Performance Analysis of FD-NOMA-based Decentralized V2X Systems

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<td>Complete List of Authors:</td>
<td>Zhang, Di; Zhengzhou University, School of Information Engineering Liu, Yuanwei; Queen Mary University of London, School of Electronic Engineering and Computer Science Dai, Linglong; Tsinghua University, Department of Electronic Engineering Bashir, Ali; Frodskaparssetur Foroya Nallanathan, Arumugam; Queen Mary University of London, Electronic Engineering and Computer Science Shim, Byonghyo; Seoul National University, Electrical and Computer Engineering;</td>
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Q (1.2), (1.7) (3.2) Performance Analysis of FD-NOMA-based Decentralized V2X Systems

Di Zhang, Member, IEEE, Yuanwei Liu, Member, IEEE, Linglong Dai, Senior Member, IEEE, Ali Kashif Bashir, Senior Member, IEEE, Arumugam Nallanathan, Fellow, IEEE, and Byonghyo Shim, Senior Member, IEEE

Abstract

In order to meet the massively connected devices, different quality of services (QoSs), various transmit rates and ultra-reliable and low latency communications (URLLC) requirements of vehicle to everything (V2X) communications, we introduce a full duplex non-orthogonal multiple access (FD-NOMA)-based decentralized V2X system model. Compared to the orthogonal frequency division multiple access (OFDMA) scheme, NOMA is insensitive to Doppler effect caused by moving vehicles. We classify the V2X communications into two scenarios and give their exact capacity expressions. To solve the computation complicated problems of the involved exponential integral functions, we give the approximate closed-form expressions with arbitrary small errors. Numerical results indicate the validity of our derivations. Our analysis has that the accuracy of our approximate expressions is controlled by the division of $\frac{1}{\sigma}$ in the urban and crowded scenario, and the truncation point $T$ in the suburban and remote scenario. Numerical results also manifest 1) Increasing the number of V2X device, NOMA power and Rician factor value yields better capacity performance. 2) Effect of FD-NOMA is determined by the FD self-interference and the channel noise. 3) FD-NOMA has better latency performance compared to other schemes.

Index Terms

Corresponding author: B. Shim {bshim@snu.ac.kr}.

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D. Zhang is with the School of Information Engineering, Zhengzhou University, Zhengzhou, 450001, China (e-mail: di_zhang@zzu.edu.cn).

Y. Liu and A. Nallanathan are with the EECS, Queen Mary University of London, London, E14NS, U.K (e-mail: {yuanwei.liu, a.nallanathan}@qmul.ac.uk).

L. Dai is with the Department of Electronic Engineering, Tsinghua University, Beijing 100084, China (e-mail: daill@tsinghua.edu.cn).

A. Bashir is with the University of the Faroe Islands, Tórshavn 100, Faroe Islands, Denmark (e-mail: AliB@setur.fo).

B. Shim is with the Information System Laboratory, Department of Electrical and Computer Engineering, Seoul National University, Seoul, 08826, Korea (e-mail: bshim@snu.ac.kr).
Vehicle communications, V2X, full duplex, non-orthogonal multiple access, capacity analysis.

I. INTRODUCTION

A. Background

There are two distinct regimes in vehicle to everything (V2X) communications, i.e., dedicated short-range communications (DSRC) [1], [2] and cellular-V2X (C-V2X) [3], [4]. DSRC was popular in the past decades. Recently, C-V2X has received much attention with explosively growing devices connecting to the wireless networks. With the help of cellular network, C-V2X can connect more V2X devices [5], [6]; it can establish the link among vehicles, smart infrastructures and pedestrians, etc. C-V2X operates in two modes. (1.10) First, in the direct communications (DC) mode, V2X devices can directly communicate with each other. Well-known examples include vehicle to vehicle (V2V), vehicle to pedestrian (V2P) communications. Second, in the network-based communications (NC) mode, cellular base station (BS) is playing the dominant role, and the V2X devices communicate with (or with the help of) the cellular, for instance, vehicle to network (V2N), vehicle to infrastructure (V2I) communications. However, the current version of C-V2X (i.e., the long term evolution V2X (LTE-V2X)) cannot fully satisfy the requirements of low latency, various quality of services (QoSs) and different transmit rates [6], [7].

Q (1.4, 2.13) In addition, the existing orthogonal frequency division multiple access (OFDMA)-based LTE-V2X systems need orthogonality. Different from the non-moving wireless communications, moving vehicle caused Doppler effect is a vital problem for OFDMA-based LTE-V2X systems [8]. As is known, the carrier frequency offset (CFO) caused by the Doppler effect will lead to inter-carrier interference (ICI) to the OFDM-based wireless communications [9]. There have been various studies to solve the CFO compensation, see, e.g., [9], [10]. However, because the oscillators can never be oscillating at the identical frequency, in OFDMA-based wireless communications, CFO side-effect always exists even for non-moving circumstance [9].

It is noticed that fifth generation (5G) technologies can be used to address the issues of low latency, various QoS and different transmit rates in V2X communications, for instance, non-orthogonal multiple access (NOMA) [11] and full duplex (FD) [12]. Compared to the OMA scheme, NOMA can accommodate more users, and these users can be with different QoS requirements [12], [13]. In addition, NOMA is insensitive to CFO effect caused by moving devices because of its non-orthogonal frequency. NOMA uses the same resource block (RB) for multiple user’s transmission, which can alleviate the spectrum bottleneck of wireless communications [14]–[16]. NOMA can pair users with different transmit rates for simultaneous transmission [17]. On the other hand, while simultaneously transmitting and receiving
information, full duplex (FD) can provide faster speed and better spectrum efficiency (SE) performances [12]. Moreover, FD can offer reliable communications [18], which is useful for V2X applications such as navigation and emergency message broadcasting.

B. Related Works and Motivations

Q (e.2) (3.4) In cellular communications, there are previous studies on FD-NOMA. For instance, it was found that FD-NOMA can significantly suppress the co-channel interferences and achieve better performance gains compared to half duplex NOMA (HD-NOMA) and orthogonal multiple access (OMA) [19]. Q (3.5) Analysis and simulation results in [20] demonstrated that rate region performance of FD-NOMA outperforms the one with NOMA. Q (1.10) (3.4) Analysis and simulation results in [21] indicated that FD-NOMA improves the 5G’s system performance compared to HD-NOMA. Based on the relaying system model, analysis and simulation results in [12] indicated that FD-NOMA outperforms HD-NOMA in terms of outage probability and ergodic sum rate in low signal to noise ratio (SNR) region, but displays an inferior performance in high SNR region.

In V2X communications, there are some existing works on NOMA-V2X and FD-V2X [22]–[24]. Based on the NOMA, the authors in [23] proposed the graph-based practical encoding and joint belief propagation (BP) decoding techniques, which can achieve any rate pair close to the capacity region. B. Di et al. in [22] employed NOMA for URLLC communications while proposing a NOMA-based mixed centralized/distributed (NOMA-MCD) scheme to reduce the resource collision. In [24], an optimal blind interference alignment scheme was proposed for the coexisting of FD and HD modes. This scheme can improve the sum rate performance in finite SNR regime. However, most of these studies on NOMA-V2X and FD-V2X communications are based on the NC mode, which is a challenge for connecting massive V2X devices because of the cellular throughput restriction. Although the authors investigated the decentralized NOMA-V2X systems in [22], there has been no capacity analysis for such a system. To the best of our knowledge, a study investigating the impact of FD-NOMA techniques on V2X systems is rare, which motivates us to develop this treatise.

Q (e.1) (2.1) (2.2) (2.4) (3.1) In literature, various channel models are used for ergodic capacity analysis, for instance, the $\kappa - \mu$ channel model [25], [26] and the $\eta - \mu$ channel model [25]. However, obtaining the closed-form capacity expression in these channel models is very difficult because of the involved infinity series operations. Authors thus employed some special conditions and methods to give the closed-form expressions, e.g., $\mu$ with positive integer values [26] and the approximate method [27]. On the other hand, the difficulty to obtain a closed-form expression with Rayleigh or Rician channel model lies in the involved
exponential integral functions. In order to solve this, some approximate methods and algorithms have been proposed, for instance, the Swamee and Ohija method for exponential integral function [28] and the fast and accurate algorithm for generalized exponential integral function [29]. However, these methods are based on some special condition (e.g., [29]), or with lower accuracy (e.g., [28]). In this paper, we give the approximate closed-form capacity expressions for both Rayleigh and Rician channel models while taming the troublesome exponential integral functions.

In this work, we propose Q (1.2), (1.7) the FD-NOMA-based decentralized V2X system model, and also provide the capacity analysis to obtain the approximate closed-form capacity expressions with high accuracy. We try to answer the following key questions.

- Can we use one solution to meet all the requirements of V2X communications? If it is not possible, what about a combination of FD-NOMA techniques?
- If the combination is feasible to satisfy the requirements of V2X communications, what about the capacity and throughput performance of the V2X systems?
- Q (2.1) (3.1) Is there any approximate expressions for the capacity expressions with arbitrary small error and low computational complexity?

C. Contributions

The main contributions of this work can be summarized as follows:

- The FD-NOMA-based decentralized V2X systems can partly offload the cellular network1. Q (1.4), (2.13) Compared to OFDMA, NOMA is insensitive to Doppler effect caused by moving vehicles. In addition, FD-NOMA can accommodate more users with different QoSs and transmit rates for simultaneous transmission and reception, which is suitable for V2X systems.
- Q (e.1) (2.1) (3.1) Based on the system model, we derive the exact system ergodic capacity expressions and their approximate closed-form expressions for both scenarios. These approximate closed-form expressions are with low computational complexity and controllable arbitrary small errors compared to the existing approximate expressions. Insights from our analysis has 1) the accuracy of our simplified approximate expression in urban and crowded scenario is controlled by the associated division of \( \frac{\pi}{2} \) (with respect to the exponential integral function \( E_1(x) \)). 2) The accuracy of our simplified approximate expression in suburban and remote scenario is controlled by the truncation point \( T \) (with respect to the exponential integral function \( E_n(x) \)).

1Besides C-V2X communications, there are other types of cellular communications, our work can not offload all the cellular network load
• It is observed from our numerical results that: 1) the analytical results coincide with the Monte-Carlo based simulation results perfectly, which demonstrates the validity of our derivations. 2) The system capacity increases with the increasing allocated power value, SNR and Rician factor values. 3) The FD self-interference and the channel noise determine the effect of FD-NOMA. 4) FD-NOMA has better latency performance compared to HD-NOMA and HD-orthogonal multiple access (HD-OMA) schemes.

D. Notations and Organization

Notations: Q (1.10) In this article, we use upper case boldface letters to denote matrices (e.g., \( A \)), and we use lower case boldface letters to denote vectors (e.g., \( a \)). In addition, we use \( A^T \) as the transpose of \( A \), \( a \cdot b \) to denote the multiply by position operation for two vector \( a \) and \( b \). On the other hand, \( A \leftrightarrow B \) means a transmit-receive pair with \( A \) and \( B \) transceivers on each side working on FD mode, \( A \rightarrow B \) the transmission procedure from \( A \) to \( B \), vice versa.

The remainder of the paper is organized as follows. In section II, the FD-NOMA-based decentralized V2X system model is proposed. We divide the V2X communications into different scenarios in this section. We analysis the system capacity of different scenarios in section III. The numerical simulations are given by section IV, and conclusion is given in section V.

II. THE FD-NOMA-BASED DECENTRALIZED V2X SYSTEMS

A. System Model

Q (e.2) (3.4) The FD-NOMA-based decentralized V2X system model is given in Fig. 1. This system is slightly different from the existing ones in the following respects. A) Different from the existing studies on FD-NOMA, here no relaying systems are used because of the vehicle's limited energy. B) V2X devices can directly communicate with each other through DC mode without the cellular's help, and the required contents are obtained from neighboring V2X caches [30]. This system model thus has shorter transmission distance and better latency performance [30]. The cellular network load is reduced too.

It is noticed that to simplify the analysis, only V2V and V2I communications are considered in the existing V2X studies, see, e.g., [22]–[24], [31]–[33]. As discussed, not only the vehicles, V2X aims to connect everything on the road. In order to cope with this trend, in our FD-NOMA-based decentralized V2X systems, all V2X devices (vehicle, pedestrian, traffic lights, etc.) are comprehensively included. The massive connected devices and their various applications are making the V2X communications more complicated. To deal with this intractable problem, in this work, we classify the V2X communications into two scenarios: 1) the urban and crowded scenario and 2) the suburban and remote scenario.
In urban and crowded scenario, Rayleigh fading can be used as the channel model. This is due to the abundant reflection and refraction links between source and destination [34]. In contrast, Rician channel model is suitable for the suburban and remote scenario because of the less obstacles, where we can always establish a dominant light of sight (LoS) path from source to destination [35].

B. Received Signal and Power Allocation Scheme

In the considered FD-NOMA-based decentralized V2X systems, the channel matrix from $M$ sources to $N$ destinations is

$$
H = \begin{bmatrix}
  h_1 \\
  h_2 \\
  h_3 \\
  \vdots \\
  h_N
\end{bmatrix} = \begin{bmatrix}
  h_{1,1} & h_{1,2} & \ldots & h_{1,M} \\
  h_{2,1} & h_{2,2} & \ldots & h_{2,M} \\
  \vdots & \vdots & \ddots & \vdots \\
  h_{N,1} & h_{N,2} & \ldots & h_{N,M}
\end{bmatrix} \in \mathbb{C}^{N \times M},
$$

(1)

where $h_{i,j}$ is the channel between source $i$ and destination $j$. In this case, the received signal can be given as

$$
y = H\sqrt{p} \cdot x + n,
$$

(2)

where $\sqrt{p} \in \mathbb{C}^{M \times 1}$ is the allocated downlink NOMA power matrix, $x \in \mathbb{C}^{M \times 1}$ is the downlink transmit signal and $n \sim \mathcal{CN}(0, \sigma^2 I_N)$ is the downlink channel noise. Under the condition that $\hat{H} = H^T$ is the uplink channel with FD mode, uplink transmit information with FD mode will be

$$
\hat{y} = \hat{H} \sqrt{\hat{p}} \cdot z + \hat{n},
$$

(3)

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where \( z \in \mathbb{C}^{N \times 1} \) is the uplink information. NOMA power and channel noise vectors thus can be given as \( \hat{p} = p^T, \hat{n} = n^T \). The total power received by destination \( n \) from all \( M \) sources is given by

\[
p_n = p_{1,n} + p_{2,n} + \ldots + p_{M,n}.
\]  
(4)

Similarly,

\[
\hat{p}_n = \hat{p}_{n,1} + \hat{p}_{n,2} + \ldots + \hat{p}_{n,M},
\]  
(5)

is the self-interference power when transmitting information to \( M \) destinations from source \( n \).

**Remark 1:** Q (e.1) (e.3) The received signal is composed of the received downlink information and its self-interference from the FD uplink. (1.10) On the other hand, transmission and reception processes in the FD-NOMA-based decentralized V2X systems are different from the centralized cellular-based communications, i.e., each V2X destination can receive information with different NOMA power vectors from multiple distributed sources. By invoking the FD-NOMA techniques for simultaneous transmission and reception, the power received and transmitted by each V2X device are \( p_n, \hat{p}_n \).

## III. Ergodic Capacity Analysis in Different Scenarios

Q (3.7)

\[
C_{\text{sum}} = \sum_{i=1}^{M} \sum_{j=1}^{N} \log_2 \left( 1 + \frac{p_{i,j} |h_{i,j}|^2}{\sum_{l=j+1}^{N} p_{i,l} |h_{i,l}|^2 + \eta \hat{p}_{i,k} |h_{i,j}|^2 + \sigma^2} \right),
\]  
(6)

\[
C_{\text{sum}} = \sum_{i=1}^{M} \sum_{j=1}^{N} \log_2 \left[ 1 + \frac{\rho \alpha_{i,j} |h_{i,j}|^2}{\rho \left( \sum_{l=j+1}^{N} \alpha_{i,l} |h_{i,l}|^2 + \eta \alpha_{i,k} |h_{i,j}|^2 \right) + 1} \right],
\]  
(7)

Q (e.1) (1.4) (3.1) In literature, capacity analysis is to reveal the intuitive and simple-to-compute capacity expressions for the wireless systems [36], [37]. In this regard, closed-form capacity expression is of great importance. Generally, capacity can be classified into two different types, i.e., the ergodic (Shannon) capacity and the outage capacity [38]. In time varying channels, on condition that the channel state information (CSI) is known at the receiver but not the transmitter, i.e., \( \gamma \) (signal to interference plus noise ratio (SINR)) is known for every time slot. In practice, this can be accomplished by some channel estimation method [38], [39]. Furthermore, the distribution of \( \gamma \) is known at both the transmitter and receiver. Ergodic capacity then is defined by data transmission going through all fading states, which is also called the Shannon capacity since it is the average of instantaneous capacity over all states. In contrast, outage capacity is used to describe...
the system performance under slowly varying channels with a constant instantaneous $\gamma$ [38], [39]. Here in this study, we adopt the ergodic capacity since V2X channels are generally the time varying channels.

In the decentralized FD-NOMA V2X systems, transmission channels are uncorrelated. In this case, the considered multiple input multiple output NOMA (MIMO-NOMA) can be treated as a sum of additive single input single output NOMA (SISO-NOMA) links. Moreover, similar to prior works [12], [40], we adopt an increasing order of the channel response, which means $|h_{i,1}|^2 \leq \ldots \leq |h_{i,j}|^2 \leq \ldots \leq |h_{i,N}|^2, \forall i \in [1, M], j \in [1, N]$, vice versa. In this case, after successive interference cancellation (SIC), NOMA co-channel interference of the $i$-th user are from the $(i + 1)$-th user to the $N$-th user [40].

According to Shannon theory [17], achievable capacity of each destination can be given by (6), see the equation in the top of next page. Here $\sum_{l=i+1}^{N} p_{i,l} |h_{m,i}|^2, \forall m \in [1, M]$ yields the co-channel interference from neighboring users after SIC, $\eta p_{i,k} |h_{m,i}|^2, \forall m \in [1, M]$ is the self-interference by FD uplink, $\sigma^2$ is the channel noise power, respectively. Q (e.3) (1.3) Additionally, $\eta$ is the coefficient of self-interference with $\eta \in [0, 1]$, which makes our expressions versatile to describe different schemes. For instance, in FD-NOMA scheme, large value of $\eta$ denotes the strong FD self-interference, and small value denotes the weak FD self-interference. On condition that $\eta = 0$, the expression reduces to the pure NOMA expression. Q (2.3)

On the basis of (6), normalizing the channel noise power value will give (7). Q (2.8) Here $\rho$ is the SNR, and we use $\alpha_{i,j}, \alpha_{i,l}, \alpha_{i,k}$ to denote the allocated NOMA power coefficient with FD transmission in line with a normalized channel noise power value. In the sequel, we adopt the normalized noise power for our analysis.

