Distributed Space-Time Trellis Code for Asynchronous Cooperative Communications under Frequency-Selective Channels

Zhimeng Zhong, *Student Member, IEEE*, Shihua Zhu, and Arumugam Nallanathan, *Senior Member, IEEE*

***Abstract*—In most cooperative communication works, perfect synchronization among relay nodes is assumed in order to achieve the cooperative diversity. However, this assumption is not realistic due to the distributed nature of each relay node. In this paper, we propose a family of distributed space-time trellis code (DSTTC) that does not require the synchronization assumption. It is shown that the proposed DSTTC has minimum memory order, and the construction of such DSTTCs is equivalent to using the generator matrices that have full row rank, regardless of sub-matrices shifting problem. Here, a sub-matrix corresponds to the generator matrix of a relay node under frequency-selective channels. We derive sufficient conditions on the code design such that the full cooperative and multipath diversities can be achieved. By further studying the diversity product, we design the DSTTCs which can achieve full diversity and the maximum coding gain through the exhaustive computer search. The newly proposed codes exhibit good properties, e.g., high energy efficiency and low synchronization cost, and can be applied to distributed wireless networks. Finally, various numerical examples are provided to corroborate the analytical studies.**

***Index Terms*—Asynchronous cooperative diversity, wireless sensor networks, distributed space-time trellis code, multiple-input multiple-output systems, cooperative communications.**

I. Introduction

THE EFFECT of fading can deteriorate the performance of the wireless communications and cause large variations in signal strength as a function of the user position. Diversities that resulted from spatial, temporal, and frequency domains, are the powerful technologies to combat fading. The spatial diversity can be exploited by equipping multiple antennas at the transmitter and/or the receiver. However, it meets much difficulty when placing multiple antennas onto a mobile terminal or a sensor node due to the size limit and the hardware complexity. It was shown recently that the spatial diversity for small terminals can be exploited if cooperation is adopted among users [1], [2], [3]. The corresponding transmission scheme is referred to as the cooperative communications [3].

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Z. Zhong and S. Zhu are with the Department of Information and Communication Engineering, Xi'an Jiaotong University, Xi'an, 710049, P. R. China (e-mail: {zmzhong, szhu}@mail.xjtu.edu.cn).

A. Nallanathan is with the Division of Engineering, King's College London, United Kingdom (e-mail: nallanathan@ieee.org).

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Different cooperative protocols have been introduced in [3]. The distributed space-time code (DSTC) protocol is proposed in [4], where space-time code (STC) is applied at each relay node to pass the data from source to destination simultaneously. Perfect synchronization among users is assumed in many cooperative diversity works [4]-[9]. Since each terminal has its own oscillator and sampling clock, it is hard to achieve perfect synchronization among these terminals. Recently, asynchronous cooperative diversity has been discussed in [10]-[18]. In [10], an intentional random delay is introduced to the relays' information packets, and a fractional spaced decision feedback equalizer (FSDFE) is applied to exploit the cooperative diversity. However, this method only gets some diversity order but cannot achieve full diversity. In [11], the idea of time reversal space-time block codes (STBC) is used and the perfect synchronization assumption is released. This transmission scheme can achieve both full cooperative diversity and full multipath diversity by sacrificing the transmission rate whenever more relay nodes are involved into the networks. In [12], the effects of synchronization errors on parallel relays are analyzed by using the early-late gate technique. Meanwhile [12], [13], [14], propose to use space-time coded orthogonal frequency division multiplexing (STC-OFDM) to combat the timing errors. Although STC-OFDM realizes full cooperative diversity, it cannot achieve full multipath diversity when the channels between the relays and the destination node are frequency-selective. In [15], the distributed threaded algebraic space-time code that achieves full cooperative diversity without synchronization is proposed. Furthermore, the authors of [16] propose a family of distributed space-time trellis code (DSTTC) that can achieve full cooperative diversity in asynchronous communication systems. The systematic constructions of such trellis code with the minimum memory sizes is developed later in [17]. Note that, the code in [15]-[17] are designed under the assumption that the channels between relays and destination are flat fading. Nonetheless, [18] proposes the space-frequency code (SFC) for asynchronous cooperative communications where OFDM is adopted to combat the timing errors. The derived SFC can achieve both full cooperative diversity and full multipath diversity. However, it is well known that OFDM is sensitive to carrier frequency offset (CFO) and has the peak-to-average power ratio (PAPR) problem.

In this paper, we use the stack construction method from [19], [20] to build a family of DSTTC with minimum memory

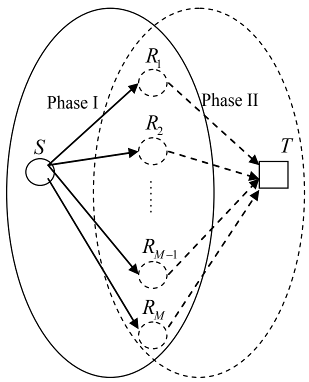

Fig. 1. Transmission Protocol.

order for asynchronous cooperative communications where the channels are considered frequency-selective. We derive sufficient conditions on the code design such that the full diversity can be achieved. Furthermore, the upper bound of the diversity product of the proposed DSTTCs is studied. We also construct DSTTCs that have full diversity order and the maximum coding gain. Simulations demonstrate that the newly proposed DSTTC can achieve both full cooperative diversity and full multipath diversity for asynchronous communications.

This paper is organized as follows. In Section II, the system model is presented and the problem is formulated. In Section III, the new family of DSTTC is developed. In Section IV, theoretical analysis on the upper bound of the diversity product is derived. Code design which gives the maximum upper bound of the diversity product is also provided in this section. Finally, simulations are conducted in section V and conclusions are made in Section VI.

Notations: Vectors and matrices are boldface small and capital letters; the transpose, complex conjugate, Hermitian of the matrix $\mathbf{A}$ are denoted by $\mathbf{A}^T$, $\mathbf{A}^*$, $\mathbf{A}^H$, respectively; $\mathrm{Tr}(\mathbf{A})$ and $\|\mathbf{A}\|_F$ are the trace and the Frobenius norm of $\mathbf{A}$; $(\mathbf{A})_{i,j}$ is the (i,j)th entry of $\mathbf{A}$.

II. System Model and Problem Formulation

Consider a system with $M+2$ nodes that communicate cooperatively as shown in Fig. 1. We assume that there is one source node S, one destination node T, and M relays $R_i, i = 1, 2, \ldots, M$. We also assume that there is no direct connection between the source and the destination (for example due to shadowing or too large separation) and that all terminals operate in half-duplex fashion [3]. We consider the *decode-and-forward* (DF) transmission protocol that consists of two phases. During phase I, S broadcasts its information to all the relays. During phase II, each relay firstly checks whether the decoding is successful according to Cyclic Redundancy Check (CRC) bits that was inserted by the source, then, if the decoding is successful, the relays will encode the information and forward the encoded data to the destination. We assume that CRC is able to detect all the packet errors [10], [16]. In this section, we use the similar denotations in [16], [17] to describe the system model.

Define $\mathcal{R}_s$ as the set of potential relays that decode successfully, where $M_s = |\mathcal{R}_s|$ is the cardinality of $\mathcal{R}_s$. Clearly, $\mathcal{R}_s$ is determined by the channel quality between the source and the relay nodes, and M_s can be considered as a random variable. If the potential relays are synchronized up to a symbol duration, T receives

$$y(n) = \sum_{1 \le i \le M, R_i \in \mathcal{R}_s} \sum_{l=0}^{L_i} h_i(l) s_i(n-l) + z(n), \quad (1)$$

where $h_i(l)$ is the lth path gain from R_i to T and is a circularly complex Gaussian random variable with variance $\sigma_i^2(l)$. In addition, L_i is the length of the channel impulse response (CIR) from R_i to T. The channel gains are normalized such that $\sum_{l=0}^{L_i} \sigma_i^2(l) = 1$ for any R_i. Moreover, $s_i(n)$ is the symbol transmitted by R_i and is encoded based on the information decoded by relay R_i, while $z(n)$ is the additive white Gaussian noise whose variance is N_0. Since the terminals are separated by sufficiently large distances, different $h_i(l)$ can be reasonably assumed independent from each other. We consider the quasi-static fading process where $h_i(l)$ is constant over one packet but may vary independently from packet to packet. Furthermore, CIR is assumed perfectly known at the destination.

