸ Dawid & Mortera 1998; Fenton, Neil & Hsu 2014.

²⁹ Koehler 1993; Thompson, Taroni & Aitken 2003.

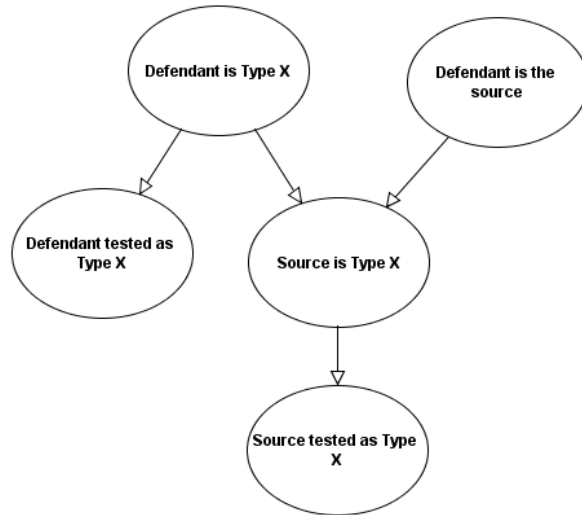


Figure 2 Bayesian network for DNA match evidence. Each node has states *true* or *false*

Moreover, there are further crucial hypotheses not shown in Figure 2 (a full version of the model is provided in ³⁰ supplementary material) such as: “Defendant was at the scene of the crime” and the ultimate hypothesis “Defendant committed the crime”. These are only omitted here because, whereas the law might accept a statistical or forensic expert reasoning probabilistically about the source of the forensic evidence, it is presupposed that any probabilistic reasoning about the ultimate hypothesis is the province of the trier of fact, i.e., the judge and/or the jury.

With or without the additional hypotheses, Figure 2 is an example of a Bayesian Network (BN). As in the simple case of Figure 1, to perform the correct Bayesian inference once we observe evidence we need to know the prior probabilities of the nodes without parents and the conditional prior probabilities of the nodes with parents. Assuming that it is possible to obtain suitable estimates of these prior probabilities, the bad news is that, even with a small number of nodes, the calculations necessary for performing correct probabilistic inference are far too complex to be done manually. Moreover, until the late 1980’s there were no known efficient computer algorithms for doing the calculations. This is the reason why, until relatively recently, only rather trivial Bayesian arguments with over simplistic assumptions could realistically be used in legal reasoning.

However, algorithmic breakthroughs in the late 1980s made it possible to perform correct probabilistic inference efficiently for a wide class of Bayesian networks and tools ³¹. These algorithms have subsequently been incorporated into widely available graphical toolsets that enable users without any statistical knowledge to build and run BN models ³². Moreover, further algorithmic breakthroughs have enabled us to model an even broader class of BNs, namely those including numeric nodes with arbitrary statistical distributions ³³. These breakthroughs are potentially crucial for modelling legal arguments. Yet, despite widely documented examples of their use for legal arguments ³⁴ BNs have been largely ignored.

³⁰ Fenton, Neil & Berger 2016.

³¹ Pearl 1988.

³² Agena Ltd, ‘AgenaRisk’ <<http://www.agenarisk.com>>.

³³ Neil, Tailor & Marquez 2007.

³⁴ Fenton & Neil 2018; Biedermann & Taroni 2006; Dawid, Mortera & Vicard 2007.

Moreover, even many experts who propose the Bayesian approach for legal reasoning continue to oversimplify their underlying legal arguments in order to ensure the computations can be carried out manually. This is an unnecessary and debilitating constraint on the use of Bayes.

4 Addressing relevant objections

4.1 Problem of unconstrained priors

The problem of how we attain priors that are not arbitrary and potentially biased is well covered in the chapter by Dahlman and Kolflaath in this volume. Here we focus on how to avoid the problem of wildly different priors: consider, for example, the extremes whereby one juror assumes that the prior probability a defendant is guilty is $\frac{1}{2}$ while another assumes it is $\frac{1}{7 \text{ billion}}$ (i.e. one over the world population). Then, whereas a minimal amount of evidence supporting the prosecution hypothesis would lead to a sufficiently high posterior probability of guilt for the first juror, even enormous amounts of evidence would not be sufficient for the second juror. The novel *opportunity prior* approach³⁵ can – in many real-world cases – address this problem.

When the police suspect someone of a crime, one of the first questions they ask is where the suspect was at the time of the crime. This question is very diagnostic: if the suspect can show he was elsewhere, then he is ruled out. If, however, the police can show the suspect was at the crime scene, then he is ruled into a relatively small set of possible perpetrators.

