# Keyhole Effect in Dual-Hop MIMO AF Relay Transmission with Space-Time Block Codes 

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#### Abstract

In this paper, the effect of keyhole on the performance of multiple-input multiple-output (MIMO) amplify-and-forward (AF) relay networks with orthogonal space-time block codes (OSTBCs) transmission is investigated. In particular, we analyze the asymptotic symbol error probability (SEP) performance of a downlink communication system where the amplifying processing at the relay can be implemented by either the linear or squaring approach. Our tractable asymptotic SEP expressions enable us to obtain both diversity and array gains. Our finding reveals that with condition $n_{\mathrm{S}}>\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)$, the linear approach can provide the full achievable diversity gain of $\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)$ when only the second hop suffers from the keyhole effect, i.e., single keyhole effect (SKE), where $n_{\mathrm{S}}, n_{\mathrm{R}}$, and $n_{\mathrm{D}}$ are the number of antennas at source, relay, and destination, respectively. However, for the case that both the source-relay and relay-destination links experience the keyhole effect, i.e., double keyhole effect (DKE), the achievable diversity order is only one regardless of the number of antennas. In contrast, utilizing the squaring approach, the overall diversity gain can be achieved as $\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)$ for both SKE and DKE. An important observation corroborated by our studies is that for satisfying the tradeoff between performance and complexity, we should use the linear approach for SKE and the squaring approach for DKE.


Index Terms-Keyhole effect, amplify-and-forward relay, multiple-input multiple-output, orthogonal space-time block code.

## I. Introduction

THE multiple-input multiple-output (MIMO) technique has been considered as a promising transmission scheme for future wireless systems as being included in the extension of the 3GPP Long Term Evolution (LTE) standard. By deploying multiple antennas at transmitter and receiver ends, MIMO systems exploit the rich scattering propagation environment of wireless links to enhance the system performance [1]. In this ideal case of rich scattering, the channel matrix is of full rank. As a consequence, a MIMO channel between an $n_{\mathrm{S}}$-antenna source and an $n_{\mathrm{D}}$-antenna destination under such ideal condition exhibits the maximum multiplexing gain of $\min \left(n_{\mathrm{S}}, n_{\mathrm{D}}\right)$ and diversity gain of $n_{\mathrm{S}} n_{\mathrm{D}}$. In some realistic

[^0]indoor and outdoor propagation conditions such as corridors, crowded subways and tunnels, scattering objects can be randomly arranged, which may cause a rank deficiency of the channel matrix. This degeneration effect, so called keyhole or pinhole effect, significantly decreases the spatial multiplexing and diversity gains in MIMO systems [2]-[4]. As a result, the keyhole effect reduces the MIMO channel capacity to that of single-input single-output (SISO) systems and the diversity gain is of $\min \left(n_{\mathrm{S}}, n_{\mathrm{D}}\right)$ order.
MIMO relay networks have attracted much interest because of their ability to remarkably increase coverage and system performance [5], [6]. However, there has been limited research in the contemporary literature investigating the keyhole effect on the performance of MIMO relay networks. To the best of the authors' knowledge, only Souihli and Ohtsuki have very recently attempted to consider the effect of keyhole for MIMO relay systems [7], [8]. In particular, by considering a downlink cellular network in which the source-to-relay channel enjoys rich scattering while the source-to-destination and relay-todestination channels suffer from the keyhole phenomenon, they have shown that using the relay can mitigate the effect of the keyhole in terms of channel capacity. However, [7] and [8] have only focused on the multiplexing gain and the decode-and-forward (DF) relay protocol. In our previous work [9], we have investigated the cooperative diversity gain for a similar downlink system with orthogonal space-time block code (OSTBC) transmission. We have shown that the antenna correlation has no impact on the diversity gain of MIMO keyhole relay channels.

The OSTBC transmission over MIMO amplify-and-forward (AF) relay networks can be subdivided into two groups depending on the signal processing at the relay: 1) Linear Approach: the relay simply amplifies the source's OSTBC matrix and forwards it to the destination without incurring any additional processing [10], [11] and 2) Squaring Approach: the relay performs the squaring technique [12] to decompose the source's OSTBC matrix into independent SISO signals and then re-encodes them by a new OSTBC matrix before forwarding to the destination [13]-[15]. It is important to note that for the linear approach, the relay can operate in either semi-blind or channel state information (CSI)-assisted modes. However, for the squaring approach, since the relay utilizes the CSI knowledge for decoupling the source's OSTBC matrix, it is reasonable to assume that the AF relay will operate using the CSI-assisted gain.

Several papers have investigated the performance of OSTBCs in AF relay systems (see e.g., [10], [11], [13]-[15]
and references therein). For the linear approach, the exact symbol error probability (SEP) for semi-blind AF relay has been derived for dual-hop fading channel in [10]. The final SEP expression in [10] is not given in closed-form expression but as an integral whose integrand is expressed in terms of hypergeometric functions. In [11], several closed-form expressions of the bit error rate (BER) for a specific number of antennas have been presented. For the squaring approach, the error rate performance has been investigated for the cases when the destination has a single antenna [14] and multiple antennas [13]. The performance of OSBTC transmission in MIMO AF relay networks with best relay selection has been addressed in [15].

In this paper, we investigate the effect of MIMO keyhole channels on the cooperative diversity gain of relay networks. We consider the downlink cooperative communication where an $n_{\mathrm{S}}$-antenna base station S using OSTBC communicates with an $n_{\mathrm{D}}$-antenna mobile station D through the assistance of an $n_{\mathrm{R}}$-antenna relay station R . To distinguish the advantage of each type of signal processing at $R$, the linear approach is assumed to operate in semi-blind mode for low complexity processing while in the squaring approach the CSI-assisted AF relay is exploited to perform some additional complicated processing. In addition, we consider both single keyhole effect (SKE) and double keyhole effect ${ }^{1}$ (DKE). For the case of SKE, the relay is assumed to be a fixed base station (BS) and placed in some strategic location, leading the source-to-relay link to enjoy rich scattering whereas the relay-to-destination link suffers poor scattering due to the mobility of the destination. For the case of DKE, due to ad-hoc relay placement or in some specific networks such as vehicle-to-vehicle communication [16], [17], we assume that both hops are subject to the keyhole effect.

Our contributions include the following:

- We develop new analytical expressions to investigate the performance of AF systems with OSTBCs and keyhole effects for downlink communication ${ }^{2}$. In particular, we investigate the system's average SEP for both linear and squaring approaches.
- We present new asymptotic results which reveal the effect of keyhole on diversity and array gains of the considered system for both linear and squaring approaches. For SKE, the relaying channel (source-relay-destination link) provides the full cooperative diversity gain of $\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)$ for both AF relaying approaches (linear and squaring). However, for the case of DKE, the linear approach cannot offer any diversity gain, i.e., its diversity order is equal to that of a SISO channel. In contrast, for the squaring approach, the full achievable diversity gain remains as $\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)$.
- Our analysis gives additional insights for the system designer that for keyhole relay channels, the deployment of more than $n_{D}$ antennas at the relay, i.e., $n_{R}>n_{D}$,

[^1]

Fig. 1. System model for a dual-hop MIMO AF relay system consisting of an $n_{\mathrm{S}}$-antenna source, an $n_{\mathrm{R}}$-antenna relay, and an $n_{\mathrm{D}}$-antenna destination. (a) SKE: $\boldsymbol{H}_{\mathrm{RD}}$ is subject to the keyhole effect and $\boldsymbol{H}_{\mathrm{SR}}$ is keyhole-free. (b) DKE: Both $\boldsymbol{H}_{\text {SR }}$ and $\boldsymbol{H}_{\mathrm{RD}}$ are subject to the keyhole effect.
yields no diversity gain. More importantly, when only the second hop suffers from the keyhole effect, the relay should deploy the linear approach to achieve the full diversity gain with low complexity. When both hops experience the keyhole effect, the squaring approach is more favorable than the linear approach because it can only offer a unit diversity order.
Notation: Vectors and matrices are written as bold lowercase and uppercase letters, respectively. Superscripts $*$ and $\dagger$ stand for complex conjugate and transpose conjugate, respectively. $I_{n}$ represents an $n \times n$ identity matrix and $\|\boldsymbol{A}\|_{\mathrm{F}}$ defines Frobenius norm of the matrix $\boldsymbol{A} . \mathbb{E}\{$.$\} and \operatorname{tr}(\cdot)$ are the expectation operator and trace of a matrix, respectively. We use the notation $x \sim \mathcal{C N}(\cdot, \cdot)$ to denote that $x$ is complex circularly symmetric Gaussian distributed. Let $x \in \mathbb{C}^{m}$ be the complex Gaussian distributed vector-variate with mean vector $\boldsymbol{v}$ and variance matrix $\boldsymbol{\Sigma}$ defined as $\boldsymbol{x} \sim \tilde{\mathcal{N}}_{m}(\boldsymbol{v}, \boldsymbol{\Sigma})$. $\mathcal{K}_{\nu}(\cdot)$ is the modified Bessel function of the second kind [18, Eq. (8.432.3)]. Finally, let $\boldsymbol{X} \in \mathbb{C}^{m \times n}$ be the complex Gaussian distributed matrix-variate defined as $X \sim$ $\tilde{\mathcal{N}}_{m, n}(\boldsymbol{M}, \boldsymbol{\Sigma}, \boldsymbol{\Phi})$ if $\operatorname{vec}\left(\boldsymbol{X}^{\dagger}\right)$ is $m n$-variate complex Gaussian distributed with mean $\operatorname{vec}\left(\boldsymbol{M}^{\dagger}\right)$ and covariance $\boldsymbol{\Sigma}^{T} \otimes \boldsymbol{\Phi}$, where $\otimes$ is the Kronecker product and $\operatorname{vec}(\boldsymbol{A})$ denotes the vector formed by stacking all the columns of $\boldsymbol{A}$ into a column vector.

## II. System and Channel Models

We consider a communication network where a source terminal $S$ communicates with a destination $D$ through the assistance of a relay station $R$ as shown in Fig. 1. In this paper, we assume no direct link between the source and destination due to high pathloss and severe shadowing.

We next describe OSTBC transmission over a dual-hop AF relay network in detail. During an $L_{c}$-symbol interval, $N \log _{2} M$ information bits are mapped to a sequence of symbols $x_{1}, x_{2}, \ldots, x_{N}$ selected from an $M$-ary phase-shift keying ( $M$-PSK) or $M$-ary quadrature amplitude modulation ( $M$-QAM) signal constellation $\mathcal{S}$ using Gray mapping with average transmit energy per symbol $\mathbb{E}\left\{\left|x_{k}\right|^{2}\right\}=P_{0}$. These symbols are then encoded with an OSTBC denoted by an $L_{c} \times n_{\mathrm{S}}$ transmission matrix $\mathcal{G}$ with code rate $\mathcal{R}_{c}$ whose elements are the linear combinations of $x_{1}, x_{2}, \ldots, x_{N}$ and their conjugate with the property that the columns of $\mathcal{G}$ are orthogonal [19].