As in the conventional STC design, full diversity is preferred for a good performance. However the following three differences should be kept in mind:

1) The number of the potential relays is random, which says that the number of rows in an STC matrix is random [17];
2) The power delay profiles of the CIRs for different relays are different;
3) The perfect synchronization is hard to realize such that the rows in the STC matrix are not symbol-aligned.

Define $L = \max_{1 \le i \le M} L_i$. The first two differences can be easily solved by finding an STC matrix $\mathbf{S}$ that has dimension $M(L+1) \times (N+L)$ and can achieve full diversity (We will give the details about the construction on $\mathbf{S}$ in the next section.), because the matrix obtained by deleting a submatrix in $\mathbf{S}$ can also achieve the full diversity [16]-[18], where N is the length of the data transmitted by relays. Here, deletion of a submatrix in $\mathbf{S}$ corresponds to the absence of one relay, which is analogous to that relay experiencing a deep fade. Hence $\mathbf{S}$ can achieve full diversity order of $M(L+1)$. It also has full diversity order $M_s + \sum_{R_i \in \mathcal{R}_s} L_i$, if $M - M_s$ relays fail to decode information correctly. Without loss of generality, we consider that M relays are all enrolled in phase II and the length of channel impulse response for each relay is L in the following. Thus (1) can be rewritten in the matrix form:

$$\mathbf{y} = \mathbf{h}\mathbf{S} + \mathbf{z}, \quad (2)$$

where $\mathbf{y}$ is the received row vector of length $(N+L)$, $\mathbf{z}$ is the noise vector of length $(N+L)$, $\mathbf{h}$ is the $1 \times M(L+1)$ vector with the form

$$\mathbf{h} \triangleq [h_1(0), \ldots, h_1(L), h_2(0), \ldots, h_2(L), \ldots, h_M(L)]. \quad (3)$$

Specifically, $\mathbf{S}$ has the structure

$$\mathbf{S} = [\mathbf{S}_1^T, \mathbf{S}_2^T, \ldots, \mathbf{S}_M^T]^T, \quad (4)$$

and

$$\mathbf{S}_i = \begin{bmatrix} s_i(1) & s_i(2) & \dots & s_i(N) & \dots & \star \\ \star & s_i(1) & \dots & s_i(N-1) & \dots & \star \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \star & \star & \dots & s_i(N-L) & \dots & s_i(N) \end{bmatrix}, \quad (5)$$

where $\star$ represent the dumb symbol. If we consider the third difference, equation (2) can be rewritten as

$$\mathbf{y} = \mathbf{h}\mathbf{S}^a + \mathbf{z}, \quad (6)$$

where the asynchronous version of $\mathbf{S}$ is defined as

$$\mathbf{S}^a = \left[\left(\mathbf{S}_1^a\right)^T, \left(\mathbf{S}_2^a\right)^T, \dots, \left(\mathbf{S}_M^a\right)^T\right]^T, \quad (7)$$

and

$$\mathbf{S}_i^a = \left[\star_{(L+1)\times\tau_i}, \quad \mathbf{S}_i, \quad \star_{(L+1)\times(\tau-\tau_i)}\right], i = 1, \dots, M, \quad (8)$$

where $\star_{m\times n}$ stands for an $m \times n$ matrix whose entries are all dumb symbols, τ_i is the timing error of relay R_i and $\tau = \max_{1\le i\le M}\{\tau_i\}$. Since fractional delays are contributed to channel dispersion [11], [17], we assume that the relative timing errors between different relays are integer multiples of the symbol duration. We also assume that these relative timing errors are known at the receiver but not known at the transmitter, because the timing errors can be estimated by sending training sequences [21]. Although the symbol synchronization is not required in the above *asynchronous cooperative communications*, in order to eliminate inter-packet interference, we assume that each packet in different enrolled relays is preceded by a preamble, whose length is not less than $L_e + L$, where L_e is the upper bound of the timing errors (one may view this upper bound as a system parameter set by the physical layer design). All the symbols in this preamble are dumb symbols $\star$.

From equation (7), one can see that the full diversity is hard to achieve since the submatrices in the STC matrix $\mathbf{S}^a$ are not symbol-aligned. Hence the performance of the conventional STC such as the orthogonal space-time block code and the delay diversity in [22] will degrade significantly in the asynchronous case. To achieve full diversity in such asynchronous transmission, we provide ways to design $\mathbf{S}$ in the next section.

III. Code Construction

In this section, we first consider the case where each element in $\mathbf{S}$ is a BPSK symbol based on the algebraic stack construction [23]. We will show that the shift-full-rank (SFR) matrices properties in [17] cannot be used to construct DSTTCs to achieve full diversity order under frequency-selective channels. Then we generalize the code design to QAM, PSK and PAM signal constellations by using the unified construction developed in [20]. Before proceeding to discuss the code construction, we first clarify some definitions which will be used for the rest of this paper.

Definition 1: *Basic vector* is defined as the vector $\mathbf{v}$ over the binary field $\mathbb{F}_2 \triangleq \{0, 1\}$ whose most left 1 corresponds to the first column. For example, $\mathbf{v} = 1011$ is a *basic vector*.

Definition 2: The length $l(\mathbf{v})$ of a binary row vector $\mathbf{v}$ is defined as the number of components between the most left and the most right 1's in $\mathbf{v}$, including the two 1's themselves. In particular, let $l(\mathbf{0}) = 0$ and the length of a vector with only one nonzero component is defined as 1. For example, the length $l(\mathbf{v})$ of $\mathbf{v} = 1011$ is 4. We define $\mathcal{V}$ to be the set of all binary row vectors with finite lengths.

Definition 3: For any vector $\mathbf{v} \in \mathcal{V}$, $\mathbf{v}^{(j)}$ denotes the row vector resulted from $\mathbf{v}$ with each component shifted j bits to the right and zeros are padded to its two ends if needed. For example, the 3 bits right-shift of the binary row vector $\mathbf{v} = 1011$ is $\mathbf{v}^{(3)} = 0001011$. To get a matrix as $\mathbf{G} = \left[\mathbf{v}^T, (\mathbf{v}^{(j)})^T, \dots\right]^T$, we need to add zeros to the right end of each row if necessary. Considering the above example, $\mathbf{G} = \left[\mathbf{v}^T, (\mathbf{v}^{(3)})^T\right]^T$ can be expressed as

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}.$$

Definition 4: For two vectors $\mathbf{v}, \mathbf{u} \in \mathcal{V}$, $\mathbf{v} \circ \mathbf{u}$ denotes their convolution and $\mathbf{v} + \mathbf{u}$ is the component-wise addition over the binary field $\mathbb{F}_2$.

A. Generator Matrix Model for DSTTC

If the source information bits are correctly detected by a relay R_i, they will be sent to a linear shift register with tapped coefficients $(g_{i,0}, g_{i,1}, \dots, g_{i,v})$, where $g_{i,d} \in \mathbb{F}_2$ for $d = 0, 1, \dots, v$, and v is the maximal memory order. Define the generator polynomial $g_i(D) = g_{i,0} + g_{i,1}D + \dots + g_{i,v}D^v$ and the corresponding coefficient vector of $g_i(D)$ as $\mathbf{g}_i = [g_{i,0}, g_{i,1}, \dots, g_{i,v}]$, where D represents the symbol delay. The proposed DSTTC is based on the idea of "virtual transmit antennas". Namely, in an ISI environment with $L + 1$ resolvable paths, a space-time system with M relays or transmit antennas is equivalent to a space-time system operating in flat-fading channel with $M(L+1)$ relays or transmit antennas [19]. Moreover, each R_i with $L + 1$ resolvable paths is equivalent to a relay with $L + 1$ antennas whose generator polynomials respectively are $g_{i0}(D), g_{i1}(D), \dots, g_{iL}(D)$, and $g_{ij}(D) = D^j g_i(D)$ for each path j [19], $j = 0, \dots, L$. Thus, the coefficient matrix of the generator polynomials $g_{i0}(D), g_{i1}(D), \dots, g_{iL}(D)$ is

$$\mathbf{G}_{M,i} \triangleq \begin{bmatrix} \mathbf{g}_{i0} \\ \mathbf{g}_{i1} \\ \vdots \\ \mathbf{g}_{iL} \end{bmatrix}_{(L+1)\times(v+L+1)}, \quad (9)$$

where $\mathbf{g}_{ij} \triangleq \mathbf{g}_i^{(j)}$ is the corresponding coefficient vector for each path j, $j = 0, \dots, L$.