Establishing opportunity is thus critical at the investigate phase. But the same logic applies at later stages of the legal process, in particular when the suspect is charged with the crime, and we must evaluate the strength of evidence against him. Information about the suspect's whereabouts in relation to the crime scene provides a starting point for building a case, before other evidence is presented. A key point, frequently neglected in formal analyses of evidence, is that case-specific information allows us to assess the probative value of opportunity evidence.

Consider an idealised case first. Suppose we know that only five people were in a room when an item of jewellery was stolen from a small boutique. Before considering any other information, the only rational (and fair) judgment is to assign each person a probability of $\frac{1}{5}$ of committing the theft. More generally, for n people in the room, each is assigned a probability of $\frac{1}{n}$. Note this doesn't mean that we think each person has an equal propensity to commit the crime; but just that given our current state of knowledge we should assign an equal probability to each potential perpetrator; anything else would be illogical and unfair.

³⁵ Fenton et al 2019; Lagnado 2021.

The ease of estimating n depends on what is known about the location and time of the crime. For a crime committed at a solitary place and during a brief time window, we can safely assume there was only a small number of possible perpetrators. By contrast, a crime in a busy high street will include a far larger number of people. Often it will be possible to get a rough estimate or establish reasonable upper bounds for n . A crucial point here is that we are estimating the number of people who were *actually* at the crime scene at the critical time, not the number of people who *could have been* there. Thus, even if we don't know who the other people are (and might never discover this), we can still assign our suspect, who was definitely at the crime scene, a probability of $1/n$. In other words, even if many individuals *could have* been one of the other $n - 1$ at the crime scene, our suspect has probability $1/n$ regardless.

This analysis does not simply see opportunity as a necessary condition for guilt. Instead it can set a reasonable (and fair) initial probability – informed by the spatiotemporal circumstances of the case, but before considering other evidence. In cases where the suspect's presence at the crime scene is uncontested, this is a major advantage because we can set the prior at $1/n$. For example, in a murder case where a man was accused of killing his wife, the fact that he was definitely with her when she was violently killed, and it was established that at most only three other people were in the vicinity, justifies an initial probability of about $1/4$ based on this opportunity information.³⁶

In many cases, however, the suspect denies being at the crime scene at the time of the crime. To apply our analysis to these contexts we introduce the notion of the *extended crime scene*, which is based on the closest proven location and time for the suspect from which he could still have got to the crime scene to commit the crime. This can include a location before or after the crime was committed. For example, it might be accepted that the suspect was at a location two miles from the crime scene, one hour after the crime took place. We use this location and time to generate the extended crime scene, which will cover all people who were in the area at most two miles from the crime and at most one hour after the crime. This gives us the number of possible perpetrators N , which includes the suspect. Based on this extended crime scene we assign a probability for the suspect committing the crime of n/N (as there are n people at the crime scene).

Estimating the number of people in the extended crime scene can be difficult, especially if the agreed locations and times are distant from the crime scene. But in many cases (including most of those considered in this book) we can set reasonable upper limits on N , and thus reasonable lower bounds on the prior probability of the suspect being at the crime scene. Moreover, we can accommodate uncertainty in these estimates by using distributions rather than point values for n and N .

In sum, the opportunity prior helps us incorporate crucial information about the spatiotemporal location of the suspect in relation to the crime scene – something that detectives do intuitively.

³⁶ Fenton et al 2020.

The analysis quantifies the value of this information, rather than simply concluding that the suspect ‘might’ have been at crime scene. It also shows us how to combine opportunity with other evidence in the case. There is plenty of scope to debate the numbers, and sometimes priors will be extremely low. But some inferential edge, however small, is better than none. Moreover, this approach also helps avoid the common objection of ‘double-counting’ statistical information about priors discussed in Section 1.

4.2 Objections to Bayes caused by misunderstandings and misuse of the likelihood ratio method

Alongside the prior probabilities the other main component in the Bayesian framework is the probabilistic evaluation of evidence. In simple cases (such as the two node BN in Figure 1) we showed in Section 3 that the strength of evidence is captured by the likelihood ratio (LR), and this is the basis for the main approach to evaluating evidence. But there are several challenges to this approach. For example, some commentators reject the probabilistic approach wholesale, claiming that the use of the LR to evaluate evidence leads to legal paradoxes³⁷. However, in³⁸ it was shown that these paradoxes are simply the result of a flawed approach to the use of the LR - most typically because it forces multiple different related hypotheses and pieces of evidence into a 2-node BN model rather than one which separates out the different hypotheses and evidence. In this section we identify the key problems with the LR approach which compromises the use of Bayes and describe how these problems are avoided.

