Optimization of Cooperative Spectrum Sensing in Cognitive Radio

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***Abstract*—In this paper, we consider cooperative spectrum sensing when two secondary users (SUs) collaborate via the relaying scheme. We investigate two cooperative sensing strategies, i.e., SUs exchange data information locally, and SUs relay information to a central controller. The relaying scheme at each SU is optimized via functional analysis with either the average or peak power constraints. For the local cooperative sensing strategy, the optimal relaying schemes look like *amplify-and-forward* (AF) in the low-signal-to-noise-ratio (SNR) region and behave like *decode-and-forward* (DF) in the high-SNR region. The fundamental performance limit using local cooperative sensing is discussed. For the global cooperative sensing strategy, we propose both coherent and noncoherent sensing, depending on whether SUs are synchronized. In the coherent case, a decentralized approach is designed, and each SU optimizes its relaying function locally. In the noncoherent case, we use linear energy combination detector to decouple the relaying function from weight coefficient optimization. Simulation results demonstrate that the proposed protocols achieve much better performance over the existing protocols.**

***Index Terms*—Cognitive radio (CR), cooperative spectrum sensing, optimal strategy, wireless relay networks.**

I. Introduction

In traditional spectrum management, frequency bands are exclusively allocated to licensed users, which induces spectrum scarcity due to the emergence of new wireless services. According to the Federal Communications Commission (FCC) [1], the current utilization of the licensed spectrum varies from 15% to 85%, whereas only 2% of spectrum would be used in the U.S. at any given moment. The concept of cognitive radio (CR) was introduced in [2] to remedy the spectrum scarcity problem. In CR, the unlicensed users could opportunistically access the spectrum assigned to the licensed users, provided that no harmful interference is caused to incumbent services. For example, the IEEE 802.22 standard for cognitive wireless regional area networks [3] aims at sharing the unused spectrums that have been allocated to the television broadcast service to bring broadband access to hard-to-reach areas.

The key technology for CR is the spectrum sensing that can find the vacant frequency band that is not currently occupied by the primary users (PUs). Existing spectrum-sensing techniques include energy detection, matched filter, and various others that can be found from [4] and the reference therein. The sensing performance with a single cognitive user greatly degrades with channel fading and shadowing. To enhance the sensing reliability, cooperative spectrum sensing has been studied in [5]. Normally, the cooperative sensing involves two successive stages: 1) sensing and 2) reporting. In the sensing stage, the techniques in [4] can be used by each secondary user (SU). In the reporting stage, all local sensing decisions are reported to a central controller through a control channel. Finally, the central controller makes the decision and informs it to all SUs. The previously described scheme requires a control channel and can be characterized as a centralized scheme.

On the other hand, a distributed cooperative sensing protocol was proposed in [6], where SUs exchange the sensing information by relaying a properly designed signal to each other. An improved technology over [6] was reported in [9] by allowing the SU to choose to relay and not based on the first sensing decision. Unfortunately, both [6] and [9] are based on intuitive relaying functions, and no optimal criterion is examined. Cooperative sensing with optimal criterion has been designed in many CR scenarios [7], [8], [10]–[13]. In [7], the channel throughput is maximized under the interference constraints to the CR network. Energy detection with the optimal detection threshold is derived in [8] and [10]. Soft combination of the observed energies from different CR users is investigated in [11] based on the Neyman–Pearson criterion.

In this paper, we propose a different way to optimize the performance of cooperative spectrum sensing by designing the relaying function. We consider two cooperative sensing strategies: 1) local cooperative sensing and 2) global cooperative sensing. In local cooperative sensing, we derive the optimal relaying function at SUs by optimizing the sensing performance under both average and peak power constraints. Interestingly, the optimal function with the average power constraint agrees with the *amplify-and-forward* (AF)-like scheme in [6] at the low-signal-to-noise (SNR) region, whereas it reduces to the *decode-and-forward* (DF)-like scheme at the high-SNR region. Moreover, we design an *estimate-and-forward* (EF) relaying function, whose performance is close to the optimal function. In global cooperative sensing, SUs first observe the signal from the PU and then transmit processed signals to a central

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controller to get the final decision. We discuss both coherent cooperation (synchronized SUs) and noncoherent cooperation (unsynchronized SUs). The former cooperation utilized a decentralized approach, where each SU locally optimizes its relaying function, whereas the latter refers to a linear energy detector. Simulation results show that the proposed strategies achieve superior performance over the existing strategies.

The rest of this paper is organized as follows: Section II provides the background. Section III presents the local cooperative sensing, and Section IV discusses the global cooperative sensing. Simulation results are demonstrated in Section V. Finally, conclusions are drawn in Section VI.

II. System Model

Consider a simple network with two SUs $\mathbb{U}_1$ and $\mathbb{U}_2$ and a single PU $\mathbb{P}$. When SUs listen to the environment, the received signal at $\mathbb{U}_i$ is

$$y_i = \theta x_p h_{pi} + w_i, \quad i = 1, 2 \tag{1}$$

where $\theta \in \{0, 1\}$ is the PU indicator; x_p belongs to the signal constellation $\mathcal{C}$; h_{pi} is the channel gain between $\mathbb{P}$ and $\mathbb{U}_i$, which is complex Gaussian with unit variance; and w_i is the additive white Gaussian noise with variance σ_i^2. All y_i, x_p, h_{pi}, and w_i are complex random variables. We further assume that the transmission power of PU is $\tilde{P}$ and θ remains static over the spectrum-sensing period.

The probability density function (pdf) of the received y_i is

$$f(y_i|\theta = 0) = \frac{1}{\pi\sigma_i^2} e^{-\frac{|y_i|^2}{\sigma_i^2}} \tag{2}$$

$$f(y_i|\theta = 1) = \sum_{x\in\mathcal{C}} \frac{1}{\pi(|x|^2 + \sigma_i^2)} e^{-\frac{|y_i|^2}{|x|^2+\sigma_i^2}} \Pr(x) \tag{3}$$

where $\Pr(x)$ denotes the probability of x being sent from PU. The optimal detector can be derived from the likelihood ratio test

$$\Lambda(y) = \frac{f(y|\theta = 1)}{f(y|\theta = 0)} = \sum_{x\in\mathcal{C}} \frac{\Pr(x)\sigma_i^2}{|x|^2 + \sigma_i^2} \exp\left(\frac{|x|^2|y|^2}{(|x|^2 + \sigma_i^2)\,\sigma_i^2}\right). \tag{4}$$

SU decides the existence of PU if $\Lambda(y)$ is greater than a threshold and not otherwise. Since $\Lambda(y)$ is a strictly increasing function of $|y|^2$, the optimal decision problem is equivalent to comparing $|y|^2$ with a threshold λ. That is, if $|y|^2 > \lambda$, the SU decides $\hat{\theta} = 1$; otherwise, $\hat{\theta} = 0$. Therefore, energy detector is proven optimal. The key measurements in spectrum sensing are the probability of correct detection and the probability of false alarm, which are defined as

$$P_d = \Pr(\hat{\theta} = 1|\theta = 1) \quad P_f = \Pr(\hat{\theta} = 1|\theta = 0). \tag{5}$$

With the optimal energy detector, it can be calculated that

$$P_f(\lambda) = \int_{\lambda}^{+\infty} \frac{1}{\sigma^2} \exp\left(-\frac{t}{\sigma^2}\right) dt = \exp\left(-\frac{\lambda}{\sigma^2}\right) \tag{6}$$

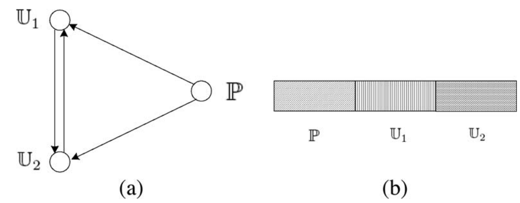


Fig. 1. Diagram of local cooperative spectrum sensing.

$$\begin{aligned} P_d(\lambda) &= \int_{\lambda}^{+\infty} \sum_{x\in\mathcal{C}} \frac{1}{|x|^2 + \sigma^2} \exp\left(-\frac{t}{|x|^2 + \sigma^2}\right) \Pr(x)\, dt \\ &= \sum_{x\in\mathcal{C}} \exp\left(-\frac{\lambda}{|x|^2 + \sigma^2}\right) \Pr(x). \end{aligned} \tag{7}$$

Practically, the false alarm probability is constrained by government regulators like the FCC [1]. Given a targeting false alarm ξ, i.e., $P_f(\lambda) = \xi$, the correct detection probability can be computed as $P_d(\lambda) = \sum_{x\in\mathcal{C}} \xi^{\sigma^2/|x|^2+\sigma^2} \Pr(x)$.

When $\mathcal{C}$ or $Pr(x)$ is not known at the SU, we may approximate $P_d(\lambda)$ from Jensen's inequality as

$$P_d(\lambda) \leq \xi^{\frac{\sigma^2}{E\{|x|^2\}+\sigma^2}} = \xi^{\frac{\sigma^2}{\tilde{P}+\sigma^2}}. \tag{8}$$

However, when $\mathcal{C}$ is a constant modulus constellation, i.e., $|x|^2 = \tilde{P}, \forall x \in \mathcal{C}$, the inequality in (8) becomes an equality.

