Delay-Tolerant Distributed Linear Convolutional Space-Time Code with Minimum Memory Length under Frequency-Selective Channels

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***Abstract*—In cooperative communication networks, the performance of distributed space-time code will be severely degraded if the timing synchronization among relay nodes is not perfect. In this letter, we propose a systematic construction of the so called distributed linear convolutional space-time code (DLCSTC) for multipath fading channels that does not require the synchronization assumption. We derive sufficient conditions on the code design such that the full cooperative and multipath diversities can be achieved under the minimum memory length constraint. Then we design DLCSTCs that both have the trace-orthonormality property and achieve the full diversity. We show that the proposed codes can also achieve the full diversity for asynchronous cooperative communications with ZF, MMSE and MMSE-DFE receivers under frequency-selective channels. Finally, various numerical examples are provided to corroborate the analytical studies.**

***Index Terms*—Asynchronous cooperative communications, linear dispersion space-time codes, distributed linear convolutional space-time code.**

I. Introduction

THE effect of fading, as a function of the user position, deteriorates the performance of wireless communications and causes large variations in signal strength. Diversities resulted from spatial, temporal, and frequency domains are powerful techniques to combat fading. Exploiting the spatial diversity can be realized by equipping multiple antennas at the transmitter and/or the receiver. However, applying multiple antennas onto a mobile terminal or a sensor node meets difficulties, such as the size limitation and the hardware complexity. Thanks to the new transmission scheme introduced in [1]–[3], the spatial diversity for small terminals can be exploited if cooperation is adopted among users. The corresponding transmission scheme is referred to as the cooperative communications [3].

There have been a number of research studies on the code design for the cooperative communication networks [4]-[9]. However, unlike the MIMO system, relay nodes are located at different places and each equipped with its own oscillator. In order to achieve the full diversity gain, synchronization is required, which could introduce a significant overhead. Recently, asynchronous cooperative diversity has been discussed in [10]-[24]. The authors of [18], [19] propose a family of distributed space-time trellis code (DSTTC) that can achieve full cooperative diversity in asynchronous communication systems. Note that, the code in [13]-[20] are designed under the assumption that the channels between relays and destination are flat fading. Then [21] proposes the space-frequency code (SFC) for asynchronous cooperative communications where OFDM is adopted to combat both the timing error and the multipath fading. The authors of [22] propose the distributed linear convolutional space-time codes (DLCSTC) which can achieve full diversity by zero-forcing (ZF), MMSE and MMSE-DFE receivers, but only the timing error is addressed in the code design, and the derived code in [22] cannot achieve full multipath diversity order. Recently, the code design of the DSTTC is extended to the case of frequency-selective channel in [23] and to MIMO relay networks over frequency-selective channel in [24]. However in [23], [24], exact code construction is only available for no more than three relay node. Moreover, the full diversity order achieved in [23], [24] depends on maximum-likelihood sequence detection (MLSD) which, in practice, is computationally prohibitive, especially when the number of relays or the constellation size is large.

In this letter, we build the general construction methods of the delay-tolerant DLCSTC where the channels are considered frequency-selective. Here, the delay-tolerant property means that the code can maintain the full diversity property under any delay profile. We derive a sufficient condition on the code design such that the full diversity can be achieved and the minimum memory length for the arbitrary relay number is attained. We also construct DLCSTCs that ensure full diversity order and trace-orthonormality constraint under frequency-selective channels. By using some recent results in [25], [26], we show that our proposed delay-tolerant DLCSTC can achieve the full diversity order with suboptimal receivers under frequency-selective channels.

This letter is organized as follows. In Section II, the system model is presented. In Section III, the design criteria for the DLCSTC is derived, and the systematic code design over frequency-selective channels is presented. Moreover, the trace-orthonormality constraint for the DLCSTC and diversity property of the DLCSTC with suboptimal receivers are also studied in this section. Finally, simulations are conducted in section IV and conclusions are made in Section V.