A. Ergodic Capacity Analysis in Urban and Crowded Scenario

We first analyze the achievable sum capacity in urban and crowded scenario. Note that in this article, we use the superscript $a$ and $c$ to distinguish different scenarios. Q (e.1) (e.3) (1.4) In urban and crowded scenario, to the destination side, PDF of instantaneous signal to interference plus noise ratio (SINR) in each time slot, say, $\gamma_{i,j}$, is given by

$$f^a(\gamma_{i,j}) = \frac{1}{\gamma_{i,j}} e^{-\frac{\gamma_{i,j}}{\gamma_{i,j}}},$$

Q (2.9) where

$$\tilde{\gamma}_{i,j} = \frac{\rho \bar{\alpha}_{i,j}}{\rho (\sum_{l=i+1}^{N} \tilde{\alpha}_{i,l} \bar{\alpha}_{i,l} + \eta \bar{\alpha}_{i,k}) + 1},$$

Q (9)
the averaged channel power gain of each destination. As is well known, ergodic capacity is achieved by experiencing all the channel fading states. That is,

\[
C_{i,j}^a = \mathbb{E} \left[ \log_2(1 + \gamma_{i,j}) \right] = \int_0^{+\infty} \log_2(1 + \gamma_{i,j}) f_a(\gamma_{i,j}) d\gamma_{i,j} = \int_0^{+\infty} \log_2(1 + \gamma_{i,j}) \frac{1}{\gamma_{i,j}} e^{-\gamma_{i,j}} d\gamma_{i,j}.
\]  

(10)

In the following theorem, we provide the exact ergodic capacity expression of the FD-NOMA-based decentralized V2X systems.

**Theorem 1:** In urban and crowded scenario, the exact achievable sum ergodic capacity of the FD-NOMA-based decentralized V2X systems is

\[
C_{\text{sum}}^a = \sum_{i=1}^{M} \sum_{j=1}^{N} e^{\gamma_{i,j}} E_1 \left[ \frac{1}{\gamma_{i,j}} \right] \log_2 e,
\]

(11)

where \( E_1(x) \) is the exponential integral function that defined as

\[
E_1(x) = \int_x^{+\infty} \frac{e^{-t}}{t} dt.
\]

(12)

Additionally, we have \( \gamma_{i,j} \) given as (9).

**Proof:** See Appendix A.

Exact ergodic capacity expression in urban and crowded scenario is provided in **Theorem 1**. Since the exponential integral function is involved, this expression thus is not given in closed-form. We thereby pursue an approximate closed-form expression of the achievable capacity in this article. As noticed, in (11), the only expression not given by closed-form is the generalized exponential integral functions. In this case, our main focus is to find out a closed-form expression of \( E_1(x) \).

**Lemma 1:** Closed-form expression (lower bound) of the generalized exponential integral function is given by

\[
E_1(x) \leq 4\pi \sum_{k=1}^{n+1} \sum_{s=1}^{t+1} a_k b_s e^{-b_k b_s x},
\]

(13)

where \( a_k, b_k \) are defined as

\[
a_k = \frac{\theta_k - \theta_{k-1}}{\pi},
\]

(14)

\[
b_k = \frac{\cot \theta_{k-1} - \cot \theta_k}{\theta_k - \theta_{k-1}}.
\]

(15)

In addition, \( \theta_k, k \in [0, n + 1] \) is given by \( 0 \leq \theta_0 < \theta_1 < ... \theta_k < ... < \theta_{n+1} = \frac{\pi}{2} \). Besides, \( a_s, b_s, \theta_s \) are defined with the same method, i.e.,

\[
a_s = \frac{\theta_s - \theta_{s-1}}{\pi},
\]

(16)
\[ b_s = \frac{\cot \theta_{s-1} - \cot \theta_s}{\theta_s - \theta_{s-1}}, \quad (17) \]

and \( 0 \leq \theta_0 < \theta_1 < \ldots \theta_s < \ldots < \theta_{t+1} = \frac{\pi}{2} \). It is also worth noting that the approximation accuracy is controlled by the division of \( \frac{\pi}{2} \) with \( \theta_k \) and \( \theta_s \) (associate with \( a_s, b_s \))^2.

**Proof:** See Appendix B.

In order to verify the tightness of this approximation, we compare the performances of the exact expression, the approximate expression and the well known Swamee and Ohija approximation. Note that the Swamee and Ohija approximation expression is given by [28]

\[ E_1(x) = (A^{-7.7} + B)^{-0.13}, \quad (18) \]

where

\[ A = \ln \left( \frac{0.56146}{x} + 0.65 \right), \quad (19) \]
\[ B = x^4 e^{7.7x} (2 + x)^{3.7}. \quad (20) \]

Here while using the approximate expression in **Lemma 1**, we divide the \( \frac{\pi}{2} \) with 1000 segments, which means, \( \theta_k - \theta_{k-1} = \frac{\pi}{2000} \). Q (1.9) The simulation results are given by Fig. 2. As noticed, the gap between the approximation and the exact form curves is large. In this case, this approximation method is better than the Swamee and Ohija approximation method, while is unsuitable to be adopted directly.

We notice from Appendix B that in our derivations, the only issue that might bring in difference is the Jensen’s inequality, i.e., in the derivations of \( Q(x) \)-function’s closed-form expression, we use

\[ \frac{\int_{\theta_{k-1}}^{\theta_k} e^{-\frac{x^2}{2 \sin^2 \theta}} \, d\theta}{\int_{\theta_{k-1}}^{\theta_k} 1 \, d\theta} \geq e^{\int_{\theta_{k-1}}^{\theta_k} \frac{x^2}{2 \sin^2 \theta} \, d\theta}. \quad (21) \]

Q (1.9) Additionally, one can see from Fig. 2 that the approximation curve displays a similar curvature to the exact curve. We can expect that a coefficient factor to the closed-form expression might improve the accuracy, i.e.,

\[ E_1'(x) = q4\pi \sum_{k=1}^{n+1} \sum_{s=1}^{t+1} a_k \sqrt{b_k} a_s e^{-b_k b_s x}. \quad (22) \]

Consequently, our task is to find out \( q \) satisfying

\[ |E_1'(x) - E_1(x)| \leq \epsilon. \quad (23) \]

\(^2\)It is worth noting that here in our analysis, the equal division of \( \frac{\pi}{2} \) is used.
Here we use $\epsilon = 0.00001$. After some manipulations, we notice that when $q = \frac{1}{4}$, the above condition is met (e.g., $|E'_1(1) - E_1(1)| = |0.2193827 - 0.2193839| = 1.2187 \times 10^{-6}$). We thus have an approximate closed-form expression of $E_1(x)$ as

$$E_1(x) \approx \pi \sum_{k=1}^{n+1} \sum_{s=1}^{t+1} a_k \sqrt{b_k a_s} e^{-b_k b_s x}.$$  \hspace{1cm} (24)

Q (1.9) We further give the comparison results of the exact, improved and approximate expressions, which is shown in Fig. 3. Compared to the approximate results, the improved approximate results coincide with the exact results perfectly, which indicates the validity of our hypothesis. Closed-form expression of $C_{\text{Ray}}^{\text{sum}}$ is given by the following corollary.

**Corollary 1:** By substituting (24) into (11), we obtain the approximate closed-form expression of the achievable capacity in urban and crowded scenario

$$C_{\text{sum}}^a \approx \pi \log_2 e \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{n+1} \sum_{s=1}^{t+1} e^{\left(\frac{1}{n_{ij}}\right)} a_k \sqrt{b_k a_s} e^{-b_k b_s \frac{1}{n_{ij}}}.$$ \hspace{1cm} (25)

**Remark 2:** Insights from **Corollary 1** is that the system ergodic capacity in urban and crowded scenario is determined by $M, N, \gamma_{i,j}$. The system capacity increases with $M, N$. The accuracy of this approximate closed-form expression is determined by $n, t$. That is, the divisions of $\frac{n}{2}$. The validity of this approximate expression will be verified by the following numerical results.
B. Ergodic Capacity Analysis in Suburban and Remote Scenario

In the subsection III.A, we obtained both exact and approximate forms of the capacity of the FD-NOMA-based decentralized V2X systems in urban and crowded scenario. In this subsection, we focus on the system capacity analysis in suburban and remote scenario.

We use $K$ as the Rician factor (which is the ratio between the deterministic and random fast-fading component). Q (e.3) (2.11) It is noticed that in Rician channel, we have

$$K = \frac{r^2}{2\omega^2}.$$  \hspace{1cm} (26)

where $r^2$ yields the channel gain of LoS component, $2\omega^2$ is the average channel power gain of all NLoS components. By defining the total average power gain as $\bar{\gamma}$ and following the prior work in [41], PDF of $\gamma_{i,j}$ can be given as

$$f_c(\gamma_{i,j}) = \frac{K + 1}{\bar{\gamma}_{i,j}} e^{-K - \frac{(K+1)\gamma_{i,j}}{\bar{\gamma}_{i,j}}} I_0 \left( 2 \sqrt{\frac{K(K+1)}{\bar{\gamma}_{i,j}}} \right).$$ \hspace{1cm} (27)

Here $I_0(\cdot)$ is the first kind modified Bessel function with zeroth order. By following a similar procedure of the previous analysis, we can obtain the system capacity expression in suburban and remote scenario, which is given by the following theorem.

**Theorem 2:** Exact ergodic capacity expression of the FD-NOMA-based decentralized V2X systems in suburban and remote scenario is given by

$$C_{sum}^c = \sum_{i=1}^{M} \sum_{j=1}^{N} e^{-K} \frac{K + 1}{\ln 2} \frac{Km+1}{\ln m} \sum_{m=0}^{\infty} \frac{K^m}{m!} \sum_{l=1}^{m+1} E_{m-l+2} \left( \frac{K + 1}{\bar{\gamma}_{i,j}} \right).$$ \hspace{1cm} (28)
Here $E_n(x)$ is the generalized exponential integral function defined as [42]
\[
E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt \quad (\text{Re}(x) > 0),
\]
(29)
where $\text{Re}(x)$ yields the real part of $x$.

**Proof:** See Appendix C.

Although we have derived the exact capacity expression in suburban and remote scenario, this expression is still intractable to use directly because of the involved infinite factorial and generalized exponential integral expressions. In order to tame this troublesome problem, we give one approximate expression with arbitrary small error by invoking the truncation method in the sequel.

We find that the following expression
\[
\sum_{m=0}^\infty \frac{K^m}{m!} \sum_{q=1}^{m+1} E_{m-q+2}(\frac{K+1}{\tilde{\gamma}_{i,j}})
\]
(30)
has an upper ceiling approximation, as shown by Corollary 2. In this case, the system capacity can be given by an approximate expression with much lower computation complexity and arbitrary small error, $\epsilon$.

**Corollary 2:** By truncating the infinite series with regard to $T$, the capacity expression is approximately given as
\[
C_{\text{sum}}^c \approx \sum_{i=1}^M \sum_{j=1}^N e^{-K \ln 2} e^{\frac{K+1}{\tilde{\gamma}_{i,j}}} \sum_{m=0}^T \frac{K^m}{m!} \sum_{q=1}^{m+1} E_{m-q+2}(\frac{K+1}{\tilde{\gamma}_{i,j}}),
\]
(31)
The truncation error is
\[
\sum_{i=1}^M \sum_{j=1}^N e^{-K \ln 2} e^{\frac{K+1}{\tilde{\gamma}_{i,j}}} \sum_{m=T+1}^\infty \frac{K^m}{m!} \sum_{q=1}^{m+1} E_{m-q+2}(\frac{K+1}{\tilde{\gamma}_{i,j}}).
\]
(32)

**Proof:** See Appendix D.

**Remark 3:** One can notice that the accuracy of the approximate expression in (31) is controlled by $T$. In other words, we may obtain an approximate expression with an arbitrary small error when
\[
\sum_{i=1}^M \sum_{j=1}^N e^{-K \ln 2} e^{\frac{K+1}{\tilde{\gamma}_{i,j}}} \sum_{m=T+1}^\infty \frac{K^m}{m!} \sum_{q=1}^{m+1} E_{m-q+2}(\frac{K+1}{\tilde{\gamma}_{i,j}}) < \epsilon.
\]
(33)
Insight from Corollary 2 is that the system capacity in suburban and remote scenario is determined by $M, N, \tilde{\gamma}_{i,j}$ and $K$. With $M, N$ increasing, the system capacity always increases. The precise effects of $\tilde{\gamma}_{i,j}, K$ to the capacity are still nonintuitive, which will be discussed in the following section.
IV. NUMERICAL RESULTS

In this section, we perform the Monte Carlo simulations to verify the validity of our analysis. We also perform simulations to exposit the effects of different parameters to the system capacity, and compare the performance between FD-NOMA and NOMA schemes based on the decentralized FD-NOMA-enabled V2X systems. Due to variable parameters, we separately explain the parameter values in the following simulations.

We first check the validity of the derived capacity expressions in (11), (25) and (31). In these simulations, for the sake of compactness, one source with multiple destinations are used, where the source employs the FD-NOMA scheme to serve these destinations. We also assume that the allocated NOMA power variance is growing linearly with a normalized noise variance value (e.g., with 4 users, the NOMA power vector is $\mathbf{a}_i = [4, 3, 2, 1]$), where $\mathbf{a}_i = [\alpha_{i,1}, ..., \alpha_{i,N}]$. Additionally, $\eta = 0.1, \alpha_{i,k} = 5$ are used. As clearly shown by Fig. 4 and Fig. 5, our analytical results and the MC results almost exactly coincide, which demonstrates the validity of our analysis. For instance, in Fig. 4, with $\rho = 15$ dB, 1 $\leftrightarrow$ 4, the Exa, MC and App results are 3.6865, 3.6866, 3.6865 Bit/S/Hz. On the other hand, under the same condition, as shown in Fig. 5, MC result and App result are 3.8458, 3.8456 Bit/S/Hz, respectively. The differences are less than 0.001 Bit/S/Hz in both scenarios. We observe that as the values of $N, \rho$ increases, the system capacity also increases. By comparing Fig. 4 and Fig. 5, we also notice that under the same condition, capacities in suburban and remote scenario always outperform the ones in urban and crowded scenario (for instance, in 1 $\leftrightarrow$ 3 case, SNR = 0 dB, $C^c_{sum} = 125% C^a_{sum};$ SNR = 30 dB, $C^c_{sum} = 103% C^a_{sum}$). This is because of the less propagation loss with a dominant LoS path between source and destination in the suburban and remote scenario.

In order to verify the benefits of our analytical expressions, we compare the consumed time of Exa, App and MC simulations in Table I for both scenarios with $\rho = 15$ dB. In this case, the eight-core 3.4 GHz processors, 16 GB memory and windows 10 64-bit operating system are used. The results are rounded off to four decimal places. As shown in Table I, the consumed time of our analytical expressions (Exa in urban and crowded scenario, App in suburban and remote scenario) are about $10^6$ times shorter than the MC simulations. In particular, the consumed time of App simulations is even shorter (about 10 times) than the Exa simulations in urban and crowded scenario.

In the next step, we check the effect of Rician factor $K$ to the system capacity in suburban and remote scenario. In order to keep $K$ as the only variable, we do some manipulations as follows: 1) we keep all variables consistent except $K$; 2) with normalized noise power value and 3 destinations, we set $\mathbf{a}_i = [1, 2, 3]$.

Fig. 4: Q (1.9) (3.8) (3.9) Comparison of the system achievable sum capacity performances of Exa, MC and App results in urban and crowded scenario. The exactly and approximate results are obtained according to (11) and (25), respectively.

Fig. 5: Q (1.9) (3.8) (3.9) Comparison of the system achievable sum capacity performances with MC and App results in suburban and remote scenario. The analytical results are obtained according to (31).

The simulation results of the system capacity vs the destination number in suburban and remote scenario is given in Fig. 6. We notice from Fig. 6 that as the $K$ increases, system capacity also increases. Q (1.10) This is because the higher $K$ brings in a stronger LoS component and a weaker multi-path propagation loss.

Besides the effects of $N, \rho, K$, the effects of $M$ and $a_i$ to the system capacity are also checked. In
TABLE I: Q (1.6) Consumed time (second) of Exa, App and MC simulations with $\rho = 15$ dB.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Exa</th>
<th>App</th>
<th>MC</th>
<th>App</th>
<th>MC</th>
<th>App</th>
<th>MC</th>
<th>App</th>
<th>MC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Urban and crowded scenario</td>
<td>0.0103</td>
<td>0.0066</td>
<td>0.0528</td>
<td>0.0072</td>
<td>0.0023</td>
<td>0.0022</td>
<td>0.0013</td>
<td>0.0012</td>
<td>0.0000</td>
</tr>
<tr>
<td>Suburban and remote scenario</td>
<td>83.0272</td>
<td>89.2913</td>
<td>83.0986</td>
<td>80.8462</td>
<td>81.7240</td>
<td>87.6330</td>
<td>91.5975</td>
<td>87.3378</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Fig. 6: Q (3.9) Comparison of the capacities with different power values and source numbers.

these simulations, our setting are given as follows: 1) a linearly growing power value with $M = 1$ (i.e., $a_1 = [0.5, 1.5, 1.5], a_2 = [1, 2, 3], a_3 = [2, 4, 6]$); 2) different NOMA power vectors with $M = 2$ (i.e., $2 \leftrightarrow 3, a_1, a_3$ denote that two sources are transmitting information to 3 destinations with FD-NOMA, where the NOMA power vector are $a_1, a_2$, respectively). The simulation results are given by Fig. 7 and Fig. 8.

Q (1.5) As shown by the solid lines in both figures, increasing the power values leads to better capacity performance, which is due to the increased SNR value. For instance, in $1 \leftrightarrow 3$ case and SNR = 20 dB, we have $C_{\text{sum}}^a(a_2) = 131\% C_{\text{sum}}^a(a_1)$. We can also confirm from both figures that as $M$ increases, the system capacities also increase.

Q (e.2) (3.5) Finally, we compare the achievable throughputs with FD-NOMA, NOMA, FD-OMA and HD-OMA schemes in different scenarios. The results are given in Fig. 9 and Fig. 10. In these simulations, carrier bandwidth $B = 100$ MHz, $a_i = [3, 2, 1], \eta = 0.1$ and $\alpha_{i,k} = 0.1, 1, 10$ are used. In order to be fair, we average the allocated power in FD-OMA and HD-OMA schemes. As shown in both figures, NOMA scheme has a better throughput performance compared to OMA scheme. Moreover, with a smaller value of $\alpha_{i,k}$,
Fig. 7: Q (3.9) Comparison of the capacities with different power values and source numbers.

Fig. 8: Q (3.9) Comparison of the capacities with different power values and source numbers.

FD-NOMA always outperforms the other schemes (HD-NOMA, FD-OMA, HD-OMA). However, the benefit of FD-NOMA decreases while $\alpha_{i,k}$ increasing. This is mainly due to the increased FD self-interferences. We also notice that even with a higher FD self-interference value, FD-NOMA outperforms NOMA in low SNR scenario (i.e., $\rho \in [0, 5]$ dB). This is due to the fact that in low SNR scenario, channel noise is the dominant factor compared to FD self-interference. In contrast, FD-NOMA self-interference becomes the dominant factor in high SNR scenario, NOMA scheme without FD self-interference thus has a better
Fig. 9: Q (e.2) (3.5) (3.9) System achievable throughput comparisons with FD-NOMA, NOMA, FD-OMA and HD-OMA schemes in urban and crowded scenario.

throughput performance. It is also worth noting that the effective transmission time is limited because of the fast moving V2X devices. FD-NOMA enabled bidirectional transmission can greatly reduce the transmission latency compared to other schemes. For example, compared to HD-NOMA and HD-OMA, FD-NOMA only needs a half latency time to transmit the same amount of data by its simultaneous transmission and reception scheme.
V. CONCLUSION

In this article, we proposed the FD-NOMA-based decentralized V2X systems. We classified the V2X communications into two typical scenarios, i.e., the urban and crowd scenario and the suburban and remote scenario, and then derived the exact system capacity expressions in both scenarios. To tackle down the capacity expression’s intractable calculations in both scenarios, we further obtained their simplified approximate expressions. Insights of our analysis are that the accuracy of our simplified approximate expression in urban and crowded scenario is determined by the associated division of $\frac{\gamma}{2}$ (with respect to exponential integral function ($E_1(x)$)), and the accuracy of simplified approximate expression in suburban and remote scenario is determined by the truncation point $T$ (with respect to generalized exponential integral function ($E_n(x)$)). Numerical results demonstrate the validity and effectiveness of our analytical results. Compared to MC method, the consumed time is greatly reduced by our Exa and App expressions. Simulation results also demonstrated that the system capacity performance can be enhanced by increasing the number of V2X devices, NOMA power and Rician factor (suburban and remote scenario), and the effectiveness of FD-NOMA is determined by the FD self-interference and the channel noise. In addition, FD-NOMA can greatly reduce the system latency compared to other schemes.

