Resource Allocation for Double IRSs Assisted Wireless Powered NOMA Networks

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Abstract—We consider an intelligent reflecting surface (IRS)-aided wireless-powered non-orthogonal multiple access (NOMA) network, in which double IRSs are adopted to boost the communication between the transmitter and receiver. We split the overall process into two phases: i) downlink wireless energy transmission (WET) in the first phase and ii) uplink wireless information transmission (WIT) in the second phase. Compared with the existing works, the double IRSs scheme is considered due to obstacles or deep shadowing between the transmitter and receiver. Specifically, we propose a scheme to maximize the sum rate by jointly optimizing the time allocation and phase shift matrices in these two phases. The optimization problem is non-trivial because of the coupling of multiple variables and the constraint of phase shift. An alternative optimization algorithm is adopted to surmount the objective step by step. The WET phase shift matrices are first obtained under given WIT phase shift matrices, and then vice versa. Numerical analysis proves the validity of the proposed scheme in increasing the sum rate and the effectiveness of double IRSs.

Index Terms—Intelligent reflecting surface (IRS), non-orthogonal multiple access (NOMA), alternate optimization.

I. INTRODUCTION

Intelligent reflecting surface (IRS) has a mass of reflecting elements, and by adjusting phase shifts of elements can control wireless environment configuration [1, 2]. Compared to traditional relays, IRS has the features of being passive and low-cost, and it can improve the spectral efficiency and energy efficiency available [3]. Furthermore, IRS has the advantages of light weight and flexible deployment, and thus, deploying IRS on a wireless communication network is an effective way to enhance signal [4]. In addition to supporting signal enhancement in wireless communication systems, IRS can also play a core transmission role in the network when there is a barrier between the transmitter and receiver [5].

On a parallel avenue, compared to wire transmission, wireless energy transmission (WET) is superior to prolong the lifespan of wireless devices [6]. In this regard, the harvest-then-transmit protocol has been widely adopted in a wireless powered communication network (WPCN) [7]. That is, the energy is firstly harvested during downlink WET and then the information is transmitted by uplink wireless information transmission (WIT) [8]. The combination of the WPCN with IRS enables a higher system performance [9]. At the same time, spectrum efficiency can be further improved through non-orthogonal multiple access (NOMA), which actively introduces interferences, and then decodes the information by successive interference cancelation (SIC) [10]. Compared with orthogonal multiple access technology, NOMA technology greatly reduces frequency spectrum resources and ensures fairness among different users in WPCN [11, 12].

Although there are already some studies on the combination of IRS, NOMA and WPCN [13, 14], studies on the application of multiple IRSs are limited. Motivated by this, we consider a framework of two IRS NOMA network powered by wireless communication to respond to a special communication environment. We deploy two IRSs on a wireless powered NOMA system to enhance signal between the transmitter and receiver. The time allocation for WET and WIT, and phase shift matrices are variables and need to be optimized, forming a non-convex problem. To address this non-trivial problem, an alternate optimization scheme is proposed. Mathematical derivation is used to solve the time allocation and the semi-definite programming (SDP) algorithm is used to solve the phase shift. The effectiveness of the proposed algorithm is validated through numerical results. In particular, it is shown that double IRSs dominate single IRS.