III. Local Cooperative Sensing

Our proposed local cooperative sensing consists of three time slots. During the first time slot, both $\mathbb{U}_i$'s listen to the environment and receive signals from $\mathbb{P}$. In the second time slot, $\mathbb{U}_1$ processes its received signal and transmits to $\mathbb{U}_2$. Similarly, $\mathbb{U}_2$ transmits a processed signal to $\mathbb{U}_1$ in the third time slot. Finally, $\mathbb{U}_1$ and $\mathbb{U}_2$ make decisions based on the received signals, respectively. A diagram of local cooperative sensing is shown in Fig. 1. Due to symmetry, we only analyze the sensing performance at $\mathbb{U}_2$. The received signal at $\mathbb{U}_2$ in the second time slot is

$$y_2^{(2)} = \theta x_p^{(2)} h_{p2}^{(2)} + f_1\left(y_1^{(1)}\right) h_{12} + w_2^{(2)} \tag{9}$$

$$\phantom{y_2^{(2)}} = \theta x_p^{(2)} h_{p2}^{(2)} + f_1\left(\theta x_p^{(1)} h_{p2}^{(1)} x + w_1^{(1)}\right) h_{12} + w_2^{(2)} \tag{10}$$

where the superscript denotes the time slot index; $f_1(\cdot)$ is the relaying function at $\mathbb{U}_1$ that maps its received signal to its transmitted signal; and h_{12} is the channel between $\mathbb{U}_1$ and $\mathbb{U}_2$, which is complex Gaussian with mean zero and variance σ_h^2.

Our objective is to optimize the relaying function $f_1(\cdot)$ under the relay power constraints. Motivated by the energy detector in Section II, we propose that $\mathbb{U}_2$ compares $\alpha|y_2^{(1)}|^2 + |y_2^{(2)}|^2$ with a threshold λ, where α is a nonnegative coefficient to be determined later. If $\alpha|y_2^{(1)}|^2 + |y_2^{(2)}|^2 > \lambda$, $\mathbb{U}_2$ decides $\hat{\theta} = 1$; otherwise, $\hat{\theta} = 0$.

A. Average Power Constraint

We first consider the average power constraint P_1 at $\mathbb{U}_1$. The optimization problem can be expressed as

$$\begin{aligned}\max_{f_1,\alpha}\quad & P_d(f_1,\alpha,\lambda)\\ \text{s.t.}\quad & P_f(f_1,\alpha,\lambda)=\xi, E\left\{\left|f_1\left(y_1^{(1)}\right)\right|^2\middle|\theta\right\}\leq P_1\\ & \theta=0,1.\end{aligned}\tag{11}$$

Given θ, we approximate $\theta x_p^{(1)}h_{p1}^{(1)}+w_1^{(1)}$ and $\theta x_p^{(2)}h_{p2}^{(2)}+w_2^{(2)}$ as Gaussian random variables with zero mean and variance $\omega_1^2=\theta\tilde{P}+\sigma_1^2$ and $\omega_2^2=\theta\tilde{P}+\sigma_2^2$, respectively. If $x_p^{(1)}$ and $x_p^{(2)}$ are from a constant modulus constellation, the approximation is exact. We can rewrite

$$y_2^{(2)}=x_2+h_{12}f(x_1)\tag{12}$$

where x_1 and x_2 are two independent Gaussian random variables with zero mean and variances ω_1^2 and ω_2^2, respectively. As an energy detector is applied at SUs, we assume that $f(x_1)$ is only a function of $|x_1|$, i.e., $f(x_1)=\sqrt{g(|x_1|^2)}=\sqrt{g(r)}$, where r is chi-square distributed with two degrees of freedom.

Assuming that $E\{|h_{12}|^2\}=\sigma_h^2$ is known at $\mathbb{U}_1$ and $\mathbb{U}_2$, conditioned on a given r, y is a Gaussian random variable with mean zero and variance $\omega_2^2+\sigma_h^2g(r)$, and $v=|y|^2$ is a chi-square random variable with two degrees of freedom. From the characteristic function approach [14], $z=\alpha|y_2^{(1)}|^2+|y_2^{(2)}|^2$ is a noncentral chi-square random variable, given $r=|x_1|^2$ with pdf

$$p(z|r)=\begin{cases}\dfrac{e^{-\frac{z}{\left(\theta\tilde{P}+\sigma_2^2\right)+\sigma_h^2g(r)}}-e^{-\frac{z}{\alpha\left(\theta\tilde{P}+\sigma_2^2\right)}}}{(1-\alpha)\left(\theta\tilde{P}+\sigma_2^2\right)+\sigma_h^2g(r),}\\ \text{if}(1-\alpha)\left(\theta\tilde{P}+\sigma_2^2\right)+\sigma_h^2g(r)\neq 0,\\ \dfrac{z}{\alpha^2\left(\theta\tilde{P}+\sigma_2^2\right)^2}e^{-\frac{z}{\alpha\left(\theta\tilde{P}+\sigma_2^2\right)}}, & \text{otherwise.}\end{cases}\tag{13}$$

The pdf of z is computed as

$$p(z)=\int p(z|r)p(r)\,dr=\int_0^{+\infty}p(z|r)\frac{1}{\omega_1^2}e^{-\frac{r}{\omega_1^2}}\,dr.\tag{14}$$

Given the threshold λ, we find that

$$\begin{aligned}P(\lambda,\theta)=&\int_\lambda^{+\infty}p(z)dz\\ =&\int_0^{+\infty}\left(\frac{\left(\theta\tilde{P}+\sigma_2^2\right)+\sigma_h^2g(r)}{(1-\alpha)\left(\theta\tilde{P}+\sigma_2^2\right)+\sigma_h^2g(r)}e^{-\frac{\lambda}{\left(\theta\tilde{P}+\sigma_2^2\right)+\sigma_h^2g(r)}}\right.\\ &\left.-\frac{\alpha\left(\theta\tilde{P}+\sigma_2^2\right)}{(1-\alpha)\left(\theta\tilde{P}+\sigma_2^2\right)+\sigma_h^2g(r)}e^{-\frac{\lambda}{\alpha\left(\theta\tilde{P}+\sigma_2^2\right)}}\right)\\ &\times\frac{1}{\omega_1^2}e^{-\frac{r}{\omega_1^2}}\,dr.\end{aligned}\tag{15}$$

Hence, the optimization problem (11) can be rewritten as

$$\begin{aligned}\max_{g,\alpha}\quad & P(\lambda,1)\\ \text{s.t.}\quad & P(\lambda,0)\leq\xi,\quad\int_0^{+\infty}\frac{g(r)}{\theta\tilde{P}+\sigma_1^2}e^{-\frac{r}{\theta\tilde{P}+\sigma_1^2}}\,dr\leq P_1\\ & \theta=0,1,\quad g(r)\geq 0,\ \forall r\geq 0.\end{aligned}\tag{16}$$

From (16), we know that considering $\theta=0$ in the average power constraint is redundant if $g(r)$ is a nondecreasing function, which is a reasonable assumption in practice. The optimal way to solve (16) is to find the optimal g for each α and then to perform a line search to find the α that achieves the best performance. In the following, we consider the case $\alpha=0$, whose solution can provide sufficient insight into what the optimal relay function looks like. We then substitute the derived relay functions into (16) and perform a line search to find the best α. When $\alpha=0$, (16) simplifies to

$$\begin{aligned}\max_g\quad & \int_0^{+\infty}e^{-\frac{\lambda}{\left(\tilde{P}+\sigma_2^2\right)+\sigma_h^2g(r)}}e^{-\frac{r}{\tilde{P}+\sigma_1^2}}\,dr\\ \text{s.t.}\quad & \int_0^{+\infty}e^{-\frac{\lambda}{\sigma_2^2+\sigma_h^2g(r)}}\frac{1}{\sigma_1^2}e^{-\frac{r}{\sigma_1^2}}\,dr\leq\xi\\ & \int_0^{+\infty}\frac{g(r)}{\tilde{P}+\sigma_1^2}e^{-\frac{r}{\tilde{P}+\sigma_1^2}}\,dr\leq P_1,\quad g(r)\geq 0,\ \forall r\geq 0.\end{aligned}\tag{17}$$

1) Lagrange Approach: Lagrange method is a conventional way to solve optimization problem [15]. Using this method, the optimal relaying function $g(r)$ can be found by maximizing the Lagrange dual function

$$\begin{aligned}L(g,\mu_1,\mu_2)=&\int_0^{+\infty}e^{-\frac{\lambda}{\left(\tilde{P}+\sigma_2^2\right)+\sigma_h^2g(r)}}e^{-\frac{r}{\tilde{P}+\sigma_1^2}}\,dr\\ &-\mu_1\left(\int_0^{+\infty}e^{-\frac{\lambda}{\sigma_2^2+\sigma_h^2g(r)}}e^{-\frac{r}{\sigma_1^2}}\,dr-\sigma_1^2\xi\right)\\ &-\mu_2\left(\int_0^{+\infty}g(r)e^{-\frac{r}{\tilde{P}+\sigma_1^2}}\,dr-\left(\tilde{P}+\sigma_1^2\right)P_1\right)\end{aligned}\tag{18}$$

where $\mu_1,\mu_2\geq 0$ are dual variables. To find the optimal $g(r)$ for each r, we take the derivative of $L(g,\mu_1,\mu_2)$ with respect to $g(r)$, which can be obtained as

$$\begin{aligned}F\left(g(r)\right)=&\frac{\partial L(g,\mu_1,\mu_2)}{\partial g}\\ =&\frac{\lambda\sigma_h^2}{\left(\tilde{P}+\sigma_2^2+\sigma_h^2g(r)\right)^2}e^{-\frac{\lambda}{\left(\tilde{P}+\sigma_2^2\right)+\sigma_h^2g(r)}}e^{-\frac{r}{\tilde{P}+\sigma_1^2}}\\ &-\frac{\mu_1\lambda\sigma_h^2}{\left(\sigma_2^2+\sigma_h^2g(r)\right)^2}e^{-\frac{\lambda}{\sigma_2^2+\sigma_h^2g(r)}}e^{-\frac{r}{\sigma_1^2}}-\mu_2e^{-\frac{r}{\tilde{P}+\sigma_1^2}}.\end{aligned}\tag{19}$$

To numerically solve (17), we consider two cases.

