Coverage Analysis of MmWave Networks With Cooperative NOMA Systems

Huailong Shen, Wenqiang Yi, Zhijin Qin, Yuanwei Liu, Fei Li, and Arumugam Nallanathan

***Abstract*—In this letter, a cooperative non-orthogonal multiple access (NOMA) method is utilized in millimeter wave (mmWave) communications to active non-line-of-sight (NLOS) users, where NOMA users are modeled with the aid of Poisson point process (PPP) and all users are severed by a central base station (BS). Closed-form expressions for outage probabilities of paired NOMA users are derived to characterize the performance of the proposed networks. Our numerical results show that the proposed system outperforms the traditional mmWave networks using non-cooperative NOMA.**

***Index Terms*—Cooperative non-orthogonal multiple access, stochastic geometry, millimeter wave communications.**

I. Introduction

Millimeter-Wave (mmWave) is a promising technique in the fifth-generation (5G) network owing to the large available bandwidth [1]. For dense networks with massive users, the severe propagation path loss and low penetration capability of mmWave signals require the help of new multiple access technologies [2]. One promising method to further enhance the spectrum efficiency is introducing non-orthogonal multiple access (NOMA) into mmWave systems [2]. Another problem for mmWave communications is the high outage probability for non-line-of-sight (NLOS) users due to the quasi-optical property [3, 4]. Most existing studies ignore the performance of NLOS users [2, 5]. However, network designers would not give up NLOS users just because these users have weak channel gain.

Cooperative NOMA is to let NOMA users with good channel conditions act as relays to assist NOMA users with weak channel conditions [6]. In this letter, we use cooperative NOMA to active NLOS users in mmWave communications. The main motivation is that the data rate at NLOS users is hard to satisfy the required quality of services and hence additional transmission can be obtained from the better performing line-of-sight (LOS) users. For example, in a Poisson point process (PPP) based spatial model [7], when the signal noise is -40 dBm and the transmit power of base stations (BSs) equals to 20 W, the probability that one BS successfully sends messages to its nearest LOS user is five times larger than that for the nearest NLOS user. This gain can be used to serve NLOS users. Compared with traditional relay techniques, the selected NLOS user has one more signal from LOS users at the relaying time slot.

The main contributions of this letter are summarized as follows: Firstly, we propose a cooperative NOMA method to active mmWave-enabled NLOS users. By using this method, the coverage performance of NLOS users has a huge improvement. Secondly, we propose two user selection schemes from the perspective of user performance enhancement and user selection fairness. Lastly, we derive a closed-form expression of outage probabilities for two paired NOMA users.

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II. System Model

In this letter, we consider a downlink mmWave-NOMA dense network where one BS located at the origin $S = (0, 0)$ serves massive single-antenna users that are located in the cell center[1]. The coverage of this BS is a circle with a radius R. BSs with massive antennas are able to generate multiple orthogonal beams in the angle domain, which is known as beam division multiple access (BDMA) techniques [8]. To simplify the analysis, a sectored antenna model [7] is applied as shown in Fig. 1 and each beam has a constant antenna gain. Since there is no interference between beams, this letter focuses on a typical beam with antenna gain M. Users are modeled according to a PPP Φ with the density λ. Due to the large user density, there exists more than one user in the typical beam. Based on this spatial model, the path loss gain $\mathrm{L}(r_k)$ of user k located at $k' \in \Phi$ is $\mathrm{L}_{\mathcal{X}}(r_k) = C_{\mathcal{X}} r_k^{-\alpha_{\mathcal{X}}}$, where r_k is the communication distance between the user k and the BS, $C_{\mathcal{X}}$ is the intercept and $\alpha_{\mathcal{X}}$ is the path loss exponent. $\mathcal{X} \in \{N, L\}$ represent NLOS and LOS links, respectively.

Due to the quasi-optical property of mmWave signals, the impact of obstacles is greater than that of path loss and small-scale fading in most mmWave communications [2]. Based on this property, we consider a LOS-NLOS pairing strategy. User A is randomly selected from the NLOS user group and user B is the nearest LOS user to BS or randomly selected from the LOS user group. The locations of user A and user B are A' and B', respectively. The decode-and-forward (DF) approach is implemented at user B. Two cooperative NOMA stages [9] are described below.

