Bayesianism: Objections and Rebuttals

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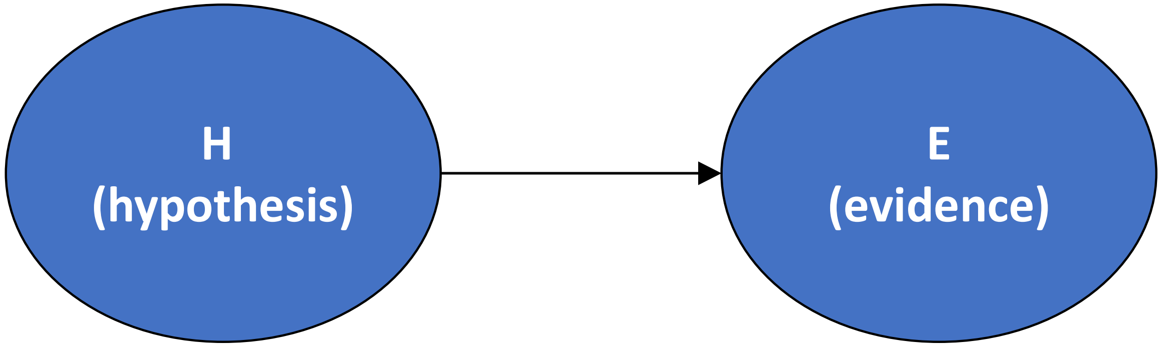
On-line seminars on chapters in:
Dahlman, Stein & Tuzet (eds.) Philosophical Foundations of Evidence Law, OUP, 2021

11 May 2021

The Legal Approach = Bayes Theorem

We have hypothesis H (e.g. H= “Fred guilty of crime” or “DNA at crime scene is from Fred”)

We now get some evidence E (Fred’s DNA matches that from crime scene)



We want to know the ‘posterior’ probability of H, i.e. $P(H|E)$

Typically we can estimate $P(E|H)$ and $P(E|\text{ not } H)$

$$P(H|E) = \frac{P(E|H) \times P(H)}{P(E)} = \frac{P(E|H) \times P(H)}{P(E|H) \times P(H) + P(E|\text{not } H) \times P(\text{not } H)}$$