APPENDIX A: PROOF OF THEOREM 1

Q (1.10) Firstly, according to the integration by parts method, we have

\[
\int_0^{+\infty} \log_2(1 + \gamma_{i,j}) \frac{1}{\gamma_{i,j}} e^{-\frac{\gamma_{i,j}}{\gamma_{i,j}}} d\gamma_{i,j} = -\int_0^{+\infty} \log_2(1 + \gamma_{i,j})(e^{-\frac{\gamma_{i,j}}{\gamma_{i,j}}})' d\gamma_{i,j} \\
= \log_2(1 + \gamma_{i,j}) e^{-\frac{\gamma_{i,j}}{\gamma_{i,j}}} \bigg|_0^{+\infty} + \int_0^{+\infty} \frac{1}{\ln 2(1 + \gamma_{i,j})} e^{-\frac{\gamma_{i,j}}{\gamma_{i,j}}} d\gamma_{i,j} \\
= \frac{1}{\ln 2} \int_0^{+\infty} \frac{1}{(1 + \gamma_{i,j})} e^{-\frac{\gamma_{i,j}}{\gamma_{i,j}}} d\gamma_{i,j} \\
= \frac{1}{\ln 2} \int_0^{+\infty} \frac{1}{\gamma_{i,j}} e^{-\frac{\gamma_{i,j}}{\gamma_{i,j}}} \left(\frac{1}{\gamma_{i,j}} + \frac{1}{\gamma_{i,j}}\right) d\gamma_{i,j} \\
= \frac{1}{\ln 2} \int_0^{+\infty} e^{-\frac{\gamma_{i,j}}{\gamma_{i,j}} + \frac{1}{\gamma_{i,j}}} \left(\frac{1}{\gamma_{i,j}} + \frac{1}{\gamma_{i,j}}\right) d\gamma_{i,j} \tag{A.1}
\]
So far the expression is still intractable. In the next step, we recall the alternative generalized exponential integral expression \[42\]

\[E_n(x) \triangleq \int_1^\infty \frac{e^{-xt}}{t^n} \, dt\]

\[= \int_0^1 e^{-\frac{x}{t}} t^{n-2} \, dt\]

\[= x^{n-1} \int_x^\infty \frac{e^{-t}}{t} \, dt, \quad x > 0, \quad (A.2)\]

By substituting (A.2) into (A.1), and further summarizing the result with \(M\) sources and \(N\) destinations, we can safely arrive the final expression.

This completes the proof.

**APPENDIX B: PROOF OF LEMMA 1**

As noticed, \(E_1(x)\) can be rewritten as

\[E_1(x) = \int_x^\infty \frac{e^{-t}}{\sqrt{t}} \, dt. \quad (B.1)\]

It is also noticed that the following equality holds \[43\]

\[
\frac{e^{-t}}{\sqrt{t}} = -\sqrt{2}e^{-t} \frac{d}{dt} \frac{d}{\sqrt{2t}}
\]

\[= -2\sqrt{\pi} \frac{d}{dt} \frac{d}{\sqrt{2\pi}} \int_{\sqrt{\pi}}^\infty e^{-\frac{t^2}{2}} \, dt
\]

\[= -2\sqrt{\pi} \frac{d}{dt} \{Q(\sqrt{2t})\}. \quad (B.2)\]

Thus in the next step, our work is to seek a closed-form expression for the \(Q\)-function. Actually, there are various closed-form expressions to capture the lower or upper bounds of the \(Q\)-function, for instance, the chernoff bound

\[Q(x) \leq e^{-\frac{x^2}{2}}, \quad x > 0, \quad (B.3)\]

the improved exponential bound

\[Q(x) \leq \frac{1}{4} e^{-x^2} + \frac{1}{4} e^{-x^2} \begin{cases} \leq & e^{-\frac{x^2}{2}}, \quad x > 0. \end{cases} \quad (B.4)\]
However, the integral is still intractable while substituting those expressions into (B.1), an alternative method is needed. According to prior work, by adopting the Craig’s form, we have [43]

\[
Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-\frac{t^2}{2}} dt \tag{B.1}
\]

Craig’s form

\[
= \frac{1}{\pi} \int_{0}^{\pi/2} e^{- \frac{\theta^2}{2\sin^2 \theta}} d\theta
\]

\[
= \frac{1}{\pi} \sum_{k=1}^{n+1} \int_{\theta_{k-1}}^{\theta_k} e^{- \frac{\theta^2}{2\sin^2 \theta}} d\theta
\]

\[
= \frac{\theta_k - \theta_{k-1}}{\pi} \sum_{k=1}^{n+1} \int_{\theta_{k-1}}^{\theta_k} e^{- \frac{\theta^2}{2\sin^2 \theta}} 1d\theta
\]

(B.5)

with \(a_k, b_k\) are defined as

\[
a_k = \frac{\theta_k - \theta_{k-1}}{\pi}, \tag{B.6}
\]

\[
b_k = \cot \theta_{k-1} - \cot \theta_k \tag{B.7}
\]

Then by substituting the Jensen’s inequality [44] to (B.5), we have the lower bound expression of \(Q\)-function as

\[
Q(x) \geq \frac{\theta_k - \theta_{k-1}}{\pi} \sum_{k=1}^{n+1} a_k e^{- \frac{\theta_{k-1}^2}{2\sin^2 \theta} 1d\theta}
\]

\[
= \frac{\theta_k - \theta_{k-1}}{\pi} \sum_{k=1}^{n+1} a_k e^{- \theta_{k-1}^2 (\cot \theta_{k-1} - \cot \theta_k \cdot \cot \theta_{k-1})} \tag{B.8}
\]

Additionally, it is worth noting that \(\theta_k, k \in [0, n+1]\) is given by \(0 \leq \theta_0 < \theta_1 < \ldots < \theta_k < \ldots < \theta_{n+1} = \frac{\pi}{2}\) [43], [44]. The approximate accuracy of this lower bound expression is controlled by the interval gap between each pair of \([\theta_{k-1}, \theta_k]\). Moreover, it is known that the following equality holds

\[
\int \sin^{-2} xd\theta = - \cot x + C. \tag{B.9}
\]

Substituting it into (B.8), we thus have

\[
Q(x) \approx \sum_{k=1}^{n+1} a_k e^{- \frac{x^2 b_k}{2}}. \tag{B.10}
\]
Finally, by substituting (B.2) into (B.1), $E_1(x)$ can be given as

$$E_1(x) = \int_x^\infty \frac{e^{-t}}{\sqrt{t}} \frac{1}{\sqrt{t}} dt$$

$$= -2\sqrt{\pi} \int_x^\infty \frac{d}{dt} \left(Q\left(\sqrt{2t}\right)\right) \frac{1}{\sqrt{t}} dt$$

$$\approx 2\sqrt{\pi} \sum_{k=1}^{n+1} a_k b_k \int_x^\infty \frac{e^{-b_k t}}{\sqrt{t}} dt$$

$$= 2\sqrt{\pi} \sum_{k=1}^{n+1} a_k b_k \int_x^\infty \frac{\sqrt{b_k} e^{-b_k t}}{b_k t} dt$$

$$= 4\pi \sum_{k=1}^{n+1} a_k \sqrt{b_k} Q\left(\sqrt{2b_k x}\right)$$

As the final inequality is obtained while substituting the approximate expression of $Q\left(\sqrt{2b_k x}\right)$, definitions of $a_s, b_s, \theta_s$ thus are similar as prior definitions of $a_k, b_k, \theta_k$, i.e., $a_s = \frac{\theta_s - \theta_{s-1}}{\pi}, b_s = \cot \frac{\theta_s - \theta_{s-1}}{\theta_s - \theta_{s-1}}$ and $0 \leq \theta_0 < \theta_1 < ... \theta_s < ... < \theta_{t+1} = \frac{\pi}{2}$.

This completes the proof.

**APPENDIX C: PROOF OF THEOREM 2**

It is noticed that the PDF of $\gamma_{i,j}$ in Rician channel condition can be given by [41]

$$f^c(\gamma_{i,j}) = \frac{K+1}{\tilde{\gamma}_{i,j}} e^{-\frac{\gamma_{i,j}(K+1)}{\tilde{\gamma}_{i,j}}} I_0\left(2\sqrt{\frac{K(K+1)\gamma_{i,j}}{\tilde{\gamma}_{i,j}}}\right),$$

(C.1)

By following a similar derivation procedure as in Theorem 1, we have the following equation

$$C_{i,j}^c = \mathbb{E}[\log_2(1 + \gamma_{i,j})]$$

$$= \int_0^{+\infty} \log_2(1 + \gamma_{i,j}) f^c(\gamma_{i,j}) d\gamma_{i,j}.$$
and substituting equations (C.4) and (C.5) into this expression, the derivations of capacity expression of the
FD-NOMA-based decentralized V2X systems can be given as:

\[ C_{i,j} = \int_{0}^{+\infty} \log_2(1 + \gamma_{i,j}) f_c^e(\gamma_{i,j}) d\gamma_{i,j} \]

\[ = \int_{0}^{+\infty} \frac{\ln(1 + \gamma_{i,j})}{\ln 2} \frac{K + 1}{\bar{\gamma}_{i,j}} e^{-K\bar{\gamma}_{i,j}} d\gamma_{i,j} \]

\[ I_0 \left( 2\sqrt{\frac{K(K+1)\gamma_{i,j}}{\bar{\gamma}_{i,j}}} \right) d\gamma_{i,j} \]

\[ = \frac{(K + 1)e^{-K}}{\bar{\gamma}_{i,j} \ln 2} \sum_{m=0}^{\infty} \frac{\left[ \frac{K(K+1)}{\bar{\gamma}_{i,j}} \right]^m}{(m!)^2} \int_{0}^{+\infty} \frac{\ln(1 + \gamma_{i,j})}{\ln 2} \frac{\gamma_{i,j}^{m+1}}{e^{K\gamma_{i,j}}} d\gamma_{i,j} \]

\[ = \frac{(K + 1)e^{-K}}{\bar{\gamma}_{i,j} \ln 2} \sum_{m=0}^{\infty} \frac{\left[ \frac{K(K+1)}{\bar{\gamma}_{i,j}} \right]^m}{(m!)^2} \frac{\Gamma(m+1)e^{\frac{K+1}{\bar{\gamma}_{i,j}}}}{m} \sum_{q=1}^{m+1} E_{m-q+2}(\frac{K + 1}{\bar{\gamma}_{i,j}}) \]

\[ = \frac{e^{-K}}{\ln 2} \frac{K^m}{m!} \sum_{q=1}^{\infty} E_{m-q+2}(\frac{K + 1}{\bar{\gamma}_{i,j}}). \]

Here the second equality is due to the modified Bessel function of the zeroth order expression [45]

\[ I_0(x) = \sum_{m=0}^{\infty} \frac{\left( \frac{x}{2} \right)^{2m}}{m!\Gamma(m+1)}. \] (C.4)

Additionally, the third equality is because [45]

\[ \int_{0}^{+\infty} \ln(1 + \kappa x)^{z-1} e^{\beta x} dx = \frac{\Gamma(z)e^{\frac{\beta x}{x}}}{x^z} \sum_{l=1}^{2} E_{z-l+1}(\frac{\beta}{x}). \] (C.5)

By further summarizing this expression with M sources and N destinations, we can finally arrive at (31).

This completes the proof.

**APPENDIX D: PROOF OF COROLLARY 2**

The remaining section after a truncation with regard to \( T \) is

\[ \sum_{i=1}^{M} \sum_{j=1}^{M} e^{-K} \frac{K+1}{\ln 2} \sum_{m=T_1}^{\infty} \frac{K^m}{m!} \sum_{q=1}^{m+1} E_{m-q+2}(\frac{K + 1}{\bar{\gamma}_{i,j}}). \] (D.1)

As shown here, approximate error mainly comes from the infinite expression series with regard to \( m \).

According to prior work in [46], [47], \( E_n(x) \) monotonically decreasing in \( n \) giving equal \( x \). In this case, by

\[ \sum_{m=T_1}^{\infty} \frac{K^m}{m!} \sum_{q=1}^{m+1} E_{m-q+2}(\frac{K + 1}{\bar{\gamma}_{i,j}}) < \sum_{m=T_1}^{\infty} \frac{K^m}{m!} \frac{(m+1)(m+2)}{2} E_1(\frac{K + 1}{\bar{\gamma}_{i,j}}). \] (D.2)
It is noticed that giving constant values of $\gamma_{i,j}$ and $K$, $E_1\left(\frac{K+1}{\gamma_{i,j}}\right)$ then becomes a constant coefficient. Consequently, we focus on the function

$$f(x) = \frac{K^x (x+1)(x+2)}{x!}.$$  \hspace{1cm} (D.3)

By some mathematical manipulations, it is found that there existing $x'$, so that $f'(x') = 0$ with $f''(x'-) > 0$, $f''(x'+) < 0$. Additionally, observation has that $f(x)$ rapidly converges to 0 after $x'$ (e.g., $f(100) = 6.9966 \times 10^{-125}$). This gives approximate capacity expression of (31) with an arbitrary small error $\epsilon$.

This completes the proof.

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Q (1.2), (1.7) (3.2) Performance Analysis of FD-NOMA-based Decentralized V2X Systems

Di Zhang, Member, IEEE, Yuanwei Liu, Member, IEEE, Linglong Dai, Senior Member, IEEE,
Ali Kashif Bashir, Senior Member, IEEE, Arumugam Nallanathan, Fellow, IEEE, and Byonghyo Shim, Senior Member, IEEE

Abstract—In order to meet the massively connected devices, different quality of services (QoSs), various transmit rates and ultra-reliable and low latency communications (URLLC) requirements of vehicle to everything (V2X) communications, Q (1.2, 1.7) we introduce a full duplex non-orthogonal multiple access (FD-NOMA)-based decentralized V2X system model, Q (1.4, 2.13) Compared to the orthogonal frequency division multiple access (OFDMA) scheme, NOMA is insensitive to Doppler effect caused by moving vehicles. We classify the V2X communications into two scenarios and give their exact capacity expressions. To solve the computation complicated problems of the involved exponential integral functions, we give the approximate closed-form expressions with arbitrary small numbers. Numerical results indicate the validness of our derivations. Q (1.10) Our analysis has that the accuracy of our approximate expressions is controlled by the division of 1 in the urban and crowded scenario, and the truncation point T in the suburban and remote scenario. Numerical results also manifest 1) Increasing the number of V2X device, NOMA power and Rician factor value yields better capacity performance. 2) Effect of FD-NOMA is determined by the FD self-interference and the channel noise. 3) FD-NOMA has better latency performance compared to other schemes.

Index Terms—Vehicle communications, V2X, full duplex, non-orthogonal multiple access, capacity analysis.

I. INTRODUCTION

A. Background

There are two distinct regimes in vehicle to everything (V2X) communications, i.e., dedicated short-range communications (DSRC) [1], [2] and cellular-V2X (C-V2X) [3], [4]. DSRC was popular in the past decades. Recently, C-V2X has received much attention with explosively growing devices connecting to the wireless networks. Q (1.10) With the help of cellular network, C-V2X can connect more V2X devices [5], [6]; it can establish the link among vehicles, smart infrastructures and pedestrians, etc. C-V2X operates in two modes. Q (1.10) First, in the direct communications (DC) mode, V2X devices can directly communicate with each other. Well-known examples include vehicle to vehicle (V2V), vehicle to pedestrian (V2P) communications. Second, in the network-based communications (NC) mode, cellular base station (BS) is playing the dominant role, and the V2X devices communicate with (or with the help of) the cellular, for instance, vehicle to network (V2N), vehicle to infrastructure (V2I) communications. However, the current version of C-V2X (i.e., the long term evolution V2X (LTE-V2X)) cannot fully satisfy the requirements of low latency, various quality of services (QoSs) and different transmit rates [6], [7]. Q (1.4, 2.13) In addition, the existing orthogonal frequency division multiple access (OFDMA)-based LTE-V2X systems need orthogonality. Different from the non-moving wireless communications, moving vehicle caused Doppler effect is a vital problem for OFDMA-based LTE-V2X systems [8]. As is known, the carrier frequency offset (CFO) caused by the Doppler effect will lead to inter-carrier interference (ICI) to the OFDM-based wireless communications [9]. There have been various studies to solve the CFO compensation, see, e.g., [9], [10]. However, because the oscillators can never be oscillating at the identical frequency, in OFDMA-based wireless communications, CFO side-effect always exists even for non-moving circumstance [9].

It is noticed that fifth generation (5G) technologies can be used to address the issues of low latency, various QoS and different transmit rates in V2X communications, for instance, non-orthogonal multiple access (NOMA) [11] and full duplex (FD) [12]. Q (1.10) (3.3) Compared to the OMA scheme, NOMA can accommodate more users, and these users can be with different QoS requirements [12], [13]. Q (1.4) (2.13) In addition, NOMA is insensitive to CFO effect caused by moving devices because of its non-orthogonal frequency. NOMA uses the same resource block (RB) for multiple user’s transmission, which can alleviate the spectrum bottleneck of wireless communications [14]–[16]. NOMA can pair users with different transmit rates for simultaneous transmission [17]. On the other hand, while simultaneously transmitting and receiving information, full duplex (FD) can provide faster speed and better spectrum efficiency (SE) performances [12]. Moreover,
FD can offer reliable communications [18], which is useful for V2X applications such as navigation and emergency message broadcasting.

B. Related Works and Motivations

Q (e.2) (3.4) In cellular communications, there are previous studies on FD-NOMA. For instance, it was found that FD-NOMA can significantly suppress the co-channel interferences and achieve better performance gains compared to half duplex NOMA (HD-NOMA) and orthogonal multiple access (OMA) [19]. Q (3.5) Analysis and simulation results in [20] demonstrated that rate region performance of FD-NOMA outperforms the one with NOMA. Q (1.10) (3.4) Analysis and simulation results in [21] indicated that FD-NOMA improves the 5G's system performance compared to HD-NOMA. Based on the relaying system model, analysis and simulation results in [12] indicated that FD-NOMA outperforms HD-NOMA in terms of outage probability and ergodic sum rate in low signal to noise ratio (SNR) region, but displays an inferior performance in high SNR region.