We assume that the channel is subject to frequency-flat fading and is perfectly known at the destination but unknown
at the source. The source will transmit the OSTBC matrix $\boldsymbol{X}=\boldsymbol{\mathcal { G }}^{T}$ to the relay in the first hop. Let us denote $P_{s}$ as the transmit power in $n_{\mathrm{S}}$ antennas at the source so that the total transmit power of an OSTBC block is $\mathbb{E}\left\{\|\boldsymbol{X}\|_{\mathrm{F}}^{2}\right\}=L_{c} P_{s}$. The signal received at the relay in the first hop is given by $\boldsymbol{Y}_{\mathrm{SR}}=\boldsymbol{H}_{\mathrm{SR}} \boldsymbol{X}+\boldsymbol{W}_{\mathrm{SR}}$, where $\boldsymbol{H}_{\mathrm{SR}} \sim \tilde{\mathcal{N}}_{n_{\mathrm{R}}, n_{\mathrm{S}}}\left(\mathbf{0}, \Omega_{1} \boldsymbol{I}_{n_{\mathrm{R}}}, \boldsymbol{I}_{n_{\mathrm{S}}}\right)$ is a random channel matrix, with $\Omega_{1}=\mathbb{E}\left\{\left|\left\{\boldsymbol{H}_{\mathrm{SR}}\right\}_{i j}\right|^{2}\right\}$ for $i=1,2, \ldots, n_{\mathrm{R}}$ and $j=1,2, \ldots, n_{\mathrm{S}}$, and $\boldsymbol{W}_{\mathrm{SR}} \sim$ $\tilde{\mathcal{N}}_{n_{\mathrm{R}}, N_{c}}\left(\mathbf{0}, N_{0} \boldsymbol{I}_{n_{\mathrm{R}}}, \boldsymbol{I}_{L_{c}}\right)$ is an additive white Gaussian noise (AWGN) matrix, with $N_{0}=\mathbb{E}\left\{\left|\left\{\boldsymbol{W}_{\mathrm{SR}}\right\}_{i j}\right|^{2}\right\}$.

## A. Linear Approach

The received signal at the relay, $Y_{\mathrm{SR}}$, is then multiplied by a fixed gain $G_{\mathrm{LA}}$, and retransmitted to the destination. The signal at the destination in the second hop is given by $Y_{\mathrm{RD}}=$ $\boldsymbol{H}_{\mathrm{RD}} G_{\mathrm{LA}} \boldsymbol{Y}_{\mathrm{SR}}+\boldsymbol{W}_{\mathrm{RD}}$, where $\boldsymbol{H}_{\mathrm{RD}} \sim \tilde{\mathcal{N}}_{n_{\mathrm{D}}, n_{\mathrm{R}}}\left(\mathbf{0}, \Omega_{2} \boldsymbol{I}_{n_{\mathrm{D}}}, \boldsymbol{I}_{n_{\mathrm{R}}}\right)$ is a random channel matrix, with $\Omega_{2}=\mathbb{E}\left\{\left|\left\{\boldsymbol{H}_{\mathrm{RD}}\right\}_{k \ell}\right|^{2}\right\}$ for $k=1,2, \ldots, n_{\mathrm{D}}$ and $\ell=1,2, \ldots, n_{\mathrm{R}}$, and $W_{\mathrm{RD}} \sim$ $\tilde{\mathcal{N}}_{n_{\mathrm{D}}, L_{c}}\left(\mathbf{0}, N_{0} \boldsymbol{I}_{n_{\mathrm{D}}}, \boldsymbol{I}_{L_{c}}\right)$ is an AWGN matrix, with $N_{0}=$ $\mathbb{E}\left\{\left|\left\{\boldsymbol{W}_{\mathrm{RD}}\right\}_{k \ell}\right|^{2}\right\}$.

Utilizing the orthogonal property of OSTBCs, the maximum likelihood (ML) decoding metric can be decomposed into a sum of $N$ terms, where each term depends exactly on one complex symbol $x_{n}, n=1,2, \ldots, N$. Consequently, the detection of $x_{n}$ is decoupled from the detection of $x_{p}$ for $n \neq p$ and the end-to-end instantaneous SNR can be written as [10]

$$
\begin{align*}
\gamma_{\mathrm{LA}}=\frac{\bar{\gamma}}{\mathcal{R}_{c} n_{\mathrm{S}}} \operatorname{tr}\left\{\left[G_{\mathrm{LA}}^{2} \boldsymbol{H}_{\mathrm{RD}}^{\dagger}\right.\right. & \left(\boldsymbol{I}_{n_{\mathrm{D}}}+G_{\mathrm{LA}}^{2} \boldsymbol{H}_{\mathrm{RD}} \boldsymbol{H}_{\mathrm{RD}}^{\dagger}\right)^{-1} \\
& \left.\left.\times \boldsymbol{H}_{\mathrm{RD}} \boldsymbol{H}_{\mathrm{SR}} \boldsymbol{H}_{\mathrm{SR}}^{\dagger}\right]\right\} \tag{1}
\end{align*}
$$

where $\bar{\gamma}=P_{s} / N_{0}$ is the average SNR and (1) is obtained from the fact that $\mathbb{E}\left\{\|\boldsymbol{X}\|_{\mathrm{F}}^{2}\right\}=L_{c} P_{s}$. Given the condition that the relay does not have full channel knowledge of the first hop, $G_{\mathrm{LA}}$ can be obtained as $G_{\mathrm{LA}}^{2}=\left[n_{\mathrm{R}}\left(\Omega_{1}+1 / \bar{\gamma}\right)\right]^{-1}[10]$, [11]. When the relay is deployed with a single antenna, i.e., $n_{\mathrm{R}}=1$, the channel matrices $\boldsymbol{H}_{\mathrm{SR}}$ and $\boldsymbol{H}_{\mathrm{RD}}$ for both hops become vectors $\boldsymbol{h}_{\mathrm{SR}}$ and $\boldsymbol{h}_{\mathrm{RD}}$, respectively. The instantaneous SNR for the linear approach given in (1) can now be simplified as

$$
\begin{equation*}
\gamma_{\mathrm{LA}}=\frac{\bar{\gamma}}{\mathcal{R}_{c} n_{\mathrm{S}}} \frac{\left\|\boldsymbol{h}_{\mathrm{SR}}\right\|_{\mathrm{F}}^{2}\left\|\boldsymbol{h}_{\mathrm{RD}}\right\|_{\mathrm{F}}^{2}}{\left\|\boldsymbol{h}_{\mathrm{RD}}\right\|_{\mathrm{F}}^{2}+1 / G_{\mathrm{LA}}^{2}} \tag{2}
\end{equation*}
$$

## B. Squaring Approach

Using the squaring approach [12]-[15] of OSTBC transmission, the received matrix at the relay can be decomposed as $y_{k}=\left\|\boldsymbol{H}_{\mathrm{SR}}\right\|_{\mathrm{F}}^{2} x_{k}+\eta_{k}$, where $k=1,2, \ldots, N$ and $\eta_{k} \sim \mathcal{C N}\left(0,\left\|\boldsymbol{H}_{\mathrm{SR}}\right\|_{\mathrm{F}}^{2} N_{0}\right)$. With the decomposed symbols $y_{k}$ at hand, the relay generates an OSTBC matrix and then amplifies it with an amplifying gain $G_{\text {SA }}$ before transmitting to the destination. For the squaring approach, it is plausible to assume that the relay operates in the CSI-assisted mode and its amplifying gain is defined as $G_{\mathrm{SA}}=\left(\left\|\boldsymbol{H}_{\mathrm{SR}}\right\|_{\mathrm{F}}\right)^{-2}$ [13]-[15] by excluding the effect of the noise. The received signal at the destination is given by $Y_{\mathrm{RD}}=H_{\mathrm{RD}} G_{\mathrm{SA}} \tilde{X}+W_{\mathrm{RD}}$, where $\tilde{X}$ is the re-encoded OSTBC at the relay after using the squaring
approach ${ }^{3}$, whose element is the linear combination of $y_{k}$ and $y_{k}^{*}$ for $k=1,2, \ldots, N$. The end-to-end SNR is given by [13][15]

$$
\begin{equation*}
\gamma_{\mathrm{SA}}=\frac{\bar{\gamma}}{\mathcal{R}_{c} n_{\mathrm{S}}} \times \frac{\left\|\boldsymbol{H}_{\mathrm{SR}}\right\|_{\mathrm{F}}^{2}\left\|\boldsymbol{H}_{\mathrm{RD}}\right\|_{\mathrm{F}}^{2}}{\left\|\boldsymbol{H}_{\mathrm{SR}}\right\|_{\mathrm{F}}^{2}+\left\|\boldsymbol{H}_{\mathrm{RD}}\right\|_{\mathrm{F}}^{2}} \tag{3}
\end{equation*}
$$

If $n_{R}=1$, it can be shown that the SNR expression given in (3) is equivalent to (2). This demonstrates that the two approaches exhibit the same performance when the system has a single-antenna relay. In the existing literature, to the best of the authors' knowledge, there is no comparison between the two approaches, even for the widely treated case of Rayleigh fading.

## C. Keyhole Models and Statistics of $\left\|\boldsymbol{H}_{\mathrm{RD}}\right\|_{\mathrm{F}}^{2}$ and $\left\|\boldsymbol{H}_{\mathrm{SR}}\right\|_{\mathrm{F}}^{2}$

In this subsection, we first introduce the keyhole model adopted in this paper and some statistical properties of the two random variables (RVs) $\left\|\boldsymbol{H}_{\mathrm{RD}}\right\|_{\mathrm{F}}^{2}$ and $\left\|\boldsymbol{H}_{\mathrm{SR}}\right\|_{\mathrm{F}}^{2}$ which will be helpful for asymptotic SEP derivation.

1) Single Keyhole Effect: For this scenario, we assume that the relay terminal together with the source BS is part of a fixed infrastructure network, i.e., the relay has been installed at a strategic location by the network operator [7]-[9]. Hence, the $S \rightarrow R$ link enjoys a rich-scattering environment, yielding the channel gain matrix $H_{\mathrm{SR}}$ to be of full rank. On the other hand, in downlink systems, D is considered as a mobile station (MS) and applicable to be in a poor scattering environment. In order to model this practical scenario of interest, we assume that the $\mathrm{R} \rightarrow \mathrm{D}$ link is a keyhole channel, i.e., $\boldsymbol{H}_{\mathrm{RD}}$ is of unit rank. With the keyhole assumption, the channel matrix $H_{\mathrm{RD}}$ can be mathematically described as [21] $\boldsymbol{H}_{\mathrm{RD}}=\Omega_{2} \boldsymbol{x}_{\mathrm{RD}} \boldsymbol{y}_{\mathrm{RD}}^{\dagger}$. Here, $x_{\mathrm{RD}} \in \mathbb{C}^{n_{\mathrm{D}}}$ and $\boldsymbol{y}_{\mathrm{RD}} \in \mathbb{C}^{n_{\mathrm{R}}}$ describe the scattering environment at the destination and relay, respectively [21].
2) Double Keyhole Effect: In some unplanned situations, the deployment of the relay may be more or less ad hoc, e.g., the relay may be placed based on a rough knowledge of the coverage issues and traffic density (hotspots) in the network. In such cases, both hops can be subject to poor scattering and thus we assume that both the $\boldsymbol{H}_{\mathrm{SR}}$ and $\boldsymbol{H}_{\mathrm{RD}}$ links of the network are under the influence of the keyhole effect. In this case, since the first-hop channel is now also under the keyhole effect, the corresponding channel matrix therefore can be expressed as $\boldsymbol{H}_{\mathrm{SR}}=\sqrt{\Omega_{1}} \boldsymbol{x}_{\mathrm{SR}} \boldsymbol{y}_{\mathrm{SR}}^{\dagger}$, where $\boldsymbol{x}_{\mathrm{SR}} \in \mathbb{C}^{n_{\mathrm{R}}}$, $\boldsymbol{y}_{\mathrm{SR}} \in$ $\mathbb{C}^{n_{s}}$ are independent random vectors constituting the scattering environment at the relay and source, respectively [21]. It is important to note that for the case of the linear approach, the current form of $\gamma_{\mathrm{LA}}$ given in (1) is too complicated to be used for performance evaluation due to the unit-rank property of $H_{\mathrm{SR}}$. Fortunately, we can utilize the unit-rank property to simplify the complex form given in (1), which facilitates our analysis. The derivation steps are shown in the sequel.