4.2.1 The notion that the LR can only be used for ‘statistically valid’ evidence

A 2010 UK Court of Appeal Ruling - known as *RvT*³⁹ - dealt the use of Bayes and the LR a devastating blow. The ruling quashed a murder conviction in which the prosecution had relied heavily on footwear matching evidence presented using Bayes and the LR. What certainly contributed to the ruling was the poorly presented evidence by the footwear expert; in particular, he did not make clear that likelihood ratios for different aspects of the evidence were multiplied together to arrive at a composite likelihood ratio. However, the ruling asserted:

“We are satisfied that in the area of footwear evidence, no attempt can realistically be made in the generality of cases to use a formula to calculate the probabilities. The practice has no sound basis”.

“It is quite clear that outside the field of DNA (and possibly other areas where there is a firm statistical base) this court has made it clear that Bayes’ theorem and likelihood ratios should not be used”

³⁷ Park et al 2010.

³⁸ de Zoete et al 2019.

³⁹ *R v T, EWCA Crim 2439, Case No 2007/03644/D2* (2010).

Numerous articles have criticised the ruling⁴⁰. In fact, the judge’s assertions essentially repeat the fundamental fallacy addressed in Section 1, which assumes that if probabilities are in any way subjective, then it is impossible to make rational and consistent conclusions from them. But, as explained in⁴¹, the idea that the statistics associated with DNA match evidence is somehow purely objective, while the statistics associated with footwear match evidence is purely subjective is a myth. All probabilities based on statistical data rely on multiple subjective assumptions about the interpretation and source of the data. Unfortunately, the ruling is having a devastating impact on the way some forensic evidence is presented with experts deliberately concealing or obfuscating their calculations.

4.2.2 LR models are inevitably over-simplified

The simplest and most common use of the LR – involving a single piece of forensic trace evidence for a single source level hypothesis – can actually be very complex as already explained in Section 2 (where Figure 2, rather than Figure 1 is the correct model). Even if we completely ignore much of the context (including issues of reliability of trace sample collection/storage and potential testing errors) the LR may still be difficult or even impossible to elicit because somehow we have to factor in to the hypothesis H_d (defendant is not the source of the DNA trace) every person other than the defendant who could have been the source (potentially every other person in the world)⁴². For example, $P(E | H_r)$ is much higher than $P(E | H_u)$ where H_r is the hypothesis “a close relative of the defendant is the source of the trace” and H_u is the hypothesis “a totally unrelated person is the source”

This means that, in reality, H_d is made up of multiple hypotheses that are difficult to articulate and quantify. The standard pragmatic solution (which has been widely criticised⁴³) is to assume that H_d represents a ‘random person unrelated to the defendant’. But not only does this raise concerns about the homogeneity of the population used for the random match probabilities, it also requires separate assumptions about the extent to which relatives can be ruled out as suspects.

It is not just the hypotheses that may need to be ‘decomposed’. In practice, even an apparently ‘single’ piece of evidence E actually comprises multiple separate pieces of evidence, and it is only when the likelihoods of these separate pieces of evidence are considered that correct conclusions about probative value of the evidence can be made.

Example 1: Consider the evidence E : “tiny matching DNA trace found”. Suppose that the DNA trace has a profile with a random match probability of 1/100 (such relatively ‘high’ match probabilities are common in low-template samples). Assuming H_p and H_d are the prosecution and defence hypotheses respectively, it would be typical to assume that

$$P(E | H_p) = 1$$

⁴⁰ Aitken et al 2011; Berger et al 2011; Morrison 2012; Nordgaard, Hedell & Ansell 2012; Redmayne et al 2011; Robertson, Vignaux & Berger 2011; Sjerps & Berger 2012.

⁴¹ Fenton et al 2020.

⁴² Balding & Steele 2015; Nordgaard, Hedell and Ansell 2012.

⁴³ Balding & Steele 2015.

and that

$$P(E | H_d) = 1/100$$

leading to a LR of 100, thus indicating quite strong support for the prosecution hypothesis. However, the evidence E actually comprises two separate pieces of evidence:

- $E1$: *tiny* DNA trace found
- $E2$: DNA trace found matches defendant

In particular, this makes clear the relevance of finding only a tiny trace of DNA when larger amounts would be expected to have been left by the person who committed the crime. So, actually $P(E | H_p)$ will be much smaller than 1, because we would expect substantial amounts of DNA to be found, rather than just a tiny trace. To elicit all the necessary individual likelihood values, and to carry out the correct Bayesian calculations needed for the overall LR in situations such as this, we again need to turn to BNs as shown in Figure 5.