1) If $F(g(r)) < 0$ for all $g(r) \geq 0$, then it is clear that we should choose $g(r) = 0$ to maximize $L(g, \mu_1, \mu_2)$, which corresponds to the boundary solution.
2) If there exists a $\tilde{g}(r)$ such that $F(\tilde{g}(r)) > 0$, then there must exist a $g(r)$ such that $F(g(r)) = 0$, because $F(\infty) < 0$, and $F(g(r))$ is a continuous function in $g(r)$. In this case, by solving $F(g(r)) = 0$, we obtain an implicit function $g(r)$, depending on λ, μ_1, and μ_2.

We then fix one of λ, μ_1, and μ_2 (e.g., μ_2) and substitute $g(r)$ obtained from the two cases into (17). By making the two constraints in (17) attain equality, we can obtain the other two parameters (e.g., λ, μ_1) as a function of the fixed parameter μ_2. Finally, substituting $g(r)$ into the objective function of (17) and optimizing over the remaining parameter μ_2, we obtain the optimal $g(r)$.

To gain insights on the structure of the optimal relaying function, we consider several important limiting scenarios here.

1) $r \gg \sigma_1^2$: Since $e^{-r/\sigma_1^2} \approx 0$ when $r \gg \sigma_1^2$, (19) reduces to

$$\frac{\lambda \sigma_h^2}{\left(\tilde{P} + \sigma_2^2 + \sigma_h^2 g(r)\right)^2} e^{-\frac{\lambda}{\left(\tilde{P}+\sigma_2^2\right)+\sigma_h^2 g(r)}} = \mu_2 \tag{20}$$

which indicates that $g(r) = C$ when $r \gg \sigma_1^2$ and that C is a constant.

2) $0 \leq r \ll \sigma_1^2$ and σ_1^2, $\sigma_2^2 \gg \tilde{P}$: When $0 \leq r \ll \sigma_1^2$, $e^{-r/\tilde{P}+\sigma_1^2} \approx 1$. With σ_1^2, $\sigma_2^2 \gg \tilde{P}$ (corresponding to the low-SNR case), (19) can then be simplified to

$$\frac{(1-\mu_1)\lambda \sigma_h^2}{\left(\tilde{P} + \sigma_2^2 + \sigma_h^2 g(r)\right)^2} e^{-\frac{\lambda}{\left(\tilde{P}+\sigma_2^2\right)+\sigma_h^2 g(r)}} = \mu_2 e^{\frac{\tilde{P}r}{\sigma_1^2\left(\tilde{P}+\sigma_1^2\right)}} \tag{21}$$

which gives

$$g(r) = -\frac{\lambda}{2\sigma_h^2 W\left(-Ae^{\frac{\tilde{P}}{2\sigma_1^2\left(\tilde{P}+\sigma_1^2\right)}}\right)} - \frac{\tilde{P}+\sigma_2^2}{\sigma_h^2} \tag{22}$$

where $W(\cdot)$ denotes Lambert's W function defined as $W(x)e^{W(x)} = x$. Since $r\tilde{P} \ll (\tilde{P} + \sigma_1^2)\sigma_1^2$, $g(r)$ can be linearized to be $g(r) = \tilde{A}r + \tilde{B}$ by using first-order Taylor series expansion, where $\tilde{A}$ and $\tilde{B}$ are two constants.

Combining both cases, the optimized relaying function can be approximated by a piecewise linear function as

$$g(r) = \begin{cases} C, & \text{if } r > \lambda_1 \\ 0, & \text{if } r \leq \lambda_2 \\ C\frac{r-\lambda_2}{\lambda_1-\lambda_2}, & \text{if } \lambda_2 < r \leq \lambda_1 \end{cases} \tag{23}$$

where $C > 0$, $\lambda_1 \geq \lambda_2 \geq 0$. and λ_1 and λ_2 are two detection thresholds at $\mathbb{U}_1$. To find C, λ_1, and λ_2, we need to substitute (23) into (17). By making the two constraints in (17) attain equality, two variables out of C, λ_1, and λ_2 can be eliminated. The objective function of (17) now only depends on the only remaining variable, which can be maximized by performing a line search. Finally, substituting the optimized C, λ_1, and λ_2 into (23) gives the optimized $g(r)$. From the simulation results in Fig. 4, we can see that, when $\sigma_1^2 = \sigma_2^2 = 10^{0.5}$, the misdetection probability $1 - \eta$ by using (23) is twice as that by directly solving (19).

Interestingly, the function (23) contains several special cases illustrated as follows:

1) DF: In (23), if we choose $\lambda_1 = \lambda_2$, we obtain

$$g(r) = \begin{cases} C, & \text{if } r > \lambda_1 \\ 0, & \text{otherwise} \end{cases} \tag{24}$$

which is similar to the DF strategy in conventional relay channels. Substituting (24) into (17), we obtain

$$\begin{aligned} \max_{C,\lambda,\lambda_1} \quad & e^{-\frac{\lambda}{\tilde{P}+\sigma_2^2}}\left(1 - e^{-\frac{\lambda_1}{\tilde{P}+\sigma_1^2}}\right) + e^{-\frac{\lambda}{\left(\tilde{P}+\sigma_2^2\right)+C\sigma_h^2}} e^{-\frac{\lambda_1}{\tilde{P}+\sigma_1^2}} \\ \text{s.t.} \quad & e^{-\frac{\lambda}{\sigma_2^2}}\left(1 - e^{-\frac{\lambda_1}{\sigma_1^2}}\right) + e^{-\frac{\lambda}{\sigma_2^2+C\sigma_h^2}} e^{-\frac{\lambda_1}{\sigma_1^2}} \leq \xi, \\ & Ce^{-\frac{\lambda_1}{\tilde{P}+\sigma_1^2}} \leq P_1. \end{aligned} \tag{25}$$

We can then convert the DF optimization problem (25) into a single parameter optimization problem by solving C and λ from the two constraints for a given λ_1 and maximizing the objective function over λ_1.

2) AF: In (23), if we choose $\lambda_1 = +\infty$, $C/\lambda_1 = A$ and $\lambda_2 = 0$, we obtain the AF-like function. To satisfy the average power constraint, we should choose $A = P_1/\tilde{P} + \sigma_1^2$, i.e., $g(r) = P_1 r/\tilde{P} + \sigma_1^2$, which agrees with the AF scheme in [6].
3) *Hybrid*: Since (23) can be considered as a combination of AF and DF, we name it as hybrid strategy in the rest of this paper.

2) Minimum-MSE Approach: So far, we have discussed how to obtain the relaying function at $\mathbb{U}_1$ via the Lagrangian function $L(g, \mu_1, \mu_2)$. We next consider another class of $g(r)$ by minimizing the average mean square error (MSE) at $\mathbb{U}_1$. The detection at $\mathbb{U}_1$ incurs false detection. Directly sending the detection at $\mathbb{U}_1$ to $\mathbb{U}_2$ will cause error prorogation. Although spectrum sensing is a detection problem, $\mathbb{U}_1$ can estimate θ rather than making a hard decision, which can be considered as sending soft detection information. We thus consider the function $\tilde{g}(r)$ to minimize the MSE between θ and $\tilde{g}(r)$, i.e.,

$$\tilde{g}(r) = \arg\min_{\tilde{g}'} E\left\{ \left|\theta - \tilde{g}'(r)\right|^2 \middle| r \right\}. \tag{26}$$

Assuming that the *a priori* probability of $\Pr(\theta = 0)$ is known to be ζ, the objective function in (26) can be written as

$$E\left\{ \left|\theta - \tilde{g}(r)\right|^2 \middle| r \right\} = \sum_{\theta \in \{0,1\}} \frac{\Pr(r|\theta)\Pr(\theta)}{\Pr(r)} \left|\theta - \tilde{g}(r)\right|^2. \tag{27}$$

Note that $\Pr(r)$ is a common factor. Therefore, minimizing (27) is equivalent to minimizing

$$\sum_{\theta\in\{0,1\}} p(r|\theta)\Pr(\theta)\left|\theta-\tilde{g}(r)\right|^2$$
$$=\frac{\zeta}{\sigma_1^2}e^{-\frac{r}{\sigma_1^2}}\tilde{g}^2(r)+\frac{1-\zeta}{\tilde{P}+\sigma_1^2}e^{-\frac{r}{\tilde{P}+\sigma_1^2}}\left(1-\tilde{g}(r)\right)^2 \quad (28)$$

which gives the result

$$\tilde{g}(r)=\frac{\frac{1-\zeta}{\tilde{P}+\sigma_1^2}e^{-\frac{r}{\tilde{P}+\sigma_1^2}}}{\frac{1-\zeta}{\tilde{P}+\sigma_1^2}e^{-\frac{r}{\tilde{P}+\sigma_1^2}}+\frac{\zeta}{\sigma_1^2}e^{-\frac{r}{\sigma_1^2}}}. \quad (29)$$

Finally, we set $g(r)=C\tilde{g}(r)$, where C is a constant to keep the average power constraint at $\mathbb{U}_1$. We can compute C and λ from the last two constraints and the first constraint in (17), respectively. When ζ is unknown, we can substitute $g(r)=C\tilde{g}(r)$ into (17) and optimize over C, ζ, λ to maximize the correct detection probability. This strategy is called EF in this paper.