Notations: Vectors and matrices are boldface small and capital letters; the transpose, Hermitian and trace of the matrix $\mathbf{A}$ are denoted by $\mathbf{A}^T$, $\mathbf{A}^H$, $\mathrm{Tr}(\mathbf{A})$ respectively; $\mathbf{I}$ is the identity matrix. $\mathbb{C}$ and $\mathbb{Z}$ denote the field of complex numbers and the ring of integerals, respectively; $\otimes$ and $\circ$ denote Kronecker product and convolution, respectively.

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II. System Model

Consider a system with $M+2$ nodes that communicate cooperatively. We assume that there is one source node S, one destination node D, and M relays $R_i, i=1,2,\ldots,M$. Each node is equipped with one antenna. We also assume that there is no direct connection between the source and the destination and that all terminals operate in half-duplex fashion [3]. We consider the *decode-and-forward* (DF) transmission protocol that consists of two phases. During phase I, S broadcasts its information to all the relays. During phase II, each relay first checks whether the decoding is successful according to Cyclic Redundancy Check (CRC) bits that was inserted by the source, then, if the decoding is successful, the relays will encode the information and forward the encoded data to the destination. Since a space-time code (STC) designed to M relays has full diversity property, it also has full diversity if $M-M_s$ relays are deleted [21]. Hence we assume that M relays are all enrolled in phase II.

Before proceeding to discuss the system model, we define the STC matrix as

$$\mathbf{X}=\left[\mathbf{X}_1^T,\mathbf{X}_2^T,\ldots,\mathbf{X}_M^T\right]^T, \tag{1}$$

$$\mathbf{X}_i=\begin{bmatrix} x_i(1) & x_i(2) & \ldots & x_i(N) & \ldots & 0\\ 0 & x_i(1) & \ldots & x_i(N-1) & \ldots & 0\\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots\\ 0 & 0 & \ldots & x_i(N-L+1) & \ldots & x_i(N)\end{bmatrix}, \tag{2}$$

where $\mathbf{x}_i=[x_i(1),\ldots,x_i(N)]^T$ is the signal sequence transmitted by relay R_i, L is the length of the channel impulse response, and N is the length of the transmission sequence. Because of the timing errors, we define the asynchronous version of the STC matrix as $\mathbf{X}_\Delta$, and $\mathbf{X}_\Delta$ of dimension $ML\times(N+L+\tau-1)$ can be expressed as

$$\mathbf{X}_\Delta=\begin{bmatrix}\mathbf{0}_{L\times\tau_1} & \mathbf{X}_1 & \mathbf{0}_{L\times(\tau-\tau_1)}\\ \vdots & \vdots & \vdots\\ \mathbf{0}_{L\times\tau_M} & \mathbf{X}_M & \mathbf{0}_{L\times(\tau-\tau_M)}\end{bmatrix}, \tag{3}$$

where the delay profile $\Delta=[\tau_1,\ldots,\tau_M]$, $\mathbf{0}_{m\times n}$ is the $m\times n$ all-zero matrix, τ_i is the timing error of relay R_i and $\tau=\max\limits_{1\le i\le M}\{\tau_i\}$. We assume that the relative timing errors between different relays are integer multiples of the symbol duration [13]-[22]. We also assume that both of these relative timing errors and the channel path gains are perfectly known at the receiver but not known at the transmitter. Although the symbol synchronization is not required in the above *asynchronous cooperative communications*, in order to eliminate inter-frame interference, we assume that each frame in different enrolled relays is preceded by a preamble, whose length is not less than L_e+L-1, where L_e is the upper bound of the timing errors. All the symbols in the preamble are zeros.