In V2X communications, there are some existing works on NOMA-V2X and FD-V2X [22]–[24]. Based on the NOMA, the authors in [23] proposed the graph-based practical encoding and joint belief propagation (BP) decoding techniques, which can achieve any rate pair close to the capacity region. D. Di et al. in [22] employed NOMA for URLLC communications while proposing a NOMA-based mixed centralized/distributed (NOMA-MCD) scheme to reduce the resource collision. In [24], an optimal blind interference alignment scheme was proposed for the coexisting of FD and HD modes. This scheme can improve the sum rate performance in finite SNR regime. However, most of these studies on NOMA-V2X and FD-V2X communications are based on the NC mode, which is a challenge for connecting massive V2X devices because of the cellular throughput restriction. Although the authors investigated the decentralized NOMA-V2X systems in [22], there has been no capacity analysis for such a system. To the best of our knowledge, a study investigating the impact of FD-NOMA techniques on V2X systems is rare, which motivates us to develop this treatise.

Q (e.1) (2.1) (2.2) (2.4) (3.1) In literature, various channel models are used for ergodic capacity analysis, for instance, the \(\kappa - \mu\) channel model [25], [26] and the \(\eta - \mu\) channel model [25]. However, obtaining the closed-form capacity expression in these channel models is very difficult because of the involved infinity series operations. Authors thus employed some special conditions and methods to give the closed-form expressions, e.g., \(\mu\) with positive integer values [26] and the approximate method [27]. On the other hand, the difficulty to obtain a closed-form expression with Rayleigh or Rician channel model lies in the involved exponential integral functions. In order to solve this, some approximate methods and algorithms have been proposed, for instance, the Swamee and Ohija method for exponential integral function [28] and the fast and accurate algorithm for generalized exponential integral function [29]. However, these methods are based on some special condition (e.g., [29]), or with lower accuracy (e.g., [28]). In this paper, we give the approximate closed-form capacity expressions for both Rayleigh and Rician channel models while taming the troublesome exponential integral functions.

In this work, we propose Q (1.12), (1.17) the FD-NOMA-based decentralized V2X system model, and also provide the capacity analysis to obtain the approximate closed-form capacity expressions with high accuracy. We try to answer the following key questions.

- Can we use one solution to meet all the requirements of V2X communications? If it is not possible, what about a combination of FD-NOMA techniques?
- If the combination is feasible to satisfy the requirements of V2X communications, what about the capacity and throughput performance of the V2X systems?
- Q (2.1) (3.1) Is there any approximate expressions for the capacity expressions with arbitrary small error and low computational complexity?

C. Contributions

The main contributions of this work can be summarized as follows:

- The FD-NOMA-based decentralized V2X systems can partly offload the cellular network. Q (1.4), (2.13) Compared to OFDMA, NOMA is insensitive to Doppler effect caused by moving vehicles. In addition, FD-NOMA can accommodate more users with different QoSs and transmit rates for simultaneous transmission and reception, which is suitable for V2X systems.

- Q (e.1) (2.1) (3.1) Based on the system model, we derive the exact system ergodic capacity expressions and their approximate closed-form expressions for both scenarios. These approximate closed-form expressions are with low computational complexity and controllable arbitrary small errors compared to the existing approximate expressions.

Insights from our analysis has 1) the accuracy of our simplified approximate expression in urban and crowded scenario is controlled by the associated division of \(\frac{2}{3}\) (with respect to the exponential integral function \(E_1(x)\)). 2) The accuracy of our simplified approximate expression in suburban and remote scenario is controlled by the truncation point \(T\) (with respect to the exponential integral function \(E_n(x)\)).

- It is observed from our numerical results that: 1) the analytical results coincide with the Monte-Carlo based simulation results perfectly, which demonstrates the validity of our derivations. 2) The system capacity increases with the increasing allocated power value, SNR and Rician factor values. 3) The FD self-interference and the channel noise determine the effect of FD-NOMA. 4) FD-NOMA has better latency performance compared to HD-NOMA and HD-orthogonal multiple access (HD-OMA) schemes.

\(^1\)Besides C-V2X communications, there are other types of cellular communications, our work can not offload all the cellular network load.
D. Notations and Organization

Notations: $Q$ (1.10) In this article, we use upper case boldface letters to denote matrices (e.g., $A$), and we use lower case boldface letters to denote vectors (e.g., $a$). In addition, we use $A^T$ as the transpose of $A$, $a \bullet b$ to denote the multiply by position operation for two vector $a$ and $b$. On the other hand, $A \leftrightarrow B$ means a transmit-receive pair with $A$ and $B$ transceivers on each side working on FD mode, $A \rightarrow B$ the transmission procedure from $A$ to $B$, vice versa.

The remainder of the paper is organized as follows. In section II, the FD-NOMA-based decentralized V2X system model is proposed. We divide the V2X communications into different scenarios in this section. We analysis the system capacity of different scenarios in section III. The numerical simulations are given by section IV, and conclusion is given in section V.

II. THE FD-NOMA-BASED DECENTRALIZED V2X SYSTEMS

A. System Model

Q (e.2) (3.4) The FD-NOMA-based decentralized V2X system model is given in Fig. 1. This system is slightly different from the existing ones in the following respects. A) Different from the existing studies on FD-NOMA, here no relaying systems are used because of the vehicle’s limited energy. B) V2X devices can directly communicate with each other through DC mode without the cellular’s help, and the required contents are obtained from neighboring V2X caches [30]. This system model thus has shorter transmission distance and better latency performance [30]. The cellular network load is reduced too.

It is noticed that to simplify the analysis, only V2V and V2I communications are considered in the existing V2X studies, see, e.g., [22]–[24], [31]–[33]. As discussed, not only the vehicles, V2X aims to connect everything on the road. In order to cope with this trend, in our FD-NOMA-based decentralized V2X systems, all V2X devices (vehicle, pedestrian, traffic lights, etc.) are comprehensively included. The massive connected devices and their various applications are making the V2X communications more complicated. To deal with this intractable problem, in this work, we classify the V2X communications into two scenarios: 1) the urban and crowded scenario and 2) the suburban and remote scenario.

In urban and crowded scenario, Rayleigh fading can be used as the channel model. This is due to the abundant reflection and refraction links between source and destination [34]. In contrast, Rician channel model is suitable for the suburban and remote scenario because of the less obstacles, where we can always establish a dominant light of sight (LoS) path from source to destination [35].

B. Received Signal and Power Allocation Scheme

In the considered FD-NOMA-based decentralized V2X systems, the channel matrix from $M$ sources to $N$ destinations is

$$H = \begin{bmatrix}
  h_1 \\
  h_2 \\
  h_3 \\
  \vdots \\
  h_N
\end{bmatrix} = \begin{bmatrix}
  h_{1,1} & h_{1,2} & \ldots & h_{1,M} \\
  h_{2,1} & h_{2,2} & \ldots & h_{2,M} \\
  \vdots & \vdots & \ddots & \vdots \\
  h_{N,1} & h_{N,2} & \ldots & h_{N,M}
\end{bmatrix} \in \mathbb{C}^{N \times M}, (1)$$

where $h_{i,j}$ is the channel between source $i$ and destination $j$. In this case, the received signal can be given as

$$y = H\sqrt{p_1} \bullet x + n,$$  \hspace{1cm} (2)

where $\sqrt{p_1} \in \mathbb{C}^{M \times 1}$, is the allocated downlink NOMA power matrix, $x \in \mathbb{C}^{M \times 1}$ is the downlink transmit signal and $n \sim \mathcal{CN}(0, \sigma^2 I_N)$ is the downlink channel noise. Under the condition that $H = H^T$ is the uplink channel with FD mode, uplink transmit information with FD mode will be

$$\hat{y} = \hat{H} \sqrt{\hat{p}} \bullet z + \hat{n},$$  \hspace{1cm} (3)

where $z \in \mathbb{C}^{N \times 1}$ is the uplink information. NOMA power and channel noise vectors thus can be given as $\hat{p} = p^T$, $\hat{n} = n^T$. The total power received by destination $n$ from all $M$ sources is given by

$$p_n = p_{1,n} + p_{2,n} + \ldots + p_{M,n}. \hspace{1cm} (4)$$

Similarly,

$$\hat{p}_n = \hat{p}_{n,1} + \hat{p}_{n,2} + \ldots + \hat{p}_{n,M}, \hspace{1cm} (5)$$

is the self-interference power when transmitting information to $M$ destinations from source $n$.

Remark 1: $Q$ (e.1) (e.3) The received signal is composed of the received downlink information and its self-interference from the FD uplink. (1.10) On the other hand, transmission and reception processes in the FD-NOMA-based decentralized V2X systems are different from the centralized cellular-based communications, i.e., each V2X destination can receive information with different NOMA power vectors from multiple
A. Ergodic Capacity Analysis in Urban and Crowded Scenario

We first analyze the achievable sum capacity in urban and crowded scenario. Note that in this article, we use the superscript $a$ and $c$ to distinguish different scenarios. Q (e.1) (e.3) (1.4) In urban and crowded scenario, to the destination side, PDF of instantaneous signal to interference plus noise ratio (SINR) in each time slot, say, $\gamma_{i;j}$, is given by

$$f^a(\gamma_{i;j}) = \frac{1}{\gamma_{i;j}} e^{-\frac{\gamma_{i;j}}{\rho_i}},$$

Q (2.9) where

$$\bar{\gamma}_{i;j} = \frac{\rho \bar{a}_{i;j}}{\rho \bar{a}_{i;j} + \eta \bar{a}_{i;k}} + 1,$$

the averaged channel power gain of each destination. As is well known, ergodic capacity is achieved by experiencing all the channel fading states. That is,

$$C_{i;j} = \mathbb{E} \left[ \log_2 (1 + \gamma_{i;j}) \right]$$

$$= \int_0^{\infty} \log_2 (1 + \gamma_{i;j}) f^a(\gamma_{i;j}) \, d\gamma_{i;j}$$

$$= \int_0^{\infty} \log_2 (1 + \gamma_{i;j}) \frac{1}{\bar{\gamma}_{i;j}} e^{-\frac{\gamma_{i;j}}{\bar{\gamma}_{i;j}}} \, d\gamma_{i;j}. $$

In the following theorem, we provide the exact ergodic capacity expression of the FD-NOMA-based decentralized V2X systems.

**Theorem 1:** In urban and crowded scenario, the exact achievable sum ergodic capacity of the FD-NOMA-based decentralized V2X systems is

$$C_{sum} = \sum_{i=1}^M \sum_{j=1}^N e^{-\frac{\gamma_{i;j}}{\rho_i}} E_1 \left[ \frac{1}{\bar{\gamma}_{i;j}} \right] \log_2 e,$$  \hspace{1cm} (11)

where $E_1(x)$ is the exponential integral function that defined as

$$E_1(x) = \int_x^{\infty} \frac{e^{-t}}{t} \, dt.$$ \hspace{1cm} (12)

Additionally, we have $\bar{\gamma}_{i;j}$ given as (9).

**Proof:** See Appendix A.

Exact ergodic capacity expression in urban and crowded scenario is provided in Theorem 1. Since the exponential integral function is involved, this expression thus is not given in closed-form. We thereby pursue an approximate closed-form expression of the achievable capacity in this article. As noticed, in (11), the only expression not given by closed-form is the generalized exponential integral functions. In this case, our main focus is to find out a closed-form expression of $E_1(x)$.

**Lemma 1:** Closed-form expression (lower bound) of the generalized exponential integral function is given by

$$E_1(x) \leq 4x \sum_{k=1}^{n+1} \sum_{s=1}^{t+1} a_k b_k e^{-b_k x},$$

where $a_k, b_k$ are defined as

$$a_k = \frac{\theta_k - \theta_{k-1}}{\pi},$$

$$b_k = \frac{\sqrt{\theta_k^2 - x^2}}{\theta_k},$$

$$\theta_k = \frac{k\pi}{n+1}.$$
Q (3.7)

\[ C_{\text{sum}} = \sum_{i=1}^{M} \sum_{j=1}^{N} \log_2 \left( 1 + \frac{p_{i,j} |h_{i,j}|^2}{\sum_{l=j+1}^{N} p_{i,l} |h_{i,l}|^2 + \eta p_{i,k} |h_{i,k}|^2 + \sigma^2} \right), \]

\[ C_{\text{sum}} = \sum_{i=1}^{M} \sum_{j=1}^{N} \log_2 \left( 1 + \frac{\rho x_{i,j} |h_{i,j}|^2}{\rho (\sum_{l=j+1}^{N} x_{i,l} |h_{i,l}|^2 + \eta x_{i,k} |h_{i,k}|^2 + \sigma^2) + 1} \right), \]

In addition, \( \theta_k, k \in [0, n + 1] \) is given by \( \theta_0 < \theta_1 < ... < \theta_k < ... < \theta_{n+1} = \frac{\pi}{2} \). Besides, \( a_s, b_s, \theta_s \) are defined with the same method, i.e.,

\[ a_s = \frac{\theta_s - \theta_{s-1}}{\pi}, \]

\[ b_s = \frac{\cot \theta_{s-1} - \cot \theta_s}{\theta_s - \theta_{s-1}}, \]

and \( 0 \leq \theta_0 < \theta_1 < ... < \theta_s < ... < \theta_{t+1} = \frac{\pi}{2} \). It is also worth noting that the approximation accuracy is controlled by the division of \( \frac{\pi}{2} \) with \( \theta_0 \) and \( \theta_s \) (associate with \( a_s, b_s \)).

Proof: See Appendix B.

In order to verify the tightness of this approximation, we compare the performances of the exact expression, the approximate expression and the well known Swamee and Ohija approximation. Note that the Swamee and Ohija approximation expression is given by [28]

\[ E_1(x) = (A^{-7.7} + B)^{-0.13}, \]

where

\[ A = \ln \left( \frac{0.56146}{x} + 0.65 \right), \]

\[ B = x^4 e^{7.7} (2 + x)^{3.7}. \]

Here while using the approximate expression in Lemma 1, we divide the \( \frac{\pi}{2} \) with 1000 segments, which means, \( \theta_k - \theta_{k-1} = \frac{\pi}{2000} \). Q (1.9) The simulation results are given by Fig. 2. As noticed, the gap between the approximation and the exact form curves is large. In this case, this approximation method is better than the Swamee and Ohija approximation method, while is unsuitable to be adopted directly.

We notice from Appendix B that in our derivations, the only issue that might bring in difference is the Jensen’s inequality, i.e., in the derivations of \( Q(x) \)-function’s closed-form expression, we use

\[ \int_{\theta_k}^{\theta_{k-1}} e^{-\frac{x^2}{\sin^2 \theta}} d\theta \geq e^{\left( \frac{x^2}{\sin^2 \theta} \right) \left( \frac{\theta_k - \theta_{k-1}}{\theta_k - \theta_{k-1}} \right)} \geq e^{\left( \frac{x^2}{\sin^2 \theta} \right) \left( \frac{\theta_k - \theta_{k-1}}{\theta_k - \theta_{k-1}} \right)}. \]

Q (1.9) Additionally, one can see from Fig. 2 that the approximation curve displays a similar curvature to the exact curve. We can expect that a coefficient factor to the closed-form expression might improve the accuracy, i.e.,

\[ E'_1(x) = q 4\pi \sum_{k=1}^{n+1} a_k \sum_{l=1}^{t+1} b_k a_s e^{-b_k b_s x}. \]

Consequently, our task is to find out \( q \) satisfying

\[ |E'_1(x) - E_1(x)| \leq \epsilon. \]

Here we use \( \epsilon = 0.00001 \). After some manipulations, we notice that when \( q = \frac{1}{2} \), the above condition is met (e.g., \( |E'_1(1) - E_1(1)| = 0.2193827 - 0.2193839 = 1.2187 \times 10^{-6} \)). We thus have an approximate closed-form expression of \( E_1(x) \) as

\[ E_1(x) \approx \pi \sum_{k=1}^{n+1} a_k \sum_{s=1}^{t+1} b_k a_s e^{-b_k b_s x}. \]

Q (1.9) We further give the comparison results of the exact, improved and approximate expressions, which is shown in Fig. 3. Compared to the approximate results, the improved approximate results coincide with the exact results perfectly, which indicates the validity of our hypothesis. Closed-form expression of \( C_{\text{sum}}^{\text{Ray}} \) is given by the following corollary.
defining the total average power gain as \( \tilde{\gamma} \) and following the prior work in [41], PDF of \( \tilde{\gamma}_{i,j} \) can be given as
\[
f(\tilde{\gamma}_{i,j}) = \frac{K + 1}{\tilde{\gamma}_{i,j}} e^{-K\frac{(K+1)\tilde{\gamma}_{i,j}}{\tilde{\gamma}_{i,j}}} I_0 \left( 2\sqrt{\frac{K(K+1)\tilde{\gamma}_{i,j}}{\tilde{\gamma}_{i,j}}} \right).
\]
(27)

Here \( I_0(\cdot) \) is the first kind modified Bessel function with zeroth order. By following a similar procedure of the previous analysis, we can obtain the system capacity expression in suburban and remote scenario, which is given by the following theorem.

**Theorem 2:** Exact ergodic capacity expression of the FD-NOMA-based decentralized V2X systems in suburban and remote scenario is given by
\[
C_{\text{sum}}^c = \sum_{i=1}^{M} \sum_{j=1}^{N} e^{-K\frac{\ln 2}{\tilde{\gamma}_{i,j}}} e^{\sum_{m=0}^{\infty} \frac{K^m}{m!} \sum_{l=1}^{m+1} E_{m-l+2} \frac{K+1}{\tilde{\gamma}_{i,j}}}.
\]
(28)

Here \( E_n(x) \) is the generalized exponential integral function defined as [42]
\[
E_n(x) = \int_1^{\infty} e^{-xt} \frac{dt}{t^n} \quad (\text{Re}(x) > 0),
\]
(29)

where \( \text{Re}(x) \) yields the real part of \( x \).

**Proof:** See Appendix C.

Although we have derived the exact capacity expression in suburban and remote scenario, this expression is still intractable to use directly because of the involved infinite factorial and generalized exponential integral expressions. In order to tame this troublesome problem, we give one approximate expression with arbitrary small error by invoking the truncation method in the sequel.

We find that the following expression
\[
\sum_{m=0}^{\infty} \frac{K^m}{m!} \sum_{q=1}^{m+1} E_{m-q+2} \frac{K+1}{\tilde{\gamma}_{i,j}}
\]
(30)

has an upper ceiling approximation, as shown by **Corollary 2**. In this case, the system capacity can be given by an approximate expression with much lower computation complexity and arbitrary small error, \( \epsilon \).

**Corollary 2:** By truncating the infinite series with regard to \( T \), the capacity expression is approximately given as
\[
C_{\text{sum}}^c \approx \sum_{i=1}^{M} \sum_{j=1}^{N} e^{-K\frac{\ln 2}{\tilde{\gamma}_{i,j}}} e^{\sum_{m=0}^{T} \frac{K^m}{m!} \sum_{q=1}^{m+1} E_{m-q+2} \frac{K+1}{\tilde{\gamma}_{i,j}}}.
\]
(31)

The truncation error is
\[
\sum_{i=1}^{M} \sum_{j=1}^{N} e^{-K\frac{\ln 2}{\tilde{\gamma}_{i,j}}} e^{\sum_{m=T+1}^{\infty} \frac{K^m}{m!} \sum_{q=1}^{m+1} E_{m-q+2} \frac{K+1}{\tilde{\gamma}_{i,j}}}.
\]
(32)

**Proof:** See Appendix D.

**Remark 3:** One can notice that the accuracy of the approximate expression in (31) is controlled by \( T \). In other words, we
may obtain an approximate expression with an arbitrary small error when
\[
\sum_{i=1}^{M} \sum_{j=1}^{N} e^{-K_{i,j}} \frac{K_{i,j}^{m+1}}{m!} \sum_{q=1}^{\infty} E_{m-q+2}^{q+2} \left( \frac{K_{i,j}}{m!} \right) < \epsilon.
\]
(33)

Insight from Corollary 2 is that the system capacity in suburban and remote scenario is determined by \( M, N, \gamma_{i,j} \) and \( K \). With \( M, N \) increasing, the system capacity always increases. The precise effects of \( \gamma_{i,j} \), \( K \) to the capacity are still unintuitive, which will be discussed in the following section.