II. SYSTEM MODEL

We consider a double-IRS-assisted NOMA system powered by wireless transmission, which includes a hybrid access point (HAP), K users and two IRSs, denoted IRS1 and IRS2. Both the HAP and users are single antenna devices, and N1 and N2 are the number of reflecting elements of IRSs, respectively. As shown in Fig. 1, the obstacle interrupts the communication between the HAP and users, and therefore both the transmission of energy and information has to rely on the two IRSs. The overall process is divided into two phases: 1) HAP broadcasts energy with power P0 to users...
in the first phase $t_0$; and 2) users transmit their information to HAP in the second phase $t_1$. We make $t_0 + t_1 = 1$ in a general way. Clearly, adjusting the phase shift matrices of IRSs can reconfigure the wireless channels of WET and WIT, and further improve the system performance. During $t_0$, the phase shift matrices of IRS$_1$ and IRS$_2$ are represented by $\Theta_1$ and $\Theta_2$, while in $t_1$ they are denoted by $\Theta_3$ and $\Theta_4$, respectively. The channels of HAP-IRS$_1$ and HAP-IRS$_2$ are defined by $r_1 \in \mathbb{C}^{1 \times N_1}$ and $r_2 \in \mathbb{C}^{1 \times N_2}$, respectively, while those between IRS$_1$ and $k$-th user, as well as IRS$_2$ and $k$-th user are represented by $h_{1,k} \in \mathbb{C}^{N_1 \times 1}$ and $h_{2,k} \in \mathbb{C}^{N_2 \times 1}$, respectively. All channels comply with Rician fading, for example, $r_1 = \sqrt{\gamma_{1,LOS}} r_{1,LOS} + \sqrt{\gamma_{1,NLOS}} r_{1,NLOS}$, where $\gamma_{1,LOS}$ is the Rician factor, $r_{1,LOS}$ and $r_{1,NLOS}$ are the line-of-sight (LoS) and non-line-of-sight (NLoS) components of Rayleigh fading, respectively.

Thereby, the $k$-th user harvests energy as

$$E_k = \eta P_{k0} |r_1 \Theta_1 h_{1,k} + r_2 \Theta_2 h_{2,k}|^2, \forall k \in [1, K].$$

(1)

where $\eta$ is a constant that represents the energy conversion efficiency. We assume the channel reciprocity, and the signal received at the HAP can be written as

$$y = \sum_{k=1}^{K} \left( h_{1,k}^H \Theta_3 r_1^H + h_{2,k}^H \Theta_4 r_2^H \right) \sqrt{P_k} s_k + n,$$

(2)

where $P_k$ is the transmitted power of $k$-th user, $s_k$ is the information of $k$-th user and $n$ represents the noise. After perfect SIC$,^1$ the signal-to-interference-plus-noise ratio of the $k$-th user is given by

$$\gamma_k = \frac{P_k \left| h_{1,k}^H \Theta_3 r_1^H + h_{2,k}^H \Theta_4 r_2^H \right|^2}{\sum_{i=k+1}^{K} P_i \left| h_{1,i}^H \Theta_3 r_1^H + h_{2,i}^H \Theta_4 r_2^H \right|^2 + \sigma^2}.$$

(3)

$^1$Perfect SIC is assumed here for simplicity. Note that the results also adapt to the case with imperfect SIC.

Note that decoding order is that users with good channel conditions are first decoded [15]. We assume that users utilize all the harvested energy for WIT, e.g., $P_k = E_k/t_1$. Thus, the rate of the $k$-th user can be expressed as

$$R_k = t_1 \log \left( 1 + \frac{E_k \left| h_{1,k}^H \Theta_3 r_1^H + h_{2,k}^H \Theta_4 r_2^H \right|^2}{\sum_{i=k+1}^{K} E_i \left| h_{1,i}^H \Theta_3 r_1^H + h_{2,i}^H \Theta_4 r_2^H \right|^2 + t_1 \sigma^2} \right),$$

(4)

where $\sigma^2$ is the variance of Gaussian white noise. Then, like [16], we can write the sum rate of the considered system as

$$R_{sum} = t_1 \log \left( 1 + \frac{\sum_{k=1}^{K} E_k \left| h_{1,k}^H \Theta_3 r_1^H + h_{2,k}^H \Theta_4 r_2^H \right|^2}{\sum_{k=1}^{K} \sum_{i=k+1}^{K} E_i \left| h_{1,i}^H \Theta_3 r_1^H + h_{2,i}^H \Theta_4 r_2^H \right|^2 + t_1 \sigma^2} \right).$$

(5)

### III. SUM RATE MAXIMIZATION

We devote to maximizing $R_{sum}$ of the considered system under the constraints of time and phase shifts. We can write the objective function as

$$\max_{\Theta, t} R_{sum}$$

(6a)

s.t. $t_0 + t_1 = 1, t_0 \in [0, 1], t_1 \in [0, 1]$  \quad (6b)