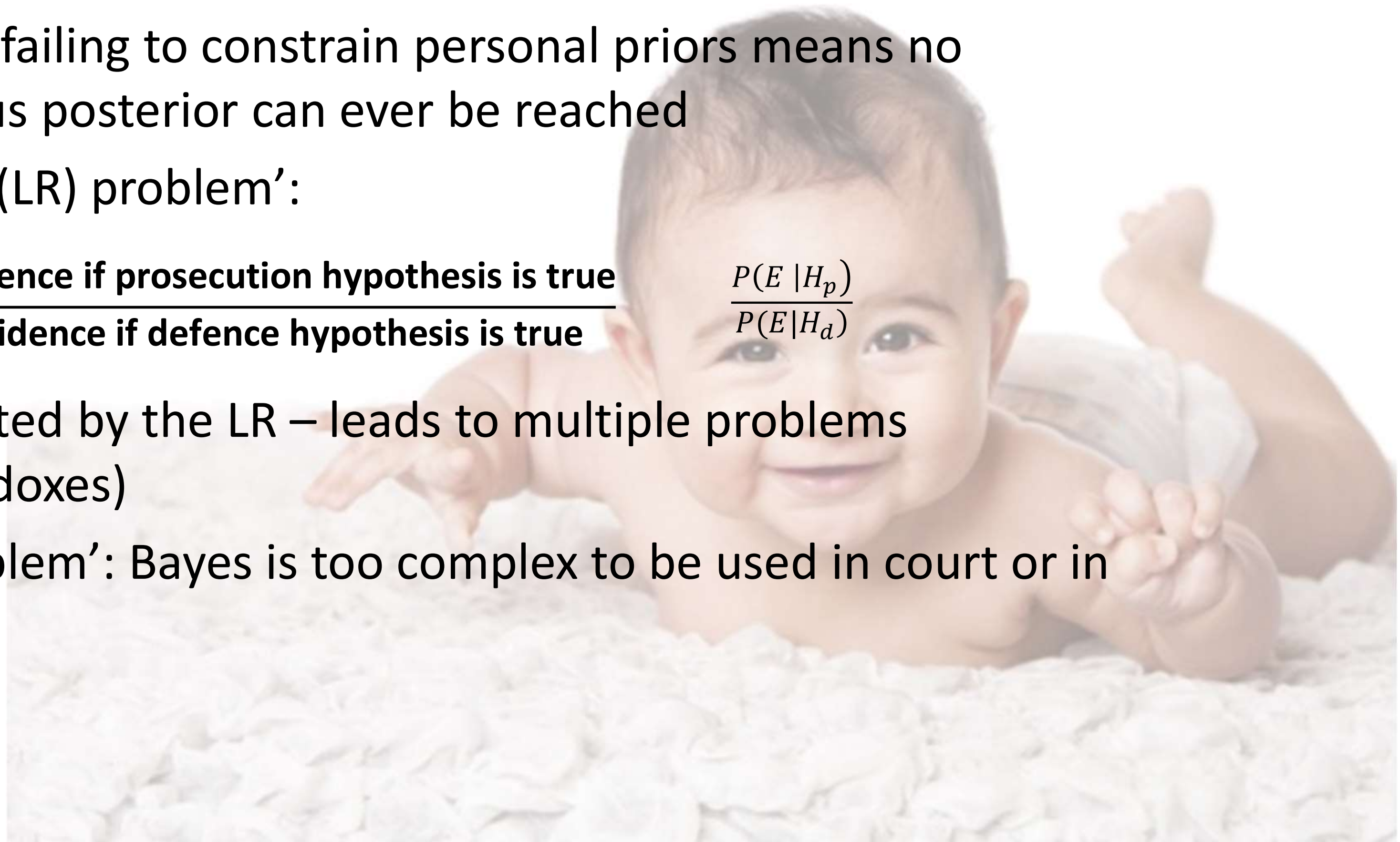
Objections to Bayes

1. The 'prior problem': failing to constrain personal priors means no reasonable consensus posterior can ever be reached
2. The 'likelihood ratio (LR) problem':

$$\frac{\textbf{Probability of evidence if prosecution hypothesis is true}}{\textbf{Probability of evidence if defence hypothesis is true}} \qquad \frac{P(E\,|H_p)}{P(E|H_d)}$$

   Bayes - as encapsulated by the LR – leads to multiple problems (including legal paradoxes)

3. The 'complexity problem': Bayes is too complex to be used in court or in legal arguments



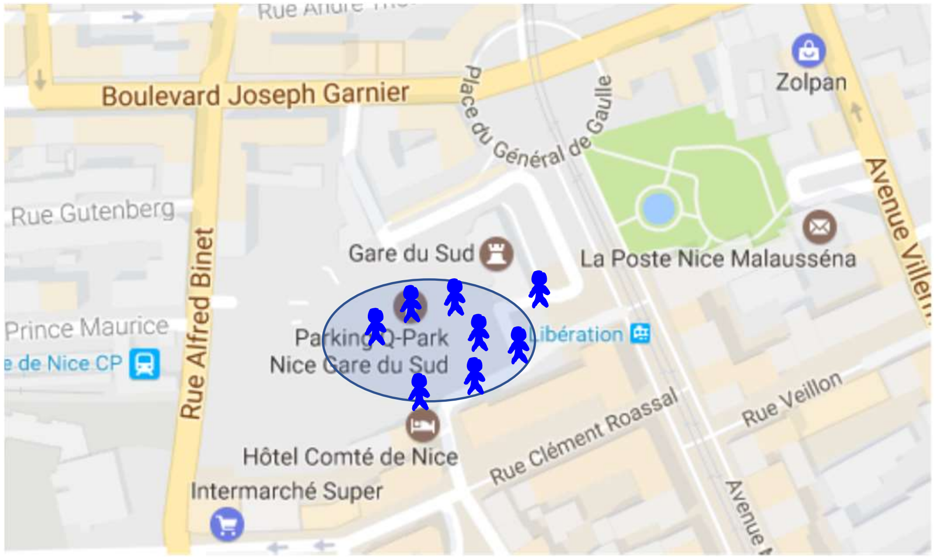
Rebuttal 1: The prior problem

- Like it or not people form their own subjective priors, so anything that makes this more explicit and rational should be welcomed
- The ‘opportunity prior’ work provides an objective prior for a large class of crimes

Opportunity prior: Crime Scene and Crime Time

Assume a crime has taken place, it was committed by one person against one other person (e.g. murder, assault, robbery). Then:

- The ***crime scene*** (CS): smallest physical area within which it is certain the crime happened.
- The ***crime time*** (CT): smallest time interval (*t*, *t'*) between which it is certain the crime took place.



Imagine we can observe people who are in the CS at any time during CT

Number of people at Crime Scene during Crime Time: (*n*)

- We generally do not know who was at CS during CT. But it is possible to estimate number of people *n* (other than the victim) who were.
- By definition the criminal is one of these *n* people.