A. Stage 1: Direct Transmission

During the first stage, the BS broadcasts one message $p_A s_A + p_B s_B$ to user A and user B based on NOMA schemes, where p_k is the power allocation coefficient, s_k is the desired message of user k, and $k \in \{A, B\}$.

We assume that $|p_A|^2 > |p_B|^2$ with $|p_A|^2 + |p_B|^2 = 1$. User A decodes its own message directly, and the signal-to-

[1]This paper focuses on enhancing NLOS users in mmWave communication via cooperative NOMA technique. A basic single cell is applied as a preliminary analysis. For other network structures, our future work will study the impact of inter-cell interference.

interference-plus-noise ratio (SINR) of user A is given by

$$\mathrm{SINR}_A = \frac{P_S M|h_A|^2|p_A|^2\mathrm{L}_N(r_A)}{P_S M|h_A|^2|p_B|^2\mathrm{L}_N(r_A)+\sigma^2}, \tag{1}$$

where h_k models the small scale Nakagami fading [7] from the BS to user k and $|h_k|^2$ is a normalized Gamma random variable. P_S is the transmit power of the BS and σ^2 is thermal noise.

When performing NOMA, user B carries out the successive interference cancellation (SIC). It first decodes the message of user A, then subtracts this message from the received signal to detect its own message. As a result, the received SINR at user B to detect its partner's information is given by

$$\mathrm{SINR}_B^A = \frac{P_S M|h_B|^2|p_A|^2\mathrm{L}_L(r_B)}{P_S M|h_B|^2|p_B|^2\mathrm{L}_L(r_B)+\sigma^2}. \tag{2}$$

If the decoding is successful, the received signal-to-noise ratio (SNR) at user B to detect its own information is

$$\mathrm{SNR}_B = \frac{P_S M|h_B|^2|p_B|^2\mathrm{L}_L(r_B)}{\sigma^2}. \tag{3}$$

B. Stage 2: Cooperative Transmission

We assume NOMA user pairs use orthogonal frequencies and hence no inter-pair interference exists. During this stage, user B retransmit the decoded messages s_A to user A. Under this situation, the received information at user A is

$$y_A^2 = \sqrt{P_t\mathrm{L}_L(r_{A,B})}s_A h_{A,B} + \sigma^2, \tag{4}$$

where P_t is the transmit power of user B, $h_{A,B}$ is the independent Nakagami fading between user A and user B. The $r_{A,B}$ is the distance from user B to user A, it can be calculated by $r_{A,B} = \sqrt{r_A^2 + r_B^2 - 2r_A r_B\cos(\theta)}$, and θ is the angle $\angle A'SB'$. Due to the high directional beamforming, we assume θ is small. Therefore, we can approximate the distance between user A and user B as $r_{A,B} \approx |r_A - r_B|$ and the path between user A and B can be a LOS or NLOS link, which respectively represent the upper bound and the lower bound of our system.

Then the received SNR for user A to decode s_A that is retransmitted from user B can be expressed as

$$\mathrm{SNR}_{A\to B}^A = \frac{P_t\mathrm{L}_L(r_{A,B})|h_{A,B}|^2}{\sigma^2}. \tag{5}$$

At the end of this stage, user A uses maximal-ratio combining (MRC) to combine the signals from the BS and user B [9]. Adding the SNR of the stage 2 and the SINR of the stage 1 together, the received SINR of user A is

$$\begin{aligned}&\mathrm{SINR}_{MRC} = \mathrm{SINR}_A + \mathrm{SNR}_{A\to B}^A\\ =&\frac{P_S M|h_A|^2|p_A|^2\mathrm{L}_N(r_A)}{P_S M|h_A|^2|p_B|^2\mathrm{L}_N(r_A)+\sigma^2} + \frac{P_t\mathrm{L}_L(r_{A,B})|h_{A,B}|^2}{\sigma^2}.\end{aligned}$$