Since $\boldsymbol{H}_{\mathrm{SR}}=\sqrt{\Omega_{1}} \boldsymbol{x}_{\mathrm{SR}} \boldsymbol{y}_{\mathrm{SR}}^{\dagger}$, the SNR given in (1) can be rewritten as

$$
\begin{equation*}
\gamma_{\mathrm{LA}}=\frac{\bar{\gamma}}{\mathcal{R}_{c} n_{\mathrm{S}}} G_{\mathrm{LA}}^{2} \Omega_{1}\left\|\boldsymbol{y}_{\mathrm{SR}}\right\|_{\mathrm{F}}^{2} \operatorname{tr}\left(A \boldsymbol{x}_{\mathrm{SR}} \boldsymbol{x}_{\mathrm{SR}}^{\dagger}\right) \tag{4}
\end{equation*}
$$

[^2]where $\boldsymbol{A}=\boldsymbol{H}_{\mathrm{RD}}^{\dagger}\left(\boldsymbol{I}_{n_{\mathrm{D}}}+G_{\mathrm{LA}}^{2} \boldsymbol{H}_{\mathrm{RD}} \boldsymbol{H}_{\mathrm{RD}}^{\dagger}\right)^{-1} \boldsymbol{H}_{\mathrm{RD}}$. Owing to the fact that $\boldsymbol{A}$ is a positive definite matrix, we can apply the eigen-decomposition yielding $\boldsymbol{A}=\boldsymbol{U} \boldsymbol{\Lambda} \boldsymbol{U}^{\dagger}$ where $\boldsymbol{U}$ is a unitary matrix and $\boldsymbol{\Lambda}$ is the diagonal matrix containing the eigenvalues of $\boldsymbol{A}$. Since $\boldsymbol{H}_{\mathrm{RD}}=\sqrt{\Omega_{2}} \boldsymbol{x}_{\mathrm{RD}} \boldsymbol{y}_{\mathrm{RD}}^{\dagger}$ is a unitrank matrix, $A$ has only one non-zero eigenvalue denoted as $\frac{1}{G_{\mathrm{LA}}^{2}+1 /\left\|\boldsymbol{H}_{\mathrm{RD}}\right\|_{\mathrm{F}}^{2}}$, leading (4) to become
\[

$$
\begin{equation*}
\gamma_{\mathrm{LA}}=\frac{\bar{\gamma} \Omega_{1}}{\mathcal{R}_{c} n_{\mathrm{S}}} \times \frac{|z|^{2}\left\|\boldsymbol{y}_{\mathrm{SR}}\right\|_{\mathrm{F}}^{2}\left\|\boldsymbol{H}_{\mathrm{RD}}\right\|_{\mathrm{F}}^{2}}{\left\|\boldsymbol{H}_{\mathrm{RD}}\right\|_{\mathrm{F}}^{2}+1 / G_{\mathrm{LA}}^{2}} \tag{5}
\end{equation*}
$$

\]

where $z$ is the first element of vector $\tilde{x}_{\mathrm{SR}}$, with $\tilde{x}_{\mathrm{SR}}=U^{\dagger} x_{\mathrm{SR}}$. Since $U$ is a unitary matrix and $x_{\text {SR }} \sim \tilde{\mathcal{N}}_{n_{\mathrm{R}}}\left(\mathbf{0}, \boldsymbol{I}_{n_{\mathrm{R}}}\right)$, we can easily see that $\tilde{x}_{\text {SR }} \sim \tilde{\mathcal{N}}_{n_{\mathrm{R}}}\left(\mathbf{0}, \boldsymbol{I}_{n_{\mathrm{R}}}\right)$ leads to $z \sim \mathcal{C} \mathcal{N}(0,1)$. As compared to (1), the new expression of $\gamma_{\mathrm{LA}}$ is more tractable.
3) Statistics of $\left\|\boldsymbol{H}_{\mathrm{RD}}\right\|_{\mathrm{F}}^{2}$ and $\left\|\boldsymbol{H}_{\mathrm{SR}}\right\|_{\mathrm{F}}^{2}$ : The statistics of $\left\|\boldsymbol{H}_{\mathrm{RD}}\right\|_{\mathrm{F}}^{2}$ and $\left\|\boldsymbol{H}_{\mathrm{SR}}\right\|_{\mathrm{F}}^{2}$ applicable under keyhole conditions will be utilized in our subsequent performance analysis. Due to symmetry between $\left\|\boldsymbol{H}_{\mathrm{RD}}\right\|_{\mathrm{F}}^{2}$ and $\left\|\boldsymbol{H}_{\mathrm{SR}}\right\|_{\mathrm{F}}^{2}$, let us denote $A B \in\{S R, R D\}$ and $\ell \in\{1,2\}$ for the sake of brevity. Since $\boldsymbol{H}_{\mathrm{AB}}=\sqrt{\Omega_{\ell}} \boldsymbol{x}_{\mathrm{AB}} \boldsymbol{y}_{\mathrm{AB}}^{\dagger}$, we then can express $\left\|\boldsymbol{H}_{\mathrm{AB}}\right\|_{\mathrm{F}}^{2}=$ $\Omega_{\ell}\left\|x_{\mathrm{AB}}\right\|_{\mathrm{F}}^{2}\left\|\boldsymbol{y}_{\mathrm{AB}}\right\|_{\mathrm{F}}^{2}$. As $\boldsymbol{x}_{\mathrm{AB}}$ and $\boldsymbol{y}_{\mathrm{AB}}$ are statistically independent, the PDF of $\left\|H_{\mathrm{AB}}\right\|_{\mathrm{F}}^{2}$ when subject to the keyhole effect is given by [4]

$$
\begin{equation*}
f_{\left\|\boldsymbol{H}_{\mathrm{AB}}\right\|_{\mathrm{F}}^{2}}(z)=\frac{2 z^{\left(n_{\mathrm{A}}+n_{\mathrm{B}}\right) / 2-1}}{\Gamma\left(n_{\mathrm{A}}\right) \Gamma\left(n_{\mathrm{B}}\right) \Omega_{\ell}^{\left(n_{\mathrm{A}}+n_{\mathrm{B}}\right) / 2}} \mathcal{K}_{n_{\mathrm{B}}-n_{\mathrm{A}}}\left(2 \sqrt{\frac{z}{\Omega_{\ell}}}\right) . \tag{6}
\end{equation*}
$$

Since $\left\|\boldsymbol{H}_{\mathrm{AB}}\right\|_{\mathrm{F}}^{2}$ is the product of two RVs, the CDF of $\left\|\boldsymbol{H}_{\mathrm{AB}}\right\|_{\mathrm{F}}^{2}$ can be written as $F_{\left\|\boldsymbol{H}_{\mathrm{AB}}\right\|_{\mathrm{F}}^{2}}(z)=$ $\int_{0}^{\infty} f_{\left\|\boldsymbol{x}_{\mathrm{AB}}\right\|_{\mathrm{F}}^{2}}(w) F_{\left\|\boldsymbol{y}_{\mathrm{AB}}\right\|_{\mathrm{F}}^{2}}\left(\frac{z}{\Omega_{\ell} w}\right) d w$. Note that the CDF of $\left\|\boldsymbol{y}_{\mathrm{AB}}\right\|_{\mathrm{F}}^{2}$ can be given in the form of elementary functions by applying [18, Eq. (8.352.4)]. Then, utilizing the result of [18, Eq. (3.471.9)], we get the CDF of $\left\|\boldsymbol{H}_{\mathrm{AB}}\right\|_{\mathrm{F}}^{2}$ as

$$
\begin{align*}
F_{\left\|\boldsymbol{H}_{\mathrm{AB}}\right\|_{\mathrm{F}}^{2}}(z)=1- & \sum_{k=1}^{n_{\mathrm{A}}-1} \frac{2}{\Gamma\left(n_{\mathrm{B}}\right) k!}\left(\frac{z}{\Omega_{\ell}}\right)^{\left(n_{\mathrm{B}}+k\right) / 2} \\
& \times \mathcal{K}_{n_{\mathrm{B}}-k}\left(2 \sqrt{\frac{z}{\Omega_{\ell}}}\right) \tag{7}
\end{align*}
$$

## III. High-SNR SEP Analysis

It is noted that for the considered SKE and DKE cases, an exact closed-form SEP analysis is not possible. Moreover, using our subsequent analysis, the exact SEP can be evaluated using numerical integration. In order to obtain key insights and how different parameters affect the system performance, we therefore investigate the high SNR SEP behavior that yields the array and diversity gains of MIMO AF systems with the keyhole effect. Specifically, in the case of SKE with LA, we use exact SEP expression [11], which is given in the form of a finite integral whose integrand is the MGF function of the SNR, to obtain the asymptotic SEP. In the cases of: (1) SKE with SA and (2) DKE with SA, we utilize an asymptotic analysis as in [22] to obtain a polynomial approximation to the exact SEP in the high SNR regime. In the case of DKE with LA, we obtain the desired diversity order result using an upper and a lower bound to the exact end-to-end SNR.

## A. Single Keyhole Effect with Linear Approach

For the dual-hop MIMO AF relaying with linear approach, from (1), the moment generating function (MGF) can be obtained from the definition $\mathcal{M}_{\mathrm{LA}}(s)=\mathbb{E}_{\gamma_{\mathrm{LA}}}\left\{e^{-s \gamma_{\mathrm{LA}}}\right\}$ as [10], [11]

$$
\begin{align*}
& \mathcal{M}_{\mathrm{LA}}(s) \\
& =\mathbb{E}_{\boldsymbol{H}_{\mathrm{RD}}}\left\{\left(\frac{\operatorname{det}\left(\boldsymbol{I}_{n_{\mathrm{D}}}+G_{\mathrm{LA}}^{2} \boldsymbol{H}_{\mathrm{RD}} \boldsymbol{H}_{\mathrm{RD}}^{\dagger}\right)}{\operatorname{det}\left(\boldsymbol{I}_{n_{\mathrm{D}}}+G_{\mathrm{LA}}^{2}\left(1+\frac{s \bar{\gamma} \Omega_{1}}{\mathcal{R}_{c} n_{\mathrm{S}}}\right) \boldsymbol{H}_{\mathrm{RD}} \boldsymbol{H}_{\mathrm{RD}}^{\dagger}\right)}\right)^{n_{\mathrm{S}}}\right\} . \tag{8}
\end{align*}
$$

Let us denote $x$ as a non-zero eigenvalue of $\boldsymbol{H}_{\mathrm{RD}} \boldsymbol{H}_{\mathrm{RD}}^{\dagger}$. Since $\boldsymbol{H}_{\mathrm{RD}}$ is of unit rank, it is easy to see that there exists only one non-zero eigenvalue of $x=\left\|\boldsymbol{H}_{\mathrm{RD}}\right\|_{\mathrm{F}}^{2}$. Using this property and from (6), we can rewrite (8) as

$$
\left.\begin{array}{rl}
\mathcal{M}_{\gamma_{\mathrm{LA}}}(s)= & \int_{0}^{\infty} \frac{2\left(\Omega_{2} \bar{\gamma}\right)^{-\left(n_{\mathrm{R}}+n_{\mathrm{D}}\right) / 2}}{\Gamma\left(n_{\mathrm{R}}\right) \Gamma\left(n_{\mathrm{D}}\right)}
\end{array} 1+\frac{G_{\mathrm{LA}}^{2} \Omega_{1} s t}{\mathcal{R}_{\mathrm{c}} n_{\mathrm{S}}\left(1+G_{\mathrm{LA}}^{2} \bar{\gamma}^{-1} t\right)}\right]^{-n_{\mathrm{S}}}
$$

To the best of the authors' knowledge, (9) has no closed-form solution. As a result, we present the asymptotic SEP for single keyhole effect with linear approach in the following Theorem.