Thus the destination node D receives $\mathbf{y}=\mathbf{h}\mathbf{X}_\Delta+\mathbf{z}$, where $\mathbf{y}$ is the received row vector, $\mathbf{z}$ is the additive white Gaussian noise vector whose variance is N_0, and $\mathbf{h}$ is the $1\times ML$ vector with the form $\mathbf{h}\triangleq[h_1(0),\ldots,h_1(L-1),\ldots,h_M(0),\ldots,h_M(L-1)]$, where $h_i(l)$ is the lth path gain from R_i to D and is a circularly complex Gaussian random variable with variance $\sigma_i^2(l)$. The channel gains are normalized such that $\sum_{l=0}^{L-1}\sigma_i^2(l)=1$ for any relay. To achieve full diversity in asynchronous cases, we will provide ways to design $\mathbf{X}$ in the following section.

III. DLCSTC over Frequency-Selective Channels

Following [22], at each relay node, the information symbol sequence $\mathbf{s}=[s_0,\ldots,s_{l-1}]^T\in\mathbb{C}^{l\times 1}$ is transformed into $\mathbf{x}_i$ through a vector $\mathbf{v}_i\triangleq[v_{i0},v_{i1},\ldots,v_{i(k-1)}]\in\mathbb{C}^{1\times k}$, i.e., $\mathbf{x}_i=\mathbf{v}_i\circ\mathbf{s}$. Such DLCSTC is a special family of distributed linear dispersion space-time code [22]. A DLCSTC encoder is composed of a set of convolution matrices, each of which is determined by a generator polynomial $p_i(z)=v_{i0}+v_{i1}z+\ldots+v_{i(k-1)}z^{k-1}$. Here we call k as *memory length*. The output of the encoder is the convolution of the information symbol sequence and the polynomial coefficients. Every relay node is assigned one of such generator polynomials. The transmitted symbols on the lth channel path of relay R_i can be equivalently generated by the generator polynomial $p_{il}(z)\triangleq z^lp_i(z)$, thus the polynomial form of the STC matrix $\mathbf{X}$ is $\mathbf{X}(z)=[p_{\mathbf{s}}(z)p_1(z),\ldots,z^{L-1}p_{\mathbf{s}}(z)p_1(z),\ldots,p_{\mathbf{s}}(z)p_M(z),\ldots,z^{L-1}p_{\mathbf{s}}(z)p_M(z)]$, where $p_{\mathbf{s}}(z)=s_0+s_1z^1+\ldots+s_{l-1}z^{l-1}$. If the asynchronous case is considered, $\mathbf{X}_\Delta$ can be equivalently generated by the generator polynomials $z^{\tau_1}p_{10}(z),\ldots,z^{\tau_1}p_{1(L-1)}(z),\ldots,z^{\tau_M}p_{M0}(z),\ldots,z^{\tau_M}\cdot p_{M(L-1)}(z)$. We define the coefficients of the generator polynomial $p_{il}(z)$ as a row vector $\mathbf{v}_{il}=[\mathbf{0}_{1\times l},v_{i0},\ldots,v_{i(k-1)},\mathbf{0}_{1\times(L-l-1)}]$. Therefore, to ensure the full diversity in the asynchronous cooperative communication, there are requirements on the generator polynomials $p_1(z),\ldots,p_M(z)$, stated in the following theorem.

Theorem 1: The DLCSTC can achieve full diversity for any delay profile if and only if any asynchronous version of the generator matrix $\mathbf{P}_M$, which is defined as $\mathbf{P}_{M,\Delta}=[\mathbf{v}_{10,\Delta}^T,\ldots,\mathbf{v}_{1(L-1),\Delta}^T,\ldots,\mathbf{v}_{M0,\Delta}^T,\ldots,\mathbf{v}_{M(L-1),\Delta}^T]^T$, where $\mathbf{v}_{ij,\Delta}\triangleq[\mathbf{0}_{1\times\tau_i},\mathbf{v}_{ij},\mathbf{0}_{1\times(\tau-\tau_i)}]$, has full row rank for any non-negative $\tau_i\in\mathbb{Z}, i=1,\ldots,M$.