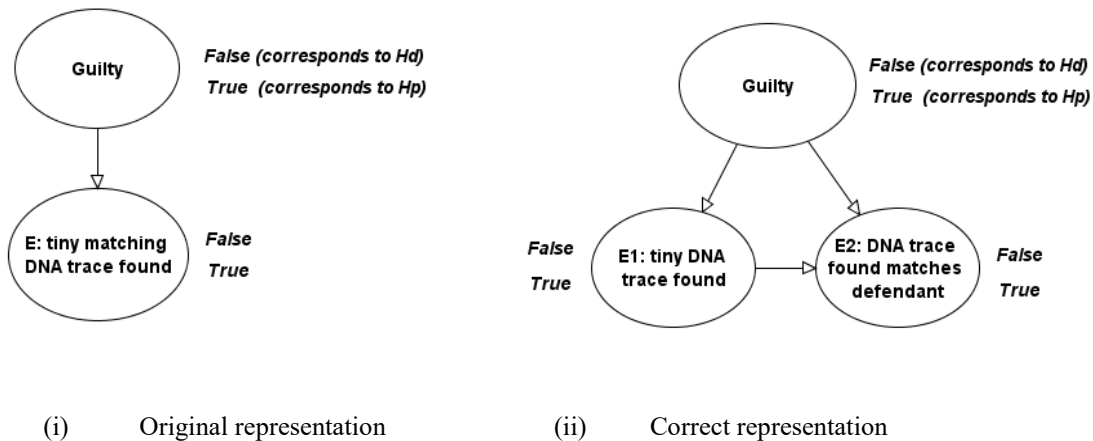


Figure 3 Modelling complex evidence in a BN

The oversimplistic model fails to capture the relevance of the fact that the trace was tiny. If the defendant were guilty it is expected that the investigator would have found significant traces of DNA. The significance of the tiny trace is properly captured by separating out $E1$ in the second model. A reasonable conditional probability table for $E1$ is shown in Table 2.

Table 1 Conditional probability table for $E1$

Guilty	False	True
False	0.5	0.999
True	0.5	0.0010

The conditional probability table for $E2$ shown in Table 3 uses the same RMP information as was used in the oversimplified model.

Table 2 Conditional probability table for E2

Guilty	False		True	
	False	True	False	True
E1: tiny DNA trace found	False	True	False	True
False	1.0	0.99	1.0	0.0
True	0.0	0.01	0.0	1.0

Calculating the overall LR manually in this case is much more complex, so we go directly to the result of running the model in a BN tool with *E2* set as true (and the prior odds of guilt set at 50:50 again). This is shown in Figure 15. The LR is just the probability of guilty divided by the probability of not guilty, which is 0.2. So the evidence supports the defence hypothesis rather than the prosecution.

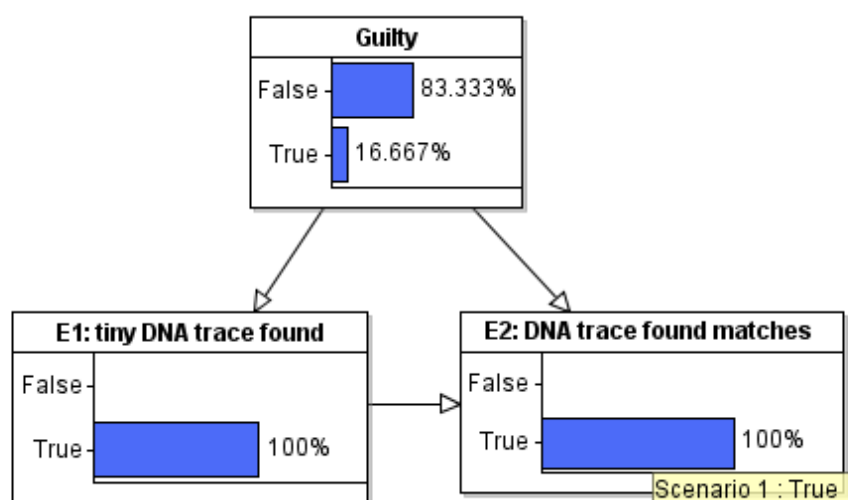


Figure 4 Posterior odds in correct model

This example also indicates the importance of taking account of absence of evidence.

We note that a frequent objection against Bayesianism is that it is practically impossible to consider *all* of the probabilistic dependencies between pieces of evidence in a case.⁴⁴ We accept that a Bayesian Network does not capture the whole complexity and, like all reasoning, Bayesian reasoning makes some simplifications. But the advantage of the Bayesian approach is that it is much more nuanced and rigorous than the alternatives proposed such as relative plausibility.⁴⁵

4.3 Bayes is too complex for lawyers and juries to understand

⁴⁴ See Allen & Pardo in this volume.

⁴⁵ Ibid.