Theorem 1: In the high SNR regime, the SEP expression for MIMO AF relaying with linear approach and single keyhole effect can be given by

$$
\begin{align*}
P_{\mathrm{e}}^{(\mathrm{SRD})} & \stackrel{(\operatorname{large} \bar{\gamma})}{\approx} c_{1}\left(\frac{\Omega_{1} \Omega_{2} G_{\mathrm{LA}}^{2} \bar{\gamma}}{\mathcal{R}_{c} n_{\mathrm{S}}}\right)^{-\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)} \\
& \times \frac{\Gamma\left(n_{\mathrm{S}}-\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)\right) \Gamma\left(\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)\right)}{\Gamma\left(n_{\mathrm{S}}\right) \Gamma\left(n_{\mathrm{R}}\right) \Gamma\left(n_{\mathrm{D}}\right)} \tag{10}
\end{align*}
$$

with the condition that $n_{\mathrm{S}}>\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)$ and $c_{1}$ is a constant depending on the modulation scheme. For example, for $M$ PSK modulation, $c_{1}$ is defined as

$$
c_{1}= \begin{cases}\frac{1}{\pi} \int_{0}^{\pi-\frac{\pi}{M}}\left(\frac{\mathrm{~g}}{\sin ^{2} \theta}\right)^{-n_{\mathrm{R}}}\left[\ln \left(\frac{\Omega_{1} \Omega_{2} G_{\mathrm{L}}^{2} \bar{\gamma} \mathrm{~g}}{\mathcal{R}_{c} n_{\mathrm{S}} \sin ^{2} \theta}\right)\right. &  \tag{11}\\ \left.-\psi\left(n_{\mathrm{R}}\right)+\psi\left(n_{\mathrm{S}}-n_{\mathrm{R}}\right)\right] d \theta, & \text { for } n_{\mathrm{R}}=n_{\mathrm{D}}, \\ \frac{\Gamma\left(\mid n_{\mathrm{D}}-n_{\mathrm{R}}\right)}{\pi} & \text { for } n_{\mathrm{R}} \neq n_{\mathrm{D}} \\ \times \int_{0}^{\pi-\frac{\pi}{M}}\left(\frac{\mathrm{~g}}{\sin ^{2} \theta}\right)^{-\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)} d \theta, & \end{cases}
$$

Proof: See Appendix A.
Since we consider downlink communication, it is reasonable to assume that $n_{\mathrm{S}}>n_{\mathrm{D}}$. As can be observed from (10), the SEP is inversely proportional to $\bar{\gamma}^{\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right) 4}$. Hence, the full achievable diversity gain provided by the fixed relay link using linear approach is $\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)$. In addition, as can be observed from (10), the array gain can be clearly obtained.

## B. Double Keyhole Effect with Linear Approach

In the case of the double keyhole channel, as can be seen from (5), the exact PDF derivation of $\gamma_{\mathrm{LA}}$ requires the statistics of multiple products and division of RVs. In the considered problem, these expressions are not trivial to obtain and do

[^3]not lend mathematical tractability for further manipulation. Therefore, we will utilize the upper and lower bounds of $\gamma_{\text {LA }}$ for the analysis instead. Using (5), the instantaneous end-to-end SNR when both hops undergo keyhole can be rewritten as $\gamma_{\mathrm{LA}}=\left(\frac{1}{\gamma_{1}}+\frac{c_{2}}{\gamma_{1} \gamma_{2}}\right)^{-1}$, where $c_{2}=\bar{\gamma} / G_{\mathrm{LA}}^{2}$, $\gamma_{1}=\frac{\Omega_{1} \bar{\gamma}}{\mathcal{R}_{c} n_{S}}|z|^{2}\left\|\boldsymbol{y}_{\mathrm{SR}}\right\|_{\mathrm{F}}^{2}$, and $\gamma_{2}=\bar{\gamma}\left\|\boldsymbol{H}_{\mathrm{RD}}\right\|_{\mathrm{F}}^{2}$. Hence, it is convenient for us to introduce the lower and upper bounds of $\gamma_{\text {LA }}$ as
\[

$$
\begin{equation*}
\gamma_{\mathrm{LA}}^{(\mathrm{lo})} \triangleq \frac{1}{2} \min \left(\gamma_{1}, \frac{\gamma_{1} \gamma_{2}}{c_{2}}\right) \leq \gamma_{\mathrm{LA}} \leq \min \left(\gamma_{1}, \frac{\gamma_{1} \gamma_{2}}{c_{2}}\right) \triangleq \gamma_{\mathrm{LA}}^{(\mathrm{up})} . \tag{12}
\end{equation*}
$$

\]

Theorem 2: In the high SNR regime, the SEP expression for MIMO AF relaying with linear approach and double keyhole effect can be lower and upper bounded as

$$
\begin{equation*}
\frac{c_{3}}{\bar{\gamma}} \leq P_{\mathrm{e}}^{(\mathrm{SRD})} \leq \frac{2 c_{3}}{\bar{\gamma}} \tag{13}
\end{equation*}
$$

where

$$
\begin{align*}
c_{3} & =\left[\frac{\epsilon}{\Omega_{2} G_{\mathrm{LA}}^{2}\left(n_{\mathrm{R}}-1\right)\left(n_{\mathrm{D}}-1\right)}+\bar{\epsilon}\right] \frac{\mathcal{R}_{c} n_{\mathrm{S}}}{\Omega_{1}\left(n_{\mathrm{S}}-1\right) \pi \mathrm{g}} \\
& \times\left[\frac{(M-1) \pi}{2 M}+\frac{\sin (2 \pi / M)}{4 M}\right] . \tag{14}
\end{align*}
$$

## Proof: See Appendix B.

Based on (13), we can observe that the slope of the SEP curve in the high SNR regime is one. In other words, when both hops experience the keyhole effect, a relay using the linear approach exhibits an unit diversity order regardless of the number of antennas.

## C. Single Keyhole Effect with Squaring Approach

With the squaring approach, the instantaneous SNR incurred by the dual-hop channel can be given in the form of the harmonic mean of two independent RVs as in (3) and rewritten as follows:

$$
\begin{equation*}
\gamma_{\mathrm{SA}}=\frac{1}{\mathcal{R}_{c} n_{\mathrm{S}}} \frac{\gamma_{2} \gamma_{3}}{\gamma_{2}+\gamma_{3}} \tag{15}
\end{equation*}
$$

where $\gamma_{3}=\bar{\gamma}\left\|\boldsymbol{H}_{\mathrm{SR}}\right\|_{\mathrm{F}}^{2}$. In this case, since the first hop, i.e., $S \rightarrow R$, is keyhole-free, the squared Frobenius norm of $H_{S R}$, $\left\|\boldsymbol{H}_{\mathrm{SR}}\right\|_{\mathrm{F}}^{2}$, is the sum of $n_{\mathrm{S}} n_{\mathrm{R}}$ independent and identically distributed exponential RVs. Then, the asymptotic PDF of $\gamma_{3}$ can be given as [22]

$$
\begin{equation*}
f_{\gamma_{3}}\left(\gamma_{3}\right) \stackrel{(\operatorname{large}}{\approx} \stackrel{\bar{\gamma})}{ } \frac{\gamma_{3}^{n_{\mathrm{S}} n_{\mathrm{R}}-1}}{\Gamma\left(n_{\mathrm{S}} n_{\mathrm{R}}\right)(\bar{\gamma})^{n_{\mathrm{S}} n_{\mathrm{R}}-1}} \tag{16}
\end{equation*}
$$

Depending on the relationship between $n_{R}$ and $n_{D}$, the asymptotic SEP expression for MIMO AF relaying with squaring approach and single keyhole effect can be presented in the following Theorem.

Theorem 3: When $n_{\mathrm{R}} \neq n_{\mathrm{D}}$, in the high SNR regime, the asymptotic SEP is written as

$$
\begin{align*}
P_{\mathrm{e}}^{(\mathrm{SRD})} & \stackrel{(\operatorname{large}}{\approx} \bar{\gamma}) \\
& \frac{\prod_{i=1}^{\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)}(2 i-1)}{\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)} \frac{\Gamma\left(\left|n_{\mathrm{D}}-n_{\mathrm{R}}\right|\right)}{\Gamma\left(n_{\mathrm{R}}\right) \Gamma\left(n_{\mathrm{D}}\right)}  \tag{17}\\
& \times\left(\frac{\mathcal{R}_{c} n_{\mathrm{S}}}{2 \mathrm{~g} \Omega_{2} \bar{\gamma}}\right)^{\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)}
\end{align*}
$$

When $n_{\mathrm{R}}=n_{\mathrm{D}}$, in the high SNR regime, the SEP can be upper and lower bounded as

$$
\begin{equation*}
\frac{c_{4}(1)}{\bar{\gamma}^{n_{\mathrm{R}}}} \leq P_{\mathrm{e}}^{(\mathrm{SRD})} \leq \frac{c_{4}(2)}{\bar{\gamma}^{n_{\mathrm{R}}}} \tag{18}
\end{equation*}
$$

where $c_{4}(\star)$ is a constant expressed as

$$
\begin{align*}
c_{4}(\star)= & \frac{1}{\pi \Gamma\left(n_{\mathrm{R}}\right)}\left(\frac{\mathcal{R}_{c} n_{\mathrm{S}}}{\Omega_{2} \mathrm{~g}}\right)^{n_{\mathrm{R}}} \int_{0}^{\pi-\pi / M}\left(\star \sin ^{2} \theta\right)^{n_{\mathrm{R}}} \\
& \times\left[\ln \left(\frac{\Omega_{2} \bar{\gamma} \mathrm{~g}}{\star \mathcal{R}_{c} n_{\mathrm{S}} \sin ^{2} \theta}\right)-\psi\left(n_{\mathrm{R}}\right)\right] d \theta \tag{19}
\end{align*}
$$

Proof: See Appendix C.
By combining (17) and (18), we see that when the fixed relay terminal uses the squaring approach, the full achievable diversity order is $\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)$.

## D. Double Keyhole Effect with Squaring Approach

In this case where both hops are subject to the keyhole effect, the PDF and CDF of $\gamma_{3}$ can be obtained from (6) and (7), respectively, by applying a Jacobian transformation. Since we consider a downlink communication scenario, i.e., $n_{\mathrm{S}}>$ $n_{\mathrm{R}}$, from (25) the asymptotic $f_{\gamma_{3}}(\gamma)$ can be derived as

$$
\begin{equation*}
\left.f_{\gamma_{3}}(\gamma) \stackrel{(\operatorname{large}}{\approx} \bar{\gamma}\right) \frac{\Gamma\left(n_{\mathrm{S}}-n_{\mathrm{R}}\right)}{\Gamma\left(n_{\mathrm{S}}\right) \Gamma\left(n_{\mathrm{R}}\right)} \frac{\gamma^{n_{\mathrm{R}}-1}}{\left(\Omega_{1} \bar{\gamma}\right)^{n_{\mathrm{R}}}} \tag{20}
\end{equation*}
$$

Again, similarly as in Sections III-A, III-B, and III-C, by distinguishing separate cases due to the approximation of the modified Bessel function of the second kind, the asymptotic SEP expression for MIMO AF relaying with squaring approach and double keyhole effect can be presented in the following Theorem.