III. Performance Evaluation

In this section, we characterize the outage performance depending on the distance distributions of user A and user B. The cumulative density function (CDF) of the distance from the BS to a random selected user A is

$$F_N(r) = \frac{\int_0^{\vartheta}\int_0^{r}(1-p(t))tdtd\theta}{B_N\vartheta R^2/2} = \frac{2\int_0^{r}(1-p(t))tdt}{B_N R^2}, \tag{6}$$

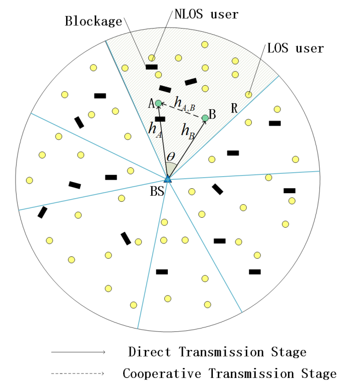

Fig. 1: A system diagram for the considered cooperative mmWave-NOMA scenario. The paired user A and user B are located in the considered beam region with the beam gain M.

where $B_N = \frac{2\int_0^R(1-p(t))tdt}{R^2}$, ϑ is the beam angel, $p(r_k)$ is the LOS probability function of the user k. Based on the rectangle Boolean scheme in [7], $p(r_k) = e^{-\beta r_k}$ and β is a parameter related to the average size and the density of the blockages. The probability density function (PDF) of the distance from the BS to user A is given by

$$f_N(r) = \frac{dF_N(r)}{dr} = \frac{2}{B_N R^2}\left(1-p(r)\right)r. \tag{7}$$

On the one side, since the nearest LOS user has the best channel condition, it has the largest performance gain to provide additional transmission to other NLOS users. On the other side, randomly selecting a LOS user helps to ensure the fairness of user selection and avoid overloading at the same user. Therefore, for user B, we consider two choices: 1) the nearest LOS user and 2) random LOS user.

1) The CDF of the distance from the BS to the nearest LOS user B can be expressed as follows

$$\begin{aligned}F_{L_{N_L}}(x) &= 1 - \Pr[\Phi_{LOS}\cap \mathrm{O}(0,x) = \emptyset]\\ &= 1-\exp\left(-\vartheta\lambda\int_0^x tp(t)\,dt\right),\end{aligned} \tag{8}$$

where Φ_{LOS} is the thining PPP of LOS points, $\mathrm{O}(0,x)$ is the circle of BS with a radius r. The probability that the BS has at least one LOS user is $B_{N_L} = F_{L_{N_L}}(R)$. The PDF of the distance from the BS to the nearest LOS user B is

$$f_{L_{N_L}}(x) = \frac{\vartheta\lambda x p(x) e^{-\vartheta\lambda\int_0^x rp(r)dr}}{B_{N_L}}. \tag{9}$$

2) The CDF of the distance from the BS to a random selected user B is as follows

$$F_{L_R}(r) = \frac{\int_0^{\vartheta}\int_0^{r}p(t)tdtd\theta}{B_L\vartheta R^2/2} = \frac{2\int_0^{r}p(t)tdt}{B_L R^2}, \tag{10}$$

where $B_L = \frac{2\int_0^R p(t)tdt}{R^2}$. The PDF of the distance from the BS to a random selected user B can be expressed as

$$f_{L_R}(r) = \frac{2p(r)r}{B_L R^2}. \tag{11}$$

1) Outage Probability of User B: There may be an outage of user B for two reasons under NOMA protocols. The first

is that user B fails to decode s_A. The second is that s_A can be decoded successfully by user B but the decoding of s_B is failed. After that, user B's outage probabilities can be calculated as follows:

$$P_B = \Pr\left[\mathrm{SINR}_B^A < \tau_A\right] + \Pr[\mathrm{SINR}_B^A > \tau_A, \mathrm{SNR}_B < \tau_B], \tag{12}$$

where $\tau_A = 2^{2R_A} - 1$ and $\tau_B = 2^{2R_B} - 1$ with R_k being the target rate at user k. To ensure the implementation of NOMA protocols, we hold the condition that $|p_A|^2 - |p_B|^2\tau_A > 0$ [9].