Proof: The proof follows the same argument of the generator matrix construction in flat-fading channels [22]. ∎

One can see that the importance of Theorem 1 lies in that, we only need to construct $p_i(z)$ such that any submatrix-shifted version $\mathbf{P}_{M,\Delta}$ of the generator matrix $\mathbf{P}_M$ has full rank. The main difference between Theorem 1 and [22, Theorem 1] is that, $p_i(z)$ here is constructed in a way to ensure that any sub-matrix-shifted version $\mathbf{P}_{M,\Delta}$ of the generator matrix $\mathbf{P}_M$ has full rank. If $L=1$, i.e., flat fading channels, the sub-matrix shifting is degraded to the row shifting. Hence [22, Theorem 1] is a special case of the proposed Theorem 1. In the following, we will give sufficient conditions under which the asynchronous versions $\mathbf{P}_{M,\Delta}$ of $\mathbf{P}_M$ always have full row rank for any delay profile.

A. Construction of the Generator Polynomials

In this section, we will study the construction of the generator polynomials. Since the memory length k determines the complexity of MLSD receiver, we will give the sufficient conditions for the generator polynomials such that the full diversity order can be achieved under the minimum memory length constraint.

Lemma 1: The necessary condition to ensure the full rank property of $\mathbf{P}_M$ is that the minimum memory length k is $(M-1)L+1$.

Proof: Since $\mathbf{P}_M$ is a matrix of dimension $ML \times (k+L-1)$, we must have $(k+L-1) \geq ML$ in order to achieve full row rank. This completes the proof. ■

For the cases $M=2$ and $M=3$, we have the following theorem:

Theorem 2: Let $\mathbf{P}_M$ be constructed by the polynomials $[p_1(z), \ldots, p_M(z)]$ for $M = 2, 3$, and $v_{i0} \neq 0, v_{i(k-1)} \neq 0, i = 1, \ldots, M$, $k = (M-1)L+1$. If $\mathbf{P}_M$ has full rank, then its asynchronous versions $\mathbf{P}_{M,\Delta}$ will also have full rank.

Proof: The proof follows the same argument of the DSTTC in [23, Theorem 3&Theorem 5]. ■

In fact, Theorem 2 extends the generator matrix construction in the binary field in [23, Theorem 3&Theorem 5] to the complex number field. It gives the generator matrix construction of the DLCSTC for the cases that the numbers of the relays are $M=2$ and $M=3$, respectively. When $M>3$, it can be easily checked that Theorem 2 will not hold. Hence we give the following theorem for the general case:

Theorem 3: For the matrix form $\mathbf{V} = \left[\mathbf{v}_1^T, \ldots, \mathbf{v}_M^T\right]^T$ of the generator polynomials set $[p_1(z), \ldots, p_M(z)]$, if $\mathbf{V} = \mathbf{G} \otimes \mathbf{e}_L$, where $\mathbf{e}_n \triangleq [1, \mathbf{0}_{1\times(n-1)}]$ and $\mathbf{G} = \left[\mathbf{g}_1^T, \ldots, \mathbf{g}_M^T\right]^T \in \mathbb{C}^{M\times M}$, is a shift-full-rank (SFR) matrix, then $\mathbf{P}_{M,\Delta}$ has full rank for any delay profile.

Proof: First of all, k takes the value of $(M-1)L+1$ because of $\mathbf{G} \in \mathbb{C}^{M\times M}$, and $\mathbf{v}_i$ takes the $\mathbf{g}_i \otimes \mathbf{e}_L$ if we ignore the last $L-1$ zeros. Then, we need prove that any row cannot be expressed as a linear combination of the other rows in $\mathbf{P}_{M,\Delta}$. It means that we need to prove $\sum_{i=1}^{M}\sum_{j=0}^{L-1} a_{ij}\mathbf{v}_{ij,\Delta} \neq \mathbf{0}$ for any $a_{ij} \in \mathbb{C}$, where there exists at least one non-zero a_{ij}. Note that $\sum_{i=1}^{M}\sum_{j=0}^{L-1} a_{ij}\mathbf{v}_{ij,\Delta}$ can be rewritten as $\sum_{i=1}^{M}[\mathbf{g}_i \otimes \mathbf{a}_i]_\Delta$, where $\mathbf{a}_i = [a_{i0}, \ldots, a_{i(L-1)}]$. Hence, we need to prove $\sum_{i=1}^{M}[\mathbf{g}_i \otimes \mathbf{a}_i]_\Delta \neq \mathbf{0}$ for any $\mathbf{a}_i \in \mathbb{C}^{1\times L}$, not all zero.