The generator matrix over frequency-selective channels is then equivalent to

$$\mathbf{G}_M = \left[\mathbf{G}_{M,1}^T, \mathbf{G}_{M,2}^T, \dots, \mathbf{G}_{M,M}^T\right]^T. \quad (10)$$

If the binary source information bits detected in the relays in one packet are $\bar{\mathbf{u}} \in \mathbb{F}_2^{1\times L_u}$, then the binary output of all the paths belongs to the set

$$\mathcal{C} =\Big\{\mathbf{C}(\bar{\mathbf{u}}) \in \mathbb{F}_2^{M(L+1)\times(v+L_u+L)} \mid \Big[(\mathbf{c}_{10}(\bar{\mathbf{u}}))^T,\ldots,(\mathbf{c}_{1L}(\bar{\mathbf{u}}))^T ,\ldots,(\mathbf{c}_{M0}(\bar{\mathbf{u}}))^T,\ldots,(\mathbf{c}_{ML}(\bar{\mathbf{u}}))^T\Big]^T, \bar{\mathbf{u}} \in \mathbb{F}_2^{1\times L_u}\Big\}, \quad (11)$$

where $\mathbf{c}_{ij}(\bar{\mathbf{u}}) = \bar{\mathbf{u}} \circ \mathbf{g}_{ij}$ is the binary output vector for the jth path of relay R_i, and

$$\mathbf{C}(\bar{\mathbf{u}}) = \Big[(\bar{\mathbf{u}} \circ \mathbf{g}_{10})^T, \cdots, (\bar{\mathbf{u}} \circ \mathbf{g}_{1L})^T, \cdots, (\bar{\mathbf{u}} \circ \mathbf{g}_{M0})^T, \cdots, (\bar{\mathbf{u}} \circ \mathbf{g}_{ML})^T\Big]^T \triangleq \bar{\mathbf{u}} \circ \mathbf{G}_M. \quad (12)$$

In turn, the DSTTC generated by $\mathbf{G}_M$ belongs to the set

$$\mathcal{S} =\{\mathbf{S} \in \mathbb{C}^{M(L+1)\times(v+L_u+L)} | (\mathbf{S})_{m,n} = (-1)^{(\mathbf{C}(\bar{\mathbf{u}}))_{m,n}}, \mathbf{C}(\bar{\mathbf{u}}) \in \mathcal{C}\}, \quad (13)$$

and possesses trellis structure. For example, if $M = 2, L = 1$ and $g_1(D) = 1+D+D^2$, $g_2(D) = 1+D^2$, then $g_{10}(D) = 1+D+D^2$, $g_{11}(D) = D+D^2+D^3$, $g_{20}(D) = 1+D^2$, $g_{21}(D) = D + D^3$. The corresponding trellis structure is shown in Fig. 2. In this code structure, if the length of information bits in one packet is L_u, the rate of the space-time trellis code $\mathcal{S}$ generated by $\mathbf{G}_M$ is $L_u/(L_u + v + L + L_e)$ bits/sec/Hz. For long data packet, the rate approaches 1 bits/sec/Hz. In the following, we investigate conditions on the generator matrix $\mathbf{G}_M$ to achieve the full diversity.

Assuming that the relative timing error of relay R_i is τ_i, i.e., τ_i dumb symbols $\star$ are padded to the left of $((i-1)(L+1)+j)$th row in the signal matrix $\mathbf{S}$, $j = 1,\ldots,L+1$, as shown in (7). If dumb symbol $\star = 1$, then this is equivalent to padding τ_i zeros to the left of the $((i-1)(L+1)+j)$th row of binary matrix $\mathbf{C}(\bar{\mathbf{u}})$, $j = 1,\ldots,L+1$. These matrices can be generated by the generator polynomials $D^{\tau_1}g_{10}(D),\ldots,D^{\tau_1}g_{1L}(D),\ldots, D^{\tau_M}g_{M0}(D),\ldots,D^{\tau_M}g_{ML}(D)$. For example, if $M = 2, L = 1$ and $g_1(D) = 1 + D + D^2$, $g_2(D) = 1 + D^2$. If the second relay R_2 has one relative timing error, i.e., $\tau_2 = 1$, then the equivalent generator polynomials for each path are $g_{10}(D) = 1 + D + D^2$, $g_{11}(D) = D + D^2 + D^3$, $Dg_{20}(D) = D + D^3$, and $Dg_{21}(D) = D^2 + D^4$, respectively. The equivalent trellis structure is shown in Fig. 2. To ensure the full diversity in the asynchronous cooperative communication, there are requirements on the tapped coefficients $g_{i,d}, i = 1,2,\ldots,M, d = 0,1,\ldots,v$, stated in the following theorem.

Theorem 1: Define the asynchronous version of the generator matrix $\mathbf{G}_M$ as

$$\mathbf{G}_M^a = [(\mathbf{G}_{M,1}^a)^T,\cdots,(\mathbf{G}_{M,M}^a)^T]^T = \Big[\Big(\mathbf{g}_{10}^{(\tau_1)}\Big)^T,\cdots,\Big(\mathbf{g}_{1L}^{(\tau_1)}\Big)^T,\cdots,\Big(\mathbf{g}_{M0}^{(\tau_M)}\Big)^T,\cdots,\Big(\mathbf{g}_{ML}^{(\tau_M)}\Big)^T\Big]^T.$$

The STC generated by $g_1(D), g_2(D),\ldots,g_M(D)$ achieves full diversity in the asynchronous cooperative communication if and only if $\mathbf{G}_M^a$ has full rank in the binary field $\mathbb{F}_2$ for arbitrary $\tau_1,\tau_2,\ldots,\tau_M$.

Proof: The proof follows the same argument of the stacking construction in flat-fading channels in [16]. ■

In Theorem 1, $\mathbf{G}_M^a$ is a submatrix-shifted version of $\mathbf{G}_M$ and the shifted amount τ_i is arbitrary for each submatrix $\mathbf{G}_{M,i}$. The importance of Theorem 1 lies in that, we only need to construct $g_i(D)$ such that any submatrix-shifted version $\mathbf{G}_M^a$ of the generator matrix $\mathbf{G}_M$ has full rank.

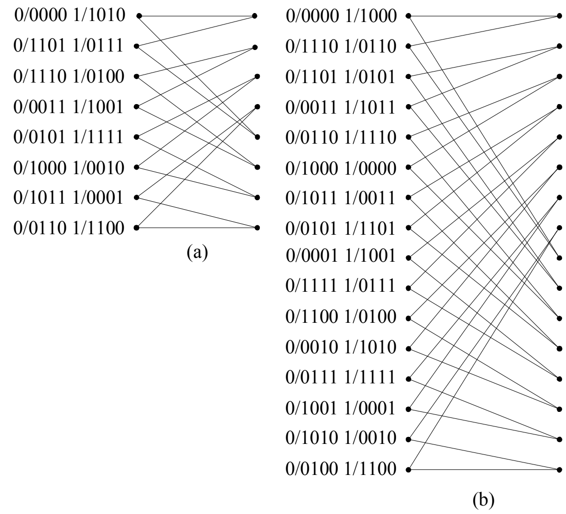

Fig. 2. (a) Trellis structure for $g_{10}(D), g_{11}(D), g_{20}(D)$, and $g_{21}(D)$. (b) Trellis structure for $g_{10}(D), g_{11}(D), Dg_{20}(D)$, and $Dg_{21}(D)$ when $\tau_1 = 0$, $\tau_2 = 1$.