$|s_{i,n}| = 1, \forall n \in [1, N_1]$  \quad (6c)

$|s_{2,n}| = 1, \forall n \in [1, N_2]$  \quad (6d)

$|s_{3,n}| = 1, \forall n \in [1, N_3]$  \quad (6e)

$|s_{4,n}| = 1, \forall n \in [1, N_2]$  \quad (6f)

where $\Theta = [\Theta_1, \Theta_2, \Theta_3, \Theta_4]$ represents the matrix of phase shift, $t = [t_0, t_1]$ is the time vector, $s_{i,n}$, $i \in [1, 4]$ is the phase shift, $s_{i,n} = [\exp(j\alpha_1), \exp(j\alpha_2), \cdots, \exp(j\alpha_N)]$, $\alpha_n \in [0, 2\pi]$. In problem (6), (6b) is the constraint of time, and (6c)-(6f) are the constraints of phase shifts. (6) is non-convex because of the constraints (6c)-(6f). Therefore, (6) cannot be solved directly. To simplify the expression, we set $\Theta_1 = [\text{diag}(\Theta_1), \text{diag}(\Theta_2)], \Theta_2 = [\text{diag}(\Theta_3), \text{diag}(\Theta_4)],$

$$g_{1,k} = \begin{bmatrix} \text{diag}(r_1) h_{1,k} \end{bmatrix}$$

and $g_{2,k} = \begin{bmatrix} \text{diag}(h_{1,k}^H r_1^H) \end{bmatrix}$, and thus, (6) can be rewritten as

$$\max_{\Theta, t} t_1 \log \left( 1 + \frac{\eta P_{00} \sum_{k=1}^{K} |\Theta_1 g_{1,k}|^2 |\Theta_2 g_{2,k}|^2}{t_1 \sigma^2} \right)$$

(7a)

s.t. (6b) – (6f).  \quad (7b)

Next, we choose an alternative optimization algorithm to solve problem (7). More specifically, the vector $t$ is optimized for a given $\Theta_1$ of WET and $\Theta_2$ of WIT, and then vice versa.

#### A. Time Allocation for Transmission

To simplify the problem, we set $a = \eta P_0 \sum_{k=1}^{K} |\Theta_1 g_{1,k}|^2 |\Theta_2 g_{2,k}|^2 / \sigma^2$ and replace $t_1$ with $1 - t_0$.
then problem (7) can be reformulated as
\[
\max_{t_0} \quad (1 - t_0) \log \left( 1 + \frac{at_0}{1 - t_0} \right) \\
\text{s.t.} \quad t_0 \in [0, 1].
\] (8a)
(8b)

Theorem 1 provides the optimal value of \(t_0\) in a semi-closed form.

**Theorem 1.** The optimal time \(t_0\) is obtained as
\[
t_0^* = \frac{1 - \exp \left( W \left( \frac{a}{e^a - 1} \right) + 1 \right)}{1 - \exp \left( W \left( \frac{a}{e^a - 1} \right) + 1 \right) - a},
\] (9)
where \(W(\cdot)\) represents the Lambert function [17].

**Proof:** We rewrite (6a) as follows
\[
g(t_0) = (1 - t_0) \log \left( 1 + \frac{at_0}{1 - t_0} \right).
\] (10)

To maximize \(g(t_0)\), we take the first derivative of \(g(t_0)\) with respect to \(t_0\). The optimal solution is obtained when \(\partial g(t_0) / \partial t_0 = 0\), that is, when the following equation holds
\[
\log \left( 1 + \frac{at_0}{1 - t_0} \right) = \frac{a}{1 - t_0} \frac{t_0}{1 + at_0}.
\] (11)
Let \(x = \frac{at_0}{1 - t_0}\), (11) can be rewritten as
\[
x \log x - x = a - 1.
\] (12)
After some transformation, we can convert (12) into
\[
\log \left( \frac{x}{e} \right) \exp \left( \log \left( \frac{x}{e} \right) \right) = \frac{a}{1 - t_0} \frac{t_0}{1 + at_0}.
\] (13)

Applying the function \(W\), for example, \(y \exp(y) = z \Rightarrow y = W(z)\) to (13), we can obtain
\[
\log \left( \frac{x}{e} \right) = W \left( \frac{a}{e^a - 1} \right)
\] (14)
\[
x = \exp \left( W \left( \frac{a}{e^a - 1} \right) + 1 \right).
\] (15)

Then, the semi-closed solution of \(t_0\) can be obtained by substituting \(x\) of (14) into \(x = \frac{a}{1 - t_0}\) [18].