Assume that that there exists one $\Delta = [\tau_1, \ldots, \tau_M]$ such that $\sum_{i=1}^{M}[\mathbf{g}_i \otimes \mathbf{a}_i]_\Delta = \mathbf{0}$. Then, any column of $\sum_{i=1}^{M}[\mathbf{g}_i \otimes \mathbf{a}_i]_\Delta$ is equal to zero. Because all columns of $\sum_{i=1}^{M}[\mathbf{g}_i \otimes \mathbf{a}_i]_\Delta$ can be expressed as the linear combinations of the columns of $\mathbf{g}_1, \ldots, \mathbf{g}_M$, there exist $a_1, \ldots, a_M \in \mathbb{C}$, not all zero, and $a_i \in \mathbf{a}_i$, such that $\sum_{i=1}^{M} a_i \mathbf{g}_{i,\bar{\Delta}} = 0$, where $\bar{\Delta} = [\tau_1 + i_1, \ldots, \tau_M + i_M]$ and $0 \leq i_1, \ldots, i_M \leq L-1$. It indicates that $\mathbf{G}$ is not an SFR matrix, which contradicts with the SFR property of $\mathbf{G}$. Consequently, the assumption made previously is incorrect, which completes the proof. ■

Since the construction method for the SFR matrix can be found in [19], [22], we can construct the DLCSTC such that the full diversity order can be ensured under frequency-selective channels for arbitrary M and L in accordance with Theorem 3. We need to mention that Theorem 3 can also be used to build the generator matrix for DSTTC in [23] when the generator matrix is constructed in the binary number field. Hence Theorem 3 provides a much general statement compared to our previous works in [23], [24]. Obviously, there are many generator polynomial sets that can be found through Theorem 2 and Theorem 3 such that full diversity can be ensured. In the following, we impose an additional constraint on the generator polynomials, with which the resulted DLCSTC is optimal from both information theoretic and detection error viewpoints.

B. Trace-Orthonormality Constraint for DLCSTC

In [22], a linear dispersion code is defined as $\mathbf{C} = \sum_{i=0}^{l-1} s_i \mathbf{C}_i$, where s_i, $i = 0, 1, \ldots, l-1$ are information symbols and $\mathbf{C}_i$, $i = 0, 1, \ldots, l-1$ are called linear dispersion matrices. In [27], it is proved that if $\mathbf{C}_i$ satisfies:

$$\mathrm{Tr}(\mathbf{C}_i\mathbf{C}_i^H) = L, \quad \mathrm{Tr}(\mathbf{C}_i\mathbf{C}_j^H) = 0, \quad i \neq j \tag{4a}$$

$$\mathbf{C}_i\mathbf{C}_i^H = M\mathbf{I}, \tag{4b}$$

then the resulted code can utilize as much spatial freedom as possible and could achieve the lower bound of the worst case pairwise error probability. This property is the *trace-orthonormality* property [27].

In [22], the authors call the equation (4a) as the *trace orthogonality constraint* which is the necessary and sufficient condition to maximize the mutual information [27]. Similar to the flat fading case in [22], we can easily prove that if the generator polynomials satisfy [22, Theorem 3], the DLCSTC also has the trace orthogonality constraint property under frequency-selective channels.

In this letter, we call (4b) as *unitary constraint* which is the necessary condition for the lower bound of the pairwise error probability to be achieved [27]. Although the unitary constraint in (4b) cannot be ensured for any delay profile, we still prefer to finding such DLCSTC with the unitary constraint in synchronous case. For the linear dispersion matrices generated by Theorem 2 and Theorem 3, to ensure the unitary constraint, it can be easily verified that we only need to find the generator polynomials such that $\|\mathbf{v}_1\|^2 = \ldots = \|\mathbf{v}_M\|^2$ with $\mathbf{v}_i\mathbf{v}_j^H = 0, 1 \leq i \neq j \leq M$.