Remarks:

- The main difference between Theorem 1 and [16, Theorem 1] is that, $g_i(D)$ here is constructed in a way to ensure that any sub-matrix-shifted version $\mathbf{G}_M^a$ of the generator matrix $\mathbf{G}_M$ has full rank. If $L = 0$, i.e., flat fading channels, each sub-matrix $\mathbf{G}_{M,i}$ in $\mathbf{G}_M$ is degraded to one row only. Hence [16, Theorem 1] is a special case of the proposed Theorem 1.
- In [17], a family of the generator matrices, called SFR matrices, is constructed such that they have full row rank no matter how their rows are shifted. However, the SFR matrices are not shift-full-rank in frequency-selective channels, since each relay has $L+1$ paths and the tapped coefficients of each path are equivalent to the right shifted versions of each relay's tapped coefficients, as shown in (9). Besides, $\mathbf{G}_M$ consists of the tapped coefficients of the relays and the right shifted versions of each relay's tapped coefficients. If $\mathbf{G}$ is an SFR matrix, we cannot ensure the matrix $\mathbf{G}_M$ obtained by adding some rows (the nonzero shifted versions of the rows in $\mathbf{G}$) to $\mathbf{G}$ is an SFR matrix. An example is given here.

  Example 1: If there are two relays and channels are flat fading, the generator matrix

  $$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

  is an SFR matrix and it has full row rank no matter how its two rows are shifted. However in frequency-selective channels, if each channel has two paths. Then, according to (9), (10), $\mathbf{G}_2$ obtained by $\mathbf{G}$ is equivalent to

  $$\mathbf{G}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

If relay R_1 has one relative timing error, the asynchronous version of $\mathbf{G}_2$ does not have full row rank. Hence $\mathbf{G}$ cannot be used for asynchronous cooperative communications to achieve full diversity over frequency-selective channels.

In the following, we give sufficient conditions under which the asynchronous versions $\mathbf{G}_M^a$ of $\mathbf{G}_M$ always have full rank.

B. Generator Matrix Construction for DSTTC

Theorem 2: A necessary condition to ensure the full rank property of $\mathbf{G}_M$ is that the maximum memory order v of the generating polynomials $g_i(D), i = 1, \ldots, M$ should not be less than $(M-1)(L+1)$.

Proof: Since $\mathbf{G}_M$ is a binary matrix of dimension $M(L+1) \times (v+L+1)$, $v+L+1$ must not be less than $M(L+1)$, i.e., $v \geq (M-1)(L+1)$ in order to achieve full row rank. ∎

For the case that $M = 2$, we have the following theorem:

Theorem 3: For any two different *basic vectors* $\mathbf{v}_1, \mathbf{v}_2$ with $l(\mathbf{v}_1) = l(\mathbf{v}_2) \geq L+2$, if the generator matrix $\mathbf{G}_2 = \left[\mathbf{v}_1^T, \left(\mathbf{v}_1^{(1)}\right)^T, \ldots, \left(\mathbf{v}_1^{(L)}\right)^T, \mathbf{v}_2^T, \left(\mathbf{v}_2^{(1)}\right)^T, \ldots, \left(\mathbf{v}_2^{(L)}\right)^T\right]^T$ has full row rank in the binary field $\mathbb{F}_2$, its asynchronous versions $\mathbf{G}_2^a$ will also have full rank in the binary field $\mathbb{F}_2$.

Proof: See proof in Appendix A. ∎

We should note that $L+2$ is the minimum length of *basic vectors* for $M = 2$ according to both Theorem 2 and Theorem 3. Hence we only need to construct the full row rank binary matrix $\mathbf{G}_2$ from two different *basic vectors* that have the minimum length $L+2$. The following theorem gives a sufficient condition to ensure that $\mathbf{G}_2$ is of full row rank.

Theorem 4: For any two different *basic vectors* $\mathbf{v}_1, \mathbf{v}_2$ with $l(\mathbf{v}_1) = l(\mathbf{v}_2) = L+2$, if $l((\mathbf{v}_1 + \mathbf{v}_2) + (\mathbf{v}_1 + \mathbf{v}_2)^{(L)}) < L+2$, then $\mathbf{G}_2 = \left[\mathbf{v}_1^T, \left(\mathbf{v}_1^{(1)}\right)^T, \ldots, \left(\mathbf{v}_1^{(L)}\right)^T, \mathbf{v}_2^T, \left(\mathbf{v}_2^{(1)}\right)^T, \ldots, \left(\mathbf{v}_2^{(L)}\right)^T\right]^T$ is of full row rank.

Proof: See proof in Appendix B. ∎

Example 2: If $L = 1$, the *basic vectors* $\mathbf{v}_1, \mathbf{v}_2$ are 101 and 111 according to Theorem 3 and Theorem 4. Then, the generator matrix $\mathbf{G}_2$ can be written as

$$\mathbf{G}_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}.$$

We also list the *basic vectors* for the cases that $L = 2, 3, 4$ in Table I.

For scenario of $M > 2$, it is intractable to find a sufficient condition for $\mathbf{G}_M$ to ensure full cooperative and multipath diversities in asynchronous cooperative communication. Normally when there is only one antenna in the destination node, the performance gain achieved by more than four transmit antennas is marginal compared with that achieved by four transmit antennas [22], [24], and the diversity order beyond 5 is not necessary for a regular signal-to-noise ratio (SNR) range [25]. In addition, if M is too large, say $M = 5$ and $L = 1$, the minimal maximum memory order v is 8 and the trellis has $2^8 = 256$ states in the synchronous case. If asynchronous case is considered, trellis has more states than in the synchronous case. The resulted decoding complexity is prohibitively high. In a nutshell, selection of the number of relay nodes depends on the application-specific performance, the complexity, and the decoding delay tradeoffs. Hence, we only consider the case that $M < 5$ in the following discussions. For the three-relay case, we have the following theorem:

TABLE I
THE BINARY VECTORS IN OCTAL WITH OPTIMAL FREE DISTANCE OR MAXIMUM WEIGHT FOR DIFFERENT M AND L UNDER FREQUENCY-SELECTIVE CHANNELS.

M	L	$\mathbf{v}_1$	$\mathbf{v}_2$	$\mathbf{v}_3$	$\mathbf{v}_4$
2	1	5	7		
	2	15	17		
	3	23	35		
	4	53	75		
3	1	25	33	37	
	2	133	145	175	
	3	577	773	777	
4	1	176	157	173	177
	2	1775	1377	1737	1777

Theorem 5: For any three different *basic vectors* $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ with $l(\mathbf{v}_1) = l(\mathbf{v}_2) = l(\mathbf{v}_3) = 2L+3$, if the generator matrix $\mathbf{G}_3 = \left[\mathbf{v}_1^T, \ldots, \left(\mathbf{v}_1^{(L)}\right)^T, \mathbf{v}_2^T, \ldots, \left(\mathbf{v}_2^{(L)}\right)^T, \mathbf{v}_3^T, \ldots, \left(\mathbf{v}_3^{(L)}\right)^T\right]^T$ has full row rank in the binary field $\mathbb{F}_2$, its asynchronous versions $\mathbf{G}_3^a$ will also be full rank in $\mathbb{F}_2$.

Proof: See proof in Appendix C. ∎

Theorem 5 is similar to Theorem 3. It shows that when there are three relays and if the generator matrix $\mathbf{G}_3$ is constructed by three different *basic vectors* $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ with $l(\mathbf{v}_1) = l(\mathbf{v}_2) = l(\mathbf{v}_3) = 2L+3$, then $\mathbf{G}_3$ being full row rank indicates that its asynchronous version $\mathbf{G}_3^a$ also has full row rank. Hence we only need to find *basic vectors* $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ with $l(\mathbf{v}_1) = l(\mathbf{v}_2) = l(\mathbf{v}_3) = 2L+3$ such that $\mathbf{G}_3$ has full row rank.