Theorem 1. The outage probability of user B P_B is given by

$$P_B \simeq \mathrm{F}_{r_B}\left(\mathcal{F}_Y\left(r_B\right), 0, R\right), \tag{13}$$

where

$$\mathcal{F}_Y(r_B) = \mathrm{G}\left(\frac{N_L r_B^{\alpha_L}\max\left(\varepsilon_A, \varepsilon_B\right)}{C_L}, N_L\right) f_{L_S}(r_B), \tag{14}$$

$$\mathrm{F}_y\left(f\left(y\right), a, b\right) = \frac{\pi\left(b-a\right)}{2n}\sum_{i=1}^{n} f\Big(\frac{\left(x_i+1\right)\left(b-a\right)}{2} + a\Big) \times \left(1 - x_i^2\right)^{1/2} \tag{15}$$

and $N_\mathcal{X}$ is the parameter of the Nakagami small scale fading. $\mathrm{G}(x, y)$ is the lower incomplete gamma function. $\varepsilon_A = \frac{\tau_A\sigma^2}{P_S M(|p_A|^2 - \tau_A|p_B|^2)}$ and $\varepsilon_B = \frac{\tau_B\sigma^2}{P_S M|p_B|^2}$, $S \in \{N_L, R\}$ represent the nearest LOS user B and random selected user B, respectively. The $x_i = \cos\left(\frac{2i-1}{2n}\pi\right)$ indicates the Gauss-Chebyshev node and the subscript y of $\mathrm{F}_y(f(y), a, b)$ is the independent variable of the function $f(y)$. The value of n is to balance the precision and complexity [9]. The equality of equation (13) *can be created only if $n \to \infty$.*

Proof: Let $X = |h_A|^2\mathrm{L}_N(r_A)$, $Y = |h_B|^2\mathrm{L}_L(r_B)$, $Z = |h_{A,B}|^2\mathrm{L}_\mathcal{X}(r_{A,B})$, equation (12) *is changed to*

$$P_B = \Pr(Y < \varepsilon_A) + \Pr(Y > \varepsilon_A, Y < \varepsilon_B). \tag{16}$$

When $\varepsilon_A < \varepsilon_B$, we have $P_B = \Pr(Y < \varepsilon_B)$. When $\varepsilon_A \geq \varepsilon_B$, we have $P_B = \Pr(Y < \varepsilon_A)$. Based on the PDF of the gamma variable $|h_B|^2$, P_B is given by

$$P_B = \Pr\left(Y < \max\left(\varepsilon_A, \varepsilon_B\right)\right) = \int_0^R \mathcal{F}_Y(r_B)dr_B. \tag{17}$$

With the aid of Gaussian-Chebyshev quadrature, which is defined as $\int_a^b f(y)dy \simeq \mathrm{F}_y\left(f\left(y\right), a, b\right)$. We obtain this theorem. ■

Remark 1. *Note that M has a negative correlation with the lower incomplete gamma function in* (14)*. We find outage probabilities of user B decrease with the increase of the beam gain M.*

2) Outage Probability of User A: Under cooperative NOMA protocols, outage encountered by user A happens in two circumstances. The first is when user B decodes s_A but fails to support the targeted rate at user A. The second is when the decoding of s_A is failed at both user A and user B. On this basis, outage probabilities of user A is as follows:

$$\begin{aligned} P_A =& \Pr(\mathrm{SINR}_{MRC} < \tau_A, \mathrm{SINR}_B^A > \tau_A) \\ &+ \Pr(\mathrm{SINR}_A < \tau_A, \mathrm{SINR}_B^A < \tau_A), \end{aligned} \tag{18}$$

Theorem 2. By utilizing MRC, the outage probability of user A can be expressed as

$$\begin{aligned} P_A \simeq& \mathrm{F}_{r_B}\left(\mathrm{F}_{r_A}\left(\mathrm{F}_x(\mathcal{F}_1(x, r_A, r_B), 0, \varepsilon_A), 0, R\right), 0, R\right) \\ &+ \mathrm{F}_{r_B}\left(\mathcal{F}_L(r_B), 0, R\right)\mathrm{F}_{r_A}\left(\mathcal{F}_N\left(r_A\right), 0, R\right), \end{aligned} \tag{19}$$