If the suspect was at CS during CT then the truly fair prior probability of guilt is 1/n

As close to 'innocent until proven guilty' as possible

We also handle the case where suspect was NOT at CS during CT by using notion of ***extended CS***

Crushing the ‘anybody in the world’ fallacy

- Suppose only TWO people Fred and Joe were at the CS during CT
- P(Fred is guilty) = ½
- Suppose only TWO people Fred and an unknown other were at the CS during CT.
- The ‘other’ can be anybody in the world. So what is P(Fred is guilty)?
- Fallacy is to assume *n* is different (e.g. much higher) in this case

Rebuttal 2: The likelihood ratio (LR) problem

- The confusion over LR and Bayes
- LR is only a measure of probative value of evidence because of Bayes – and only when the prosecution and defence hypotheses are mutual exclusive and exhaustive
- LR models are typically over-simplified (to avoid the required a full causal Bayesian network)
- The LR for source level hypotheses tells us nothing about offense level hypotheses
- Confusion about LR being expressed on a verbal scale

The confusion over LR and Bayes

R v T judgement

"It is quite clear that outside the field of DNA (and possibly other areas where there is a firm statistical base) this court has made it clear that Bayes' theorem and likelihood ratios should not be used"

Response by: CGG Aitken and many other signatories, 'Expressing Evaluative Opinions: A Position Statement' (2011) 51 Science and Justice 1

"It is regrettable that the judgment confuses the Bayesian approach with the use of Bayes' Theorem. The Bayesian approach does not necessarily involve the use of Bayes' Theorem."

Likelihood Ratio (LR) as a measure of probative value

It is because of Bayes' Theorem and ONLY because of Bayes Theorem that the LR can be considered meaningfully to be a measure of 'probative value of evidence'.

Bayes Theorem:

Posterior odds of *H* = LR x Prior odds of *H*

LR > 1: means *E* supports prosecution hypothesis H

(as the 'posterior odds' in favour of H ***increase*** in this case)

LR <1: means *E* supports defence hypothesis

(as the 'posterior odds; in favour of H ***decrease*** in this case)

LR = 1: means *E* has no probative value

(as 'posterior odds' are in favour of H are ***unchanged*** in this case)

Likelihood Ratio: Need for mutually exclusive and exhaustive hypotheses

If the hypotheses are not mutually exclusive and exhaustive then it is possible that

- LR>1 but the evidence supports the defence hypothesis
- LR=1 but the evidence is still probative

LR>1 ….but the evidence supports the defence hypothesis

A lottery has 10 tickets numbered 1 to 10

Fred buys 3 tickets and gets numbers 3, 4 and 5. Jane buys 2 tickets and gets numbers 1 and 6

The winning ticket is drawn but is blown away in the wind. However, a totally reliable eye-witness asserts that the winning ticket was a number between 4 and 10.

Fred claims he must have won and sues the organisers.

The prosecution hypothesis H_f is "Fred won the raffle" (i.e H_f : "winning ticket was 3, 4, or 5").

Fred's lawyer provides the following argument to support the claim:

> We have two alternative hypotheses. Either Fred won the lottery (H_f) or Jane won the lottery (H_j).
>
> We have the evidence E that the winning ticket was a number between 4 and 10.
>
> $P(E \mid H_f) = 2/3$ because if Fred won then there is a 2/3 chance the winning number was 4 or 5
>
> $P(E \mid H_j) = 1/2$ because if Jane won then there is a 1/2 chance the winning number was 6
>
> Hence, the LR is 2/3 divided by 1/2 which is equal to 4/3. As the LR>1, the evidence supports H_f

BUT: While the evidence supports H_f over H_j it does NOT support H_f

The defence hypothesis is *not* H_f "winning ticket = 1,2,6,7,8,9, or 10"

> $P(E \mid \textit{not } H_f) = 5/7$
>
> Hence, LR of H_f against *not* H_f is 2/3 divided by 5/7 which is equal to 14/15. As the LR<1 the evidence supports *not* H_f

The probability of H_f drops from a prior of 0.3 to a posterior of 0.286 after getting the evidence E

LR=1 …but the evidence is probative

A lottery has 10 tickets numbered 1 to 10

Fred buys 3 tickets and gets numbers 3, 4 and 5. Jane buys 3 tickets and gets numbers 1, 2 and 6

The winning ticket is drawn but is blown away in the wind. However, a totally reliable eye-witness asserts that the winning ticket was less than 7.