IV. NUMERICAL RESULTS

In this section, we perform the Monte Carlo simulations to verify the validity of our analysis. We also perform simulations to exhibit the effects of different parameters to the system capacity, and compare the performance between FD-NOMA and NOMA schemes based on the decentralized FD-NOMA-enabled V2X systems. Due to variable parameters, we separately explain the parameter values in the following simulations.

We first check the validity of the derived capacity expressions in (11), (25) and (31). In these simulations, for the sake of compactness, one source with multiple destinations are used, where the source employs the FD-NOMA scheme to serve these destinations. Q (1.3) We also assume that the allocated NOMA power variance is growing linearly with a normalized noise variance value (e.g., with 4 users, the NOMA power vector is \( \mathbf{a} = [4, 3, 2, 1] \)), where \( \mathbf{a} = [a_{1}, ..., a_{N}] \). Additionally, \( \eta = 0.1, \alpha_{s,k} = 5 \) are used. As clearly shown by Fig. 4 and Fig. 5, our analytical results\(^3\) and the MC results almost exactly coincide, which demonstrates the validity of our analysis. For instance, in Fig. 4, with \( \rho = 15 \) dB, 1 \( \leftrightarrow \) 4; the Exa, MC and App results are 3.6865, 3.6866, 3.6865 Bit/S/Hz. On the other hand, under the same condition, as shown in Fig. 5, MC result and App result are 3.8458, 3.8456 Bit/S/Hz, respectively. The differences are less than 0.001 Bit/S/Hz in both scenarios. We observe that as the values of \( N, \rho \) increases, the system capacity also increases. Q (1.5) By comparing Fig. 4 and Fig. 5, we also notice that under the same condition, capacities in suburban and remote scenario always outperform the ones in urban and crowded scenario (for instance, in 1 \( \leftrightarrow \) 3 case, \( \text{SNR} = 0 \) dB, \( C_{\text{sum}}^{a} = 125\%C_{\text{sum}}^{a} \); \( \text{SNR} = 30 \) dB, \( C_{\text{sum}}^{a} = 130\%C_{\text{sum}}^{a} \)). This is because of the less propagation loss with a dominant LoS path between source and destination in the suburban and remote scenario.

In order to verify the benefits of our analytical expressions, we compare the consumed time of Exa, App and MC simulations in Table I for both scenarios with \( \rho = 15 \) dB. In this case, the eight-core 3.4 GHz processors, 16 GB memory and windows 10 64-bit operating system are used. The results are rounded off to four decimal places. As shown in Table I, the consumed time of our analytical expressions (Exa in urban and crowded scenario, App in suburban and remote scenario) are about 10\(^6\) times shorter than the MC simulations. In particular,

\(^3\)Exa: exact, App: approximate, MC: Monte Carlo.
TABLE I: Q (1.6) Consumed time (second) of Exa, App and MC simulations with $\rho = 15$ dB.

<table>
<thead>
<tr>
<th>System achievable sum capacity (Bit/S/Hz)</th>
<th>Exa</th>
<th>App</th>
<th>MC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Urban and crowded scenario</td>
<td>0.0103</td>
<td>0.0066</td>
<td>0.0528</td>
</tr>
<tr>
<td>Suburban and remote scenario</td>
<td>0.0002</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

Fig. 6: Q (3.9) Comparison of the capacities with different power values and source numbers.

Fig. 7: Q (3.9) Comparison of the capacities with different power values and source numbers.

Fig. 8: Q (3.9) Comparison of the capacities with different power values and source numbers.

system capacity also increases. Q (1.10) This is because the higher $K$ brings in a stronger LoS component and a weaker multi-path propagation loss.

Besides the effects of $N$, $\rho$, $K$, the effects of $M$ and $a_i$ to the system capacity are also checked. In these simulations, our setting are given as follows: 1) a linearly growing power value with $M = 1$ (i.e., $a_1 = [0.5, 1, 1.5]$, $a_2 = [1, 2, 3]$, $a_3 = [2, 4, 6]$); 2) different NOMA power vectors with $M = 2$ (i.e., $2 \leftrightarrow 3, a_1, a_3$ denote that two sources are transmitting information to 3 destinations with FD-NOMA, where the NOMA power vector are $a_1, a_2$, respectively). The simulation results are given by Fig. 7 and Fig. 8. Q (1.5) As shown by the solid lines in both figures, increasing the power values leads to better capacity performance, which is due to the increased SNR value. For instance, in $1 \leftrightarrow 3$ case and SNR = 20 dB, we have $C_{sum}(a_2) = 131\% C_{sum}(a_1)$. We can also confirm from both figures that as $M$ increases, the system capacities also increase.

Q (e.2) (3.5) Finally, we compare the achievable throughputs with FD-NOMA, NOMA, FD-OMA and HD-OMA schemes in different scenarios. The results are given in Fig. 9 and Fig. 10. In these simulations, carrier bandwidth $B = 100$ MHz, $a_i = [3, 2, 1]$, $\eta = 0.1$ and $\alpha_{k,i} = 0.1, 1, 10$ are used. In order to be fair, we average the allocated power in FD-OMA and HD-OMA schemes. As shown in both figures, NOMA scheme has a better throughput performance compared to OMA scheme. Moreover, with a smaller value of $\alpha_{k,i}$, FD-NOMA always outperforms the other schemes (HD-NOMA, FD-OMA, HD-OMA). However, the benefit of FD-NOMA decreases while $\alpha_{k,i}$ increasing. This is mainly due to the increased FD self-interferences. We also notice that even with a higher FD self-interference value, FD-NOMA outperforms NOMA in low SNR scenario (i.e., $\rho \in [0, 5]$ dB). This is due to the fact that in low SNR scenario, channel noise is the dominant factor.
compared to FD self-interference. In contrast, FD-NOMA self-interference becomes the dominant factor in high SNR scenario, NOMA scheme without FD self-interference thus has a better throughput performance. It is also worth noting that the effective transmission time is limited because of the fast moving V2X devices. FD-NOMA enabled bidirectional transmission can greatly reduce the transmission latency compared to other schemes. For example, compared to HD-NOMA and HD-OMA, FD-NOMA only needs a half latency time to transmit the same amount of data by its simultaneous transmission and reception scheme.

Fig. 9: Q (e2) (3.5) (3.9) System achievable throughput comparisons with FD-NOMA, NOMA, FD-OMA and HD-OMA schemes in urban and crowded scenario.

Fig. 10: Q (e2) (3.5) (3.9) System achievable throughput comparisons with FD-NOMA, NOMA, FD-OMA and HD-OMA schemes in suburban and remote scenario.

V. CONCLUSION

In this article, we proposed the FD-NOMA-based decentralized V2X systems. We classified the V2X communications into two typical scenarios, i.e., the urban and crowd scenario and the suburban and remote scenario, and then derived the exact system capacity expressions in both scenarios. To tackle down the capacity expression’s intractable calculations in both scenarios, we further obtained their simplified approximate expressions. Insights of our analysis are that the accuracy of our simplified approximate expression in urban and crowded scenario is determined by the associated division of $\frac{\pi}{2}$ (with respect to exponential integral function $(E_1(x))$, and the accuracy of simplified approximate expression in suburban and remote scenario is determined by the truncation point $T$ (with respect to generalized exponential integral function $(E_n(x))$). Numerical results demonstrate the validity and effectiveness of our analytical results. Compared to MC method, the consumed time is greatly reduced by our Exa and App expressions. Simulation results also demonstrated that the system capacity performance can be enhanced by increasing the number of V2X devices, NOMA power and Rician factor (suburban and remote scenario), and the effectiveness of FD-NOMA is determined by the FD self-interference and the channel noise. In addition, FD-NOMA can greatly reduce the system latency compared to other schemes.

APPENDIX A: PROOF OF THEOREM 1

Q (1.10) Firstly, according to the integration by parts method, we have

$$\int_0^{+\infty} \log_2 (1 + \gamma_{i,j}) \frac{1}{\gamma_{i,j}} e^{-\frac{\gamma_{i,j}}{\gamma_{i,j}}} d\gamma_{i,j}$$

$$= \int_0^{+\infty} \log_2 (1 + \gamma_{i,j}) (e^{-\frac{\gamma_{i,j}}{\gamma_{i,j}}} - 0) d\gamma_{i,j}$$

$$= \log_2 (1 + \gamma_{i,j}) e^{-\frac{\gamma_{i,j}}{\gamma_{i,j}}} \bigg|_0^{+\infty} + \int_0^{+\infty} \frac{1}{\ln 2 (1 + \gamma_{i,j})} e^{-\frac{\gamma_{i,j}}{\gamma_{i,j}}} d\gamma_{i,j}$$

$$= \frac{1}{\ln 2} \int_0^{+\infty} \frac{1}{(1 + \gamma_{i,j})} e^{-\frac{\gamma_{i,j}}{\gamma_{i,j}}} d\gamma_{i,j}$$

$$= \frac{1}{\ln 2} \int_0^{+\infty} \frac{1}{\gamma_{i,j}} (e^{-\frac{\gamma_{i,j}}{\gamma_{i,j}}} - 0) d\gamma_{i,j}$$

$$= \frac{1}{\ln 2} \int_0^{+\infty} \frac{1}{\gamma_{i,j}} \left( e^{-\frac{\gamma_{i,j}}{\gamma_{i,j}}} + \frac{\gamma_{i,j}}{\gamma_{i,j}} \right) d\gamma_{i,j}$$

$$= e^{-\frac{1}{\gamma_{i,j}}} \int_0^{+\infty} \frac{1}{\gamma_{i,j}} \left( e^{-\frac{\gamma_{i,j}}{\gamma_{i,j}}} + \frac{\gamma_{i,j}}{\gamma_{i,j}} \right) d\gamma_{i,j}$$

So far the expression is still intractable. In the next step, we recall the alternative generalized exponential integral expression [42]

$$E_n(x) \triangleq \int_1^{\infty} \frac{e^{-xt}}{t^n} dt$$

$$= \int_0^{1} e^{-xt} t^{n-2} dt$$

$$= x^{n-1} \int_x^{\infty} \frac{e^{-t}}{t} dt, x > 0,$$  

$$E_n(x) \triangleq \int_1^{\infty} \frac{e^{-xt}}{t^n} dt$$

$$= \int_0^{1} e^{-xt} t^{n-2} dt$$

$$= x^{n-1} \int_x^{\infty} \frac{e^{-t}}{t} dt, x > 0,$$
By substituting (A.2) into (A.1), and further summarizing the result with $M$ sources and $N$ destinations, we can safely arrive at the final expression.

This completes the proof.

**APPENDIX B: PROOF OF LEMMA 1**

As noticed, $E_1(x)$ can be rewritten as

$$E_1(x) = \int_{x}^{\infty} e^{-t} \frac{1}{\sqrt{t}} dt. \quad (B.1)$$

It is also noticed that the following equality holds [43]

$$\frac{e^{-t}}{\sqrt{t}} = -\sqrt{2e^{-t}} d\sqrt{2t} \quad (B.2)$$

Thus in the next step, our work is to seek a closed-form expression for the $Q$-function. Actually, there are various closed-form expressions to capture the lower or upper bounds of the $Q$-function, for instance, the Chernoff bound [13].

The improved exponential bound

$$Q(x) \leq e^{-\frac{x^2}{2}}, x > 0, \quad (B.3)$$

provides the lower bound expression of the $Q$-function, where $\pi$ is noticed that the following equality holds [43]

$$E(x) = \pi \int_{x}^{\infty} e^{-t} \frac{1}{\sqrt{t}} dt$$

is also noticed that the following equality holds [43]

$$\frac{e^{-t}}{\sqrt{t}} = -\sqrt{2e^{-t}} d\sqrt{2t} \quad (B.2)$$

Thus in the next step, our work is to seek a closed-form expression for the $Q$-function. Actually, there are various closed-form expressions to capture the lower or upper bounds of the $Q$-function, for instance, the Chernoff bound [13].

**APPENDIX C: PROOF OF THEOREM 2**

It is noticed that the PDF of $\gamma_{i,j}$ in Rician channel condition can be given by [41]

$$f^c(\gamma_{i,j}) = K + 1 \frac{1}{\gamma_{i,j}} e^{-\frac{K}{\gamma_{i,j}}} I_0 \left( 2 \sqrt{\frac{K(K + 1)}{\gamma_{i,j}}} \right), \quad (C.1)$$

By following a similar derivation procedure as in Theorem 1, we have the following equation

$$C_{i,j}^c = E[\log_2(1 + \gamma_{i,j})] = \int_{0}^{\infty} \log_2(1 + \gamma_{i,j}) f^c(\gamma_{i,j}) d\gamma_{i,j}. \quad (C.2)$$
and substituting equations (C.4) and (C.5) into this expression, the derivations of capacity expression of the FD-NOMA-based decentralized V2X systems can be given as:

\[ C_{i,j}^c = \int_0^\infty \log_2(1 + \gamma_{i,j}) f(x) dx. \]

By some mathematical manipulations, it is found that there exists \( x' \), so that \( f'(x') = 0 \) with \( f''(x') > 0, f''(x') < 0 \). Additionally, observation has that \( f(x) \) rapidly converges to 0 after \( x' \) (e.g., \( f(100) = 6.9966 \times 10^{-125} \)). This gives approximate capacity expression of (31) with an arbitrary small error \( \epsilon \).

This completes the proof.

REFERENCES


Dear Professor Zhang,

We appreciate you and the anonymous reviewers very much for the valuable comments on this manuscript. The paper is seriously revised according but not limited to these comments. The concerns will be replied item by item in this response letter. In this response letter, all the comments are typeset in

Comment: *italic blue font.*

Our responses are written in plain font. The reproduced changes are given with *sans serif typeface.* In addition, in the revised manuscript, the rephrased sentences and changes are marked by *plain blue font.* The related questions are marked by Q *(e,b)* and Q *(a,b)* in the revised manuscript, where *e* represents editor, *a* reviewer *a* and *b* the comment item given by the editor or reviewer.

Thank you and the anonymous reviewers again for your time and valuable comments on this manuscript.

Sincerely yours,

Di Zhang, Yuanwei Liu, Linglong Dai, Ali Kashif Bashir, Arumugam Nallanathan, and By-onghyo Shim
Evaluation: We receive three expert reviews for this paper. Overall, we agree that this paper is interesting and has technical contributions in applying decentralized FD-NOMA and content caching in V2X systems. However, the paper has room to improve.

Response: We thank the editor very much for your time on handling our submission and the valuable comments to improve the quality of it. We have seriously revised the manuscript according to these comments and the comments from anonymous reviewers in this updated version. We will first answer the comments from the editor in this response letter.

Comment 1: First, the author may want to enhance the analysis of the obtained results and how the analysis results can be made use of.

Response: We thank the editor very much for this valuable comment. The analysis of our obtained results have been enhanced. In addition, for our capacity analysis and the related figures to verify the correctness of our derivation, as pointed out by the original work of Claude E. Shannon [1], the capacity analysis is to reveal the intuitive and simple-to-compute characteristics of the communication systems [2]. In this regard, the closed-form capacity expression is of great importance.

On the other hand, it is known that the capacity expressions of Rayleigh and Rician channel models are troublesome to use directly because of the involved exponential integral functions (un-generalized exponential integral function for Rayleigh channel case and generalized exponential integral function for Rician channel case). Although some approximate and asymptotic expressions are given in previous studies, for instance, the widely used Swamee and Ohija method for exponential integral function. But these approximate expressions are still time consuming or based on some special conditions, and mostly not given in closed-form. In this work, we give approximate closed-form expressions for the involved exponential integral functions. In addition, as is shown in the simulation results, our approximate closed-form expressions can achieve arbitrary small errors compared to existing approximate methods, and our approximate closed-form expressions are of less computation complexity, which consumes much less time compared to the existing methods.

We reproduce our revisions here for the editor’s convenience.

Q (e.1) (2.1) (2.2) (2.4) (3.1) In literature, various channel models are used for ergodic capacity analysis, for instance, the $\kappa - \mu$ channel model [3], [4] and the $\eta - \mu$ channel model [3]. However, obtaining the closed-form capacity expression in these channel models is very difficult because of the involved infinity series operations. Authors thus employed some special conditions and methods to give the closed-form expressions, e.g., $\mu$ with positive integer values [4] and the approximate method [5]. On the other hand, the difficulty to obtain a closed-form expression with Rayleigh or Rician channel model lies in the involved exponential integral functions. In order to solve this, some approximate methods and algorithms have been proposed, for instance, the Swamee and Ohija method for exponential integral function [6] and the fast and accurate algorithm for generalized exponential integral function [7]. However, these methods are based on some special condition (e.g., [7]), or with lower accuracy (e.g.,
In this paper, we give the approximate closed-form capacity expressions for both Rayleigh and Rician channel models while taming the troublesome exponential integral functions.

Based on the system model, we derive the exact system ergodic capacity expressions and their approximate closed-form expressions for both scenarios. These approximate closed-form expressions are with low computational complexity and controllable arbitrary small errors compared to the existing approximate expressions.

In literature, capacity analysis is to reveal the intuitive and simple-to-compute capacity expressions for the wireless systems [1], [2]. In this regard, closed-form capacity expression is of great importance. Generally, capacity can be classified into two different types, i.e., the ergodic (Shannon) capacity and the outage capacity [8]. In time-varying channels, on condition that the channel state information (CSI) is known at the receiver but not the transmitter, i.e., \( \gamma \) (signal to interference plus noise ratio (SINR)) is known for every time slot. In practice, this can be accomplished by some channel estimation method [8], [9]. Furthermore, the distribution of \( \gamma \) is known at both the transmitter and receiver. Ergodic capacity then is defined by data transmission going through all fading states, which is also called the Shannon capacity since it is the average of instantaneous capacity over all states. In contrast, outage capacity is used to describe the system performance under slowly varying channels with a constant instantaneous \( \gamma \) [8], [9]. Here in this study, we adopt the ergodic capacity since V2X channels are generally the time-varying channels.

The received signal is composed of the received downlink information and its self-interferences from the FD uplink. On the other hand, transmission and reception processes in the FD-NOMA-based decentralized V2X systems are different from the centralized cellular-based communications, i.e., each V2X destination can receive information with different NOMA power vectors from multiple distributed sources. By invoking the FD-NOMA techniques for simultaneous transmission and reception, the power received and transmitted by each V2X device are \( p_n, \hat{p}_n \).

In urban and crowded scenario, to the destination side, PDF of instantaneous signal to interference plus noise ratio (SINR) in each time slot, say, \( \gamma_{i,j} \), is given by

\[
f(\gamma_{i,j}) = \frac{1}{\bar{\gamma}_{i,j}} e^{-\frac{\gamma_{i,j}}{\bar{\gamma}_{i,j}}},
\]

where we have

\[
\bar{\gamma}_{i,j} = \frac{\rho \bar{\alpha}_{i,j}}{\rho (\sum_{l=i+1}^{N} \bar{\alpha}_{l,j} + \eta \bar{\alpha}_{i,k}) + 1}.
\]

the averaged channel power gain of each destination.