When $M > 3$, it can be easily checked that the asynchronous versions of the generator matrix may not be full rank even if the generator matrix itself is full rank. According to Theorem 1 and Theorem 2, through the exhaustive computer search, we can find the binary vectors that make all the asynchronous versions of the generator matrix full rank for arbitrary M. The search results are displayed in Table I for different length L and $M = 4$, as well as those for $M = 2, 3$ which are found from Theorem 3, Theorem 4 and Theorem 5.

It should be mentioned that the above results are obtained for BPSK signals only. For QAM, PSK and PAM signals, a general result can also be obtained based on the unified construction method [20, Theorem 4].

Theorem 6: Let K, U be two positive integers. Define

$$\{\mathcal{C}_{i,j} | 0 \leq i \leq U-1, 0 \leq j \leq K-1\}$$

as the collection of UK sets of $M(L+1) \times (v+L+L_u)$ binary matrices generated from $\mathbf{G}_M$ using (12) with UK independent binary vectors $\bar{\mathbf{u}}^{(i,j)}$ of dimension L_u. Let θ be a primitive 2^K-th root of unity. Let $0 \neq \eta \in 2\mathbb{Z}[\theta]$, where $2\mathbb{Z}[\theta]$ is the ideal generated by 2 in $\mathbb{Z}[\theta]$ with $\mathbb{Z}[\theta]$ being the ring of algebraic integers in the cyclotomic number field $\mathbb{Q}(\theta)$. Let

$$f : \mathcal{C}_{0,1} \times \mathcal{C}_{0,2} \times \cdots \mathcal{C}_{U-1,K-1} \rightarrow \mathcal{S} \subset \mathbb{C}^{M(L+1)\times(v+L+L_u)}$$

be the mapping defined as

$$(\mathbf{C}_{0,0}, \mathbf{C}_{0,1}, \ldots, \mathbf{C}_{U-1,K-1}) \rightarrow \kappa \sum_{i=0}^{U-1} \eta^i \theta^{\sum_{j=0}^{K-1} 2^j \mathbf{C}_{i,j}},$$

where κ is a nonzero complex number, $\mathbf{C}_{i,j}$ is a matrix in the binary matrix set $\mathcal{C}_{i,j}$, and $\mathbb{C}$ is the complex number field. The multiplication and exponential of $\mathbf{C}_{i,j}$ to θ are carried out entry by entry. If the generator matrix $\mathbf{G}_M$ satisfies the condition in Theorem 1, the STC $\mathcal{S}$ generated from the above mapping f could achieve full diversity for asynchronous cooperative communications under frequency-selective channels.

This theorem can be proved similarly to [20, Theorem 4] by noting that the timing errors in relays can be mapped to the shift bits in $\mathbf{G}_M$, and the dumb symbol sent by relays is $\star = \kappa \sum_{i=0}^{U-1} \eta^i$.

IV. Diversity Product Analysis

The analysis for the exact diversity product of every asynchronous version is intractable. In the following, we consider the upper bound of the diversity product. Suppose $\mathbf{S}$, $\bar{\mathbf{S}}$ are two different code matrices and define $\triangle\mathbf{S} \triangleq \mathbf{S} - \bar{\mathbf{S}}$. We assume that $\lambda_1, \ldots, \lambda_{M(L+1)}$ are the eigenvalues of $\triangle\mathbf{S}(\triangle\mathbf{S})^H$. The diversity product of the proposed DSTTC is defined as

$$\zeta(\mathcal{S}) = \min_{\mathbf{S}\neq\bar{\mathbf{S}}, \mathbf{S}, \bar{\mathbf{S}}\in\mathcal{S}} \left(\prod_{i=1}^{M(L+1)} \lambda_i \right)^{1/M(L+1)}. \quad (14)$$

From the arithmetic-mean geometric-mean inequality we know

$$\zeta(\mathcal{S}) \leq \min_{\mathbf{S}\neq\bar{\mathbf{S}}, \mathbf{S}, \bar{\mathbf{S}}\in\mathcal{S}} \left(\frac{\sum_{i=1}^{M(L+1)} \lambda_i}{M(L+1)} \right) = \left(\frac{\mathrm{Tr}(\triangle\mathbf{S}(\triangle\mathbf{S})^H)}{M(L+1)} \right)$$
$$= \left(\frac{\| \triangle \mathbf{S} \|_F^2}{M(L+1)} \right) \triangleq \zeta^{up}(\mathcal{S}). \quad (15)$$

For asynchronous cases, there is $\zeta^{up}(\mathcal{S}^a) = \zeta^{up}(\mathcal{S})$ [18]. For BPSK modulation, $\zeta^{up}(\mathcal{S}^a)$ is upper bounded by [20], [26] as:

$$\zeta^{up}(\mathcal{S}^a) \leq \frac{4}{M(L+1)} \min_{\mathbf{C}(\bar{\mathbf{u}})\neq 0} \omega(\mathbf{C}(\bar{\mathbf{u}}))$$
$$= \frac{4}{M(L+1)} \min_{\bar{u}\neq 0} \omega(\bar{\mathbf{u}} \circ \mathbf{G}_M^a) \leq \frac{4}{M(L+1)} \omega(\mathbf{G}_M^a), \quad (16)$$

where ω is the Hamming weight of a nonzero code matrix. Equation (16) is also held for 2^U-PAM or 4^U-QAM constellation [20]. For a special case where $\bar{\mathbf{u}}$ has weight of 1, $\omega(\bar{\mathbf{u}} \circ \mathbf{G}_M^a)$ is equal to $\omega(\mathbf{G}_M^a)$. Hence the minimum weight must be no greater than the weight for this special case which proves the last inequality in (16).

The first inequality in (16) shows that the diversity product is upper bounded by the minimum Hamming free distance of the rate $1/M$ convolutional code generated by the corresponding generator polynomials $[g_1(D), \ldots, g_M(D)]$. However, the optimal Hamming free distance convolutional codes may not satisfy the full diversity property in asynchronous cooperative communications. The second inequality in (16) shows that the larger weight of the generator matrix is, the greater the upper bound of the diversity product will be. While we cannot ensure that the larger the weight is, the better the performance will be, there is indeed a trend that the diversity product improves when the weight of $\mathbf{G}_M$ increases, which has been verified in [26], [27]. Therefore, under the requirement for the minimum memory order, if the optimal Hamming free distance convolutional codes do not have the full diversity property. Then, we always wish to choose an $M(L+1) \times M(L+1)$ binary matrix as a generator matrix $\mathbf{G}_M$ whose weight is as large as possible. Consequently, we need to find those binary vectors that determine the weight of the generator matrix. Through the exhaustive computer search, we give some binary vectors in Table I that can ensure full diversity order and have the optimal hamming free distance or maximum weight for the different length L and M under the constraint of the minimum memory order.

V. Simulation Results

In this section, we evaluate the performance of our DSTTC through various numerical examples. In all the examples, we assume that a frame contains 60 information bits and the channels are quasi-static Rayleigh fading channel. Unless otherwise stated, we assume that no errors occur in phase I transmission and all the power delay profiles are uniform. We also assume that there is only one antenna in the destination node and the random delays are uniformly selected from the set $\{0, 1, \ldots, L_e\}$. Unless otherwise mentioned, we assume that $L_e = 2$. In addition, BPSK modulation is used. Here, although the rates of the synchronous case and the asynchronous cases are different, the frame length in our simulations makes the performance comparison of these cases reasonable.

We first compare the frame error rate (FER) performances for delay diversity (DD) method in [22], the space-time trellis code generated by SFR matrix (SFR code) in *example 1* and our DSTTC in *example 2*. Two relays with $L_1 = L_2 = 1$ are assumed. Both the asynchronous and synchronous transmissions are considered. The FER performances versus SNR of these three codes are shown in Fig. 3. We see that our DSTTC outperforms the DD method in both the asynchronous and synchronous cases. That is to say, our DSTTC can achieve full diversity order of 4, which agrees well with the theoretical studies. In addition, the performance of the DD method degrades significantly for asynchronous case since its code error matrix does not have full rank and the achievable diversity order for DD in asynchronous case is 2. It is also seen that the performance of the code generated by SFR matrix degrades for asynchronous case, because it can only achieve the diversity order of 3 under frequency-selective channels. On the other hand, our proposed DSTTC has the same complexity with the DD method and the space-time trellis code generated by SFR matrix due to the same number of states used.