3) Determination of α: After obtaining $g(r)$ with $\alpha=0$, we can approximate $y_2^{(2)}$ as Gaussian and the log-likelihood ratio is

$$\ln\frac{p\left(y_2^{(1)}|\theta=1\right)p\left(y_2^{(2)}|\theta=1\right)}{p\left(y_2^{(1)}|\theta=0\right)p\left(y_2^{(2)}|\theta=0\right)}$$
$$=\frac{\tilde{P}+\sigma_2^2}{\sigma_2^2}\left|y_2^{(1)}\right|^2+\frac{\tilde{P}+\sigma_2^2+\sigma_h^2E\left\{g(r)|\theta=1\right\}}{\sigma_2^2+\sigma_h^2E\left\{g(r)|\theta=0\right\}}\left|y_2^{(2)}\right|^2. \quad (30)$$

Thus, we have

$$\alpha=\frac{\left(\tilde{P}+\sigma_2^2\right)\left(\sigma_2^2+\sigma_h^2E\left\{g(r)|\theta=0\right\}\right)}{\left(\tilde{P}+\sigma_2^2+\sigma_h^2E\left\{g(r)|\theta=1\right\}\right)\sigma_2^2}. \quad (31)$$

B. Peak Power Constraint

Peak power constraint is another commonly used power constraint. The peak power constraint $\hat{P}_1$ at $\mathbb{U}_1$ requires that $|f_1(x)|^2\leq\hat{P}_1, \forall x$. In this case, similar to (17), the optimization problem can be expressed as

$$\max_g \int_0^{+\infty} e^{-\frac{\lambda}{\left(\tilde{P}+\sigma_2^2\right)+\sigma_h^2g(r)}}e^{-\frac{r}{\tilde{P}+\sigma_1^2}}dr$$
$$\text{s.t.} \int_0^{+\infty} e^{-\frac{\lambda}{\sigma_2^2+\sigma_h^2g(r)}}\frac{1}{\sigma_1^2}e^{-\frac{r}{\sigma_1^2}}dx\leq\xi$$
$$0\leq g(r)\leq\hat{P}_1. \quad (32)$$

By adopting the Lagrange approach and relaxing the first inequality in (32), the Lagrange dual function can be written as

$$\phi(\mu)=\max_g \int_0^{+\infty} e^{-\frac{\lambda}{\left(\tilde{P}+\sigma_2^2\right)+\sigma_h^2g(r)}}e^{-\frac{r}{\tilde{P}+\sigma_1^2}}dr$$
$$-\mu\int_0^{+\infty} e^{-\frac{\lambda}{\sigma_2^2+\sigma_h^2g(r)}}e^{-\frac{r}{\sigma_1^2}}dr$$
$$\text{s.t.}0\leq g(r)\leq\hat{P}_1 \quad (33)$$

where $\mu\geq 0$ is a dual variable. Due to the peak power constraint, optimization of $g(r)$ at different r's is decoupled. Therefore, $\phi(\mu)$ can be obtained by solving $g(r)$ for each r, i.e.,

$$\max_{g(r)} e^{-\frac{\lambda}{\left(\tilde{P}+\sigma_2^2\right)+\sigma_h^2g(r)}}e^{-\frac{r}{\tilde{P}+\sigma_1^2}}-\mu e^{-\frac{\lambda}{\sigma_2^2+\sigma_h^2g(r)}}e^{-\frac{r}{\sigma_1^2}} \quad (34)$$

such that $0\leq g(r)\leq\hat{P}_1$. The derivative of the objective function in (34) with respect to $g(r)$ can be obtained as

$$F\left(g(r)\right)=\frac{\lambda\sigma_h^2}{\left(\tilde{P}+\sigma_2^2+\sigma_h^2g(r)\right)^2}e^{-\frac{\lambda}{\left(\tilde{P}+\sigma_2^2\right)+\sigma_h^2g(r)}}e^{-\frac{r}{\tilde{P}+\sigma_1^2}}$$
$$-\frac{\mu\lambda\sigma_h^2}{\left(\sigma_2^2+\sigma_h^2g(r)\right)^2}e^{-\frac{\lambda}{\sigma_2^2+\sigma_h^2g(r)}}e^{-\frac{r}{\sigma_1^2}}. \quad (35)$$

To solve (34), we need to find the roots of $F(g(r))=0$. The roots, together with two boundary points $g(r)=0$ and $g(r)=\hat{P}_1$, are substituted back into the objective function in (34), and that which attains the largest value of the objective function is chosen to be the optimal $g(r)$. The roots of $F(g(r))=0$ can be obtained by solving

$$\frac{\left(\sigma_2^2+\sigma_h^2g(r)\right)^2}{\left(\tilde{P}+\sigma_2^2+\sigma_h^2g(r)\right)^2}=\mu e^{-\frac{\tilde{P}r}{\sigma_1^2\left(\tilde{P}+\sigma_1^2\right)}}e^{-\frac{\tilde{P}\lambda}{\left(\tilde{P}+\sigma_2^2+\sigma_h^2g(r)\right)\left(\sigma_2^2+\sigma_h^2g(r)\right)}}. \quad (36)$$

Note that, when $r\gg\sigma_1^2$, $\mu e^{-\lambda/\sigma_2^2+\sigma_h^2g(r)}e^{-r/\sigma_1^2}\approx 0$, and the maximum is attained at $g(r)=\hat{P}_1$. On the other hand, when $r\ll\sigma_1^2$, the first term in the objective function of (34) can be approximated as a constant, and the maximum is achieved at $g(r)=0$. This reminds us of the DF-like strategy as in (24). The following theorem shows that, under certain conditions, the DF strategy is indeed optimal with peak power constraint.

Theorem 1: Define $f(x)$ as follows:

$$f(x)=e^{-\frac{r_1}{\sigma_1^2+x}}e^{-\frac{\lambda}{\sigma_2^2+A+x}}+e^{-\frac{r_2}{\sigma_1^2+x}}e^{-\frac{\lambda}{\sigma_2^2+B+x}}$$
$$-e^{-\frac{r_1}{\sigma_1^2+x}}e^{-\frac{\lambda}{\sigma_2^2+x}}-e^{-\frac{r_2}{\sigma_1^2+x}}e^{-\frac{\lambda}{\sigma_2^2+\hat{P}_1+x}} \quad (37)$$

and define $\mathcal{S}$ to be the set of x^* satisfying $f'(x^*)=0$. If, for any r_1, r_2, A, and B such that $0\leq r_1<r_2$, $0<A<B<\tilde{P}$, and $f(0)=0$, then we have $f(x^*)<0\,\forall x^*\in\mathcal{S}$, and DF strategy is optimal to (32).

Proof: Without loss of generality, we assume that $|h_{12}|=1$. Note that fixing λ gives a unique DF strategy such that the first constraint of (32) is satisfied with equality. For any function g satisfying the first constraint of (32) with equality and for a given r_1 with $A = g(r_1) > 0$, there must exist r_2 and $B = g(r_2)$ such that $f(0) = 0$ because otherwise, the equality cannot hold. Substituting this g into the objective function of (32) and comparing with the DF function at r_1 and r_2, the objective function is increased by $f(\tilde{P})$ due to the use of g. If $f(\tilde{P}) \leq 0\, \forall r_1, r_2, A$, and B, then the DF strategy is optimal. Note that $f(0) = 0$ and $f(+\infty) \rightarrow 0$, and therefore, $f'(x)$ is continuously functioning. If $\forall x^*$ such that $f'(x^*) = 0$, we have $f(x^*) \leq 0$. As the maximum of $f(x)$ is achieved at the saddle points in $\mathcal{S}$ or at the boundary $x = 0, +\infty$, we have $f(x) \leq 0$, for any $x \geq 0$. Therefore, $f(\tilde{P}) \leq 0$.

We can also characterize the performance limit by using cooperative sensing, e.g., the cooperative gain. If we choose

$$g(r) = \begin{cases} \hat{P}_1, & \text{if } r > \lambda_1 \\ 0, & \text{otherwise,} \end{cases} \tag{38}$$

as the DF relaying function, then (32) reduces to

$$\begin{aligned} \max_{\lambda, \lambda_1} \quad & e^{-\frac{\lambda}{\left(\tilde{P}+\sigma_2^2\right)}} \left(1 - e^{-\frac{\lambda_1}{\tilde{P}+\sigma_1^2}}\right) + e^{-\frac{\lambda}{\left(\tilde{P}+\sigma_2^2\right)+\sigma_h^2 \hat{P}_1}} e^{-\frac{\lambda_1}{\tilde{P}+\sigma_1^2}} \\ \text{s.t.} \quad & e^{-\frac{\lambda}{\sigma_2^2}} \left(1 - e^{-\frac{\lambda_1}{\sigma_1^2}}\right) + e^{-\frac{\lambda}{\sigma_2^2+\sigma_h^2 \hat{P}_1}} e^{-\frac{\lambda_1}{\sigma_1^2}} = \xi. \end{aligned} \tag{39}$$

We are interested in the performance of DF when $\hat{P}_1 \rightarrow +\infty$. There are two possible choices of λ to satisfy the constraint in (39) according to whether the first term or the second term in (39) vanishes.