Comment 2: Second, the authors may want to discuss the difference between the system performance analysis of FD-NOMA in this paper and the existing works of throughput performances of wireless communications with FD-NOMA, and clarify the settings of content caching in simulation results.

Response: We thank the editor very much for this valuable comment. We have added the difference of the existing works about full duplex non-orthogonal multiple access (FD-NOMA) and our study here about the FD-NOMA in decentralized vehicle to everything (V2X) communication systems. In addition, in Fig. 9 and Fig. 10 of this updated version, we compared the FD-NOMA, half duplex-NOMA (HD-NOMA), FD-orthogonal multiple access (FD-OMA) and (HD-OMA) for their achievable throughput and latency performance.

On the other hand, with respect to the content caching, a lot of our prior studies and studies from other groups have been done on its benefits see, e.g., [10], [11]. Here we adopt the
suggestions from reviewer 1 and reviewer 3 and delete the content caching related contents in this updated version. The title is changed as well according to these comments.

We reproduce our corrections here for the editor’s consideration.

Q (e.2) (1.2), (1.7) (3.2) Performance Analysis of FD-NOMA-based Decentralized V2X Systems

Q (e.2) (1.2) (3.4) In cellular communications, there are previous studies on FD-NOMA. For instance, it was found that FD-NOMA can significantly suppress the co-channel interferences and achieve better performance gains compared to half duplex NOMA (HD-NOMA) and orthogonal multiple access (OMA) [12]. Q (3.5) Analysis and simulation results in [13] demonstrated that rate region performance of FD-NOMA outperforms the one with NOMA. Q (1.10) (3.4) Analysis and simulation results in [14] indicated that FD-NOMA improves the 5G’s system performance compared to HD-NOMA. Based on the relaying system model, analysis and simulation results in [15] indicated that FD-NOMA outperforms HD-NOMA in terms of outage probability and ergodic sum rate in low signal to noise ratio (SNR) region, but displays an inferior performance in high SNR region.

Q (e.2) (3.4) The FD-NOMA-based decentralized V2X system model is given in Fig. 3. This system is slightly different from the existing ones in the following respects. A) Different from existing studies on FD-NOMA, here no relaying systems are used because of the vehicle’s limited energy. B) V2X devices can directly communicate with each other through DC mode without the cellular’s help, and the required contents are obtained from neighboring V2X caches [10]. This system model thus has shorter transmission distance and better latency performance [10]. The cellular network load is reduced too.

Q (e.2) (3.5) Finally, we compare the achievable throughputs with FD-NOMA, NOMA, FD-OMA and HD-OMA schemes in different scenarios. The results are given in Fig. 17 and Fig. 18. In these simulations, carrier bandwidth $B = 100$ MHz, $a_i = [3, 2, 1], \eta = 0.1$ and $\tilde{\alpha}_{i,k} = 0.1, 1, 10$ are used. In order to be fair, we average the allocated power in FD-OMA and HD-OMA schemes. As shown by both figures, NOMA scheme has a better throughput performance compared to OMA scheme. Moreover, with a smaller value of $\alpha_{i,k}$, FD-NOMA always outperforms the other schemes (HD-NOMA, FD-OMA, HD-OMA). However, the benefit of FD-NOMA decreases with $\alpha_{i,k}$ increasing. This is mainly because of the increased FD self-interferences. We also notice that even with a much higher FD self-interferences, FD-NOMA outperforms NOMA in low SNR scenario (i.e., $\rho \in [0, 5]$ dB). This is due to the fact that in low SNR scenario, channel noise is the dominant factor compared to FD self-interference. In contrast, FD-NOMA self-interference becomes the dominant factor in high SNR scenario, NOMA scheme without FD self-interference thus has a better throughput performance. It is also worth noting that the effective transmission time is limited because of the fast moving V2X devices. FD-NOMA enabled bidirectional transmission can greatly reduce the transmission latency compared to other schemes. For example, compared to HD-NOMA and HD-OMA, FD-NOMA only needs a half latency time to transmit the same amount of data by its simultaneous transmission and reception scheme.

Comment 3: Finally, many equations and theorems in this paper need more explanations to make them clear.

Response: We thank the editor very much for this suggestion. We have seriously double checked the related contents and corrected them accordingly. We reproduce our corrections here for the editor’s convenience.

Q (e.1) (e.3) The received signal is composed of the received downlink information and its self-interferences from the FD uplink. (1.10) On the other hand, transmission and reception
processes in the FD-NOMA-based decentralized V2X systems are different from the centralized cellular-based communications, i.e., each V2X destination can receive information with different NOMA power vectors from multiple distributed sources. By invoking the FD-NOMA techniques for simultaneous transmission and reception, the power received and transmitted by each V2X device are $p_n, \hat{p}_n$.

Additionally, $\eta$ is the coefficient of self-interference with $\eta \in [0, 1]$, which makes our expressions versatile to characterize different schemes. For instance, in FD-NOMA scheme, large value of $\eta$ denotes the strong FD self-interference, and small value denotes the weak FD self-interference. On condition that $\eta = 0$, the expression reduces to the pure NOMA expression. Q (2.3) On the basis of (6), normalizing the channel noise power value will give (7). Q (2.8) Here $\rho$ is the SNR, and we use $\alpha_{i,j}, \alpha_{i,k}, \alpha_{i,l}$ to denote the allocated NOMA power coefficient with FD transmission in line with a normalized channel noise power value. In the sequel, we adopt the normalized noise power for our analysis.

Q (e.1) (e.3) (1.4) In urban and crowded scenario, to the destination side, PDF of instanta-

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**Fig. 1:** Q (e.2) (3.5) (3.9) System achievable throughput comparisons with FD-NOMA, NOMA, FD-OMA and HD-OMA schemes in urban and crowded scenario.

**Fig. 2:** Q (e.2) (3.5) (3.9) System achievable throughput comparisons with FD-NOMA, NOMA, FD-OMA and HD-OMA schemes in suburban and remote scenario.
neous signal to interference plus noise ratio (SINR) in each time slot, say, $\gamma_{i,j}$, is given by

$$f^{\alpha}(\gamma_{i,j}) = \frac{1}{\gamma_{i,j}} e^{-\frac{\alpha_{i,j}}{\gamma_{i,j}}}$$  \hspace{1cm} (3)

Q (2.9) where we have

$$\tilde{\gamma}_{i,j} = \frac{\rho_{i,j}}{\rho(\sum_{l=i+1}^{N} \tilde{\alpha}_{i,l} + \eta \tilde{\alpha}_{i,k}) + 1}$$  \hspace{1cm} (4)

the averaged channel power gain of each destination.

Q (e.3) (2.11) It is noticed that in Rician channel, we have

$$K = \frac{r^2}{2\omega^2}.$$  \hspace{1cm} (5)

where $r^2$ yields the channel gain of LoS component, $2\omega^2$ is the average channel power gain of all the NLoS components.
II. IEEE Transactions on Communications
    Paper-TCOM-TPS-18-0960
    Authors’ Response to Reviewer 1

Comment 1: **Strong Points:**
- Mathematical derivation is thorough and clear.
- References are up to date.
- Figures are clear.

**Response:** We thank the reviewer very much for this evaluation. We also appreciate the reviewer very much for your valuable and constructive comments to further improve the quality of this manuscript. We have revised the paper in line with these comments, thereby avoiding potential errors and improving the quality of this manuscript. The comments will be answered item by item in this response letter. We thank the reviewer again for your time and valuable comments to improve the quality of this submission.

Comment 2: **What is the impact of caching content on the storage of devices? This needs to be elaborated on.**

**Response:** We thank the reviewer very much for this comment. In our prior studies, we have already investigated the impact of content caching in wireless communications [10], [11], [16], etc. In addition, there are also various studies on the benefits of content caching technology in wireless communications, e.g., [17]. Compared to the conventional wireless networks, content caching can reduce the transmission distance from content server to receiver. In addition, content caching can reduce the network load. For instance, in our previous study, we compared the energy consumption and energy efficiency (EE) performances between content caching and the conventional network (without content caching) in [10].

Because the comparison between content caching and conventional network without content caching has been investigated. Here we adopt the reviewer’s suggestion (1.7) and remove the content caching related texts. Related title and main texts are revised accordingly. We mark our revisions in the main text with blue color and reproduce them here for the reviewer’s convenience.

Q (e.2) (1.2), 1.7) Performance Analysis of FD-NOMA-Based Decentralized V2X Systems.
Q (e.2) (1.2), (1.7) FD-NOMA-based decentralized V2X system model (or systems).
Q (1.2, 1.7) the FD-NOMA-based decentralized V2X system model, and also provide the capacity analysis to obtain the approximate closed-form capacity expressions with high accuracy.

Comment 3: **How are the values of η and α_k chosen? This needs to be justified.**

**Response:** We thank the reviewer very much for this and following comments. Here α_k is a typo, it has been corrected as α_{i,k}. We reproduce the related contents here for the reviewer’s reference.

Q (e.3) (1.3) Additionally, η is the coefficient of self-interference with η ∈ [0, 1], which
makes our expressions versatile to character different schemes. For instance, in FD-NOMA scheme, large value of $\eta$ denotes the strong FD self-interference, and small value denotes the weak FD self-interference. On condition that $\eta = 0$, the expression reduce to the pure NOMA expression. Q (2.3) On the basis of (7), normalizing the channel noise power value will give (8). Q (2.8) Here $\rho$ is the SNR, and we use $\alpha_{i,j}, \alpha_{i,l}, \alpha_{i,k}$ to denote the allocated NOMA power coefficient with FD transmission in line with a normalized channel noise power value. In the sequel, we adopt the normalized noise power for our analysis.

Q (1.3) We also assume that the allocated NOMA power variance is growing linearly with a normalized noise variance value (e.g., with 4 users, the NOMA power vector is $a_i = [4, 3, 2, 1]$), where $a_i = [\alpha_{i,1}, ..., \alpha_{i,N}]$. Additionally, $\eta = 0.1, \alpha_{i,k} = 5$ are used.

**Comment 4:** How is the impact of vehicle mobility taken into consideration? It is not enough to assume just one-time slot/instance.

**Response:** We thank the reviewer very much for this comment. Indeed, it is a good question. We answer this question as follows.

- **The Impact of Moving Vehicle**

  Firstly, for orthogonal frequency division multiple access (OFDMA)-based wireless communication systems, the vehicle mobility will result in carrier frequency offset (CFO). It is because of the frequency mismatch and the Doppler effect caused by the moving transceiver. The CFO side effect is a vital issue for wireless communications based on the (OFDMA) [18]. For these schemes, the orthogonality can not be maintained if the transmitter and receiver cannot be perfectly synchronized. This mismatch results in inter-carrier interference (ICI) [18]. In reality, because of the oscillators in the transmitter and the receiver can never be oscillating at identical frequency, CFO side-effects always exists even if there is no Doppler effect.

  Secondly, even the Doppler effect exists, if the vehicle is moving with a low speed, the CFO side effect is small and can be compensated. For instance, it is noticed in remote and suburban areas, typically, the vehicle is moving with a lower speed (e.g., less than 100 Km/h). The speed is even lower in urban and crowded areas (around 40 ～ 80 Km/h). Assuming the carrier frequency 5.9 GHz (we may also employ other carrier frequencies, here we just use this value as an example), we can calculate as follows

  \[
  \Delta f = \frac{\Delta v}{c} f_0 \left( \frac{100 + 100 \text{ (Km/h)}}{3 \times 10^8 \text{ (m/s)}} \right) \times 5.9 \times 10^9 \text{Hz} \leq \frac{55.555556}{3 \times 10^8} \times 5.9 \times 10^9 \text{Hz} \approx 1093 \text{Hz} \approx 1 \text{KHz}. \tag{6}
  \]

  One can find from this calculation, even the vehicles are traveling with a speed of 100 Km/s, the CFO value is still within the range of CFO estimator [19], which can be compensated by various methods (see, e.g., [20], [21]).

  Indeed, the study of CFO compensation is a good topic for OFDMA-based wireless communications, and there are various work on this, for example, [20], [21]. However, it is worth noting that here we are talking about FD-NOMA-based decentralized V2X communications. We don’t need to have perfectly synchronized time between transmitter and receiver because we are using the NOMA, a scheme with non-orthogonal frequencies, not the OFDMA. In addition, even the CFO exists, we may also employ some compensation methods if the vehicle speed is low.

- **The One Time Slot Related Question**
While talking about the system capacity, there are two types, i.e., the ergodic (Shannon) capacity and the outage capacity. In time varying channel, the instantaneous signal to interference plus noise ratio (SINR) is varying from time slot to time slot. If its distribution is known, we can obtain the system capacity on condition that data transmission goes through all fading states. This is the ergodic capacity [8] [22]. On the other hand, outage capacity is used for slowly varying channels when the instantaneous SINR is constant for a large number of symbols [8].

Here in our considered V2X communications, the channels are time varying channels. We thus adopt the ergodic capacity to analysis the system performance. The instantaneous SINR in each time slot and its probability distribution function are thus used.

We reproduce our revisions in the updated manuscript for the reviewer’s convenience.

Q (1.4) (2.13) In addition, NOMA is insensitive to CFO effect caused by moving devices because of its non-orthogonal frequency.

Q (1.4, 2.13) In addition, the existing orthogonal frequency division multiple access (OFDMA)-based LTE-V2X systems need orthogonality. Different from the non-moving wireless communications, moving vehicle caused Doppler effect is a vital problem for OFDMA-based LTE-V2X systems [23]. As is known, the carrier frequency offset (CFO) caused by the Doppler effect will lead to inter-carrier interference (ICI) to the OFDM-based wireless communications [18]. There have been various studies to solve the CFO compensation, see, e.g., [18], [24]. However, because the oscillators can never be oscillating at the identical frequency, in OFDMA-based wireless communications, CFO side-effect always exists even for non-moving circumstance [18].

Q (1.4, 2.13) Compared to orthogonal frequency division multiple access (OFDMA) scheme, NOMA is insensitive to Doppler effect caused by moving vehicles.

Q (e.1) (1.4) (3.1) In literature, capacity analysis is to reveal the intuitive and simple-to-compute capacity expressions for the wireless systems [1], [2]. In this regard, closed-form capacity expression is of great importance. Generally, capacity can be classified into two different types, i.e., the ergodic (Shannon) capacity and the outage capacity [8]. In time varying channels, on condition that the channel state information (CSI) is known at the receiver but not the transmitter, i.e., γ (signal to interference plus noise ratio (SINR)) is known for every time slot. In practice, this can be accomplished by some channel estimation method [8], [9]. Furthermore, the distribution of γ is known at both the transmitter and receiver. Ergodic capacity then is defined as data transmission going through all fading states, which is also called the Shannon capacity since it is the average of instantaneous capacity over all states. In contrast, outage capacity is used to describe the system performance under slowly varying channels with a constant instantaneous γ [8], [9]. Here in this study, we adopt the ergodic capacity since V2X channels are generally the time varying channels.

Q (e.1) (e.3) (1.4) In addition, to the destination side, PDF of instantaneous signal to interference plus noise ratio (SINR) in each time slot, say, γi,j, is given by

\[ f^\gamma(\gamma_{i,j}) = \frac{1}{\bar{\gamma}_{i,j}} e^{-\frac{\gamma_{i,j}}{\bar{\gamma}_{i,j}}}, \]  

where we have

\[ \bar{\gamma}_{i,j} = \frac{\rho \tilde{\alpha}_{i,j}}{\rho (\sum_{l=i+1}^{N} \tilde{\alpha}_{l,j} + \eta \tilde{\alpha}_{i,k}) + 1}, \]  

the averaged channel power gain of each destination.
Comment 5: Analysis of the obtained results is not thorough and is intuitive. For example, it is expected that the performance will be better in a suburban and remote environment. Also, when we increase the power, the performance improves due to better SNR. A more thorough analysis of the results is needed. How much better is the performance in %?

Response: We thank the reviewer very much for this comment. Indeed, by increasing the transmission power, the capacity performance increases due to better signal to noise ratio (SNR). We have better capacity performance in suburban and remote environment compared to urban and crowded environment. Thus is mainly due to the light of sight (LoS) paths between transmitter and receiver.

We have revised our manuscript according to this comment. We reproduce our corrections here for the reviewer’s convenience.

Q (1.5) By comparing Fig. 4 and Fig. 5, we also notice that under the same condition, capacities in suburban and remote scenario always outperform the ones in urban and crowded scenario (for instance, in $1 \leftrightarrow 3$ case, SNR = 0 dB, $C_{\text{sum}} = 125\% C_{\text{sum}}^a$, SNR = 30 dB, $C_{\text{sum}}^c = 103\% C_{\text{sum}}^a$). This is because of the less propagation loss with a dominant LoS path between source and destination in the suburban and remote scenario.

Q (1.5) As shown by the solid lines in both figures, increasing the power values leads to better capacity performance, which is due to the increased SNR value. For instance, in $1 \leftrightarrow 3$ case and SNR = 20 dB, we have $C_{\text{sum}}^a(a_2) = 131\% C_{\text{sum}}^a(a_1)$. We can also confirm from both figures that as $M$ increasing, the system capacities also increase.

Comment 6: What is the unit in Table I?

Response: We are sorry for this. Here the unit is second. We have added the missing unit and reproduce the related caption here for the reviewer’s convenience. We thank the reviewer again for this careful check to further improve the readability of this submission.

Q (1.6) Consumed time (second) of Exa, App and MC simulations with $\rho = 15$ dB.

Comment 7: How is the concept of content caching considered? The simulation environment does not discuss this at all. This claim needs to be revised or removed.

Response: We thank the reviewer very much for this comment. As stated before, the benefits of content caching have been investigated a lot in our prior work and also other studies, see, e.g., [10], [11]. Here we adopt the reviewer’s suggestion and delete the content caching related contents in this updated version. Many thanks again for this suggestion to further improve the quality of our submission.

We reproduce our corrections here for the reviewer’s convenience.

Q (e.2) (1.2), (1.7) Performance Analysis of FD-NOMA-Based Decentralized V2X Systems.
Q (e.2) (1.2), (1.7) FD-NOMA-based decentralized V2X system model (or systems).
Q (1.2, 1.7) the FD-NOMA-based decentralized V2X system model, and also provide the capacity analysis to obtain the approximate closed-form capacity expressions with high accuracy.
Comment 8: Figure 1 needs work. It looks cartoon-like. Please use provide a more professional system model.

Response: We thank the reviewer very much for this comment. We have revised Figure 1 in this updated version. In addition, we reproduce our correction here for the reviewer’s convenience (Fig. 3 in this response letter).

Fig. 3: Q (1.8) The $M \leftrightarrow N$ FD-NOMA-based decentralized V2X system model. The communications among V2X devices can be accomplished by FD-NOMA working on the DC mode.

Comment 9: Discussion of Fig.2 and Fig.3 are inversed. Fig.2 shows the comparison gap between the exact, approximation, and the Swamee and Ohija methods. Please fix this.

Response: We thank the reviewer very much for the careful check to avoid potential typos of our submission. Indeed, here the labels of Fig. 2 and Fig. 3 are inversed.