Corollary 1: For $M=2$, if $v_{10} = v_{1(k-1)} = 0.5$, $v_{20} = \pm 0.5$, $v_{2(k-1)} = -v_{20}$, and $v_{11} = \ldots = v_{1(k-2)} = v_{21} = \ldots = v_{2(k-2)} = 0$ where $k = L+1$, then the DLCSTC can ensure full diversity, possess trace orthogonality property for any delay profile, and meet with the unitary constraint in synchronous case.

Corollary 1 can be immediately proved following Theorem 2 and [22, Theorem 3]. With the help of the exhaustive computer search based on Theorem 2, Theorem 3, and [22, Theorem 3], we can find the generator polynomials that ensure full diversity, trace orthogonality and unitary properties for different L and M. Some code examples are displayed in Table I.

C. Full Diversity with Suboptimal Receivers

To decode the DLCSTC, the optimal decoding method is a Viterbi algorithm and its complexity grows exponentially with the number of trellis states. Therefore, it is necessary to investigate the suboptimal equalizers for DLCSTC. Based on the design criterion for the linear dispersion space-time

TABLE I
TRACE-ORTHONORMALITY CODE DESIGN EXAMPLES

	$\mathbf{v}_i$
M=2, L=2	$\frac{1}{2}[1,0,1]$, $\frac{1}{2}[1,0,-1]$
M=2, L=3	$\frac{1}{2}[1,0,0,0,1]$, $\frac{1}{2}[1,0,0,0,-1]$
M=3, L=2	$\frac{1}{3}[1,0,1,0,1]$, $\frac{1}{3\sqrt{2}}[1,0,-2,0,1]$, $\frac{1}{\sqrt{6}}[1,0,0,0,-1]$
M=3, L=3	$\frac{1}{3}[1,0,0,1,0,0,1]$, $\frac{1}{3\sqrt{2}}[1,0,0,-2,0,0,1]$, $\frac{1}{\sqrt{6}}[1,0,0,0,0,0,-1]$
M=4, L=2	$\frac{1}{4}[1,0,-1,0,-1,0,1]$, $\frac{1}{4}[1,0,1,0,-1,0,-1]$, $\frac{1}{4}[1,0,-1,0,1,0,-1]$, $\frac{1}{4}[1,0,1,0,1,0,1]$
M=4, L=3	$\frac{1}{4}[1,0,0,-1,0,0,-1,0,0,1]$, $\frac{1}{4}[1,0,0,1,0,0,-1,0,0,-1]$, $\frac{1}{4}[1,0,0,-1,0,0,1,0,0,-1]$, $\frac{1}{4}[1,0,0,1,0,0,1,0,0,1]$

codes that can achieve full spatial diversity order with ZF and MMSE receivers in [26] and [22, Theorem 5], we can immediately prove in accordance with the same argument of [22, Theorem 5] that if the DLCSTC's generator polynomials satisfy the criteria in Theorem 1, then the DLCSTC can achieve the full diversity order with ZF, MMSE and MMSE-DFE receivers under frequency-selective channels for any delay profile, provided that the maximum delay τ is finite.

D. Relays with Multiple Antennas

The framework described in the previous subsections still works for the case where the relays have multiple antennas, provided that each extra antenna is treated as a different relay node. However, it is difficult to find the code such that both full diversity order and trace-orthonormality constraint can be ensured through computer search when the number of relays is large. In such case, if we assume that each relay has K antennas, then in the delay profiles one should only consider the shift of the sub-matrix with KL rows.¡¡Therefore, we can treat the extra antenna as channel paths which is similar to what the delay diversity does. Thus we only need to find M generator polynomials such that the full diversity order and trace-orthonormality constraint can be ensured.