Next we show the performance of the proposed DSTTC in *example 2* under the synchronous case and the asynchronous case with $L_e = 2$ and $L_e = 4$ in Fig. 4. We see that the performance of our proposed DSTTC improves when the relative timing errors range increases. This is because that the case with asynchronous transmission or larger L_e has a

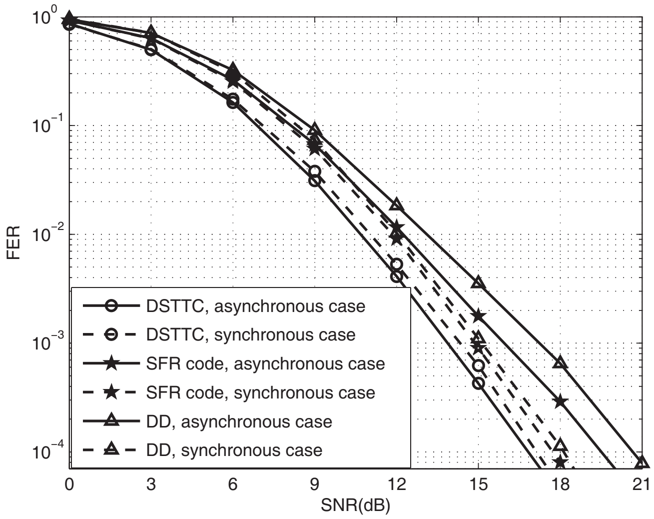

Fig. 3. Comparison of FER for delay diversity, SFR code and DSTTC.

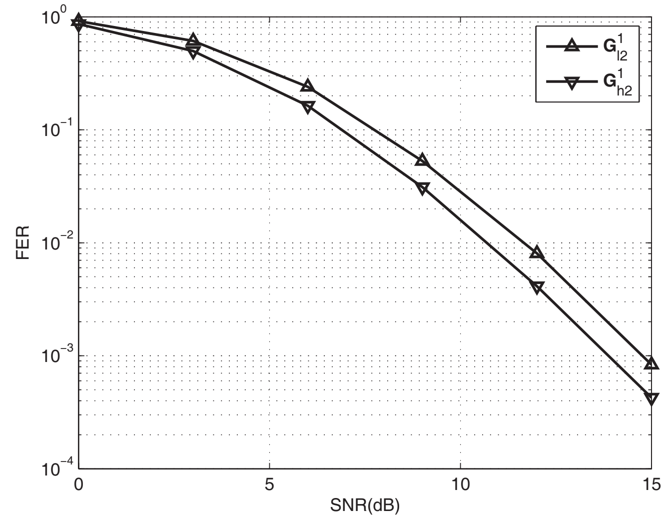

Fig. 5. Comparison of FER for DSTTC generated by $\mathbf{G}_{l2}^1$ and $\mathbf{G}_{h2}^1$.

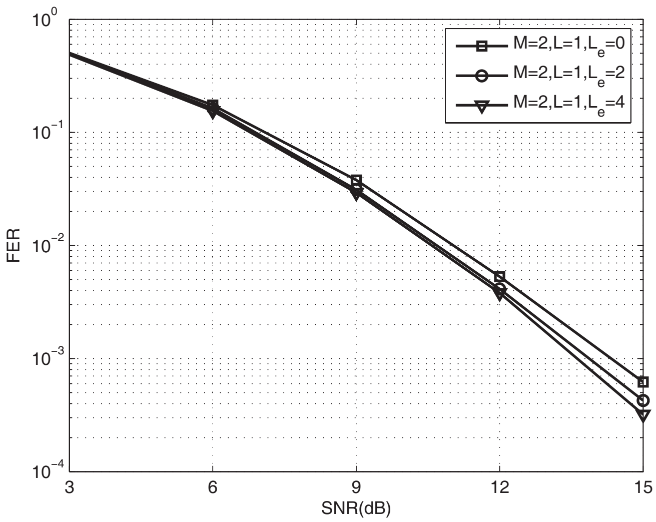

Fig. 4. FER Comparison of DSTTC for different L_e.

larger memory order. According to Eq.(8-2-36) in [28], the hamming free distance is upper bounded by the memory order, so the upper bound of the Hamming free distance in the asynchronous case is higher than that of the synchronous case. Thus the asynchronous case or larger L_e case will have bigger coding gain than the synchronous case or smaller L_e.

To check the performance of the DSTTC with optimal Hamming free distance, we show, in Fig. 5, the performance of the DSTTCs generated from the generator matrices $\mathbf{G}_{l2}^1$ and $\mathbf{G}_{h2}^1$ constructed by the polynomials $[1, 5]$ and the optimal free distance convolutional code for $M = 2, L = 1$ from Table I, respectively. We can easily check that both of them can achieve full diversity without symbol synchronization. From Fig. 5, we see that $\mathbf{G}_{h2}^1$ outperforms $\mathbf{G}_{l2}^1$. To verify the relationship between the weight of the generator matrix and the diversity product, in Fig. 6 we compare the DSTTCs for $M = 3, L = 1$, which are generated from three generator matrices $\mathbf{G}_{l3}^1$, $\mathbf{G}_{m3}^1$ and $\mathbf{G}_{h3}^1$ constructed by the polynomials $[1, 21, 24]$, $[7, 21, 24]$ and $[35, 27, 37]$, respectively, where $\omega(\mathbf{G}_{l3}^1) = 10, \omega(\mathbf{G}_{m3}^1) = 14$ and $\omega(\mathbf{G}_{h3}^1) = 26$. To verify that these three DSTTCs can achieve full diversity without synchronization assumption, we show the performance of delay diversity in the synchronous case in Fig. 6 as a benchmark. We also show the performance of the SFR code constructed by the polynomials $[20, 12, 3]$ in the asynchronous case. In Fig. 6 we see that both of our DSTTCs and the delay diversity method have identical slopes, which agrees well with the theoretical studies. We also observe from Fig. 6 that the FER performance of the SFR code has a smaller slope than those of the DSTTCs and delay diversity, since the SFR code cannot achieve full diversity order. Moreover, the DSTTC, which is constructed from the generator matrix with a larger weight, gives better performance, confirming our diversity product analysis.

Finally, we show the proposed DSTTC when errors may occur during phase I transmission in Fig. 7. We assume that there are 4 relays. The channel power profiles for the channels from $R_1, \ldots, R_4$ to T are $[2/3, 1/3]$, $[3/4, 1/4]$, $[1/2, 1/2]$, and $[1/2, 1/2]$, respectively. We use the code for $M = 4, L = 1$ in Table I. Those relays which can detect the entire packet correctly, encode the information by its generator polynomial and forward the encoded packet to T in phase II. The SNR in phase I is denoted by SNR_{sr}. The asynchronous case is considered. As a benchmark, we also show the SFR code generated by the polynomials $[100, 140, 120, 17]$. Firstly, we see that the performances of the DSTTC degrade as SNR_{sr} decreases. This is because that less potential relays may participate in phase II transmission and the achieved full diversity order $M_s + \sum_{R_i \in \mathcal{R}_s} L_i$ is decreased when SNR_{sr} decreases. From Fig. 7 we also see that the DSTTC performs better than the SFR code, since the code generated by SFR matrix cannot achieve full diversity in asynchronous scenarios.