Theorem 4: In the high SNR regime, the asymptotic SEP expression can be derived for the cases $n_{\mathrm{R}}>n_{\mathrm{D}}$ and $n_{\mathrm{R}}<n_{\mathrm{D}}$, respectively, as follows:

$$
\begin{equation*}
\left.P_{\mathrm{e}}^{(\mathrm{SRD})} \stackrel{(\operatorname{large}}{\approx} \bar{\gamma}\right) \frac{\prod_{i=1}^{n_{\mathrm{D}}}(2 i-1) \Gamma\left(n_{\mathrm{R}}-n_{\mathrm{D}}\right)}{\Gamma\left(n_{\mathrm{R}}\right) \Gamma\left(n_{\mathrm{D}}+1\right)}\left(\frac{\mathcal{R}_{c} n_{\mathrm{S}}}{2 \mathrm{~g} \Omega_{2} \bar{\gamma}}\right)^{n_{\mathrm{D}}} \tag{21}
\end{equation*}
$$

$$
\begin{align*}
& P_{\mathrm{e}}^{(\mathrm{SRD})}\stackrel{(\operatorname{large}}{\approx} \bar{\gamma}) \\
& \quad \prod_{i=1}^{n_{\mathrm{R}}}(2 i-1)  \tag{22}\\
& \Gamma\left(n_{\mathrm{R}}+1\right)\left.\frac{\Gamma\left(n_{\mathrm{D}}-n_{\mathrm{R}}\right)}{\Gamma\left(n_{\mathrm{D}}\right) \Omega_{2}^{n_{\mathrm{R}}}}+\frac{\Gamma\left(n_{\mathrm{S}}-n_{\mathrm{R}}\right)}{\Gamma\left(n_{\mathrm{S}}\right) \Omega_{1}^{n_{\mathrm{R}}}}\right] \\
& \quad \times\left(\frac{\mathcal{R}_{c} n_{\mathrm{S}}}{2 \mathrm{~g} \bar{\gamma}}\right)^{n_{\mathrm{R}}}
\end{align*}
$$

When $n_{\mathrm{R}}=n_{\mathrm{D}}$, in the high SNR regime, the SEP expression can be lower and upper bounded as

$$
\begin{equation*}
\frac{c_{5}(1)}{\bar{\gamma}^{n_{\mathrm{R}}}} \leq P_{\mathrm{e}}^{(\mathrm{SRD})} \leq \frac{c_{5}(2)}{\bar{\gamma}^{n_{\mathrm{R}}}} \tag{23}
\end{equation*}
$$

where $c_{5}(\star)$ is a constant given by

$$
\begin{align*}
c_{5}(\star) & =\frac{\left(\mathcal{R}_{c} n_{\mathrm{S}}\right)^{n_{\mathrm{R}}}}{\pi \Gamma\left(n_{\mathrm{R}}\right)} \int_{0}^{\pi-\pi / M}\left(\frac{\star \sin ^{2} \theta}{\mathrm{~g}}\right)^{n_{\mathrm{R}}} \\
& \times\left[\frac{\ln \left(\frac{\Omega_{1} \bar{\gamma} \mathrm{~g}}{\star \mathcal{R}_{c} n_{\mathrm{S}} \sin ^{2} \theta}\right)-\psi\left(n_{\mathrm{R}}\right)}{\Omega_{2}^{n_{\mathrm{R}}}}+\frac{\Gamma\left(n_{\mathrm{S}}-n_{\mathrm{R}}\right)}{\Omega_{1}^{n_{\mathrm{R}}}}\right] d \theta \tag{24}
\end{align*}
$$

Proof: See Appendix D.


Fig. 2. SEP of $(4,2,2)$ and (5, 4, 3)-MIMO AF relay systems for 8-PSK modulation versus SNR when only the second hop experiences the keyhole effect and the relay uses the linear approach.

By combining (21), (22), and (23), we can observe that the diversity order is $\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)$. Therefore, compared with the unit diversity order achieved with the linear approach (cf. Section III-B), a larger diversity order can be achieved with the squaring approach.

## IV. Numerical Results and Discussion

In this section, we investigate the keyhole effect on the SEP performance of OSTBC MIMO AF relaying with the help of the analytical derivation developed in Section III. To illustrate the correctness of the asymptotic result, we also provide exact numerical results, which are obtained from numerical integration. For example, in the case of fixed relay with the linear approach, from (9), we can evaluate the exact SEP using numerical integration techniques. However, these SEP expressions are in the form of double or triple integrals and do not reveal any insight into the system performance. Moreover, simulation results also confirm our analytical results but are not shown here to avoid clutter in the figures. In all examples, results are shown for 8-PSK modulation and unit variances ( $\Omega_{1}=\Omega_{2}=1$ ). We consider different numbers of antennas at $\mathrm{S}, \mathrm{R}, \mathrm{D}$ denoted as $\left(n_{\mathrm{S}}, n_{\mathrm{R}}, n_{\mathrm{D}}\right)$, where $n_{\mathrm{S}}>\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)$. In addition, the OSTBCs are generated from [20], e.g., the code-rates for the case of two and three transmit antennas are one and $3 / 4$, respectively.

For the linear approach, the SEP when only the second hop experiences the keyhole effect is shown in Fig. 2, where we compare asymptotic SEP with exact numerical results for two different antenna configurations, e.g., $(4,2,2)$ and $(5,4,3)$. As expected, the asymptotic curves exactly converge to the exact curves in the high SNR regime. We can see that the slopes of SEP curves are regulated by the minimum number of antennas at the relay and destination. Fig. 3 displays the SEP performance with DKE and the squaring approach. We have plotted the upper asymptotic bounds for the two different MIMO AF relay systems as in the above example. Clearly, as


Fig. 3. SEP of $(4,2,2)$ and $(5,4,3)$-MIMO AF relay systems for 8-PSK modulation versus SNR when both hops experience keyhole effect and the relay uses the linear approach.


Fig. 4. SEP of $(4,3,2)$ and $(5,4,4)$-MIMO AF relay systems for 8-PSK modulation versus SNR when only the second hop experiences the keyhole effect and the relay uses the squaring approach.
observed from the plots, the upper bound becomes very tight in the high SNR regime and can be considered as the asymptotic SEP. For further comparison, the two systems exhibit the same diversity gain as expected. We see that as the number of antennas increases, i.e., from $(4,2,2)$ to $(5,4,3)$, the slopes of SEP curves remain unchanged and equal to one, which agrees with our analytical conclusion in Section III-C.

For the squaring approach, Fig. 4 shows the SEP performance of the two different MIMO AF relay systems, i.e., $(4,3,2)$ and $(5,4,4)$, when only the second hop experiences the keyhole effect. The SEP performance improves as the $\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)$ increases. Again, we observe an excellent agreement between the asymptotic and exact curves in the high SNR regime. In Fig. 5, we plot the average SEP when both hops


Fig. 5. SEP of $(4,2,3),(5,4,3)$, and $(5,4,4)$-MIMO AF relay systems for 8 -PSK modulation versus SNR when both hops experience the keyhole effect and the relay uses the squaring approach.


Fig. 6. SEP of $\left(6, n_{\mathrm{R}}, 2\right)$ and $\left(7, n_{\mathrm{R}}, 3\right)$-MIMO AF relay systems for 8-PSK modulation versus SNR when only the second hop experiences the keyhole effect. For comparison, SEPs for direct link with keyhole and single antenna relaying link are also shown.
are subject to keyhole fading. Three specific system configurations, i.e., $(5,4,4),(5,4,3)$, and $(4,2,3)$, corresponding to the three conditions in the Section III-D (i.e., $n_{R}=n_{\mathrm{D}}, n_{\mathrm{R}}>n_{\mathrm{D}}$, and $n_{\mathrm{R}}<n_{\mathrm{D}}$ ), are used. In the three cases, our asymptotic results are seen to converge to the respective exact curves for relatively high SNR levels ( $\mathrm{SNR}>20 \mathrm{~dB} \mathrm{)} .\mathrm{As} \mathrm{expected}$, best performance among the three examples is achieved for (5, 4, 4).

To further demonstrate the cooperative diversity derived in the previous section, we plot the SEP for $\left(6, n_{\mathrm{R}}, 2\right)$ with the linear approach and varying $n_{R}$ from three to five and for $\left(7, n_{\mathrm{R}}, 3\right)$ with the squaring approach and varying $n_{\mathrm{R}}$ from four to six. For comparison, we also plot the SEP for single-
hop non-cooperative communications with keyhole and the SEP for single antenna relay. Here, we assume that $R$ is located half-way between S and D , which results in the channel mean power for the direct link as $\Omega_{0}=1 / 16$. As can be observed from Fig. 6, increasing the number of antennas at relays while keeping $n_{\mathrm{D}}$ unchanged results in no diversity enhancement. This observation is in line with our analytical result derived in the previous section as the full achievable diversity gain is $\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)$. In fact, under such a severe condition it is unnecessary to deploy a large number of antennas at the relay, but only requiring $n_{\mathrm{R}}=n_{\mathrm{D}}$. More importantly, we can see that direct communication, i.e., $\left(n_{\mathrm{S}}, n_{\mathrm{D}}\right)=(7,3)$, exhibits the same diversity gain as MIMO AF relaying with the squaring approach, i.e., $\left(7, n_{R}, 3\right)$. However, relaying link with the squaring approach significantly outperforms the direct link in terms of array gain. Also, when $\mathrm{SNR} \leq 35 \mathrm{~dB}$ the SEP of direct communication is inferior to that of MIMO AF relay with the linear approach, which clearly highlights the advantage of using relays in keyhole fading.

## V. Conclusion

We have investigated the effect of keyhole on the performance of downlink AF relay systems with OSTBC transmission. In particular, we have investigated the effect of keyhole on the SEP performance when the AF relay operates in either linear or squaring approaches. Our tractable asymptotic SEP expressions reveal both diversity and array gains and provide important insights for radio system designers. For the case when only the second hop suffers from the keyhole effect, we have suggested to apply the linear approach at the relay for obtaining the full achievable diversity gain of $\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)$ while keeping low complexity for the relay. In contrast, for a more severe scenario where both hops are keyhole channels, the squaring approach should be used to keep the diversity order as $\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)$ since the linear approach does not offer any diversity gain.

## Appendix A: Proof of Theorem 1

To study the high SNR performance, we make use of the fact that for small values of $z, \mathcal{K}_{n}(z)$ can be approximated as [25]

$$
\mathcal{K}_{n}(z) \stackrel{(\operatorname{small} z)}{\approx}\left\{\begin{array}{cl}
\ln \left(\frac{2}{z}\right) & \text { for } n=0  \tag{25}\\
\frac{\Gamma(|n|)}{2}\left(\frac{2}{z}\right)^{|n|} & \text { for } n \neq 0
\end{array}\right.
$$

By substituting (25) into (9), we obtain an asymptotic $\mathcal{M}_{\gamma_{\text {LA }}}(s)$ in the high SNR regime. Note that depending on the value of $n_{\mathrm{R}}$ and $n_{\mathrm{D}}, \mathcal{K}_{n_{\mathrm{D}}-n_{\mathrm{R}}}(\cdot)$ can be differently approximated as in (25). Therefore, we will separately investigate two cases as follows: For the case $n_{\mathrm{R}}=n_{\mathrm{D}}$, replacing $\mathcal{K}_{n_{\mathrm{D}}-n_{\mathrm{R}}}\left(2 \sqrt{\frac{t}{\bar{\gamma} \Omega_{2}}}\right)$ by $\frac{1}{2} \ln \left(\frac{\bar{\gamma} \Omega_{2}}{t}\right)$ and neglecting the small term relative to $\bar{\gamma}$ in the integrand of (9) results in

$$
\begin{align*}
\mathcal{M}_{\gamma \mathrm{LA}}(s) & \stackrel{(\operatorname{large}}{\approx} \bar{\gamma}) \\
& \frac{\left(\Omega_{2} \bar{\gamma}\right)^{-n_{\mathrm{R}}}}{\Gamma\left(n_{\mathrm{R}}\right) \Gamma\left(n_{\mathrm{D}}\right)}\left[\int_{0}^{\infty} \frac{\ln \left(\Omega_{2} \bar{\gamma}\right) t^{n_{\mathrm{R}}-1}}{\left(1+\frac{G_{\mathrm{LA}}^{2} \Omega_{1} s}{\mathcal{R}_{c} n_{\mathrm{S}}} t\right)^{n_{\mathrm{S}}}} d t\right.  \tag{26}\\
& \left.-\int_{0}^{\infty} \frac{t^{n_{\mathrm{R}}-1} \ln (t)}{\left(1+\frac{G_{\mathrm{LA}}^{2} \Omega_{1} s}{\mathcal{R}_{c} n_{\mathrm{S}}} t\right)^{n_{\mathrm{S}}}} d t\right]
\end{align*}
$$