1) Choose λ proportional to $\hat{P}_1$. The first term in the constraint and that in the objective function go to zero as $\hat{P}_1 \rightarrow +\infty$, respectively. We find that the correct detection probability is $\xi^{\sigma_2^2/\tilde{P}+\sigma_2^2}$, which is the same as that in (8) without cooperation. Hence, this case is not optimal.
2) Choose λ proportional to σ_2^2. In this case, we can rewrite (39) as

$$\begin{aligned} \max_{\lambda, \lambda_1} \quad & e^{-\frac{\lambda}{\left(\tilde{P}+\sigma_2^2\right)}} \left(1 - e^{-\frac{\lambda_1}{\tilde{P}+\sigma_1^2}}\right) + e^{-\frac{\lambda_1}{\tilde{P}+\sigma_1^2}} \\ \text{s.t.} \quad & e^{-\frac{\lambda}{\sigma_2^2}} \left(1 - e^{-\frac{\lambda_1}{\sigma_1^2}}\right) + e^{-\frac{\lambda_1}{\sigma_1^2}} = \xi. \end{aligned} \tag{40}$$

Therefore, the maximum correct detection probability is

$$\eta^* = \max_{\lambda_1 \geq 0} \left(\frac{\xi - e^{-\frac{\lambda_1}{\sigma_1^2}}}{1 - e^{-\frac{\lambda_1}{\sigma_1^2}}} \right)^{\frac{\sigma_2^2}{\tilde{P}+\sigma_2^2}} + e^{-\frac{\lambda_1}{\tilde{P}+\sigma_1^2}}. \tag{41}$$

If $\lambda_1 \rightarrow +\infty$, (41) reduces to (8). Therefore, η^* is always greater than or equal to (8) without cooperation, and the difference between η^* and (8) is the cooperative gain. Note that η^* does not depend on $\hat{P}_1$, and thus, η^* is the fundamental limit of local cooperative sensing, which cannot be improved by increasing SUs' transmission power. The fundamental limit also holds for average power constraint.

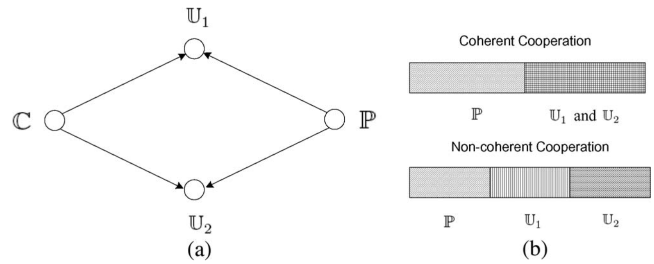


Fig. 2. Diagram of global cooperative spectrum sensing.

Remarks:

1) Since (17) is not a convex optimization problem, the solution from solving the Lagrange dual problem may not be optimal. Nevertheless, we find that the solution works well in most practical scenarios from the simulation results in Section V.
2) The hybrid processing function in (23) can be further extended to

$$g(r) = \begin{cases} C, & \text{if } r > \lambda_1 \\ 0, & \text{if } r \leq \lambda_2 \\ C\frac{r-\lambda_3}{\lambda_1-\lambda_2}, & \text{if } \lambda_2 < r \leq \lambda_1 \end{cases} \tag{42}$$

where C, λ_1, and λ_2 are defined to be the same as in (23), and $\lambda_3 \geq \lambda_2$ is an additional parameter. By choosing $\lambda_3 = \lambda_2$, (42) reduces to (23). Thus, (42) is expected to achieve a better performance than (23) due to an extra degree of freedom. However, using (42) requires more parameters to be optimized.
3) Different from [6], where only the signal in the cooperative time slot is used for sensing detection, the proposed protocol makes use of signals received in both the first and second time slots, which requires that the PU's activity remains unchanged during the sensing period.

IV. Optimization of Global Cooperative Sensing

Another possible class of cooperative spectrum-sensing protocol is global cooperative sensing [5], where the sensing is performed in two successive stages: 1) sensing and 2) reporting. Different from [5], where each SU makes hard sensing decisions, we consider each SU reporting soft information to the central controller. A diagram of global cooperative sensing is shown in Fig. 2. Unlike local cooperative sensing, where SUs individually exchange information and make the sensing decision, sensing is done solely at the central controller in the global cooperative approach, where SUs do not exchange information. In this section, we assume that the PU's activity remains static over the spectrum-sensing period. We consider coherent and noncoherent cooperations, where SUs are simultaneously synchronized and transmit over the same frequency to the central controller in the former case, and they transmit over different times and/or frequencies in the latter case.

A. Coherent Cooperation

Our coherent cooperation sensing protocol also contains two successive stages. In the sensing stage, each SU receives signal from the PU $\mathbb{P}$, whereas, in the reporting stage, each SU sends a transformation of its received signal. The cooperation is coherent as we assume that the secondary network is fully synchronized.

We consider a simple secondary network with two SUs, i.e., $\mathbb{U}_1$, $\mathbb{U}_2$, and one central controller $\mathbb{C}$. The SUs transmit to the central controller via a separate control channel. Our approach readily extends to the general case with more than two SUs. The received signal at each SU is given by (1). Assume that the relaying function at each SU is $f_i(\cdot)$, $i=1,2$, and the channel fading gain between $\mathbb{U}_i$ and $\mathbb{C}$ is h_{ic}, which is a complex Gaussian random variable with mean zero and variance σ_{hic}^2. The received signal at $\mathbb{C}$ can be written as

$$y_c = \sum_{i=1}^{2} h_{ic} f_i(\theta x_p h_{pi} + w_i) + z_c \tag{43}$$

where θ, x_p, h_{pi}, and w_i are defined as in (1), and z_c is the additive White Gaussian noise (AWGN) with zero mean and variance σ_c^2. The central controller employs an energy detector. From the same intuition and approximation as in (13), we can rewrite (43) as

$$y_c = \sum_{i=1}^{2} h_{ic} \sqrt{g_i\left(|\theta x_p h_{pi} + w_i|^2\right)} + z_c \tag{44}$$

where g_i is the equivalent relaying function at $\mathbb{U}_i$ operating on the energy of the received signal. Hence, given θ and $r_i = |\theta x_p h_{pi} + w_i|^2$, $i=1,2$, y_c is a complex Gaussian random variable with mean zero and variance $\sum_{i=1}^{2} \sigma_{hic}^2 g_i(r_i) + \sigma_c^2$. The pdf of $z = |y_c|^2$ is

$$\begin{aligned} p(z) &= \int_0^{+\infty}\int_0^{+\infty} p(y_c|r_1,r_2)p(r_1)p(r_2)\,dr_1\,dr_2 \\ &= \int_0^{+\infty}\int_0^{+\infty} \frac{e^{-\frac{z}{\sum_{i=1}^{2}\sigma_{hic}^2 g_i(r_i)+\sigma_c^2}}}{\sum_{i=1}^{2}\sigma_{hic}^2 g_i(r_i)+\sigma_c^2} \\ &\quad \times \frac{1}{\theta\tilde{P}+\sigma_1^2} e^{-\frac{r_1}{\theta\tilde{P}+\sigma_1^2}} \frac{1}{\theta\tilde{P}+\sigma_2^2} e^{-\frac{r_2}{\theta\tilde{P}+\sigma_2^2}}\,dr_1\,dr_2. \end{aligned} \tag{45}$$