This is essentially the argument that was used in the case of ⁴⁶. This was a rape case (discussed in detail in ⁴⁷) in which the only prosecution evidence was that the defendant's DNA matched that of a swab sample taken from the victim. The defence evidence included an alibi and the fact that the defendant did not match the victim's description of her attacker. At trial the prosecution had emphasised the very low random match probability (1 in 200 million) of the DNA evidence. The defence argued that if statistical evidence was to be used in connection with the DNA evidence, it should also be used in combination with the defence evidence and that Bayes Theorem was the only rational method for doing this. The defence called a Bayesian expert (Prof Peter Donnelly) who explained how, with Bayes, the posterior probability of guilt was much lower when the defence evidence was incorporated. The appeal rested on whether the judge misdirected the jury as to the evidence in relation to the use of Bayes and left the jury unguided as to how that theorem could be used in properly assessing the statistical and non-statistical evidence in the case. The Appeal was successful and a retrial was ordered, although the Court was scathing in its criticism of the way Bayes was presented, stating:

“The introduction of Bayes' theorem into a criminal trial plunges the jury into inappropriate and unnecessary realms of theory and complexity deflecting them from their proper task.

The task of the jury is ... to evaluate evidence and reach a conclusion not by means of a formula, mathematical or otherwise, but by the joint application of their individual common sense and knowledge of the world to the evidence before them”

At the retrial it was agreed by both sides that the Bayesian argument should be presented in such a way that the jury could perform the calculations themselves (a mistake in our view). The jury were given a detailed questionnaire to complete to enable them to produce their own prior likelihoods, and calculators to perform the necessary Bayesian calculations from first principles. Adams was, however, again convicted. A second appeal was launched and was also unsuccessful, with the Court not only scathing about the use of Bayes in the case but essentially ruling against its future use.

The ruling against the use of Bayes in *R v Adams* is especially damaging because it rules against the very use where Bayes has the greatest potential to simplify and clarify complex legal arguments. The fact that the complex presentation of Bayes in the case was (rightly) considered to be its death knell is especially regrettable given that in 1996 the tools for avoiding this complexity were already widely available.

The idea that different pieces of (possibly competing) evidence about a hypothesis H are combined to update our belief in H is central to all legal proceedings. Yet, although Bayes is the perfect formalism for this type of reasoning, it is difficult to find **any** well reported examples of the successful use of Bayes in combining diverse evidence in a real case. While the spectacular failure in the above Adams case has not helped, a major reason for this is to do with the lack of awareness of tools for building and running BN models that enable us to do Bayesian inference for legal arguments involving diverse related evidence.

Despite the multiple publications applying BNs to legal arguments, even many Bayesian statisticians are either unaware of these breakthroughs or are reluctant to use the available

⁴⁶ *R v Adams* [1996] 2 Cr App R 467, [1996] Crim LR 898, CA and *R v Adams* [1998] 1 Cr App R 377.

⁴⁷ Donnelly 2005.

technology. Yet, if one tries to use Bayes theorem ‘manually’ to represent a legal argument one of the following results is inevitable:

1. To ensure the calculations can be easily computed manually, the argument is made so simple that it no longer becomes an adequate representation of the legal problem.
2. A non-trivial model is developed and the Bayesian calculations are written out and explained from first principles and the net result is to totally bemuse legal professionals and jurors. This was, of course, the problem in *R v Adams*. In ⁴⁸ we show other examples where statisticians provide unnecessarily complex arguments).

The manual approach is also not scalable since it would otherwise mean having to explain and compute one of the BN inference algorithms, which even professional mathematicians find daunting.

5 Conclusions and the way forward

That fallacies of probabilistic reasoning (such as the prosecutor’s fallacy) continue to be made in legal proceedings is a sad indictment of the lack of impact made by statisticians in general (and Bayesians in particular) on legal practitioners. This is despite the fact that the issue has been extensively documented by multiple authors including ⁴⁹ and has even been dealt with in populist books such as ⁵⁰. There is almost unanimity among the authors of these works that a basic understanding of Bayesian probability is the key to avoiding probabilistic fallacies. Indeed, Bayesian reasoning is explicitly recommended in works such as ⁵¹, although there is less of a consensus on whether or not experts are needed in court to present the results of all but the most basic Bayesian arguments ⁵².

We argue that the way forward is to use BNs to present probabilistic legal arguments since this approach avoids much of the confusion surrounding both the over-simplistic LR and more complex models represented formulaically and computed manually. Unfortunately, it is precisely because BNs are assumed by legal professionals to be ‘part of those same problems’ that they have made little impact. Yet, ultimately, any use of probability – even if it is based on frequentist statistics – relies on a range of subjective assumptions. The objection to using subjective priors may also be calmed by the fact that it may be sufficient to consider a range of probabilities, rather than a single value for a prior. BNs are especially suited to this since it is easy to change the priors and do sensitivity analysis ⁵³.

A basic strategy for presenting BNs to legal professionals is described in detail in ⁵⁴ and is based on the *calculator analogy*. This affirms that since we now have efficient and easy-to-use

⁴⁸ Fenton, Neil & Berger 2016.