**B. The Optimal Phase Shift of WET**

Next, we solve (7) with respect to \(\Theta\). Due to the non-convexity, we adopt the alternative optimization algorithm. Furthermore, we first fix \(\theta_2\) to optimize \(\theta_1\), and then vice versa. Let us solve (7) with respect to \(\theta_1\), which can be re-worked as
\[
\max_{\theta_1} \sum_{k=1}^{K} b_k |\theta_1 g_{1,k}|^2
\] (15a)
\[
\text{s.t.} \quad (6e), (6f),
\] (15b)

where \(b_k = \eta_0 t_0 |\theta_2 g_{2,k}|^2\) is a constant.

To proceed, we expand \(\sum_{k=1}^{K} b_k |\theta_1 g_{1,k}|^2\), and get
\[
\sum_{k=1}^{K} b_k \theta_1 g_{1,k} g_{1,k}^H \theta_1^H.
\] (16)

Next, \(G_1 = \sum_{k=1}^{K} b_k g_{1,k} g_{1,k}^H\), and put it into (16), thus (16) can be reformulated as
\[
\theta_1 G_1 \theta_1^H.
\] (17)

Following the property of trace, we can get
\[
\theta_1 G_1 \theta_1^H = \text{Tr}(G_1 \theta_1^H \theta_1) = \text{Tr}(G_1 \Phi_1),
\] (18)
where \(\Phi_1 = \theta_1^H \theta_1\) and \(\text{Tr}(\bullet)\) represents the tracing of a matrix. Then (15) can be converted to
\[
\max_{\Phi_1} \text{Tr}(G_1 \Phi_1)
\] (19a)
\[
\text{s.t.} \quad \Phi_1(n,n) = 1, n \in [1, N_1 + N_2] \quad \text{(19b)}
\]
\[
\text{rank}(\Phi_1) = 1. \quad \text{(19c)}
\]

To solve (19), we remove the non-convex constraint (19d), and then (19) converts to semi-definite program, which can be conquered by CVX toolbox [19]. What should be noted is that the obtained solution may violate the constraint \(\text{rank}(\Phi_1) > 1\), and this can be amended by Gaussian randomization [20].

**C. The Optimal Phase Shift of WIT**

Next, we optimize \(\theta_2\) by fixing the vectors \(t\) and \(\theta_1\). Then we can rewrite (7) as
\[
\max_{\theta_2} \sum_{k=1}^{K} c_k |\theta_2 g_{2,k}|^2
\] (20a)
\[
\text{s.t.} \quad (6e), (6f),
\] (20b)

where \(c_k = \eta_0 t_0 |\theta_1 g_{1,k}|^2\). And \(\sum_{k=1}^{K} c_k |\theta_2 g_{2,k}|^2\) can be expanded as
\[
\sum_{k=1}^{K} c_k \theta_2 g_{2,k} g_{2,k}^H \theta_2^H.
\] (21)

\(G_2 = \sum_{k=1}^{K} c_k g_{2,k} g_{2,k}^H\), (19) can be changed to
\[
\theta_2 G_2 \theta_2^H.
\] (22)

Similarly, (22) can be converted to
\[
\theta_2 G_2 \theta_2^H = \text{Tr}(G_2 \theta_2^H \theta_2) = \text{Tr}(G_2 \Phi_2),
\] (23)
where \(\Phi_2 = \theta_2^H \theta_2\). Then, problem (20) can be converted to
\[
\max_{\Phi_2} \text{Tr}(G_2 \Phi_2)
\] (24a)
\[
\text{s.t.} \quad \Phi_2 = \theta_2^H \theta_2 \quad \text{(24b)}
\]
\[
\Phi_2(n,n) = 1, n \in [1, N_1 + N_2] \quad \text{(24c)}
\]
\[
\text{rank}(\Phi_2) = 1. \quad \text{(24d)}
\]