VI. Conclusions

In this paper, we propose a novel family of DSTTC that achieves full cooperative and multipath diversities under asynchronous cooperative communications. The new DSTTC tolerates imperfect synchronization among relay nodes. We give sufficient conditions to construct such family of DSTTC with the minimum memory order. By further studying the

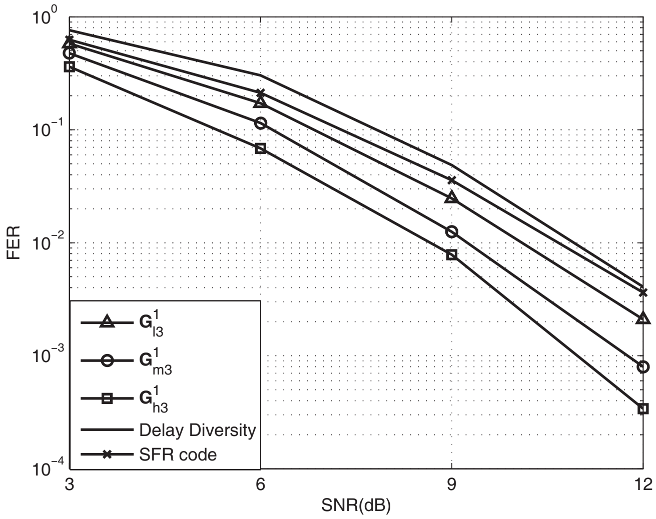

Fig. 6. Comparison of FER for delay diversity, SFR code and DSTTCs generated by $\mathbf{G}^1_{l3}$, $\mathbf{G}^1_{m3}$, and $\mathbf{G}^1_{h3}$.

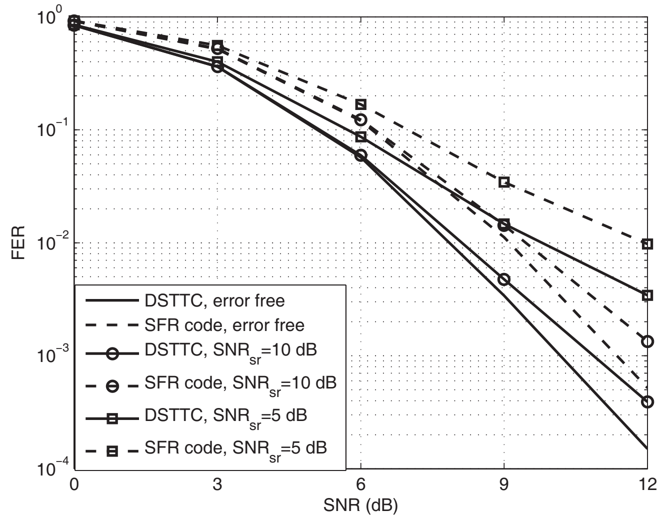

Fig. 7. Comparison of FER for DSTTC and SFR code when errors may occur in phase I.

diversity product, some optimal DSTTCs which give full diversity and maximum upper bound of the coding gain are found through the exhaustive computer search. Finally, various numerical examples are provided to corroborate the analytical studies. The proposed new technique can be applied to distributed wireless networks, e.g., sensor networks, to enhance transmission energy efficiency and to reduce synchronization cost.

Appendix A
Proof of Theorem 3

Suppose that the coefficient matrices of relay R_1 and relay R_2 are generated from $\mathbf{v}_1$ and $\mathbf{v}_2$, respectively. Without loss of generality, we define τ_1 and τ_2 as the timing errors of R_1 and R_2 respectively, and assume $\tau_1 \le \tau_2$. It is obvious that the row permutation does not affect the full diversity requirement of the asynchronous cooperative communication. Thus the case of $\tau_2 \le \tau_1$ is equivalent to the case of $\tau_1 \le \tau_2$. If $\tau_2 > \tau_1 + L$, the last $L+1$ rows in $\mathbf{G}_2^a$ are shifted by more bits compared with the first $L+1$ rows in $\mathbf{G}_2^a$. Remembering that $l(\mathbf{v}_1) = l(\mathbf{v}_2) \ge L+2$, the first $L+1$ rows cannot be obtained by the linear combination of the last $L+1$ rows and are linearly independent from each other. On the other hand, none of the last $L+1$ rows can be the linear combination of the first $L+1$ rows in $\mathbf{G}_2^a$. Clearly, $\mathbf{G}_2^a$ has full row rank. On the other hand, if $\tau_1 \le \tau_2 \le \tau_1 + L$, the first $2L+\tau_1-\tau_2+2$ rows in $\mathbf{G}_2^a$ belong to the matrix whose rows are all right shifted from $\mathbf{G}_2$ by τ_1 bits. Then the submatrix containing the first $2L+\tau_1-\tau_2+2$ rows in $\mathbf{G}_2^a$ must be full rank since $\mathbf{G}_2$ is a full rank matrix. Furthermore, the last $\tau_2 - \tau_1$ rows in $\mathbf{G}_2^a$ are shifted by more bits with respect to the first $2L+\tau_1-\tau_2+2$ rows in $\mathbf{G}_2^a$. Hence, none of the last $\tau_2 - \tau_1$ rows can be the linear combination of the first $2L+\tau_1-\tau_2+2$ rows. Moreover, none of the $2L+\tau_1-\tau_2+2$ rows can be the linear combination of the last $\tau_2-\tau_1$ rows in $\mathbf{G}_2^a$. From all the above, $\mathbf{G}_2^a$ must be full rank.

Appendix B
Proof of Theorem 4

According to the elementary row operation, the rank of $\mathbf{G}_2$ is equivalent to the rank of the matrix $\left[\mathbf{v}_1^T, \ldots, \left(\mathbf{v}_1^{(L)}\right)^T, (\mathbf{v}_1 + \mathbf{v}_2)^T, \ldots, \left(\mathbf{v}_1^{(L)} + \mathbf{v}_2^{(L)}\right)^T\right]^T$. Obviously, $\mathbf{v}_1, \ldots, \mathbf{v}_1^{(L)}$ are linearly independent from each other. Meanwhile, $\mathbf{v}_1 + \mathbf{v}_2, \ldots, \mathbf{v}_1^{(L)} + \mathbf{v}_2^{(L)}$ are also linearly independent from each other. We define two vector sets $\mathcal{V}_1 = \{\mathbf{v}_1, \ldots, \mathbf{v}_1^{(L)}\}$ and $\mathcal{V}_2 = \{\mathbf{v}_1 + \mathbf{v}_2, \ldots, \mathbf{v}_1^{(L)} + \mathbf{v}_2^{(L)}\}$. Note that $l(\mathbf{v}_1) = l(\mathbf{v}_2) = L+2$, and the length of the rows in $\mathcal{V}_2$ must be less than $L+2$. Hence no row in $\mathcal{V}_2$ is the linear combination of the rows in $\mathcal{V}_1$. The minimum length of any linear combination of the rows in $\mathcal{V}_1$ is $L+2$, because $l(\mathbf{v}_1) = l(\mathbf{v}_2) = L+2$. Therefore, if any linear combination of the rows in $\mathcal{V}_1$ can be obtained by the linear combination of the rows in $\mathcal{V}_2$, the length of the linear combination of the rows in $\mathcal{V}_2$ must not be less than $L+2$. However, the linear combination with maximum length in $\mathcal{V}_2$ is $(\mathbf{v}_1 + \mathbf{v}_2) + (\mathbf{v}_1 + \mathbf{v}_2)^{(L)}$. Since we have $l((\mathbf{v}_1 + \mathbf{v}_2) + (\mathbf{v}_1 + \mathbf{v}_2)^{(L)}) < L+2$, the row in $\mathcal{V}_1$ cannot be obtained through linear combination of the rows in $\mathcal{V}_2$. As a result, no row in the matrix $\left[\mathbf{v}_1^T, \ldots, \left(\mathbf{v}_1^{(L)}\right)^T, (\mathbf{v}_1 + \mathbf{v}_2)^T, \ldots, \left(\mathbf{v}_1^{(L)} + \mathbf{v}_2^{(L)}\right)^T\right]^T$ can be obtained from the linear combination of other rows in this matrix. Therefore, $\left[\mathbf{v}_1^T, \ldots, \left(\mathbf{v}_1^{(L)}\right)^T, (\mathbf{v}_1 + \mathbf{v}_2)^T, \ldots, \left(\mathbf{v}_1^{(L)} + \mathbf{v}_2^{(L)}\right)^T\right]^T$ is full rank. Namely, $\mathbf{G}_2$ is full rank.