The first and second integral in the above equation can be easily computed by applying the results of [18, Eq. (3.194.3)] and [26, Eq. (2.6.4.7)], respectively, with the condition that $n_{\mathrm{S}}>n_{\mathrm{R}}$. After several algebraic manipulations, the MGF of $\gamma_{\text {LA }}$ can be written as

$$
\begin{align*}
& \left.\left.\mathcal{M}_{\gamma_{\mathrm{LA}}}(s) \stackrel{(\operatorname{large} \bar{\gamma})}{\approx}\left(\frac{\Omega_{1} \Omega_{2} G_{\mathrm{LA}}^{2} \bar{\gamma} s}{\mathcal{R}_{c} n_{\mathrm{S}}}\right)^{-n_{\mathrm{R}} \Gamma\left(n_{\mathrm{S}}-n_{\mathrm{R}}\right)} \frac{\Gamma\left(n_{\mathrm{S}}\right) \Gamma\left(n_{\mathrm{R}}\right)}{\mathcal{R}_{c} n_{\mathrm{S}}}\right)-\psi\left(n_{\mathrm{R}}\right)+\psi\left(n_{\mathrm{S}}-n_{\mathrm{R}}\right)\right] .
\end{align*}
$$

For the case $n_{\mathrm{R}} \neq n_{\mathrm{D}}$, we can find the MGF of $\gamma_{\mathrm{LA}}$ as

$$
\begin{align*}
& \left.\mathcal{M}_{\gamma_{\mathrm{LA}}}(s) \stackrel{(\operatorname{large}}{\approx} \bar{\gamma}^{\gamma}\right) \\
& \times \int_{0}^{\infty} t^{\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)-1}\left(1+\frac{G_{\mathrm{LA}}-\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)}{\mathcal{R}_{c} n_{\mathrm{S}}} t\right)^{-n_{\mathrm{S}}} d t . \tag{28}
\end{align*}
$$

Note that the integral (28) converges when $n_{\mathrm{S}}>\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)$ which then yields

$$
\begin{align*}
& \mathcal{M}_{\gamma_{\mathrm{LA}}}(s) \stackrel{\left(\operatorname{large}{ }_{2} \bar{\gamma}\right)}{\approx}\left(\frac{\Omega_{1} \Omega_{2} G_{\mathrm{LA}}^{2} \bar{\gamma} s}{\mathcal{R}_{c} n_{\mathrm{S}}}\right)^{-\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)} \\
& \times \frac{\Gamma\left(n_{\mathrm{S}}-\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)\right) \Gamma\left(\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)\right) \Gamma\left(\left|n_{\mathrm{D}}-n_{\mathrm{R}}\right|\right)}{\Gamma\left(n_{\mathrm{S}}\right) \Gamma\left(n_{\mathrm{R}}\right) \Gamma\left(n_{\mathrm{D}}\right)} \tag{29}
\end{align*}
$$

By combining the two cases and using the fact that the exact SEP of $M$-PSK modulation is given by [11, Eq. (21)]

$$
\begin{equation*}
P_{\mathrm{e}}^{(\mathrm{SRD})}=\frac{1}{\pi} \int_{0}^{\pi-\frac{\pi}{M}} \mathcal{M}_{\gamma_{\mathrm{LA}}}\left(\frac{\mathrm{~g}}{\sin ^{2} \theta}\right) d \theta \tag{30}
\end{equation*}
$$

where $\mathrm{g}=\sin ^{2}\left(\frac{\pi}{M}\right)$, the asymptotic SEP of a dual-hop channel, i.e., $S \rightarrow R \rightarrow D$ link, when the fixed relay terminal operates in linear mode can be expressed as (10), which completes our proof.

## Appendix B: Proof of Theorem 2

We first derive the upper bound and note that the derivation of lower bound can be followed accordingly. By definition, the CDF of $\gamma_{\mathrm{LA}}^{(\mathrm{up})}$ is given by $F_{\gamma_{\text {LA }}^{\text {(up) }}}(\gamma)=\operatorname{Pr}\left(\gamma_{1}<\gamma, \gamma_{1}<\frac{\gamma_{1} \gamma_{2}}{c_{2}}\right)+$ $\underbrace{\operatorname{Pr}\left(\frac{\gamma_{1} \gamma_{2}}{c_{2}}<\gamma, \frac{\gamma_{1} \gamma_{2}}{c_{2}}<\gamma_{1}\right)}$. In addition, we have $\mathcal{L}=\mathcal{L}^{\mathcal{L}}\left(\gamma_{1}<\gamma, \gamma_{2}<c_{2}\right)+\operatorname{Pr}\left(\gamma_{1}>\gamma, \gamma_{1} \gamma_{2}<\gamma c_{2}\right)$. As a result, the CDF of $\gamma_{\mathrm{LA}}^{(\mathrm{up})}$ can be rewritten as $F_{\gamma_{\text {LA }}^{\text {(up) }}}(\gamma)=\underbrace{\operatorname{Pr}\left(\gamma_{1}<\gamma, \gamma_{2}>c_{2}\right)+\operatorname{Pr}\left(\gamma_{1}<\gamma, \gamma_{2}<c_{2}\right)}_{=\operatorname{Pr}\left(\gamma_{1}<\gamma\right)}$ $+\quad \operatorname{Pr}\left(\gamma_{1}>\gamma, \gamma_{1} \gamma_{2}<\gamma c_{2}\right)$, which then yields $F_{\gamma_{\text {LA }}^{(\text {PP })}}(\gamma)=F_{\gamma_{1}}(\gamma)+\operatorname{Pr}\left(\gamma<\gamma_{1}<\frac{\gamma c_{2}}{\gamma_{2}}\right)$. Since further manipulations of the exact $F_{\left.\gamma_{\mathrm{LA}}^{(\text {(up) }}\right)}(\gamma)$ is cumbersome, we asymptotically apply the following identity

$$
\begin{gather*}
F_{\gamma_{\text {LA }}^{(\text {up) })}}(\gamma) \\
\stackrel{(\operatorname{large} \underset{\sim}{\gamma})}{ } \operatorname{Pr}\left(\gamma_{2}<c_{2}\right) \operatorname{Pr}\left(\frac{\gamma_{1} \gamma_{2}}{c_{2}}<\gamma\right)  \tag{31}\\
+\operatorname{Pr}\left(\gamma_{2} \geq c_{2}\right) \operatorname{Pr}\left(\gamma_{1}<\gamma\right)
\end{gather*}
$$

Since $\gamma_{1}$ is the product of exponent and chi-square RVs, its PDF and CDF are given by, respectively

$$
\begin{align*}
f_{\gamma_{1}}(x) & =\frac{2 x^{\left(n_{\mathrm{S}}-1\right) / 2}}{\Gamma\left(n_{\mathrm{S}}\right)\left[\Omega_{1} \bar{\gamma} /\left(\mathcal{R}_{c} n_{\mathrm{S}}\right)\right]^{\left(n_{\mathrm{S}}+1\right) / 2}} \\
& \times \mathcal{K}_{n_{\mathrm{S}}-1}\left(2 \sqrt{\frac{x}{\Omega_{1} \bar{\gamma} /\left(\mathcal{R}_{c} n_{\mathrm{S}}\right)}}\right)  \tag{32}\\
F_{\gamma_{1}}(x) & \left.=1-\frac{2 x^{n_{\mathrm{S}} / 2}}{\Gamma\left(n_{\mathrm{S}}\right)\left[\Omega_{1} \bar{\gamma} /\left(\mathcal{R}_{c} n_{\mathrm{S}}\right)\right.}\right]^{n_{\mathrm{S}} / 2} \\
& \times \mathcal{K}_{n_{\mathrm{S}}}\left(2 \sqrt{\frac{x}{\Omega_{1} \bar{\gamma} /\left(\mathcal{R}_{c} n_{\mathrm{S}}\right)}}\right) \tag{33}
\end{align*}
$$

Moreover, the statistics of $\gamma_{2}$ can be obtained immediately from the PDF and CDF of $\left\|H_{\mathrm{RD}}\right\|_{\mathrm{F}}^{2}$ given in (6) and (7), respectively. Our aim now is to calculate the MGF of $\gamma_{\mathrm{LA}}^{(\mathrm{up})}$. To do so, we differentiate (31) with respect to $\gamma$ to obtain the PDF of $\gamma_{\text {LA }}^{(\text {up })}$ as

$$
\begin{equation*}
f_{\gamma_{\mathrm{LA}}^{\text {(up) }}}\left(\gamma \mid \gamma_{2}\right)=\frac{\epsilon c_{2}}{\gamma_{2}} f_{\gamma_{1}}\left(\left.\frac{c_{2} \gamma}{\gamma_{2}} \right\rvert\, \gamma_{2}\right)+\bar{\epsilon} f_{\gamma_{1}}(\gamma) \tag{34}
\end{equation*}
$$

where $\epsilon=F_{\gamma_{2}}\left(c_{2}\right)$ can be deduced from (7) and $\bar{\epsilon}=1-\epsilon$. As the MGF of $\gamma_{\mathrm{LA}}^{(\mathrm{up})}$ is the Laplace transform of its PDF, we have

$$
\begin{equation*}
\mathcal{M}_{\gamma_{\mathrm{LA}}^{(\text {up) }}}(s)=\epsilon \int_{0}^{\infty} \mathcal{M}_{\gamma_{1}}\left(\frac{s \gamma_{2}}{c_{2}}\right) f_{\gamma_{2}}\left(\gamma_{2}\right) d \gamma_{2}+\bar{\epsilon} \mathcal{M}_{\gamma_{1}}(s) \tag{35}
\end{equation*}
$$

where the first summand in (35) is obtained by exchanging the order of double integral, which is originated from the marginal PDF, with respect to the two variables $\gamma$ and $\gamma_{2}$; and $\mathcal{M}_{\gamma_{1}}(s)$ is the MGF of $\gamma_{1}$ shown as follows:

$$
\begin{equation*}
\mathcal{M}_{\gamma_{1}}(s)=\frac{1}{\Omega_{1} \bar{\gamma} s /\left(\mathcal{R}_{c} n_{\mathrm{S}}\right)} \Psi\left(1,2-n_{\mathrm{S}}, \frac{1}{\Omega_{1} \bar{\gamma} s /\left(\mathcal{R}_{c} n_{\mathrm{S}}\right)}\right) \tag{36}
\end{equation*}
$$

where (36) follows from (32) and $\Psi(a, b ; z)$ is the confluent hypergeometric function [18, Eq. (9.211.4)]. Since $n_{\mathrm{S}} \geq 2$, from (36) and using the fact that $\Psi(a, b ; z) \stackrel{(\operatorname{small} z)}{\approx} \frac{\Gamma(1-b)}{\Gamma(1+a-b)}$ if $b \leq 0$ [27], we get

$$
\begin{equation*}
\left.\mathcal{M}_{\gamma_{1}}(s) \stackrel{(\text { large }}{\approx} \bar{\gamma}^{\prime}\right) \frac{1}{\left(n_{\mathrm{S}}-1\right) \Omega_{1} \bar{\gamma} s /\left(\mathcal{R}_{c} n_{\mathrm{S}}\right)} \tag{37}
\end{equation*}
$$