Fred claims he must have won and sues the organisers, arguing that the evidence supports H_f is “Fred won the raffle”

This time the Defence lawyer argues the evidence provides no probative value to support H_f as follows

> We have two alternative hypotheses. Either Fred won the lottery (H_f) or Jane won the lottery (H_j).
>
> We have the evidence E that the winning ticket was a number less than 7.
>
> $P(E \mid H_f) = 1$ because if Fred won then it is certain the winning number was less than 7
>
> $P(E \mid H_j) = 1$ because if Jane won then it is certain the winning number was less than 7
>
> Hence the LR = 1 proving the evidence has no probative value

BUT: While the evidence provides no support for H_f over H_j it ***does*** support H_f

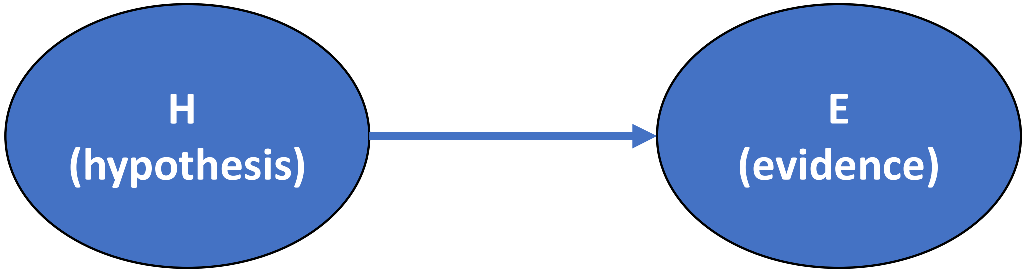
not H_f is the hypothesis “winning ticket = 1,2,6,7,8,9, or 10”

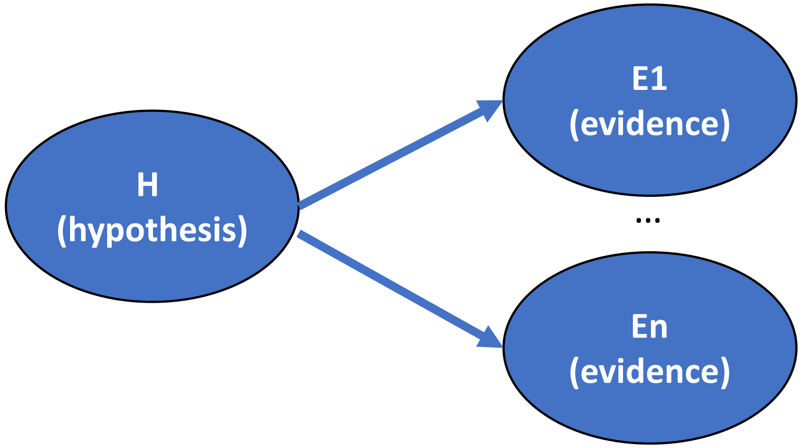
> so $P(E \mid \textit{not } H_f) = 3/7$
>
> Hence, LR of H_f *not* H_f is 1 divided by 3/7 which is equal to 7/3. As the LR>1 the evidence supports H_f

The probability of H_f increases from a prior of 0.3 to a posterior of 0.5 after getting the evidence E