Appendix C
Proof of Theorem 5

Let the coefficient matrices of relay R_1, relay R_2 and relay R_3 be constructed from $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$, respectively. Without loss of generality, we define τ_1, τ_2, τ_3 as the timing errors of R_1, R_2, R_3 and $\tau_1 \le \tau_2 \le \tau_3$ is assumed. Other cases with different order of τ_1, τ_2, τ_3 are equivalent to that with $\tau_1 \le \tau_2 \le \tau_3$ due to the row permutation operation. According to Theorem 3 and bearing in mind that $\mathbf{G}_3$ is full rank, the asynchronous versions $\mathbf{G}_2^a$ of $\mathbf{G}_2 = \left[\mathbf{v}_2^T, \ldots, \left(\mathbf{v}_2^{(L)}\right)^T, \mathbf{v}_3^T, \ldots, \left(\mathbf{v}_3^{(L)}\right)^T\right]^T$ must be full rank. Namely, the last $2L+2$ rows in $\mathbf{G}_3^a$ are linearly independent. The discussion is then divided into two cases.

1) $L+\tau_1 < \tau_2$: In this case, the last $2L+2$ rows are shifted by more bits compared with the first $L+1$ rows. Thus the linear combination of the first $L+1$ rows in $\mathbf{G}_3^a$ cannot be obtained by the linear combination of the last $2L+2$ rows, and the first $L+1$ rows are linearly independent with each other. As we have mentioned earlier, the last $2L+2$ rows in $\mathbf{G}_3^a$ are linearly independent. Hence, $\mathbf{G}_3^a$ is full rank.
2) $\tau_1 \leq \tau_2 \leq L+\tau_1$: In this case, some rows in the last $2L+2$ rows are shifted the same amount as some other rows in the first $L+1$ rows. Define p as the minimum value such that at least one row in the last $2L+2$ rows is shifted the same amount as pth row, $1 \leq p \leq L+1$. Then, none of the last $2L+2$ rows is shifted the same amount as 1st,$\ldots,(p-1)$th rows in $\mathbf{G}_3^a$. Since the first $(p-1)$ rows in $\mathbf{G}_3^a$ are shifted less compared with the pth to the $(3L+3)$th rows, the first $(p-1)$ rows cannot be obtained from the linear combination of other rows and are linearly independent from each other. Since no more than two rows are shifted the same amount as the pth row, we further consider the following two subcases:
   a) If there is only one row which is shifted the same amount as the pth row, this row must be the asynchronous version of $\mathbf{v}_2$, i.e., $\mathbf{v}_2^{(\tau_2)}$. In this subcase, the pth row to the $(2L+2)$th row belong to the matrix whose rows are all right shifted from those in $\mathbf{G}_3$ by $\tau_1+p-1=\tau_2$ bits. Since $\mathbf{G}_3$ is full rank, the rows in the row set $\mathcal{P} = \{p, p+1, \ldots, 2L+2\}$ are linearly independent. If some rows in the last $L+1$ rows are shifted the same amount as one row in $\mathcal{P}$, these rows also belong to the matrix whose rows are all right shifted from those in $\mathbf{G}_3$ by τ_1+p-1 bits. We define the row set of these rows as $\mathcal{Q}$ and the set of other rows in the last $L+1$ rows as $\bar{\mathcal{Q}}$. At the same time, the rows in $\mathcal{Q}$ and the rows in $\mathcal{P}$ are linearly independent. Since $\tau_1 \leq \tau_2 \leq \tau_3$, the rows in $\bar{\mathcal{Q}}$ are shifted by more bits compared to the other rows in $\mathbf{G}_3^a$. Thus the linear combination of the rows in $\bar{\mathcal{Q}}$ cannot be obtained from the linear combination of the other rows in $\mathbf{G}_3^a$, and the rows in $\bar{\mathcal{Q}}$ are linearly independent from each other. Hence, $\mathbf{G}_3^a$ is full rank.
   b) If there are two rows which are shifted the same amount as the pth row, these two rows must be the asynchronous versions of $\mathbf{v}_2$ and $\mathbf{v}_3$, i.e., $\mathbf{v}_2^{(\tau_2)}$ and $\mathbf{v}_3^{(\tau_3)}$. In this subcase, the $\{p, p+1, \ldots, 3L+3\}$th rows belong to the matrix whose rows are all right shifted from those in $\mathbf{G}_3$ by $\tau_1+p-1=\tau_2=\tau_3$ bits. Hence, these rows are full rank and $\mathbf{G}_3^a$ is also full rank .

From all above discussions, if $\mathbf{G}_3$ is full rank, any asynchronous version $\mathbf{G}_3^a$ is full rank.

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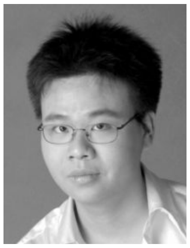

Zhimeng Zhong (S’07) received his B.Sc. degree in electrical and electronics engineering from Xi’an Jiaotong University, China in 2002. He received the M.Sc. degree in electrical and electronics engineering from Xi’an Jiaotong University, China in 2005. He is currently pursuing the Ph.D degree in the School of Electronics and Information Engineering, Xi’an Jiaotong University, China. His current research interests are in digital wireless communications, distributed space-time code, cooperative communications, multiple-input multiple-output(MIMO) systems.

Shihua Zhu received the B.Sc. degree in Radiotechnics in 1982 from Xi’an Jiaotong University, China, and the M.Sc. degree in Telecommunication Systems in 1984, the Ph.D. degree in Electronic Systems Engineering in 1987, both from the University of Essex, UK. Since 1987, he has been with Xi’an Jiaotong University where he is now a professor in the Department of Information and Communications engineering and is the vice president of the university. He is also an advisory panelist of the National Natural Science Foundation of China, an advisory panelist of the State Teaching Guidance Commission of China (electronics and information session), is on the Academic Board of China Institute of Communications, and is the vice chairman of the Shaanxi Provincial Communications Society. He worked in the Department of Information Engineering, the Chinese University of Hong Kong as a visiting researcher between May 1993 and August 1993, March 1994 and August 1994, June 1995 and September 1995. He worked in the Department of Electrical and Computer Engineering, University of Waterloo, Canada as a visiting associate Professor between June 1996 and December 1996, visiting Professor between October 1998 and October 1999. His research interests are in mobile communications, digital switching and computer networks. He has involved in more than 30 research projects including some supported by the National Science Foundation of China and by the National High Technology Research and Development (863) Project of China. He won more than 10 research, teaching and book awards. He has authored four books and published more than 130 papers.

Arumugam Nallanathan (S’97.M’00.SM’05) received the B.Sc. with honors from the University of Peradeniya, Sri-Lanka, in 1991, the CPGS from the Cambridge University, United Kingdom, in 1994 and the Ph.D. from the University of Hong Kong, Hong Kong, in 2000, all in Electrical Engineering. He was an Assistant Professor in the Department of Electrical and Computer Engineering, National University of Singapore, Singapore from August 2000 to December 2007. Currently, he is a Senior Lecturer in the Department of Electronic Engineering at King’s College London, United Kingdom. His research interests include OFDM systems, ultra-wide bandwidth (UWB) communication and localization, MIMO systems, and cooperative diversity techniques. In these areas, he has published over 120 journal and conference papers. He is a co-recipient of the Best Paper Award presented at 2007 IEEE International Conference on Ultra-Wideband (ICUWB’2007). He currently serves on the Editorial Board of IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY, JOHN WILEY’S WIRELESS COMMUNICATIONS AND MOBILE COMPUTING and EURASIP JOURNAL OF WIRELESS COMMUNICATIONS AND NETWORKING as an Associate Editor. He served as a Guest Editor for EURASIP JOURNAL OF WIRELESS COMMUNICATIONS AND NETWORKING Special issue on UWB Communication Systems-Technology and Applications. He also served as a technical program committee member for more than 25 IEEE international conferences. He currently serves as the General Track Chair for IEEE VTC’2008-Spring, Co-Chair for the IEEE GLOBECOM’2008 Signal Processing for Communications Symposium, and IEEE ICC’2009 Wireless Communications Symposium.