⁴⁹ Kaye 2001; Anderson, Schum & Twining 2005; Edwards 1991; Evett 1995; Fenton & Neil 2000; Jowett 2001; Balding & Donnelly 1994; Koehler 1993; Redmayne; Thompson & Schumann 1987; Freckelton & Selby 2005; Murphy 2003.

⁵⁰ Gigerenzer 2002; Haigh 2003.

⁵¹ Dahlman 2020; Evett 1995; Finkelstein & Levin 2001; Good 2001; Redmayne; Saks & Thompson 2003; Robertson & Vignaux 1995.

⁵² Robertson & Vignaux.

⁵³ Fenton & Neil 2018.

⁵⁴ Fenton & Neil 2011.

BN tools there should be no more need to explain the Bayesian calculations in a complex argument than there should be any need to explain the thousands of circuit level calculations used by a regular calculator to compute a long division.

Only the simplest Bayesian legal argument (a single hypothesis and a single piece of evidence) can be easily computed manually; inevitably we need to model much richer arguments involving multiple pieces of possibly linked evidence. While humans must be responsible for determining the prior probabilities (and the causal links) for such arguments, it is simply wrong to assume that humans must also be responsible for understanding and calculating the revised probabilities that result from observing evidence. The Bayesian calculations quickly become impossible to do manually, but any BN tool enables us to do these calculations instantly.

The results from a BN tool can be presented using a range of assumptions including different priors. What the legal professionals (and perhaps even jurors if presented in court) should never have to think about is how to perform the Bayesian inference calculations. They do, of course, have to consider the prior assumptions needed for any BN model. But these are precisely what have to be considered in weighing up any legal argument. The BN simply makes this all explicit rather than hidden, which is another clear benefit of the approach.

We recognise that there are significant technical challenges to overcome to make the construction of BNs for legal reasoning easier, but the lack of a systematic, repeatable method for modelling legal arguments as BNs has been addressed by using common idioms and an approach for building complex arguments from these.

Proper use of Bayesian reasoning has the potential to improve the efficiency, transparency and fairness of criminal and civil justice systems. It can help experts formulate accurate and informative opinions; help courts determine admissibility of evidence; help identify which cases should be pursued; and help lawyers to explain, and jurors to evaluate, the weight of evidence during a trial. It can also help identify errors and unjustified assumptions entailed in expert opinions.

6 References

Agena Ltd, 'AgenaRisk' <<http://www.agenarisk.com>>

Aitken CGG and many other signatories, 'Expressing Evaluative Opinions: A Position Statement' (2011) 51 Science and Justice 1

Anderson T, Schum DA and Twining W, *Analysis of Evidence* (Cambridge University Press 2005)

Balding DJ and Donnelly P, 'Prosecutor's Fallacy and DNA Evidence' (1994) October Criminal Law Review 711

Balding DJ and Steele CD, *Weight-of-Evidence for Forensic DNA Profiles* (Wiley-Blackwell 2015)

Berger CEH and others, 'Evidence Evaluation: A Response to the Court of Appeal Judgement in R v T' (2011) 51 Science and Justice 43

Berger D, 'Improving Legal Reasoning Using Bayesian Probability Methods' (Queen Mary University of London 2014)

Biedermann A and Taroni F, 'Bayesian Networks and Probabilistic Reasoning about Scientific Evidence When There Is a Lack of Data' (2006) 157 Forensic Science International 163