We have corrected this in this updated version. We also reproduce the corrections here for the reviewer’s convenience (Fig. 4 and Fig. 5 in this response letter).

Q (1.9) The simulation results are given by Fig. 4. As noticed, the gap between the approximation and the exact form curves is large. In this case, this approximation method is better than the Swamee and Ohija approximation method, while is unsuitable to be adopted directly.

Q (1.9) Additionally, one can see from Fig. 4 that the approximation curve displays a similar curvature to the exact curve.

Q (1.9) We further give the comparison results of the exact, improved and approximate expressions, which is shown by Fig. 5. Compared to the approximate results, the improved approximate results coincide with the exact results perfectly, which indicates the validity of our hypothesis. Closed-form expression of $C_{\text{Ray}}$ is given by the following corollary.
Comment 10: Perform English proofreading of the manuscript. There are multiple grammatical and typo mistakes. For example, Abstract, line 6: "our analyzes" "our analysis"
Introduction, page 1 line 23: "...which is a challenging for seamless..." "which is challenging for seamless..."
Introduction, page 1 line 34: "C-V2X can connects..." "C-V2X can connect..."
Introduction, page 1 line 53: "it is because that NOMA..." "It is because NOMA..."
These are just a few examples. There are many more mistakes that need to be fixed. Please proofread the paper to fix all grammatical and typo mistakes.

Response: Many thanks for this careful check to avoid potential typos and syntax errors in our submission. We have carefully double checked the manuscript. We reproduce our corrections
here for the reviewer’s convenience.

Q (1.10) Insights from our analysis has that the accuracy of our approximate expressions is controlled by the associated division of $\frac{1}{\gamma}$ in urban and crowded scenario, and the truncation point $T$ in suburban and remote scenario.

Q (1.10) the limited range of RSU leads to frequent handover, which is challenging for seamless connections.

Q (1.10) With the help of cellular network, C-V2X can connect more V2X devices [25], [26];

Q (1.10) First, in the direct communications (DC) mode, V2X devices can directly communicate with each other. Well-known examples include vehicle to vehicle (V2V), vehicle to pedestrian (V2P) communications. Second, in the network-based communications (NC) mode, cellular base station (BS) is playing the dominant role, and the V2X devices communicate with (or with the help of) the cellular, for instance, vehicle to network (V2N), vehicle to infrastructure (V2I) communications.

Q (1.10) (3.3) Compared to the OMA scheme, NOMA can accommodate more users, and these users can be with different QoS requirements [15], [27].

Q (1.10) Analysis and simulation results in [14] indicated that FD-NOMA can improve the 5G’s system performance compared to HD-NOMA. Based on the relaying system model, analysis and simulation results in [15] indicated that FD-NOMA outperforms HD-NOMA in terms of outage probability and ergodic sum rate in low signal to noise ratio (SNR) region, but displays an inferior performance in high SNR region.

Q (1.10) In this article, we use upper case boldface letters to denote matrices (e.g., $\mathbf{A}$), and we use lower case boldface letters to denote vectors (e.g., $\mathbf{a}$).

Q (e.3) (1.10) On the other hand, transmission and reception processes in the FD-NOMA-based decentralized V2X systems are different from the centralized cellular-based communications.

Q (1.10) This is because the higher $K$ brings a stronger LoS component and a weaker multipath propagation loss.

Q (1.10) Firstly, according to the integration by parts method, we have

\[
\begin{align*}
\int_0^{+\infty} \log_2(1 + \gamma_{i,j}) & \frac{1}{\gamma_{i,j}} e^{-\frac{1}{\gamma_{i,j}}} d\gamma_{i,j} \\
= -\int_0^{+\infty} \log_2(1 + \gamma_{i,j}) (e^{-\frac{1}{\gamma_{i,j}}})' d\gamma_{i,j} \\
= \log_2(1 + \gamma_{i,j}) e^{-\frac{1}{\gamma_{i,j}}} \bigg|_0^{+\infty} + \int_0^{+\infty} \frac{1}{\ln(2(1 + \gamma_{i,j}))} e^{-\frac{1}{\gamma_{i,j}}} d\gamma_{i,j} \\
&= \frac{1}{\ln 2} \int_0^{+\infty} \frac{1}{1 + \gamma_{i,j}} e^{-\frac{1}{\gamma_{i,j}}} d\gamma_{i,j} \\
&= \frac{1}{\ln 2} \int_0^{+\infty} \frac{1}{\gamma_{i,j}} (e^{-\frac{1}{\gamma_{i,j}}}) d\gamma_{i,j} \\
&= \frac{1}{\ln 2} \int_0^{+\infty} \frac{1}{\gamma_{i,j}} (\frac{1}{\gamma_{i,j}} + \frac{\gamma_{i,j}}{\gamma_{i,j}}) d\gamma_{i,j} \\
&= \frac{1}{\ln 2} \int_0^{+\infty} e^{-\frac{1}{\gamma_{i,j}}} \frac{1}{\gamma_{i,j}} d\gamma_{i,j}
\end{align*}
\]
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Authors’ Response to Reviewer 2

**Comment 1:** If the channels just follow Rayleigh or Rician distributions, then the sum-rate capacities in (9) and (10) will return to compute the moments of Gaussian distributions which are all well-known in previous works. The authors should give more motivations. I personally feel that the contributions are not clearly stated and not enough.

**Response:** We thank the reviewer very much for this valuable comment to re-elaborate and highlight our contributions.

In the calculation, Q-function is only involved in the approximate closed-form expression of the exponential integral function. In generalized exponential integral function, we cannot adopt this method to pursue its approximate closed-form expression. The truncation method is adopted therein. We successfully obtain an approximate closed-form expression with arbitrary small error while reducing its computation complexity. We will answer this question in details as follows:

Firstly, according to prior work and as calculated in Q (1.4), the CFO side effect of low speed vehicle communications (below 100 Km/s) is small, which can be compensated ( [8], [9]). With vehicle moving, the instantaneous SINR is varying from time slot to time slot. We thus adopt the ergodic capacity in our analysis, which is obtained by transmissions go over all fading states. For the used Rayleigh and Rician channel models, as stated in the following comments (Q (2.2)), indeed, we may also employ the \( \kappa - \mu \) and many other channel models (e.g., \( \eta - \mu \)) in the analysis. But one should notice that these channel models are some generalized channel models of Rayleigh and Rician channel models.

Secondly, while calculating the ergodic capacity of FD-NOMA-based decentralized V2X systems in urban and crowded area scenario, the exponential integral function is involved. As is known, it is troublesome and time-consuming to directly calculate the exponential integral function. We thus seek an approximate closed-form expression with low computation complexity in our analysis. We first convert it into the multiplication of \(-2 \sqrt{\pi} \) and \( \frac{1}{\mu} \{Q(\sqrt{2\mu})\} \), and elaborate the approximate closed-form expression of the involved Q-function. It is worth noting that by substituting the obtained approximate closed-form expression of Q-function, we need to make sure that the differential calculation can be successfully eliminated, i.e., equation (B.11). In this case, the approximate closed-from expression of ergodic capacity is obtained. Our purpose is not to simplify pursue a closed-form expression of Q-function. We also make sure that by substituting it into (B.2), the differential calculation can be eliminated. In literature, there are indeed various studies on the approximate closed-form expressions of the exponential integral function. However, as is shown in Fig. 2, most of them are of low accuracies, or not given in closed-form (for instance, \( E_1(x) = -\gamma - \ln z - \sum_{k=1}^{\infty} \frac{x^k}{k!} \), \( |\text{Arg}(x)| < \pi \)). The benefit of our analysis is that: an approximate closed-form expression for the ergodic capacity in urban and crowded area scenario is obtained with arbitrary small error and low computational complexity.

Thirdly, while calculating the approximate closed-form expressions in suburban and remote area scenario, the modified Bessel function of zeroth order expression

\[
I_0(x) = \sum_{m=0}^{\infty} \frac{(\frac{x}{2})^{2m}}{m! \Gamma(m + 1)},
\]

and the generalized exponential integral function

\[
E_\mu(x) = \int_{1}^{\infty} \frac{e^{-x t}}{t^\mu} dt \quad \text{(Re}(x) > 0)
\]
are involved. We cannot simplify to pursue the approximate Q-function to solve this problem of approximate closed-form expression. In order to give approximate closed-form expression with arbitrary small error, we adopt the truncation method with regard to \( T \). Simulations and calculations both show that our method is of low computation complexity and arbitrary small error, e.g., \( f(100) = 6,9966 \times 10^{-125} \).

We reproduce our revisions in the updated version here for the reviewer’s convenience.

**Q (2.1)** Is there any approximate expressions for the capacity expressions with arbitrary small error and low computational complexity?

**Q (e.1) (2.1) (2.2) (2.4) (3.1)** In literature, various channel models are used for ergodic capacity analysis, for instance, the \( \kappa - \mu \) channel model [3], [4] and the \( \eta - \mu \) channel model [3]. However, obtaining the closed-form capacity expression in these channel models is very difficult because of the involved infinity series operations. Authors thus employed some special conditions and methods to give the closed-form expressions, e.g., \( \mu \) with positive integer values [4] and the approximate method [5]. On the other hand, the difficulty to obtain a closed-form expression with Rayleigh or Rician channel model lies in the involved exponential integral functions. In order to solve this, some approximate methods and algorithms have been proposed, for instance, the Swamee and Ohija method for exponential integral function [6] and the fast and accurate algorithm for generalized exponential integral function [7]. However, these methods are based on some special condition (e.g., [7]), or with lower accuracy (e.g., [6]). In this paper, we give the approximate closed-form capacity expressions for both Rayleigh and Rician channel models while taming the troublesome exponential integral functions.

**Q (e.1) (2.1) (3.1)** Based on the system model, we derive the exact system ergodic capacity expressions and their approximate closed-form expressions for both scenarios. These approximate closed-form expressions are with low computational complexity and controllable arbitrary small errors compared to the existing approximate expressions.

**Comment 2:** Many references on studying channel capacity for vehicular communications are missing and not discussed, for example: [r1] N. Bhargav, S. L. Cotton, and D. E. Simmons, Secrecy Capacity Analysis Over \( \kappa - \mu \) Fading Channels: Theory and Applications, IEEE TCOM, 2016.

**Response:** We thank the reviewer very much for this suggestion to enrich the discussion of this submission. We have added these antecedent studies in this updated version. We will also answer the reviewer’s comments as follows.

Firstly, it is known that when channel amplitude follows \( a \) distribution, we say that the channel is \( a \) channel model, for instance, the Rayleigh channel model is because that its channel amplitude follows the Rayleigh distribution. The \( \kappa - \mu \) distribution and \( \eta - \mu \) distribution were proposed by Prof. Michel Daoud Yacoub in [3]. The \( \kappa - \mu \) and \( \eta - \mu \) fading channel models are because their channel amplitudes follow the \( \kappa - \mu \) distribution or \( \eta - \mu \) distribution. Both of them are general physical fading models that can describe, parameterize, and fully characterize the corresponding signal in terms of measurable physical parameters. In \( \kappa - \mu \) fading channels, the Rician and Nakagami-\( m \) fading channels are special cases. In contrast, \( \eta - \mu \) fading channel model includes Hoyt (Nakagami-\( q \)) and the Nakagami-\( m \) fading channel models as its special cases.

However, as stated by the original work in [3], adopting the general fading models of \( \kappa - \mu \) or \( \eta - \mu \) in the analysis has less difference compared to adopting the Rayleigh or Rician channel model. The Rayleigh and Rician channel models are their special cases. In addition, in urban and crowded area, the transmissions are rich scattered, in which the Rayleigh channel model itself is already a good model to characterize this. In contrast, the Rician with light-of-sight (LoS)
dominant path from transmitter to receiver is a good model to be adopted in suburban and remote areas. We can see a lot of antecedent studies on the channel measurement and coverage area predicting saying that their data matches the Rayleigh and Rician channel models, see, e.g., [28], [29].

Actually, we indeed adopt the $\kappa - \mu$ and various other channel models in our antecedent studies, for instance, in [5], we investigated the effective rate of multiple-input single-output (MISO) systems based on the $\kappa - \mu$ channel model. The closed-form expression for the achievable rate is derived by our analysis and verified by our simulations. In [30], we adopt the correlated Rician/Gamma fading channels to analysis the achievable sum rate, symbol error rate and outage probability of cooperative transmission mechanism in small cell networks (SCNs). In [31], we adopt the Nakagami-$m$ channel model to analysis the outage probability and ergodic sum rate at high signal-to-noise ratio (SNR) regime based on a system model with NOMA-based relaying networks with transceiver hardware impairments.

The difficulty to adopt the Rayleigh and Rician channel models for analysis lies in the infinity exponential integral functions. No matter the exponential integral function or the generalized exponential integral function. In literature, there are indeed some prior studies on their approximate expressions, for instance, the mentioned widely used Swamee and Ohija method for exponential integral function. But the prior approximate expressions are still time consuming or not given in closed-form. In order to solve these problems, in this work, we give the approximate closed-form expressions for both exponential integral function and generalized exponential integral function. As is shown by our simulation results, our approximate closed-form expressions are of less computation complexity and consume much less time.

We reproduce the revisions in this updated version for the reviewer’s convenience.

Comment 3: After equation (2), the noise vector is distributed as $n \sim CN(0, \sigma^2I_M)$. However, the value $\sigma^2$ only appears in equation (9), while many other equations the values are 1. What is happening here?

Response: We thank the reviewer very much for this comment. Here in equation (9) (equation 7 in the updated version), the value $\sigma^2$ is used to denote the channel noise power. In our analysis, we normalized this value to be 1. Thus in many other equations, the values are 1. We highlight this in the updated version and reproduce it here for the reviewer’s convenience.
Response: We thank the reviewer very much for this comment and providing us this valuable documents. We didn’t find related channel model in this document. The mentioned modeling methodology and modeling results in section II and section III are about the baseline for the existing and future number of fatalities and serious injuries by year, type of road and mode of transport, in the absence of cellular-intelligent transport systems (C-ITS) [pp. 8-21].

In OFDM-based vehicle communications with lower speed vehicles, we typically use the Rayleigh, Rician, Nakagami-m, Nakagami-q, $\kappa - \mu$, $\eta - \mu$ channel models for the analysis. There are indeed some on-going studies about the mmWave channels, but according Professor Andrea Goldsmith’s reports, up to now, no matured channel model has been reported (see: https://web.stanford.edu/class/ee359/pdfs/lecture2.pdf). In higher-speed vehicle communication scenario (mostly for the high-speed railway communication scenario), other than the mentioned channel models, the dropper shift needs to be taken into account as well. As discussed before in Q (1.4), a lot of antecedent studies are about the Doppler compensations. Even with OFDM-based vehicle communications, if the vehicle speed is low, we can still compensate this.

Here in this study, we are talking about the FD-NOMA-based decentralized V2X communications. We don’t need to take the Doppler side-effect into consideration because of the NOMA is based on non-orthogonal frequencies and because of the lower speed of vehicles in our considered systems. As answered by Q (2.1), another purpose of this study is to pursue the approximate closed-form expressions for exponential integral function and the generalized exponential integral functions in Rayleigh channel model and Rician channel model. In addition, we indeed investigate the system capacity performance based on various channel models in our prior work, see, e.g., [5], [30], [31], etc. We give detailed closed-form expressions for these channel models, and these expressions are verified by Monte-Carlo based simulations therein.

We reproduce the revisions in this updated version for the reviewer’s convenience.

Q (e.1) (2.1) (2.2) (2.4) (3.1) In literature, various channel models are used for ergodic capacity analysis, for instance, the $\kappa - \mu$ channel model [3], [4] and the $\eta - \mu$ channel model [3]. However, obtaining the closed-form capacity expression in these channel models is very difficult because of the involved infinity series operations. Authors thus employed some special conditions and methods to give the closed-form expressions, e.g., $\mu$ with positive integer values [4] and the approximate method [5]. On the other hand, the difficulty to obtain a closed-form expression with Rayleigh or Rician channel model lies in the involved exponential integral functions. In order to solve this, some approximate methods and algorithms have been proposed, for instance, the Swamee and Ohija method for exponential integral function [6] and the fast and accurate algorithm for generalized exponential integral function [7]. However, these methods are based on some special condition (e.g., [7]), or with lower accuracy (e.g., [6]). In this paper, we give the approximate closed-form capacity expressions for both Rayleigh and Rician channel models while taming the troublesome exponential integral functions.

Comment 5: In equation (11), $a$ is Rayleigh distributed. However, this explanation is ambiguous. In simulations, how can you define $a$? A similar issue is observed in equation (33).
**Response:** We thank the reviewer very much for this comment. We are referring [32, Chapter 2] on these expressions. As these contents have nothing to do with our following analysis, we delete them in this updated version. We thank the reviewer again for this valuable comment to further improve the quality and avoid ambiguous expressions for our potential readers.

**Comment 6:** In equation (11), how is $\theta_n(t)$ defined? Is it picked up randomly in $[-\pi, \pi]$? The authors should elaborate more.

**Response:** We thank the reviewer very much for this comment to improve the quality of this submission. As state by [32, chapter 2.2.3], the $\theta_n(t)$ follows a uniform distribution over $[-\pi, \pi]$. As equation (11-13) and equation (33-34) are not used in our analysis, we delete them in the updated version. We thank the reviewer again for this careful check to improve the quality and avoid ambiguous expression of our submission.

**Comment 7:** In equation (13), $\varsigma$ is the common variance. What does common mean? Does it mean that we also have uncommon variance in Rayleigh fading channels?

**Response:** We thank the reviewer very much for this comment. The definition here comes from [32, chapter 2.2.3]. The term "common variance" in this context refers to the fact that the variance of the two components ($h_1(t)^2$ and $h_0(t)^2$) are identical. As stated before, we have deleted equation (11-13) since they are not used in the following analysis.

**Comment 8:** In equation (14), what is the relationship between $\rho$, $\alpha_{i,j}$, and power coefficients in equations (9) and (10). I am asking this question since $\rho$ and $\alpha_{i,j}$ are not defined.

**Response:** We thank the reviewer very much for this comment. The definitions of $\rho, \alpha_{i,j}$ are given after equation (10) (equation (8) in this updated version). Since they have been defined before, we omit their definitions here after equation (14) (equation (10) in this updated version). We reproduce the definitions here for the reviewer’s convenience.

Q (e.3) (2.8) Here $\rho$ is the SNR, and we use $\alpha_{i,j}, \alpha_{i,j}, \alpha_{i,k}$ to denote the allocated NOMA power coefficient with FD transmission in line with a normalized channel noise power value. In the sequel, we adopt the normalized noise power for our analysis.

**Comment 9:** In equation (16), $\tilde{\alpha}_{i,j}$ are not defined.

**Response:** We thank the reviewer very much for this comment. As stated by its following text, here $\tilde{\alpha}_{i,j}$ is the averaged channel power gain for each destination. We reproduce the text here for the reviewer’s convenience.

Q (2.9) where

$$\tilde{y}_{i,j} = \frac{\rho \tilde{\alpha}_{i,j}}{\rho(\sum_{l=i+1}^{N} \tilde{\alpha}_{i,l} + \eta \tilde{\alpha}_{i,k}) + 1},$$

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the averaged channel power gain of each destination.

Comment 10: In Theorem 1, the authors state that the exact achievable rate is obtained in (18), but it is not the case because of the integration. Please elaborate more.