IV. SIMULATION RESULTS

In this section, we evaluate the performance of our DLCSTC through various numerical examples. In all the examples, we assume that a frame contains 130 information symbols and the channels are quasi-static Rayleigh frequency-selective fading channels with $L = 2$ and the uniform power delay profile. We also assume that there is only one antenna in all nodes, and the random delays are uniformly selected from the set $\{0, 1, \ldots, L_e\}$, $L_e = 2$, and QPSK modulation is used unless otherwise stated. The tap lengths of the feedforwrd and feedback filters of the MMSE-DFE receiver are respectively 40 and 20. Moreover, the generator polynomials for the DLCSTC used in all the examples are shown in Table I.

We first compare the bit error rate (BER) performance for several code schemes: DLCSTC, the linear convolutional space-time code generated by SFR set (SFR-STC) in [22] and the delay diversity (DD) code. Two relays are assumed. The generator polynomials for the SFR-STC are $[1/2, 1/2, 0]$ and $[1/2, 0, -1/2]$. Both of the DLCSTC and SFR-STC use the MLSD and MMSE-DFE receivers, and the DD code only uses the MLSD receiver. Moreover, both of the synchronous case and asynchronous case are considered for the DD code, and only the asynchronous case is considered for the other two codes. From Fig. 1, we can see that both the MLSD and MMSE-DFE receivers for DLCSTC and the synchronous DD code achieve the identical slope in the high SNR region, which confirms our proposed codes can achieve the full diversity with MLSD and MMSE-DFE receivers. Moreover, the DD code has a significant diversity gain loss due to the timing errors. Although the MLSD performs the best, we can also see that our DLCSTC with MMSE-DFE receiver outperforms the SFR-STC with both MLSD and MMSE-DFE receivers in Fig. 1. This is because that the SFR-STC cannot achieve full diversity order under frequency-selective channels. On the other side, the proposed DLCST with MLSD receiver has the same complexity with the DD code and the SFR-STC due to the same number of states used.

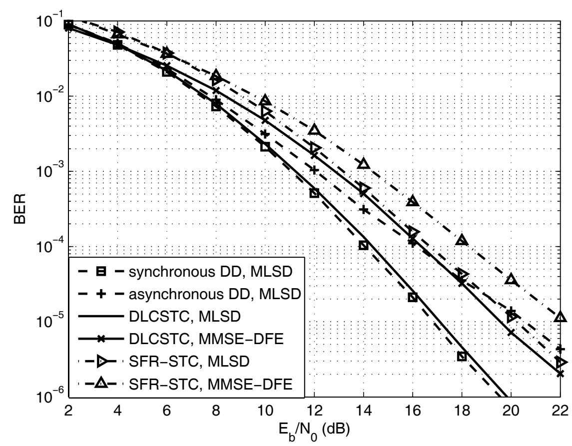

Fig. 1. BER performance for two relay nodes.

Next we show the performance of the proposed DLCSTC in the synchronous case and the asynchronous cases with $L_e = 2$ and $L_e = 4$ in Fig. 2. Two relays are assumed. We see that the performance of our proposed DLCSTC achieves the identical slope in different timing errors range, which confirms that the DLCSTC has the delay-tolerant property in any delay profile. Moreover, we can also see that the DLCSTC with MLSD receiver in the synchronous case gives a better performance than the other asynchronous cases, because the former always

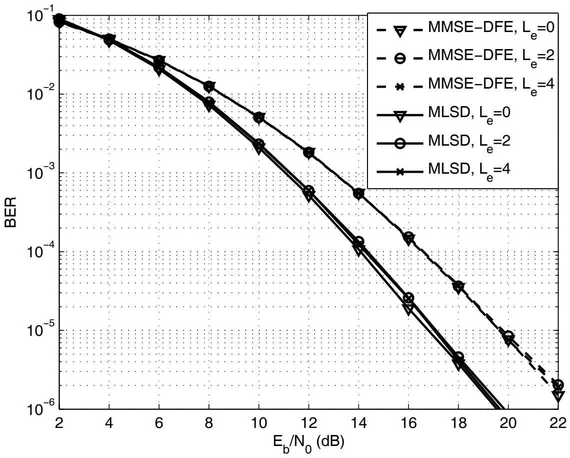

Fig. 2. BER performance comparison of DLCSTC for different L_e.