where

$$\begin{aligned} \mathcal{F}_1(x, r_A, r_B) =& (1 - \mathcal{F}_Y(r_B))f_X(x)f_N(r_A)\mathrm{G}\Big(N_\mathcal{X} \\ &\times |r_A - r_B|^{\alpha_\mathcal{X}}\frac{\left(\tau_A - \frac{P_S Mx|p_A|^2}{P_S Mx|p_B|^2 + \sigma^2}\right)\sigma^2}{P_t C_\mathcal{X}}, N_\mathcal{X}\Big) f_{L_S}(r_B), \end{aligned} \tag{20}$$

$$f_X(x) = \frac{x^{N_N - 1}}{\Gamma(N_N)}\Big(\frac{N_N r_A^{\alpha_N}}{C_N}\Big)^{N_N} e^{-\frac{N_N r_A^{\alpha_N} x}{C_N}}, \tag{21}$$

$$\mathcal{F}_\mathcal{X}(r_k) = \mathrm{G}\Big(\frac{N_\mathcal{X}\varepsilon_k r_k^{\alpha_\mathcal{X}}}{C_\mathcal{X}}, N_\mathcal{X}\Big) f_\mathcal{X}(r_k), \tag{22}$$

and $\Gamma(x)$ is the gamma function.

Proof: With the aid of the proof in **Theorem 1***, equation* (18) *can be expressed as*

$$P_A = \underbrace{\Pr\left(Z < \frac{\left(\tau_A - \frac{P_S Mx|p_A|^2}{P_S Mx|p_B|^2 + \sigma^2}\right)\sigma^2}{P_t}, X < \varepsilon_A, Y > \varepsilon_A\right)}_{\Xi_1} + \underbrace{\Pr\left(X < \varepsilon_A\right)\Pr(Y < \varepsilon_A)}_{\Xi_2}, \tag{23}$$

Based on the PDF of X, Y, and Z, we obtain that

$$\Xi_1 = \int_0^R\int_0^R\int_0^{\varepsilon_A} \mathcal{F}_1(x, r_A, r_B)dx dr_A dr_B, \tag{24}$$

$$\Xi_2 = \int_0^R \mathcal{F}_N(r_A)dr_A \int_0^R \mathcal{F}_Y(r_B)dr_B. \tag{25}$$

With the aid of Gaussian-Chebyshev quadrature, we have this theorem. ■

Assumption 1: In mmWave communications, the received power from NLOS paths is negligible [2]. We assume user A obtains information only from user B via LOS transmission.

Based on *Assumption 1*, we are able to ignore the NLOS part before "+" in (12). After that, the received SINR of user A can be obtained as:

$$\mathrm{SINR}_{MRC} = \frac{P_t\mathrm{L}_L(r_{A,B})|h_{A,B}|^2}{\sigma^2}. \tag{26}$$

Corollary 1. *Based on* Assumption 1*, we can simplify Ξ_1 as*

$$\begin{aligned} \widetilde{\Xi}_1 \simeq& \mathrm{F}_{r_B}\Big(\mathrm{F}_{r_A}\Big(\mathrm{G}\Big(\frac{N_L\tau_A\sigma^2|r_A - r_B|^{\alpha_L}}{P_t C_L}, N_L\Big) f_L(r_B) \\ &\times (1 - \mathcal{F}_Y(r_B))f_N(r_A), 0, R\Big), 0, R\Big). \end{aligned} \tag{27}$$

Proof: Due to ignoring the NLOS part, the simplified Ξ_1 can be derived via $\widetilde{\Xi}_1 = \Pr\left(Z < \frac{\tau_A\sigma^2}{P_t}, Y > \varepsilon_A\right)$. ■

Corollary 1 can by used in **Theorem 2** to simply the outage probability of user A P_A.

IV. Numerical Results

Numerical results are provided in this section to evaluate the outage performance of the considered networks. The reference distance is $d_0 = 1$ m, which means $C_L = C_N$. We assume the average LOS range of the network $1/\beta = R_L/\sqrt{2}$ m. The LOS disc range is $R_L = 80$ m, the density of users is $\lambda = 1/(r_c^2\pi) = 1/(5^2\pi)$ m^{-2}. The path loss law for LOS is $\alpha_L = 2$, $N_L = 3$. The path loss law for NLOS is $\alpha_N = 4$, $N_N = 2$. The carrier frequency is $f_m = 28$ GHz, $|p_A|^2 = 0.6$, $|p_B|^2 = 0.4$, bandwidth per resource block is $B = 100$MHz, and $P_S = 20$W, $P_t = 4$W.