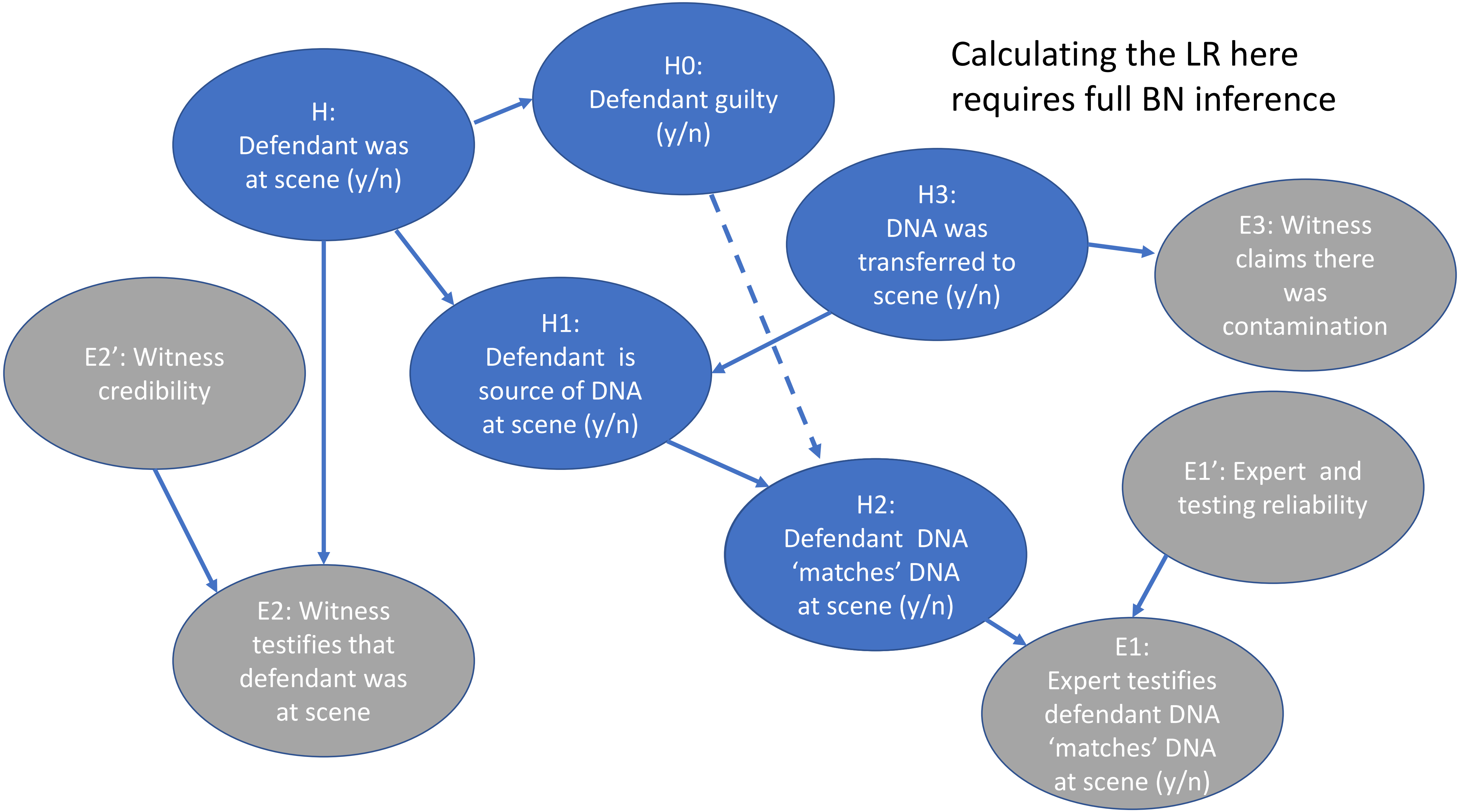
Likelihood Ratio: oversimplistic model

Wrongly encourages experts to over-simplify the evidence by combining multiple hypotheses into a single hypothesis and a single piece of evidence





or, a single hypothesis H and multiple pieces of evidence that are independent conditional on H



Calculating the LR here requires full BN inference

Rebuttal 3: The complexity problem

R v Adams Case Ruling:

"The introduction of Bayes' theorem into a criminal trial plunges the jury into inappropriate and unnecessary realms of theory and complexity deflecting them from their proper task.

…The task of the jury is … to evaluate evidence and reach a conclusion not by means of a formula, mathematical or otherwise, but by the joint application of their individual common sense and knowledge of the world to the evidence before them"

Rebuttal 3: The complexity problem

- Lack of awareness of tools for building and running BN models
- Cannot ‘do Bayes’ manually
- Certainly cannot do Bayes manually in court – as shown by spectacular failure in Adams case
- Use of BNs is the way forward, but presents its own challenges

Conclusions

- Common objections to Bayes arise from a misunderstanding of what Bayes is, together with an oversimplified and often incorrect use of the LR
- This include ‘paradoxes’ that supposedly invalidate the use of Bayes and the LR (the paradoxes unravel when the problems are properly cast as causal Bayesian networks)
- Attempts to ‘do Bayes in court’ from first principle are doomed to failure
- Any serious attempt to do Bayesian reasoning about evidence requires a (non-trivial) BN

Reading and follow-up

Fenton, N. E., and Lagnado, D (2021) "Bayesianism: Objections and Rebuttals", in G. Tuzet, C. Dahlman en A. Stein (eds.) Philosophical Foundations of Evidence Law. Oxford University Press, to appear

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de Zoete, J., Fenton, N. E., Noguchi, T., & Lagnado, D. A. (2019). "Countering the 'probabilistic paradoxes in legal reasoning' with Bayesian networks". Science & Justice 59 (4), 367-379 10.1016/j.scijus.2019.03.003