Response: We thank the reviewer very much for this comment. It is not the literal exact meaning. Here we are using the word 'exact expression' as contrasted with the following ‘approximate capacity expression’. In literature, we cannot have integration involved in the 'closed-form expression', but it is fine in the 'exact expression'. This is because the exact capacity expression is derived from the original instantaneous SINR expression with regards to the definition of ergodic capacity.

Comment 11: After equation (34), $2\sigma^2$ is the average channel power gain. What is the difference of this $\sigma$ value and the $\sigma$ value after equation (2)?

Response: We thank the reviewer very much for this comment to improve the readability and avoid the ambiguity of this submission. Indeed, $\sigma$ is multi-defined in the submission. We have corrected the second $\sigma$ as $\omega$. Please check the updated version for your kind consideration. We also reproduce our corrections here for the reviewer’s convenience.

$Q$ (e.3) (2.11) It is noticed that in Rician channel, we have

$$K = \frac{r^2}{2\omega^2}, \quad (13)$$

where $r^2$ yields the channel gain of LoS component, $2\omega^2$ is the average channel power gain of all the NLoS components.

Comment 12: In simulation results, many parameters are missing, for example the so-called Rayleigh distributed, $\theta_n(t)$? Besides, powers should have a measure unit. How are users located? Do you include shadowing effects in simulation?

Response: We thank the reviewer very much for this comment. We will answer this question as follows:

As stated before, we are referring [32] with respect to $\theta_n(t)$. However, since the equations (11-13) and equations (33-34) are not used in the analysis, we delete them in the updated version.

For the power parameters, as stated after equation (10) (equation (8) in this updated version), we are normalizing the channel noise power value in this study. In this case, power values used here are the multiple time value of the channel noise power, but not the real power value with unit. We thus don’t need to give their measure unit. The normalization method has been widely used in the analysis of wireless communications, see, e.g., [33].

Response: We thank you and Reviewer 1 in Q (1.4) very much for this valuable comment. As pointed out in Q (1.4), this is indeed a good question.

One should notice that the CFO side effect caused by Doppler effect is vital for wireless communications with OFDMA, but not a severe problem for wireless communications with NOMA. This is because that in communications with OFDMA, therein the rigorous orthogonal sub carriers needs to be maintained. The mismatch between carrier frequency will result in ICI [18]. In reality, we cannot perfectly eliminate the CFO side-effects even there is no Doppler effect. This is because of the oscillators in the transmitter and the receiver can never be oscillating at identical frequency. In order to solve the CFO’s side-effects, various studies have been done to compensate the CFO, for instance, [18], [20], [21]. In addition, for the analysis based on OFDMA, we generally take the Doppler shift effect into consideration when vehicles are moving with a relatively high-speed (e.g., the wireless communications for high-speed railway [18]). As calculate in Q (1.4), in lower speed transmission with OFDMA, we can assume that the Doppler effect can be compensated.

However, in this study, our decentralized V2X communications is based on the FD-NOMA, not the FD-OFDMA. The rigorous orthogonal sub carriers is not necessary because NOMA transmission procedure is based on the non-orthogonal frequencies. We thus don’t need to take the Doppler effect into consideration. This is another benefit of NOMA technology.

We reproduce our revisions in the updated manuscript for the reviewer’s convenience.

Q (1.4) (2.13) In addition, NOMA is insensitive to CFO effect caused by moving devices because of its non-orthogonal frequency.

Q (1.4, 2.13) Compared to orthogonal frequency division multiple access (OFDMA) scheme, NOMA is insensitive to Doppler effect caused by moving vehicles.

Q (1.4, 2.13) In addition, the existing orthogonal frequency division multiple access (OFDMA)-based LTE-V2X systems need orthogonality. Different from the non-moving wireless communications, moving vehicle caused Doppler effect is a vital problem for OFDMA-based LTE-V2X systems [23]. As is known, the carrier frequency offset (CFO) caused by the Doppler effect will lead to inter-carrier interference (ICI) to the OFDM-based wireless communications [18]. There have been various studies to solve the CFO compensation, see, e.g., [18], [24]. However, because the oscillators can never be oscillating at the identical frequency, in OFDMA-based wireless communications, CFO side-effect always exists even for non-moving circumstance [18].

Q (1.4, 2.13) NOMA is insensitive to Doppler effect caused by moving vehicles because of the non-orthogonal carrier frequencies.
Comment 1: With regard to the technical contribution of the paper, the reviewer has the following comments: The title of this paper is Performance Analysis of a propose scheme. Perhaps it should reveal how the analysis results can be made use of.

Response: We thank the reviewer very much for this comment.

In regards to the proposed FD-NOMA-based decentralized V2X system model, it can partly offload the wireless network load. In addition, the FD-NOMA scheme can accommodate more users with different QoS and transmission rate requirements compared to the OMA scheme. Compared to OFDMA, FD-NOMA-based communications can resist the Doppler effect caused by the moving transceivers. We also reveal that the FD-NOMA scheme can reduce the system latency performance.

In regards to the meaning of our capacity analysis here in this work, as stated by the original work by Claud E. Shannon in [1], the performance analysis (capacity analysis) is to reveal the intuitive quality of the system capacity. In this case, the closed-form capacity expressions are of great importance. According to the co-authored survey paper from Professor Jeffrey G. Andrews’s group in the University of Texas-Austin [2], since the wireless networks are typically difficult to characterize, the intuitive and simple-to-compute qualities of capacity analysis is a popular choice for a large number of wireless systems, e.g., the direct-sequence and frequency hopping spread spectrum, the interference cancellation, the spectrum sharing in unlicensed, overlaid and cognitive radio networks, etc. Actually, after Claud E. Shannon’s work, a great deal of endeavours are on the capacity (ergodic capacity, outage capacity, etc.) analysis, especially on the closed-form capacity expressions, see, e.g., [34], [35], [36].

In addition, our analysis also reveals that FD-NOMA has better throughput and latency performances compared to the existing schemes (NOMA, OMA). As is known, 5G is claiming for higher transmission rate and URLLC communications in order to connect more devices and providing various QoS services. The URLLC characteristic is also vital for V2X systems while providing wireless access to moving unmanned vehicles on the road. For instance, while encountering emergency situation, 5G-enabled URLLC service can guide the unmanned vehicles to brake or keep away from the disaster place.

We reproduce our revisions here for the reviewer’s convenience.

Q (2.1) (3.1) Is there any approximate expressions for the capacity expressions with arbitrary small error and low computational complexity?

Q (e.1) (2.1) (2.2) (2.4) (3.1) In literature, various channel models are used for ergodic capacity analysis, for instance, the $\kappa - \mu$ channel model [3], [4] and the $\eta - \mu$ channel model [3]. However, obtaining the closed-form capacity expression in these channel models is very difficult because of the involved infinity series operations. Authors thus employed some special conditions and methods to give the closed-form expressions, e.g., $\mu$ with positive integer values [4] and the approximate method [5]. On the other hand, the difficulty to obtain a closed-form expression with Rayleigh or Rician channel model lies in the involved exponential integral functions. In order to solve this, some approximate methods and algorithms have been proposed, for instance, the Swamee and Ohija method for exponential integral function [6] and the fast and accurate algorithm for generalized exponential integral function [7]. However, these methods are based on some special condition (e.g., [7]), or with lower accuracy (e.g., [6]). In this paper, we give the approximate closed-form capacity expressions for both Rayleigh and Rician channel models while taming the troublesome exponential integral functions.

Q (e.1) (1.4) (2.1) (3.1) In literature, capacity analysis is to reveal the intuitive and simple-
to-compute capacity expressions for the wireless systems [1], [2]. In this regard, closed-form capacity expression is of great importance. Generally, capacity can be classified into two different types, i.e., the ergodic (Shannon) capacity and the outage capacity [8]. In time varying channels, on condition that the channel state information (CSI) is known at the receiver but not the transmitter, i.e., $\gamma$ (signal to interference plus noise ratio (SINR)) is known for every time slot. In practice, this can be accomplished by some channel estimation method [8], [9]. Furthermore, the distribution of $\gamma$ is known at both the transmitter and receiver. Ergodic capacity then is defined as data transmission going through all fading states, which is also called the Shannon capacity since it is the average of instantaneous capacity over all states. In contrast, outage capacity is used to describe the system performance under slowly varying channels with a constant instantaneous $\gamma$ [8], [9]. Here in this study, we adopt the ergodic capacity since V2X channels are generally the time varying channels.

Q (e.1) (2.1) (3.1) Based on the system model, we derive the exact system ergodic capacity expressions and their approximate closed-form expressions for both scenarios. These approximate closed-form expressions are with low computational complexity and controllable arbitrary small errors compared to the existing approximate expressions.

Q (1.4) (3.1) In literature, capacity analysis is to reveal the intuitive and simple-to-compute capacity expressions for the wireless systems [1], [2]. In this regard, closed-form capacity expression is of great importance.

**Comment 2:** Has decentralized FD-NOMA and content caching not been proposed before? What are the implementation costs (e.g. computations, time) of content caching? How does content caching make a difference for decentralized FD-NOMA?

**Response:** We thank the reviewer very much for this comment. The benefits of content caching on better EE, robust and reliability communication performances have been investigated by our prior studies [10], [16], [37] and many other studies (e.g., [17]). Here we adopt the reviewer 1’s suggestion and delete the related contents in this updated version. The title of this submission has been changed too. Many thanks again for this suggestion to further improve the quality of our submission.

Q (1.2), (1.7) (3.2) Performance Analysis of FD-NOMA-based Decentralized V2X Systems

**Comment 3:** The first contribution claims that FD-NOMA can accommodate more users - As compared to which scheme?

**Response:** We thank the reviewer very much for this comment. This is compared to the OMA scheme. We have added this in the updated version. We also reproduce this correction here for the reviewer’s convenience.

Q (1.10) (3.3) Compared to the OMA scheme, NOMA can accommodate more users, and these users can be with different QoS requirements [15], [27].

**Comment 4:** This paper claims that there is no work on the system performance analysis of FD-NOMA, i.e., the capacity and throughput performances of wireless communications with FD-NOMA. But what about [*] M. F. Kader, S. Y. Shin and V. C. M. Leung, "Full-Duplex Non-Orthogonal Multiple Access in Cooperative
Response: We thank the reviewer very much for this comment. These studies are indeed providing valuable insights to the FD-NOMA applications in 5G. We have removed this phase in this updated version and clarify the differences between the existing works and our study here. It is also worth noting that these studies, as well as the study of [15], are based on the relaying systems. Here in our studies, the communication is based on the decentralized V2X system model. We are truly sorry we cannot compare the results from these two studies and our studies here. We can only compare the results under the same condition. However, as pointed out by your following comments in Q (3.5), we have added the comparison of our proposal with various existing schemes.

In addition, another purpose of this work is to pursue the approximate closed-form expressions for the involved exponential integral functions. We give the approximate closed-form expressions for achievable system capacities in both urban and crowded area scenario and suburban and remote area scenario. Simulation results demonstrate the validness of our analysis. Compared to existing methods, our approximate expressions are of low computational complexity and can be with arbitrary small error.

In the revised version, as the antecedent studies in this area, we indeed have added the comparison of these two studies as our references and marked them with blue color in the introduction section. We reproduce the corrections here for the reviewer’s convenience.

Q (e.2) (3.4) In cellular communications, there are previous studies on FD-NOMA. For instance, it was found that FD-NOMA can significantly suppress the co-channel interferences and achieve better performance gains compared to half duplex NOMA (HD-NOMA) and orthogonal multiple access (OMA) [12]. Q (3.5) Analysis and simulation results in [13] demonstrated that rate region performance of FD-NOMA outperforms the one with NOMA. Q (1.10) (3.4) Analysis and simulation results in [14] indicated that FD-NOMA improves the 5G’s system performance compared to HD-NOMA. Based on the relaying system model, analysis and simulation results in [15] indicated that FD-NOMA outperforms HD-NOMA in terms of outage probability and ergodic sum rate in low signal to noise ratio (SNR) region, but displays an inferior performance in high SNR region.

Q (e.2) (3.4) The FD-NOMA-based decentralized V2X system model is given in Fig. 3. This system is slightly different from the existing ones in the following respects. A) Different from existing studies on FD-NOMA, here no relaying systems are used because of the vehicle’s limited energy. B) V2X devices can directly communicate with each other through DC mode without the cellular’s help, and the required contents are obtained from neighboring V2X caches [10]. This system model thus has shorter transmission distance and better latency performance [10]. The cellular network load is reduced too.

Comment 5: This paper presents results of FD-NOMA that are compared with only NOMA (Figs. 9-10), but not with other related works.

Response: We thank the reviewer very much for this comment. We revised the manuscript according to this comment by comparing the FD-NOMA, HD-NOMA, FD-OMA and HD-OMA schemes in Fig. 9 and Fig. 10. It is worth noting that due to different system models, we cannot compare our results with some antecedent studies. But we indeed compare the difference
between this proposal and existing ones in the introduction section. We reproduce our revisions here for the reviewer’s consideration.

Q (e.2) (3.5) Finally, we compare the achievable throughputs with FD-NOMA, NOMA, FD-OMA and HD-OMA schemes in different scenarios. The results are given in Fig. 17 and Fig. 18. In these simulations, carrier bandwidth $B = 100$ MHz, $\alpha_i = [3, 2, 1], \eta = 0.1$ and $\tilde{\alpha}_{i,k} = 0.1, 1, 10$ are used. In order to be fair, we average the allocated power in FD-OMA and HD-OMA schemes. As shown by both figures, NOMA scheme has a better throughput performance compared to OMA scheme. Moreover, with a smaller value of $\alpha_{i,k}$, FD-NOMA always outperforms the other schemes (HD-NOMA, FD-OMA, HD-OMA). However, the benefit of FD-NOMA decreases with $\alpha_{i,k}$ increasing. This is mainly because of the increased FD self-interferences. We also notice that even with a much higher FD self-interferences, FD-NOMA outperforms NOMA in low SNR scenario (i.e., $\rho \in [0, 5]$ dB). This is due to the fact that in low SNR scenario, channel noise is the dominant factor compared to FD self-interference. In contrast, FD-NOMA self-interference becomes the dominant factor in high SNR scenario, NOMA scheme without FD self-interference thus has a better throughput performance. It is also worth noting that the effective transmission time is limited because of the fast moving V2X devices. FD-NOMA enabled bidirectional transmission can greatly reduce the transmission latency compared to other schemes. For example, compared to HD-NOMA and HD-OMA, FD-NOMA only needs a half latency time to transmit the same amount of data by its simultaneous transmission and reception scheme.

Comment 6: The paper claims that [*] is one of its most closely related works, but [*] is not in its list of references.

Response: We thank the reviewer very much for this comment. We have added this in the updated version. Kindly check the re-submission with reference item [21].
Comment 7: In terms of paper organization, the reviewer would like to make the following comment: After the section title III. THE CAPACITY ANALYSIS IN DIFFERENT SCENARIOS, at least a paragraph and eqs. (7) and (8) are missing. Without introducing the section, it is straight away followed by eqs. (9) and (10).

Response: We thank the reviewer very much for this comment. We are truly sorry for this typos. Here the equations mark numbers are wrong. You know, with IEEE \texttt{LATEX} templet, we need to name the input serial number and output serial number while using the figure command to write the long equations. We are sorry we mistakenly write (9) and (10) here. We have corrected this in this updated version. Please check the latest version for your kind consideration. We also reproduce our corrections here for the reviewer’s convenience.

\begin{align}
Q(3.7)  
C_{\text{sum}} = & \sum_{i=1}^{M} \sum_{j=1}^{N} \log_2 \left( 1 + \frac{p_{i,j} |h_{i,j}|^2}{\sum_{l=j+1}^{N} p_{i,l} |h_{i,l}|^2 + \eta \hat{p}_{i,k} |h_{i,j}|^2 + \sigma^2} \right), \\
C_{\text{sum}} = & \sum_{i=1}^{M} \sum_{j=1}^{N} \log_2 \left[ 1 + \frac{\rho \alpha_{i,j} |h_{i,j}|^2}{\rho \sum_{l=j+1}^{N} \alpha_{i,l} |h_{i,l}|^2 + \eta \alpha_{i,k} |h_{i,j}|^2 + 1} \right].
\end{align}

Comment 8: The Abstract summarizes the main findings of the paper. In terms of readability and clarity of the paper, perhaps the paper could benefit from the following suggestions: In Figs. 2 and 3, if the horizontal axis is in logarithmic scale and focus is given to $0 < x < 2$, perhaps clearer differentiation can be revealed. Please look closely at the legends used in Fig. 3. Of course we cannot see the overlapping between the improved and exact results. We can only observe that exact results are not presented.
**Response:** We thank the reviewer very much for this valuable comment to improve the readability of this submission. We have revised the related figures in the updated version. Please check the re-submission for your consideration.

In addition, as shown by the following figures (Fig. 8 - Fig. 10), we compare different cases with $0 < x < 2$, $0 < x < 2.5$, $0 < x < 3$. We finally choose $0 < x < 3$. This is because that we also want to display the fact that with $x$ value growing, these approximate methods approach to zero. If we choose $0 < x < 2$ and $0 < x < 2.5$, there will be still some gaps between these methods and the $x$-axis. The reason we didn’t use logarithmic y-axis is because that we want to show the difference between different approximate methods. However, we make some change to Fig. 3 by zooming in one point with $x = 1$. As is clearly shown here (Fig. 7 in this response letter), the exact value is about $0.21933$, and the value with our improved method is $0.219384$, the difference is about $0.000001$.

We reproduce our revisions here for the reviewer’s convenience (Fig. 10 - Fig. 11 in this response letter).

![Comparison of Exact, Approximation, and Swamee and Ohija-Based Expressions](image)

**Comment 9:** With reference to the figures of results (Figs. 4-10), it will be much clearer to the readers if different colours, different markers are applied to the graphs in a figure.

**Response:** We thank the reviewer very much for this valuable comment. The figures have been revised according to this comment to highlight the difference with different colors and marks, thereby improving the quality and readability of this submission. We reproduce our revisions here for the reviewer’s convenience (Fig. 10 - Fig. 18 in this response letter).
Fig. 9: Comparison of the exact, approximation and Swamee and Ohija-based expressions (0 < x < 2.5).

Fig. 10: Q (1.9) (3.8) (3.9) Comparison of the exact, approximation and Swamee and Ohija-based expressions.
Fig. 11: Q (1.9) (3.8) (3.9) Comparison of the exact, improved and approximate expressions.

Fig. 12: Q (3.9) Comparison of the system achievable sum capacity performances of Exa, MC and App results in urban and crowded scenario. The exactly and approximate results are obtained according to (12) and (26), respectively.
Fig. 13: Q (3.9) Comparison of the system achievable sum capacity performances with MC and App results in suburban and remote scenario. The analytical results are obtained according to (32).

Fig. 14: Q (3.9) Comparison of the capacities with different power values and source numbers.
Fig. 15: Q (3.9) Comparison of the capacities with different power values and source numbers.

Fig. 16: Q (3.9) Comparison of the capacities with different power values and source numbers.
Fig. 17: Q (e.2) (3.5) (3.9) System achievable throughput comparisons with FD-NOMA, NOMA, FD-OMA and HD-OMA schemes in urban and crowded scenario.

Fig. 18: Q (e.2) (3.5) (3.9) System achievable throughput comparisons with FD-NOMA, NOMA, FD-OMA and HD-OMA schemes in suburban and remote scenario.
REFERENCES