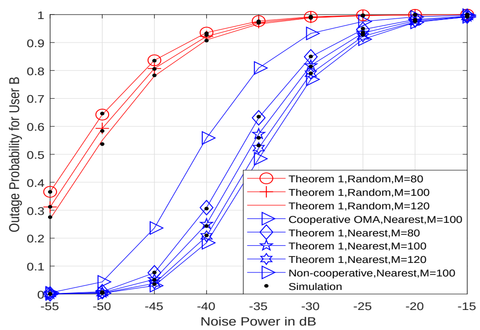

Fig. 2: Outage probability of user B with different M versus noise.

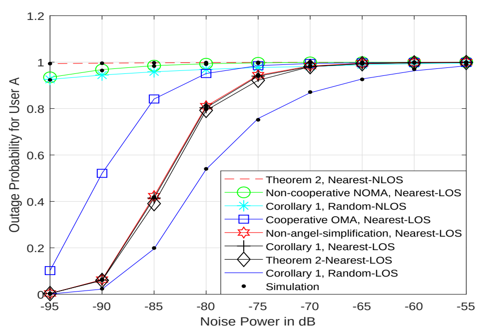

Fig. 3: Outage probability of user A versus noise and the comparison of outage probability with non-cooperative NOMA.

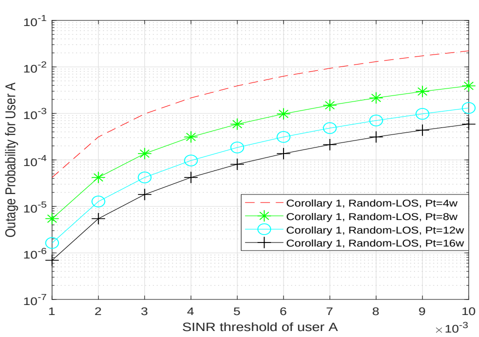

Fig. 4: Outage probability of user A with different transmit power of user B versus SINR thresholds of user A.

Fig. 2 plots the outage probability of user B versus noise. Since theoretical results match simulations perfectly, this figure validates the accuracy of the proposed analytical expressions. Moreover, Fig. 2 demonstrates that the outage probability of user B decreases with the increase of M as discussed in **Remark 1**. Fig. 2 also shows that the random selected user B always obtains a higher outage probability than the nearest LOS user B. Compared with the cooperative OMA scheme, the proposed cooperative NOMA scheme with the nearest LOS user performs better.

Fig. 3 plots the outage probability of user A versus noise for both non-cooperative NOMA and cooperative NOMA. The figure demonstrates the outage probability of NLOS users with cooperative NOMA is significantly lower than that of NLOS users with non-cooperative NOMA. The reason is that cooperative NOMA in the high SINR region ensures reliable reception for the NLOS users. This phenomenon shows that applying cooperative NOMA in mmWave communications is helpful and essential. Fig. 3 shows that when the path between users A and B is a LOS path, the outage probability of user A is much lower than when the path between users A and B is a NLOS path. Although the performance is not good in the case of the NLOS path between users, it performs very well in the case of the LOS path between users, so we can consider adopting our protocol in the case of the LOS path between users in the future. Fig. 3 also illustrates that our simplification of the distance between users is reasonable.

Fig. 4 plots the outage probability of user A with different transmit power of user B versus SINR thresholds of user A. This figure demonstrates that the outage probability of user A decreases as the transmit power of user B increases. This is owing to the high transmit power at user B contributes to the reliable reception at user A.

V. Conclusion

In this letter, we have proposed a cooperative mmWave-NOMA method to enhance the performance of blocked users. A spatial model based on stochastic geometry has been provided to analyze outage probabilities. The analytical and numerical results have demonstrated that the proposed cooperative NOMA system outperforms the traditional non-cooperative NOMA system in mmWave communications. For actual BDMA impact and multiple beam user pairing for sparse networks, we will study them in future work.

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