By substituting (37) and (6) into (35), the integral can be approximated as

$$
\begin{align*}
& \int_{0}^{\infty} \mathcal{M}_{\gamma_{1}}\left(\frac{s \gamma_{2}}{c_{2}}\right) f_{\gamma_{2}}\left(\gamma_{2}\right) d \gamma_{2} \\
& \stackrel{(\operatorname{large}}{\approx} \bar{\gamma}) \\
& \times \int_{0}^{\infty} \frac{2 \mathcal{R}_{c} n_{\mathrm{S}}}{\Omega_{1} G_{\mathrm{LA}}^{2}\left(\Omega_{2} \bar{\gamma}\right)^{\left(n_{\mathrm{R}}+n_{\mathrm{D}}\right) / 2}\left(n_{\mathrm{S}}-1\right) \Gamma\left(n_{\mathrm{R}}\right) \Gamma\left(n_{\mathrm{D}}\right) s} \\
& =\frac{\gamma_{2}^{\left(n_{\mathrm{R}}+n_{\mathrm{D}}\right) / 2-2} \mathcal{K}_{n_{\mathrm{D}}-n_{\mathrm{R}}}\left(2 \sqrt{\frac{\gamma_{2}}{\Omega_{2} \bar{\gamma}}}\right) d \gamma_{2}}{\Omega_{1} \Omega_{2} G_{\mathrm{LA}}^{2}\left(n_{\mathrm{S}}-1\right)\left(n_{\mathrm{R}}-1\right)\left(n_{\mathrm{D}}-1\right) \bar{\gamma} s} \tag{38}
\end{align*}
$$

where (38) is obtained by making use of [18, Eq. (6.561.16)]. Next, combining (37) and (38) with (35), the high SNR expression of the MGF of $\gamma_{\mathrm{LA}}^{(\mathrm{up})}$ can be rewritten as
$\mathcal{M}_{\gamma_{\mathrm{LA}}^{(\mathrm{LPP})}}(s)=\left[\frac{\epsilon}{\Omega_{2} G_{\mathrm{LA}}^{2}\left(n_{\mathrm{R}}-1\right)\left(n_{\mathrm{D}}-1\right)}+\bar{\epsilon}\right] \frac{\mathcal{R}_{c} n_{\mathrm{S}}}{\Omega_{1}\left(n_{\mathrm{S}}-1\right) \bar{\gamma} s}$.

The asymptotic MGF of $\gamma_{\text {LA }}^{(10)}$ can be easily obtained from (39) by utilizing the fact that $\mathcal{M}_{\gamma_{\mathrm{LA}}^{(\mathrm{Io})}}(s)=\mathcal{M}_{\gamma_{\mathrm{LA}}^{(\mathrm{LP})}}\left(\frac{1}{2} s\right)$. Hence, the asymptotic SEP can be shown in (13), which finalizes the proof.

## Appendix C: Proof of Theorem 3

We first consider for the case $n_{\mathrm{R}} \neq n_{\mathrm{D}}$. From (25) and (6), the asymptotic PDF of $\gamma_{2}$ can be given by

$$
\begin{equation*}
f_{\gamma_{2}}\left(\gamma_{2}\right) \stackrel{(\text { large }}{\approx} \bar{\gamma}^{\gamma} \frac{\Gamma\left(\left|n_{\mathrm{D}}-n_{\mathrm{R}}\right|\right)}{\Gamma\left(n_{\mathrm{R}}\right) \Gamma\left(n_{\mathrm{D}}\right)} \frac{\gamma_{2}^{\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)-1}}{\left(\Omega_{2} \bar{\gamma}\right)^{\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)}} \tag{40}
\end{equation*}
$$

Following the same approach as in [22] ${ }^{5}$, the SEP can be asymptotically obtained by the first non-zero high order derivative of $f_{Z}(z)$ as

$$
\begin{equation*}
P_{\mathrm{e}}^{(\mathrm{SRD})} \stackrel{(\operatorname{larg} \mathrm{g}}{\approx} \stackrel{\prod_{i=1}^{d+1}(2 i-1)}{(d+1)!}\left(\frac{\mathcal{R}_{c} n_{\mathrm{S}}}{2 \mathrm{~g}}\right)^{d+1} \frac{\partial^{d}}{\partial z^{d}} f_{Z}(0) \tag{41}
\end{equation*}
$$

where $Z=\frac{\gamma_{2} \gamma_{3}}{\gamma_{2}+\gamma_{3}}$. Our main objective now is to study the behavior of $f_{Z}(z)$ in the high SNR regime. Let us write the PDF of $Z$ as

$$
\begin{equation*}
f_{Z}(z)=f_{\gamma_{2}}(z)+\int_{0}^{\infty} \frac{\partial\left[F_{\gamma_{3}}\left(\frac{z^{2}}{x}+z\right) f_{\gamma_{2}}(z+x)\right]}{\partial z} d x \tag{42}
\end{equation*}
$$

Applying the Leibniz rule, the $d$-th order derivative of $f_{Z}(z)$ at zero value can be expressed as [18, Eq. 0.42]

$$
\begin{align*}
& \frac{\partial^{d}}{\partial z^{d}} f_{Z}(0)=\underbrace{\frac{\partial^{d}}{\partial z^{n}} f_{\gamma_{2}}(0)}_{\mathcal{I}_{1}}+ \\
& \underbrace{\left.\left.\int_{0}^{\infty} \sum_{i=0}^{d+1}\binom{d+1}{i} \frac{\partial^{i} F_{\gamma_{3}}\left(\frac{z^{2}}{x}+z\right)}{\partial z^{i}}\right|_{z=0} \frac{\partial^{d+1-i} f_{\gamma_{2}}(z+x)}{\partial z^{d+1-i}}\right|_{z=0} d x}_{\mathcal{I}_{2}} \tag{43}
\end{align*}
$$

The first summand $\mathcal{I}_{1}$ can be easily obtained from (40) as
$\mathcal{I}_{1}=\left\{\begin{array}{cl}\frac{\Gamma\left(\left|n_{\mathrm{D}}-n_{\mathrm{R}}\right|\right)}{\Gamma\left(n_{\mathrm{R}}\right) \Gamma\left(n_{\mathrm{D}}\right)} \frac{\Gamma\left(\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)\right)}{\left(\Omega_{2} \bar{\gamma}\right)^{\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)}} & \text { if } d=\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)-1, \\ 0 & \text { if } d<\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)-1 .\end{array}\right.$

To calculate the second summand $\mathcal{I}_{2}$, we use the $i$-th order derivative of the composite function [18, Eq. 0.430] which results in

$$
\begin{align*}
&\left.\frac{\partial^{i} F_{\gamma_{3}}\left(t=\frac{z^{2}}{x}+z\right)}{\partial z^{i}}\right|_{z=0}=\sum_{u=1}^{i} \sum_{v=0}^{u-1}\binom{u}{v} \frac{(-1)^{v}}{u!} t^{v}(0) \\
& \times\left.\left.\frac{\partial^{u} F_{\gamma_{3}}(t)}{\partial t^{u}}\right|_{z=0} \frac{\partial^{i} t^{u-v}}{\partial z^{u-v}}\right|_{z=0} \tag{45}
\end{align*}
$$

It is observed that only $v=0$ makes $t^{v}(0)=\left.\left(\frac{z^{2}}{x}+z\right)^{v}\right|_{\gamma=0}$ be non-zero. From (16), the highest order of $F_{\gamma_{3}}(t)$ is $n_{\mathrm{S}} n_{\mathrm{R}}$

[^4]which allows us to select $u=n_{\mathrm{S}} n_{\mathrm{R}}$. Hence, (45) becomes
\[

$$
\begin{align*}
& \left.\frac{\partial^{i} F_{\gamma_{3}}\left(t=\frac{z^{2}}{x}+z\right)}{\partial z^{i}}\right|_{z=0} \\
& =\left.\frac{1}{\Gamma\left(n_{\mathrm{S}} n_{\mathrm{R}}+1\right)(\bar{\gamma})^{n_{\mathrm{S}} n_{\mathrm{R}}-1}} \frac{\partial^{i} t^{n_{\mathrm{S}} n_{\mathrm{R}}}}{\partial z^{n_{\mathrm{S}} n_{\mathrm{R}}}}\right|_{z=0} . \tag{46}
\end{align*}
$$
\]

It is obvious that $\mathcal{I}_{2}$ equals to zero when $d \leq \min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)-1$. This is because with $i \leq d+1 \leq \min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)<n_{\mathrm{S}} n_{\mathrm{R}}$, (46) equals to zero. Hence, we will select $d=\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)-1$. By jointly considering the aforementioned results, we can obtain the asymptotic SEP as in (17).

Next we consider for the case $n_{\mathrm{R}}=n_{\mathrm{D}}$ by introducing the lower and upper bound of $\gamma_{S A}$ as

$$
\begin{align*}
\gamma_{\mathrm{SA}}^{(\mathrm{lo})} \triangleq & \frac{1}{2 \mathcal{R}_{c} n_{\mathrm{S}}} \min \left(\gamma_{2}, \gamma_{3}\right) \\
& \leq \gamma_{\mathrm{SA}} \leq \frac{1}{\mathcal{R}_{c} n_{\mathrm{S}}} \min \left(\gamma_{2}, \gamma_{3}\right) \triangleq \gamma_{\mathrm{SA}}^{(\text {up) })} \tag{47}
\end{align*}
$$

As observed from (47), the bounds in this case involve two independent RVs as opposed to (12) which is related to two dependent RVs. Hence, it is important to note that although we use the lower and upper bounds as in Section III-B, the mathematical derivations will proceed differently. We are interested in deriving the MGF of $\gamma_{\mathrm{SA}}^{(\mathrm{up})}$ since the MGF of $\gamma_{\mathrm{SA}}^{(\mathrm{lo})}$ can be followed accordingly. Let us denote $T=\min \left(\gamma_{2}, \gamma_{3}\right)$. It is easy to see that as

$$
\begin{align*}
\mathcal{M}_{\gamma_{\mathrm{SA}}^{(\mathrm{up)}}}(s) & =\mathcal{M}_{\gamma_{2}}\left(\frac{s}{\mathcal{R}_{c} n_{\mathrm{S}}}\right)+\mathcal{M}_{\gamma_{3}}\left(\frac{s}{\mathcal{R}_{c} n_{\mathrm{S}}}\right) \\
& -\int_{0}^{\infty} f_{\gamma_{2}}(t) F_{\gamma_{3}}(z) e^{-\frac{s t}{\mathcal{R}_{c} n_{\mathrm{S}}}} d t \\
& -\int_{0}^{\infty} f_{\gamma_{3}}(t) F_{\gamma_{2}}(t) e^{-\frac{s t}{\mathcal{R}^{n_{\mathrm{S}}}}} d t \tag{48}
\end{align*}
$$

To proceed, we substitute (7) into (48) and make use of the fact that $\gamma_{3}$ is the sum of $n_{\mathrm{S}} n_{\mathrm{R}}$ independent exponential variates. After several manipulations, the MGF of $\gamma_{\mathrm{SA}}^{(\mathrm{up})}$ can be reexpressed as

$$
\begin{align*}
\mathcal{M}_{\gamma_{\mathrm{SA}}(\mathrm{up)}}(s) & =\underbrace{\int_{0}^{\infty} \mathcal{A} f_{\gamma_{2}}(t) e^{-\frac{s t}{\mathcal{R}_{c} n_{\mathrm{S}}} d t}}_{\mathcal{I}_{3}} \\
& +\underbrace{\int_{0}^{\infty} \mathcal{B} f_{\gamma_{3}}(t) e^{-\frac{s t}{\mathcal{R}_{c} n_{\mathrm{S}}} d t}}_{\mathcal{I}_{4}} \tag{49}
\end{align*}
$$

where $\mathcal{A}$ and $\mathcal{B}$ are, respectively, given by

$$
\begin{align*}
\mathcal{A} & =\sum_{k=0}^{n_{\mathrm{S}} n_{\mathrm{R}}-1}\left(\frac{t}{\Omega_{1} \bar{\gamma}}\right)^{k} \frac{e^{-\frac{t}{\Omega_{1} \bar{\gamma}}}}{k!}, \\
\mathcal{B} & =\sum_{k=0}^{n_{\mathrm{R}}-1} \frac{2}{\Gamma\left(n_{\mathrm{D}}\right) k!}\left(\frac{t}{\Omega_{2} \bar{\gamma}}\right)^{\left(n_{\mathrm{D}}+k\right) / 2} \mathcal{K}_{n_{\mathrm{D}}-k}\left(2 \sqrt{\frac{t}{\Omega_{2} \bar{\gamma}}}\right) . \tag{50}
\end{align*}
$$