Given the detection threshold λ and the false alarm probability ξ, by assuming average power constraint, the optimization problem becomes

$$\begin{aligned} &\max_g \int\int_0^{+\infty} e^{-\frac{\lambda}{\sum_{i=1}^{2}\sigma_{hic}^2 g_i(r_i)+\sigma_c^2}} e^{-\frac{r_1}{\tilde{P}+\sigma_1^2}} e^{-\frac{r_2}{\tilde{P}+\sigma_2^2}}\,dr_1\,dr_2 \\ &\text{s.t.} \int\int_0^{+\infty} e^{-\frac{\lambda}{\sum_{i=1}^{2}\sigma_{hic}^2 g_i(r_i)+\sigma_c^2}} \frac{1}{\sigma_1^2\sigma_2^2} e^{-\frac{r_1}{\sigma_1^2}-\frac{r_2}{\sigma_2^2}}\,dr_1\,dr_2 \le \xi \\ &\int_0^{+\infty} \frac{g_i(ri)e^{-\frac{ri}{\tilde{P}+\sigma_i^2}}}{\tilde{P}+\sigma_i^2}\,dr_i \le P_i,\ \int_0^{+\infty}\frac{g_i(r_i)e^{-\frac{r_i}{\sigma_i^2}}}{\sigma_i^2}\,dr_i \le P_i \end{aligned} \tag{46}$$

where $i=1,2$. Since $g_1(r_1)$ and $g_2(r_2)$ can be separated, (46) is hard to solve, even with a central controller. We instead consider two suboptimal approaches. The first approach is a parametric and centralized approach. We can assume that $g_i(r_i)$ attains a specific form such as AF, DF, and EF. We then optimize over the relevant parameters in each strategy. This approach requires that the optimization is performed at the central controller, and it then informs all the SUs about the optimized functions that they should use. The second approach is a decentralized approach, where each SU optimizes its relaying function by assuming other SUs experiencing the same noise variance σ_i^2, channel fading variance σ_{hic}^2, and power P_i. By using the DF strategy and considering the general case with N SUs, the ith SU needs to solve

$$\begin{aligned} &\max_g \sum_{\mathbf{b}\in\{0,1\}^N} e^{-\frac{\lambda}{\sum_{j=1}^{N} b_j\sigma_{hic}^2 C_i+\sigma_c^2}} \prod_{j=1}^{N}\left(b_j-(2b_j-1)e^{-\frac{\lambda_i}{\tilde{P}+\sigma_i^2}}\right) \\ &\text{s.t.} \sum_{\mathbf{b}\in\{0,1\}^N} e^{-\frac{\lambda}{\sum_{j=1}^{N} b_j\sigma_{hic}^2 C_i+\sigma_c^2}} \prod_{j=1}^{N}\left(b_j-(2b_j-1)e^{-\frac{\lambda_i}{\sigma_i^2}}\right)=\xi \\ &C_i = e^{\frac{\lambda_i}{\tilde{P}+\sigma_i^2}} P_i. \end{aligned} \tag{47}$$

Note that (47) can be simplified to contain only a single parameter, which is easy to solve at each SU. Optimization under peak power constraint can be similarly performed.

B. Noncoherent Cooperation

In noncoherent cooperation, each SU asynchronously sends its signal to the central controller at different times or frequencies. Thus, the central controller receives N different signals from N SUs, i.e.,

$$y_{ic} = h_{ic} f_i(\theta x_p h_{pi} + w_i) + z_{ic}, \quad i=1,2,\ldots,N \tag{48}$$

where h_{ic}, θ, x_p, and h_{pi} are defined as before, and w_i and z_{ic} are the AWGN with zero mean and variance σ_{iw}^2, σ_c^2, respectively. The problem is how we can combine y_{ic}'s to achieve the best sensing performance. Motivated by the energy detector, we consider using the test statistic

$$z = \sum_{i=1}^{N}\alpha_i|y_{ic}|^2 = \sum_{i=1}^{N}\alpha_i u_i \tag{49}$$

where $u_i = |y_{ic}|^2$, and $\alpha_i \ge 0$ is the weighting coefficient. Applying the same approximation as in (13), y_{ic} is a complex Gaussian random variable with mean zero and variance $\sigma_{hic}^2 g_i(r_i) + \sigma_c^2$, given θ and $r_i = |\theta x_p h_{pi} + w_i|^2$, $i=1,2$. We thus obtain the mean of u_i as

$$m_{i,\theta} = E_{r_i|\theta}\left\{\sigma_{hic}^2 g_i(r_i) + \sigma_c^2\right\} \tag{50}$$

and the variance of u_i is

$$\omega_{i,\theta}^2 = E_{r_i|\theta}\left\{\left(\sigma_{hic}^2 g_i(r_i) + \sigma_c^2\right)^2\right\} - m_{i,\theta}^2. \tag{51}$$

According to Lyapunov's central limit theorem [16], if N is large, the test statistic z is asymptotically normally distributed with mean $m_\theta = \sum_{i=1}^N \alpha_i m_{i,\theta}$ and variance $\omega_\theta^2 = \sum_{i=1}^N \alpha_i^2 \omega_{i,\theta}^2$. Given the false alarm probability ξ, we determine the threshold from

$$P_f(\lambda) = Q\left(\frac{\lambda - m_0}{\omega_0}\right) = \xi \Rightarrow \lambda = Q^{-1}(\xi)\omega_0 + m_0 \tag{52}$$

where $Q(\cdot)$ is the Q-function. The correct detection probability is

$$P_d(\alpha, g) = Q\left(\frac{\lambda - m_1}{\omega_1}\right) = Q\left(\frac{Q^{-1}(\xi)\omega_0 + m_0 - m_1}{\omega_1}\right). \tag{53}$$

Therefore, we need to solve

$$\min_{\alpha, g} \frac{\delta\omega_0 + m_0 - m_1}{\omega_1} \tag{54}$$

subject to the power constraint, where $\delta = Q^{-1}(\xi)$. To solve (54), we decouple the optimization over α and g. Like in the coherent case, each SU assumes that all the other SUs have the same noise variance σ_i^2, channel fading variance σ_{hic}^2, power P_i, and weight α_i. SU i thus needs to compute g_i from

$$\min_{g_i} \frac{\delta\omega_{i,0} + \sqrt{N}\,(m_{i,0} - m_{i,1})}{\omega_{i,1}}. \tag{55}$$

Instead of directly solving (55), we consider the following problem:

$$\min_{g_i} \delta\omega_{i,0} + \sqrt{N}(m_{i,0} - m_{i,1}) - \beta\omega_{i,1} \tag{56}$$

where $\beta \geq 0$ is a parameter. By using the same Lagrangian approach, we find that the optimal solution to (56) has the form

$$g_i(r_i) = \frac{1 + Ae^{-\frac{r_i\tilde{P}}{\sigma_{iw}^2\left(\tilde{P}+\sigma_{iw}^2\right)}}}{B + Ce^{-\frac{r_i\tilde{P}}{\sigma_{iw}^2\left(\tilde{P}+\sigma_{iw}^2\right)}}} - \frac{\sigma_c^2}{\sigma_{hic}^2}. \tag{57}$$

Given B and C, A can be determined by the average power constraint. Therefore, a suboptimal solution to (56) can be found by substituting (57) into (56) and performing a 2-D search. Note that (55) can be locally solved at each SU.

To compute α_i, let $\tilde{\alpha}_i = \alpha_i\omega_{i,0}$. Given $m_{i,\theta}$, $\omega_{i,\theta}^2$, we can rewrite (54) as

$$\min_{\tilde{\alpha}} \frac{\delta\sqrt{\sum_{i=1}^N \tilde{\alpha}_i^2} + \sum_{i=1}^N \tilde{\alpha}_i \frac{m_{i,0}-m_{i,1}}{\omega_{i,0}}}{\sum_{i=1}^N \tilde{\alpha}_i^2 \frac{\omega_{i,1}^2}{\omega_{i,0}^2}}. \tag{58}$$

Define $\mathbf{x} = [\tilde{\alpha}_1, \ldots, \tilde{\alpha}_N]^T$, $\mathbf{a} = [m_{1,1} - m_{1,0}/\omega_{1,0}, \ldots, m_{N,1} - m_{N,0}/\omega_{N,0}]^T$, and $\mathbf{\Lambda} = \mathrm{diag}\{\omega_{1,1}^2/\omega_{1,0}^2, \ldots, \omega_{N,1}^2/\omega_{N,0}^2\}$. Since the value of (58) is invariant to scaling $\tilde{\alpha}_i$ by a constant, (58) is equivalent to

$$\min_{\mathbf{x}} \frac{\delta - \mathbf{a}^T\mathbf{x}}{\sqrt{\mathbf{x}^T\mathbf{\Lambda}\mathbf{x}}}, \quad \text{s.t.} \quad \mathbf{x}^T\mathbf{x} = 1. \tag{59}$$

In practice, the correct detection probability is usually greater than 0.5. By the property of Q-function, we know $\delta - \mathbf{a}^T\mathbf{x} < 0$. Therefore, (59) is equivalent to

$$\min_{\mathbf{x}} \frac{\delta - \mathbf{a}^T\mathbf{x}}{\epsilon} \quad \text{s.t. } \mathbf{x}^T\mathbf{x} = 1, \quad \mathbf{x}^T\mathbf{\Lambda}\mathbf{x} \leq \epsilon^2, \quad \delta - \mathbf{a}^T\mathbf{x} < 0. \tag{60}$$

Defining $\tilde{\mathbf{x}} = \mathbf{x}/\epsilon$, we obtain

$$\min_{\tilde{\mathbf{x}}} \delta\|\tilde{\mathbf{x}}\|_2 - \mathbf{a}^T\tilde{\mathbf{x}} \quad \text{s.t. } \tilde{\mathbf{x}}^T\mathbf{\Lambda}\tilde{\mathbf{x}} \leq 1, \quad \delta\|\tilde{\mathbf{x}}\|_2 - \mathbf{a}^T\tilde{\mathbf{x}} < 0. \tag{61}$$

It can be easily verified that (61) is a convex optimization problem, which can be efficiently solved using the interior point method [15]. If (61) is infeasible, then the correct detection probability is less than 0.5. In this case, $\delta - \mathbf{a}^T\mathbf{x} \geq 0$, and (59) is equivalent to

$$\min_{\mathbf{x}} \frac{(\delta - \mathbf{a}^T\mathbf{x})^2}{\mathbf{x}^T\mathbf{\Lambda}\mathbf{x}} \quad \text{s.t.} \quad \mathbf{x}^T\mathbf{x} = 1. \tag{62}$$

Since $(\delta - \mathbf{a}^T\mathbf{x})^2 \leq 2(\delta^2 + (\mathbf{a}^T\mathbf{x})^2)$, instead of dealing with

$$\min_{\mathbf{x}} \frac{\mathbf{x}^T(\delta^2\mathbf{I}_N + \mathbf{a}\mathbf{a}^T)\mathbf{x}}{\mathbf{x}^T\mathbf{\Lambda}\mathbf{x}} \quad \text{s.t.} \quad \mathbf{x}^T\mathbf{x} = 1 \tag{63}$$

which can be readily solved using the Rayleigh quotient [17], i.e., the solution is the eigenvector corresponding to the minimum eigenvalue of $\mathbf{\Lambda}^{1/2}(\delta^2\mathbf{I}_N + \mathbf{a}\mathbf{a}^T)\mathbf{\Lambda}^{1/2}$. After obtaining $\mathbf{x}$ from (63), we substitute it into (59) and compare its value with that obtained from an all-one $\mathbf{x}$. Finally, the approximate solution is that which attains a larger value in (59). To evaluate the performance of the proposed algorithm, 1000 random instances are generated such that $\delta - \mathbf{a}^T\mathbf{x} \geq 0$. The solution using the approximation (63) is compared with both that from solving (59) locally around the approximation and that from the all-one vector. Finally, the candidate that attains a larger value of (59) will be selected. We can see from Fig. 3 that, with probability greater than 85%, the approximate solution attains a value less than twice of that by local search.