For problem (24), just like (19), we can remove the non-convex constraint (24d) and solve it directly by using the CVX toolbox.
D. Convergence Analysis

There is an upper bound of the sum rate which can be found by taking the maximum eigenvalue of the matrix in ideal circumstances [16]. \( \Theta^n \) is the \( n \)-th solution of the proposed algorithm, and during the iteration process, \( R_{\text{sum}}( \Theta^n ) \) satisfies

\[
R_{\text{sum}}( \Theta^n ) \leq R_{\text{sum}}( \Theta^{n+1} ). \tag{25}
\]

As shown in (25), the sum rate increases or remains unchanged with iterations, and therefore the proposed algorithm converges.

IV. Numerical Results

In this section, numerical results are presented to demonstrate the effectiveness of the proposed scheme. In the simulation, we consider a two-dimensional device layout. More specifically, the coordinates of IRS\(_1\), IRS\(_2\) and HAP are (0 m, 30 m), (0 m, -30 m) and (-40 m, 0 m), respectively. \( K \) users are located within a circle with a center location (20 m, 0 m) and a radius of 10 m randomly. In addition, the path-loss model is \( P_L = \beta d^{-\alpha} \), where \( \beta = -20 \) dB, \( \alpha \) and \( d \) are the path-loss exponent and the distance of individual links. The path loss exponents between HAP and IRS\(_1\), HAP and IRS\(_2\) are 3, while those between users and IRS\(_1\), users and IRS\(_2\) are 2.8. The Rician factor is set to 5 dB. For convenience, we set \( N = N_1 = N_2 \). Other parameter settings are: \( N = 100 \), \( K = 5 \), \( N_0 = -154 \) dBm and \( P_0 = 18 \) dBm.

We make a comparison among the results of the proposed algorithm, the upper bound and the other three benchmark cases. It should be noted that SDP represents the result obtained by the proposed algorithm. The three benchmarks are: 1) the case of equal time distribution; 2) the case of random phase shifts of IRSs, that is each element of IRSs has a random phase; 3) the case of a single IRS.

We simulate the sum rate versus \( N \) of each IRS in Fig. 2. In order to make the simulation effect more obvious, the Rician factor of the figure is set as 1 dB. All cases show upward trends as the number of \( N \) increases, which shows that a larger number of reflecting elements can bring better performance to the system. The method we adopted has better effects than the three benchmarks except for the upper bound case, which demonstrates the validity of the method. Furthermore, benchmark3 is inferior to the proposed scheme, showing the superiority of double IRSs. However, the performance of the case of equal time is the worst, it indicates that reasonable time allocation in wireless powered network has a great impact on system performance to a certain extent.

In Fig. 3, we demonstrate the sum rate versus the transmitted power \( P_0 \) at HAP in different cases. The proposed scheme, benchmark2 and benchmark3 almost have the same growth trends. Compared with other cases, the rate of increase of benchmark1 is smaller, which also shows the importance of optimal time allocation. Besides, the proposed algorithm has the best performance, which highlights the role of optimizing the variables for maximizing sum rate.

We demonstrate the time for WET versus \( N \) of each IRS in Fig. 4. Regarding time allocation, more time for WIT will enhance the system performance. In all cases, times decrease as \( N \) increases. Compared with benchmark2 and benchmark3, the proposed scheme has the shortest time.
V. CONCLUSION

In this correspondence, we consider a double IRS-assisted NOMA system powered by wireless transmission and propose an optimization scheme to maximize the performance of the considered system. There is no direct solution for the objective function because of the coupled variables and non-convex constraints. We optimize the variables iteratively through an alternative algorithm. Numerical analyses certify the effectiveness and correctness of the proposed scheme and prove the superiority of double IRSs.

REFERENCES