When $n_{\mathrm{R}}=n_{\mathrm{D}}$, using (25) and (6), an asymptotic expression for the PDF of $\gamma_{2}$ can be obtained as

$$
\begin{equation*}
f_{\gamma_{2}}(t) \stackrel{(\operatorname{large} \bar{\gamma})}{\approx} \frac{t^{n_{\mathrm{R}}-1} \ln \left(\frac{\Omega_{2} \bar{\gamma}}{t}\right)}{\Gamma\left(n_{\mathrm{R}}\right)^{2}\left(\Omega_{2} \bar{\gamma}\right)^{n_{\mathrm{R}}}} \tag{51}
\end{equation*}
$$

Substituting (50) and (51) into (49) yields

$$
\begin{align*}
& \mathcal{I}_{3} \stackrel{(\operatorname{large} \bar{\gamma})}{\approx} \sum_{k=0}^{n_{\mathrm{S}} n_{\mathrm{R}}-1} \frac{1}{\Gamma\left(n_{\mathrm{R}}\right)^{2} \Omega_{1}^{k} \Omega_{2}^{n_{\mathrm{R}}} k!\bar{\gamma}^{n_{\mathrm{R}}+k}} \\
& \quad \times \int_{0}^{\infty} t^{n_{\mathrm{R}}+k-1} e^{-\left(\frac{1}{\Omega_{1} \bar{\gamma}}+\frac{s}{\mathcal{R}_{c} n_{\mathrm{S}}}\right) t} \ln \left(\frac{\Omega_{2} \bar{\gamma}}{t}\right) d t, \\
& \quad \stackrel{(\operatorname{large} \bar{\gamma})}{\approx} \sum_{k=0}^{n_{\mathrm{S}} n_{\mathrm{R}}-1} \frac{\Gamma\left(n_{\mathrm{R}}+k\right)\left[\ln \left(\frac{\Omega_{2} \bar{\gamma} s}{\mathcal{R}_{c} n_{\mathrm{S}}}\right)-\psi\left(n_{\mathrm{R}}+k\right)\right]}{\Gamma\left(n_{\mathrm{R}}\right)^{2} \Omega_{1}^{k} \Omega_{2}^{n_{\mathrm{R}}} k!} \\
& \quad \times\left(\frac{\mathcal{R}_{c} n_{\mathrm{S}}}{\bar{\gamma} s}\right)^{n_{\mathrm{R}}+k}, \tag{52}
\end{align*}
$$

where (52) is obtained by using $\left.\frac{1}{\Omega_{1} \bar{\gamma}} \stackrel{(\text { large }}{<} \bar{\gamma}\right) \frac{1}{\mathcal{R}_{c} n_{\mathrm{S}}}$ and [18, Eq. (4.352.1)]. Similarly, we get the following approximation for $\mathcal{I}_{4}$ :

$$
\begin{equation*}
\left.\mathcal{I}_{4} \stackrel{(\operatorname{large}}{\approx} \bar{\gamma}\right) \sum_{k=0}^{n_{\mathrm{R}}-1} \frac{\Gamma\left(n_{\mathrm{R}}-k\right) \Gamma\left(n_{\mathrm{S}} n_{\mathrm{R}}+k\right)}{\Gamma\left(n_{\mathrm{R}}\right) \Gamma\left(n_{\mathrm{S}} n_{\mathrm{R}}\right) \Omega_{2}^{k} \Omega_{1}^{n_{\mathrm{S}} n_{\mathrm{R}}} k!}\left(\frac{\mathcal{R}_{c} n_{\mathrm{S}}}{\bar{\gamma} s}\right)^{n_{\mathrm{S}} n_{\mathrm{R}}+k} . \tag{53}
\end{equation*}
$$

Since $\mathcal{I}_{3}$ and $\mathcal{I}_{4}$ are proportional to $\left(\frac{1}{\bar{\gamma}}\right)^{n_{\mathrm{R}}+k}$ and $\left(\frac{1}{\bar{\gamma}}\right)^{n_{\mathrm{S}} n_{\mathrm{R}}+k}$ with $k$ being an integer, respectively, $\mathcal{I}_{4}$ can be neglected as compared to $\mathcal{I}_{3}$. Then by selecting the index $k=0$ (higher values of $k$ can be omitted) for $\mathcal{I}_{3}$, the asymptotic MGF of $\gamma_{\mathrm{SA}}^{(\text {up })}$ is given by

$$
\begin{equation*}
\mathcal{M}_{\gamma_{\mathrm{SA}}^{(\mathrm{up)}}}(s) \stackrel{(\operatorname{large} \bar{\gamma})}{\approx} \frac{\ln \left(\frac{\Omega_{2} \bar{\gamma} s}{\mathcal{R}_{c} n_{\mathrm{S}}}\right)-\psi\left(n_{\mathrm{R}}\right)}{\Gamma\left(n_{\mathrm{R}}\right) \Omega_{2}^{n_{\mathrm{R}}}}\left(\frac{\mathcal{R}_{c} n_{\mathrm{S}}}{\bar{\gamma} s}\right)^{n_{\mathrm{R}}} \tag{54}
\end{equation*}
$$

Using (54), the SEP behavior in the high SNR regime pertaining to the case of $n_{R}=n_{\mathrm{D}}$ can be given by (18), which concludes our proof.

## Appendix D: Proof of Theorem 4

When $n_{R}>n_{\mathrm{D}}$, by following the same approach as in Section III-C, we can easily obtain (21). For the case $n_{R}<n_{\mathrm{D}}$, we can obtain $\mathcal{I}_{1}$ given in (43) as

$$
\mathcal{I}_{1}=\left\{\begin{array}{cl}
\frac{\Gamma\left(n_{\mathrm{D}}-n_{\mathrm{R}}\right)}{\Gamma\left(n_{\mathrm{D}}\right)\left(\Omega_{2} \bar{\gamma}\right)^{n_{\mathrm{R}}}} & \text { if } d=n_{\mathrm{R}}-1  \tag{55}\\
0 & \text { if } d<n_{\mathrm{R}}-1
\end{array}\right.
$$

In Section III-C, $\mathcal{I}_{2}$ given in (43) is zero. In contrast, $\mathcal{I}_{2}$ now becomes non-zero and the index $i$ can be selected as $i=d+1=n_{\mathrm{R}}$, which then allows us to rewrite the right-hand-side of (45) as $\frac{\Gamma\left(n_{\mathrm{S}}-n_{\mathrm{R}}\right)}{\Gamma\left(n_{\mathrm{S}}\right)\left(\Omega_{1} \bar{\gamma}\right)^{n_{\mathrm{R}}}}$. In addition, the term $\left.\frac{\partial^{d+1-i} f_{\gamma_{2}}(z+x)}{\partial z^{d+1-i}}\right|_{z=0}$ of $\mathcal{I}_{2}$ given in (43) now becomes $f_{\gamma_{2}}(x)$ and is given by $\mathcal{I}_{2}=\frac{\Gamma\left(n_{\mathrm{S}}-n_{\mathrm{R}}\right)}{\Gamma\left(n_{\mathrm{S}}\right)\left(\Omega_{1} \bar{\gamma}\right)^{n_{\mathrm{R}}}}$. Utilizing this result and pulling (55) together with (43) and (41), the asymptotic SEP is expressed as (22). The derivation for the case $n_{R}=n_{\mathrm{D}}$ follows the same pattern as in Section III-C and therefore is omitted for brevity. Specifically, the result of $\mathcal{I}_{3}$ can be shown as

$$
\begin{equation*}
\left.\mathcal{I}_{3} \stackrel{(\operatorname{large}}{\approx} \bar{\gamma}\right) \frac{\left[\ln \left(\frac{\Omega_{1} \bar{\gamma} s}{\mathcal{R}_{c} n_{\mathrm{S}}}\right)-\psi\left(n_{\mathrm{R}}\right)\right]}{\Gamma\left(n_{\mathrm{R}}\right) \Omega_{2}^{n_{\mathrm{R}}}}\left(\frac{\mathcal{R}_{c} n_{\mathrm{S}}}{\bar{\gamma} s}\right)^{n_{\mathrm{R}}} . \tag{56}
\end{equation*}
$$

In contrast to the derivation in Section III-C, $\mathcal{I}_{4}$ can not be neglected. In fact, it is comparable with $\mathcal{I}_{3}$ and can be expressed as

$$
\begin{equation*}
\left.\mathcal{I}_{4} \stackrel{(\operatorname{large}}{\approx} \bar{\gamma}\right) \frac{\Gamma\left(n_{\mathrm{S}}-n_{\mathrm{R}}\right)}{\Gamma\left(n_{\mathrm{R}}\right) \Omega_{1}^{n_{\mathrm{R}}}}\left(\frac{\mathcal{R}_{c} n_{\mathrm{S}}}{\bar{\gamma} s}\right)^{n_{\mathrm{R}}} . \tag{57}
\end{equation*}
$$

which then allows us to obtain (23), which finally completes the proof.

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[^0]:    Paper approved by N. Al-Dhahir, the Editor for Space-Time, OFDM and Equalization of the IEEE Communications Society. Manuscript received September 14, 2011; revised April 6 and July 11, 2012.
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    Digital Object Identifier 10.1109/TCOMM.2012.110616.110616

[^1]:    ${ }^{1}$ In referring here to SKE, we imply that the first hop is assumed to be keyhole-free and subject to Rayleigh fading. In contrast, for DKE both hops experience the keyhole effect. For a single-antenna relay system, the keyhole channel for mobile relay particularizes to a double fading channel [16], [17].
    ${ }^{2}$ In this paper, we have assumed that $n_{\mathrm{S}}>\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)$ for all cases. Since the considered system is downlink, i.e., $n_{\mathrm{S}}>n_{\mathrm{D}}$, the above condition is always satisfied. As a result, downlink communication implicitly indicates that $n_{\mathrm{S}}>\min \left(n_{\mathrm{R}}, n_{\mathrm{D}}\right)$.

[^2]:    ${ }^{3}$ In this paper, we apply the general OSTBC construction with the minimum delay and maximum achievable rate proposed in [20]. The code rates at $S$ and R are given by $\frac{\left\lceil\log _{2}\left(n_{\mathrm{S}}\right)\right\rceil+1}{2^{\left\lceil\log _{2}\left(n_{\mathrm{S}}\right)\right\rceil}}$ and $\frac{\left\lceil\log _{2}\left(n_{\mathrm{R}}\right)\right\rceil+1}{2^{\left\lceil\log _{2}\left(n_{\mathrm{R}}\right)\right\rceil}}$, respectively, where $\lceil\cdot\rceil$ denotes the ceiling function.

[^3]:    ${ }^{4}$ Although the asymptotic SEP is given in the form of the product of polynomial and logarithm functions with respect to the average $\operatorname{SNR} \bar{\gamma}$ [23], [24], the diversity order is solely determined by the polynomial term since $\frac{\log \log \bar{\gamma}}{\log \bar{\gamma}}$ can be neglected as $\bar{\gamma} \rightarrow \infty$.

[^4]:    ${ }^{5}$ Note that in different relay system contexts, the adopted asymptotic approach has been widely used in the existing literature to characterize the diversity and the array gains. See for example [28, Appendix] and [29].