- Casscells W and Graboys TB, 'Interpretation by Physicians of Clinical Laboratory Results' (1978) 299 *New England Journal of Medicine* 999
- Cook R and others, 'A Hierarchy of Propositions: Deciding Which Level to Address in Casework' (1998) 38 *Science and Justice* 231
- Cosmides L and Tooby J, 'Are Humans Good Intuitive Statisticians after All? Rethinking Some Conclusions from the Literature on Judgment under Uncertainty' (1996) 58 *Cognition* 1
- Dahlman C, 'De-Biasing Legal Fact-Finders with Bayesian Thinking' (2020) 12 *Topics in Cognitive Science* 1115
- Dawid AP and Mortera J, 'Forensic Identification with Imperfect Evidence' (1998) 85 *Biometrika* 835 <<http://biomet.oxfordjournals.org/content/85/4/835.short>> accessed 25 January 2015
- Dawid AP, Mortera J and Vicard P, 'Object-Oriented Bayesian Networks for Complex Forensic DNA Profiling Problems' (2007) 169 *Forensic Science International* 195
- de Zoete J and others, 'Resolving the "Probabilistic Paradoxes in Legal Reasoning" with Causal Bayesian Models' [2019] *Science & Justice*
- Donnelly P, 'Appealing Statistics' (2005) 2 *Significance* 46
- Edwards W, 'Influence Diagrams, Bayesian Imperialism, and the Collins Case: An Appeal to Reason' (1991) 13 *Cardozo Law Review* 1025
- ENFSI, 'Guideline for Evaluative Reporting in Forensic Science' <http://enfsi.eu/wp-content/uploads/2016/09/m1_guideline.pdf>
- Evetts IW, 'Avoiding the Transposed Conditional' (1995) 35 *Science and Justice* 127
- Faigman DL and Baglioni AJ, 'Bayes' Theorem in the Trial Process' (1988) 12 *Law and Human Behavior* 1 <<http://dx.doi.org/10.1007/BF01064271>>
- Fenton N and others, 'On the Limitations of Probabilistic Claims about the Probative Value of Mixed DNA Profile Evidence' <<http://arxiv.org/abs/2009.08850>> accessed 21 September 2020
- Fenton NE and others, 'When "Neutral" Evidence Still Has Probative Value (with Implications from the Barry George Case)' (2014) 54 *Science and Justice* 274
- , 'The Opportunity Prior: A Proof-Based Prior for Criminal Cases' (2019) 18 *Law, Probability and Risk* 237
- Fenton NE and Neil M, 'The Jury Fallacy and the Use of Bayesian Nets to Simplify Probabilistic Legal Arguments' (2000) 36 *Mathematics Today (Bulletin of the IMA)* 180
- , 'Avoiding Legal Fallacies in Practice Using Bayesian Networks' (2011) 36 *Australian Journal of Legal Philosophy* 114
- , *Risk Assessment and Decision Analysis with Bayesian Networks* (2nd edn, CRC Press, Boca Raton 2018)
- Fenton NE, Neil M and Berger D, 'Bayes and the Law: Supplementary Material' (2016)
- , 'Bayes and the Law' (2016) 3 *Annual Review of Statistics and Its Application* 51 <<http://www.annualreviews.org/doi/pdf/10.1146/annurev-statistics-041715-033428>> accessed 4 April 2016
- Fenton NE, Neil M and Hsu A, 'Calculating and Understanding the Value of Any Type of Match Evidence When There Are Potential Testing Errors' (2014) 22 *Artificial Intelligence and Law* 1
- Fienberg SE, 'Bayesian Models and Methods in Public Policy and Government Settings' (2011) 26 *STATISTICAL SCIENCE* 212

- Fienberg SE and Finkelstein MO, 'Bayesian Statistics and the Law' in JM Bernardo and others (eds), *Bayesian Statistics 5* (Oxford University Press 1996)
- Finkelstein MO, *Basic Concepts of Probability and Statistics in the Law* (Springer Science & Business Media 2009)
- Finkelstein MO and Levin BA, *Statistics for Lawyers* (Springer 2001)
- Forrest AR, 'Sally Clark - a Lesson for Us All' (2003) 43 *Science & Justice*: 63
- Freckelton I and Selby H, *Expert Evidence: Law, Practice, Procedure and Advocacy* (3rd edn, Lawbook Co 2005)
- Gastwirth JL, 'Statistical Science in the Courtroom' 2000.
- Gigerenzer G, *Reckoning with Risk: Learning to Live with Uncertainty* (Penguin Books 2002)
- Gillies D, 'Varieties of Propensity' (2000) 51 *British Journal for the Philosophy of Science* 807
- Good PI, *Applying Statistics in the Courtroom: A New Approach for Attorneys and Expert Witnesses* (CRC Press 2001)
- Haigh J, *Taking Chances: Winning with Probability* (2nd edn, Oxford University Press 2003)
- Hill R, 'Reflections on the Cot Death Case' (2005) 2 *Significance*
- Jowett C, 'Lies, Damned Lies, and DNA Statistics: DNA Match Testing, Bayes' Theorem, and the Criminal Courts' (2001) 41 *Med Sci. Law* 194
- Kaye DH, 'Two Fallacies About DNA Databanks for Law Enforcement' (2001) 67 *Brooklyn Law Review* 179
- Kaye DH, 'Revisiting "Dreyfus": A More Complete Account of a Trial by Mathematics' (2007) 91 *Minnesota Law Review* 825 <<http://ssrn.com/abstract=944244>>
- Koehler JJ, 'Probabilities in the Courtroom: An Evaluation of the Objections and Policies' in DK Kagehiro and WS Laufer (eds), *Handbook of Psychology and Law* (Springer-Verlag 1992)
- Koehler JJ, 'Error and Exaggeration in the Presentation of DNA Evidence at Trial' (1993) 34 *Jurimetrics* 21
- , 'The Base Rate Fallacy Myth' (1993) 4 *Psychology*
- Lagnado DA, *Explaining the Evidence*. (Cambridge University Press 2021)
- Lauritzen SL and Spiegelhalter DJ, 'Local Computations with Probabilities on Graphical Structures and Their Application to Expert Systems (with Discussion)'
- Meester R and others, 'On the (Ab)Use Of Statistics in the Legal Case against the Nurse Lucia de B.' (2007) 5 *Law, Probability and Risk* 233
- Morrison GM, 'The Likelihood Ratio Framework and Forensic Evidence in Court: A Response to RvT' (2012) 16 *International Journal of Evidence and Proof*
- Murphy P, *Evidence, Proof, and Facts: A Book of Sources* (Oxford University Press 2003)
- Neil M, Tailor M and Marquez D, 'Inference in Hybrid Bayesian Networks Using Dynamic Discretization' (2007) 17 *Statistics and Computing* 219 <<http://dx.doi.org/10.1007/s11222-007-9018-y>>
- Nordgaard A, Hedell R and Ansell R, 'Assessment of Forensic Findings When Alternative Explanations Have Different Likelihoods-"Blame-the-Brother"-Syndrome.' (2012) 52 *Science & Justice: journal of the Forensic Science Society* 226 <<http://www.sciencedirect.com/science/article/pii/S1355030611001468>> accessed 24 April 2014