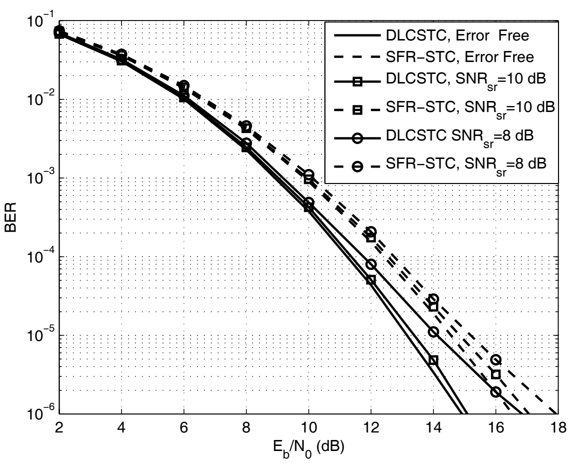

Fig. 3. BER Comparison for DLCSTC and SFR-STC with MLSD receiver when errors may occur in phase I.

satisfies the unitary constraint in (4b).

Finally, we show the proposed DLCSTC when errors may occur during phase I transmission in Fig. 3 and Fig. 4. We assume that there are 4 relays with $L = 2$. MLSD and MMSE-DFE receivers are respectively used in Fig. 3 and Fig. 4, and the constellation is BPSK. Those relays which can detect the entire packet correctly, encode the information by its generator polynomial and forward the encoded packet to D in phase II. The SNR in phase I is denoted by SNR_{sr}. The asynchronous case is considered. As benchmark, we also show the SFR-STC code generated by $1/4[1,-1,-1,1,0,0,0]$, $1/4[1,1,-1,-1,0,0,0]$, $1/4[0,0,0,1,-1,1,-1]$, $1/4[0,0,0,1,1,1,1]$. From Fig. 3 and Fig. 4, we see that the performance of the DLCSTC degrades as SNR_{sr} decreases. This is because that less potential relays may participate in phase II transmission and the achieved full diversity order is decreased when SNR_{sr} decreases. We also see that the DLCSTC performs better than the SFR-STC, since the linear convolutional code generated by SFR matrix cannot achieve full diversity in asynchronous scenarios.

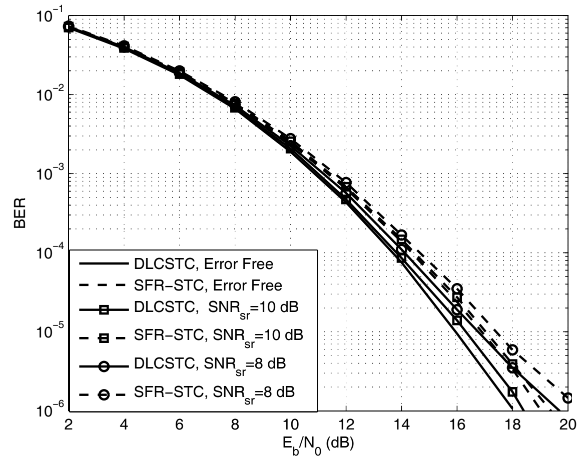

Fig. 4. BER Comparison for DLCSTC and SFR-STC with MMSE-DFE receiver when errors may occur in phase I.

V. Conclusions

In this letter, we propose a distributed linear convolutional space-time code with minimum memory length that achieves full cooperative and multipath diversities for asynchronous cooperative communications. The new DLCSTC tolerates imperfect synchronization among relay nodes. We give sufficient conditions to construct such DLCSTC. It is also shown that the delay-tolerant DLCSTC can achieve the full diversity with suboptimal receivers.

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