V. Simulation Results

In this section, simulation results are provided to corroborate the proposed theoretical results. Unless otherwise mentioned, we choose the received PU's power $\tilde{P} = 1$ at each SU.

A. Local Cooperative Sensing

In local cooperative sensing, we choose $\sigma_h^2 = E\{\sigma_h^2\} = 1$. We compare it with the cooperative spectrum-sensing strategies in

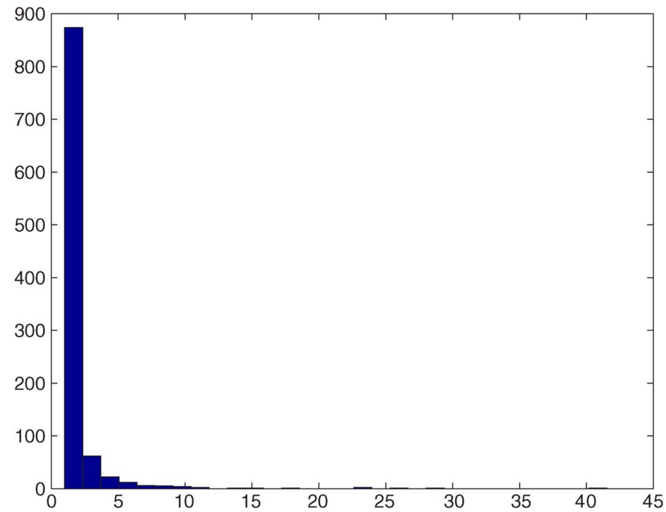
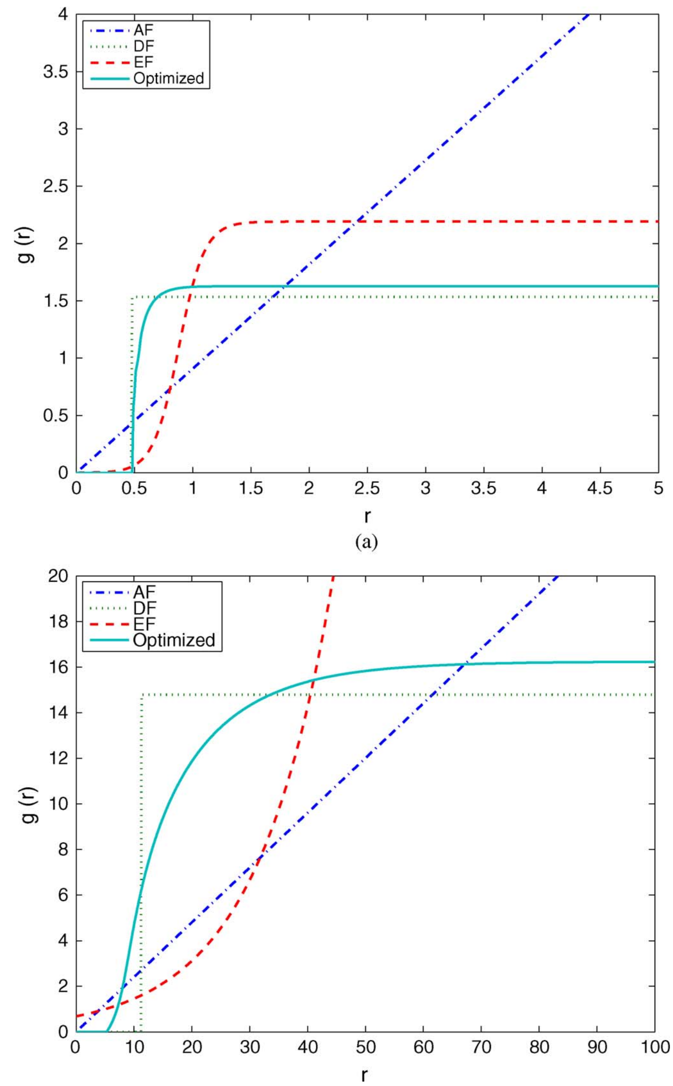


Fig. 3. Accumulative curve of the ratio between the approximate solution using (63) and the solution of (59) by local search.



(a)

(b)

Fig. 4. Comparison of relaying functions $g(r)$ under average power constraint at $\mathbb{U}_2$ with $P_1 = 1$. (a) $\sigma_1^2 = \sigma_2^2 = 0.1$. (b) $\sigma_1^2 = \sigma_2^2 = 10^{0.5}$.

[6] and [9], because the former strategy is similar to the AF scheme, whereas the latter strategy is close to DF.

1) $\alpha = 0$.

Fig. 4 compares different relaying functions $g(r)$ under the average power constraint at $\mathbb{U}_2$ with $P_1 = 1$, $\sigma_1^2 = \sigma_2^2 = 0.1$. The strategy in [9] is also included. It can be seen that, when noise variance is small, the optimized relaying function looks like DF, whereas it is like AF when noise variance is large, which agrees with the analysis in Section III-A. The EF function looks like the optimized function for both SNR extremes.

Fig. 5 shows the misdetection probability $1 - P_d$ corresponding to Fig. 4 under the average power constraint. We also include the curve without cooperation in Section II. As expected, the optimized relaying function performs better than all other strategies. At $P_f = 0.2$, with $\sigma_1^2 = \sigma_2^2 = 0.1$, the $1 - P_d$ of the optimized strategy is only 29% of the AF, whereas it is 40% of DF and 49% of the strategy in [9]. With $\sigma_1^2 = \sigma_2^2 = 10^{0.5}$, the $1 - P_d$ of the optimized strategy becomes 54% of the AF, whereas it is 55% of DF and 55% of the strategy in [9]. DF performs better than AF when noise variance is small or false alarm probability P_f is large. EF performs between DF and AF. The hybrid strategy performs very close to DF in all cases, and it performs better than DF when noise variance is large.

In Fig. 6, we compare the performance under the peak power constraint $\hat{P}_1 = 1$ with that under the average power constraint and that without cooperation. We only show the performance of DF, because we find that DF is optimal in the considered cases. It is observed that DF under the average power constraint performs worse than that under the peak power constraint, because the average power consumption under the peak power constraint is less than that under the average power constraint. When $\sigma_1^2 = \sigma_2^2 = 0.1$ and $P_f = 0.5$, the DF Average achieves a 62.51% $1 - P_d$ of that without cooperation, whereas the DF Peak achieves a 68.40% $1 - P_d$ of that without cooperation. When $\sigma_1^2 = \sigma_2^2 = 10^{0.5}$, the incorrect detection probability reduction of DF Average and DF Peak over no cooperation decreases to 96.98% and 99.27%, respectively.

Fig. 7 shows the performance of different strategies under the average power constraint with $\sigma_1^2 = 0.1$, $\sigma_2^2 = 1$, $\alpha = 0.1$, and various P_1 and P_2 at $\mathbb{U}_1$ and $\mathbb{U}_2$, respectively. The DF strategy performs better than all other strategies shown in the figure. When $\mathbb{U}_1$ power is 14 dB, DF's correction detection probability η is 2.311, 1.239, and 1.083 times higher than no cooperation, AF, and that in [9], respectively. Interestingly, unlike AF and that in [9], which may cause noise amplification, DF and EF always perform better than that without cooperation due to optimization. From Fig. 7(b), we can see that the performance gain at the SU with a smaller noise variance over the user with a larger noise variance is small. Except hybrid and DF, all other schemes' performance degrade as the power of $\mathbb{U}_1$ increases, which is due to the noise amplification. DF's correction detection probability η is only 1.017 times higher than that with no cooperation.