Park RC and others, 'Bayes Wars Redivivus — An Exchange' (2010) 8 International Commentary on Evidence 1

Pearl J, *Probabilistic Reasoning in Intelligent Systems* (Morgan Kaufmann 1988)

Pettigrew R, *Dutch Book Arguments* (Cambridge University Press 2020)

Puch-Solis R and others, 'PRACTITIONER GUIDE NO 2: Assessing the Probative Value of DNA Evidence, Guidance for Judges, Lawyers, Forensic Scientists and Expert Witnesses' (Royal Statistical Society 2012) <<http://www.rss.org.uk/uploadedfiles/userfiles/files/Practitioner-Guide-2-WEB.pdf>>

Redmayne M, 'DNA Evidence, Probability and the Courts'

Redmayne M and others, 'Forensic Science Evidence in Question' [2011] Criminal Law Review 347

Robertson B, Vignaux GA and Berger CEH, 'Extending the Confusion about Bayes' (2011) 74 The Modern Law Review 444

Robertson B and Vignaux T, *Interpreting Evidence: Evaluating Forensic Science in the Courtroom* (John Wiley and Son Ltd 1995)

——, 'Don't Teach Statistics to Lawyers!' 541

Royal Statistical Society, 'Statistics and the Law Working Group' (2015)

<http://www.rss.org.uk/RSS/Influencing_Change/Statistics_and_the_law/RSS/Influencing_Change/Current_projects_sub/Statistics_and_the_law.aspx>

Saks MJ and Thompson WC, 'Assessing Evidence: Proving Facts' in David Carson and Ray Bull (eds), *Handbook of Psychology in Legal Contexts (Second Edition)* (2003)

Sjerps M and Berger C, 'How Clear Is Transparent? Reporting Expert Reasoning in Legal Cases' (2012) 11 Law, Probability and Risk 317

Spiegelhalter DJ, 'Court of Appeal Bans Bayesian Probability (and Sherlock Holmes)' (2013) <<http://understandinguncertainty.org/court-appeal-bans-bayesian-probability-and-sherlock-holmes>>

Thompson WC and Schumann EL, 'Interpretation of Statistical Evidence in Criminal Trials' (1987) 11 Law and Human Behavior 167 <<http://dx.doi.org/10.1007/BF01044641>>

Thompson WC, Taroni F and Aitken CGG, 'How the Probability of a False Positive Affects the Value of DNA Evidence' (2003) 48 Journal of Forensic Sciences 47

Tillers P and Gottfried J, 'Case Comment - United States v. Copeland, 369 F. Supp. 2d 275 (E.D.N.Y.2005): A Collateral Attack on the Legal Maxim That Proof beyond a Reasonable Doubt Is Unquantifiable?' (2007) 5 Law, Probability & Risk 5: 135

Tillers P and Green E (eds), *Probability and Inference in the Law of Evidence: The Uses and Limits of Bayesianism* (Kluwer Academic Publishers 1988)

Tribe LH, 'Trial by Mathematics' (1971) 84 Harvard Law Review 1329

Vosk T and Emery AF, *Forensic Metrology: Scientific Measurement and Inference for Lawyers, Judges, and Criminalists* (CRC Press 2014) <<http://books.google.com/books?hl=en&lr=&id=SzTNBQAAQBAJ&pgis=1>> accessed 23 December 2014

Ziliak ST and McCloskey DN, *The Cult of Statistical Significance* (The University of Michigan Press 2008)

Nulty & Ors v Milton Keynes Borough Council EWCA Civ 15 (24 January 2013)

People v Collins, 438 P 2d 33 (68 Cal 2d 319 1968)

R v Adams [1996] 2 Cr App R 467, [1996] Crim LR 898, CA and R v Adams [1998] 1 Cr App R 377