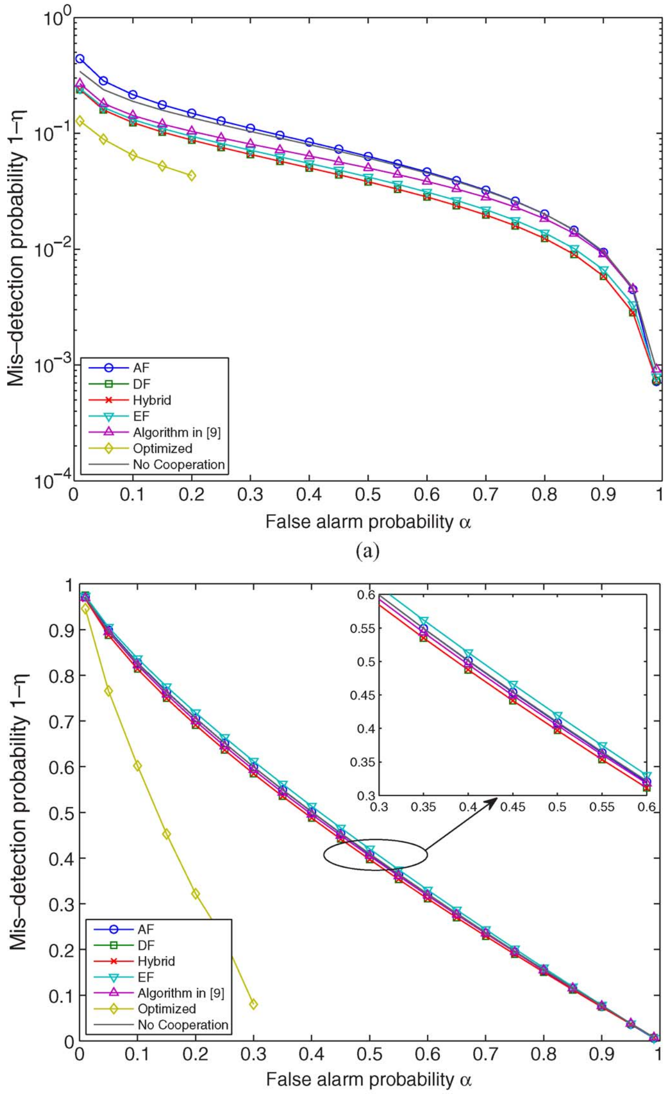


Fig. 5. Comparison of misdetection probability $1-\eta$ at $\mathbb{U}_2$ with different false alarm probabilities α under the average power constraint under the average power constraint at $\mathbb{U}_2$ with $P_1 = 1$. (a) $\sigma_1^2 = \sigma_2^2 = 0.1$. (b) $\sigma_1^2 = \sigma_2^2 = 10^{0.5}$

2) $\alpha \neq 0$.

We compare different strategies with $\alpha \neq 0$ with noncooperative scheme introduced in Section II. Fig. 8 shows the performance of different strategies at $\mathbb{U}_2$ under the average power constraint with $\sigma_1^2 = 0.1$, $\sigma_2^2 = 1$, and $P_1 = 5$. To fairly compare with the noncooperative scheme, we consider a modified noncooperative scheme that performs sensing when the total power of the PU is $3\tilde{P}$, because the cooperative sensing requires three time slots. In cooperative strategies, α is optimized by performing a line search after obtaining the corresponding processing functions for the second time slot. We can see that cooperation with $\alpha \neq 0$ significantly improves the detection probability. When the false alarm probability is 0.5, the misdetection probability of DF is only 33.87% of that with the noncooperative scheme using one time slot, and even AF attains a 43.69% smaller misdetection probability over the noncooperative scheme.

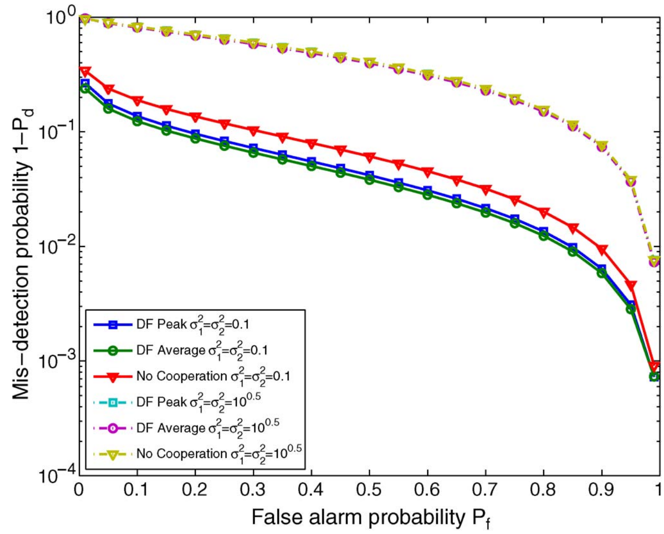


Fig. 6. Performance comparison between DF under the peak power constraint $\hat{P}_1 = 1$ and DF under the average power constraint $P_1 = 1$.

B. Global Cooperative Sensing

In this section, we give simulation results for global cooperative sensing schemes in Section IV. Fig. 9 compares global coherent and noncoherent cooperative strategies with $N = 3$ SUs, $\sigma_{hic}^2 = 1$, $P_i = 1$, and $\sigma_i^2 = \sigma_1^2\ \forall i = 1, 2, 3$. Due to symmetry, $\alpha_i = 1$, $\forall i = 1, \ldots, N$ in (49). We choose N to be small, because the optimal performance can be obtained in both cases[1] to facilitate comparison with suboptimal strategies. Only the DF strategy is considered. We find that the optimal noncoherent strategy performs slightly better than the optimal coherent strategy when σ_1^2 is either small or large, which suggests that it is beneficial to obtain independent copies of signals from different SUs rather to obtain an aggregated signal. When the false alarm probability is 0.5 and $\sigma_i = \sigma_c = 0.1$, the noncoherent optimal strategy achieves a 60.60% reduction in misdetection probability over the coherent optimal strategy. Even though $N = 3$ is a small number, the performance by using Gaussian approximation is fairly close to the optimal performance when the noise variance is large. We also find that the performance of noncooperative sensing is close to the cooperative counterparts in low SNR. This is because of the noise amplification.

Fig. 10 shows the performance of global cooperative sensing schemes over fading channels with $N = 3$ SUs, where h_{ic} is a complex Gaussian random variable with zero mean and variance 1. $P_i = 1$, $\sigma_i^2 = 0.1$, $\forall i = 1, 2, 3$, and $\sigma_c^2 = 0.1$. The curves are obtained after averaging more than 30 fading realizations. The performance of local optimization, where each SU assumes that all the other users use the same processing function, is compared with the global optimization solution. We find the local optimization performs close to the global optimization. In the noncoherent strategy, α_i in (49) is optimized using Gaussian approximation (53), where g_i is solved by using local or global optimization, assuming $\alpha_i = 1\ \forall i = 1, \ldots, N$.

[1]The performance of the noncoherent strategy can be obtained from a noncentral chi-square random variable.

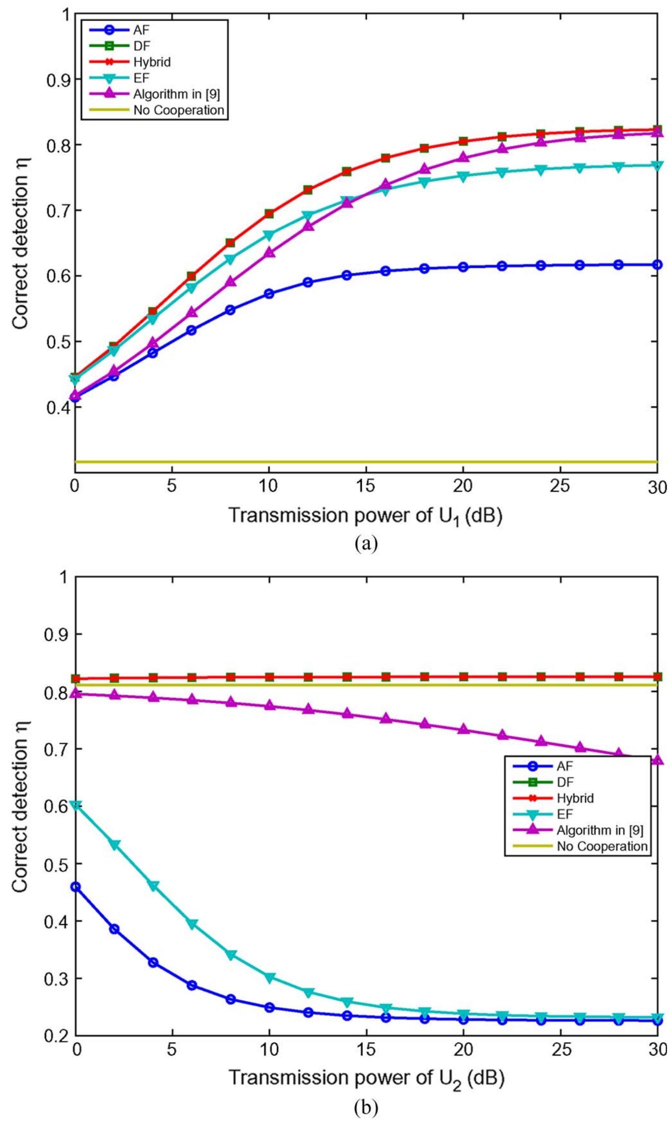


Fig. 7. Performance comparison of different strategies under the average power constraint with $\sigma_1^2 = 0.1$, $\sigma_2^2 = 1$, $\xi = 0.1$, and various P_1 and P_2 at (a) $\mathbb{U}_1$ and (b) $\mathbb{U}_2$, respectively.

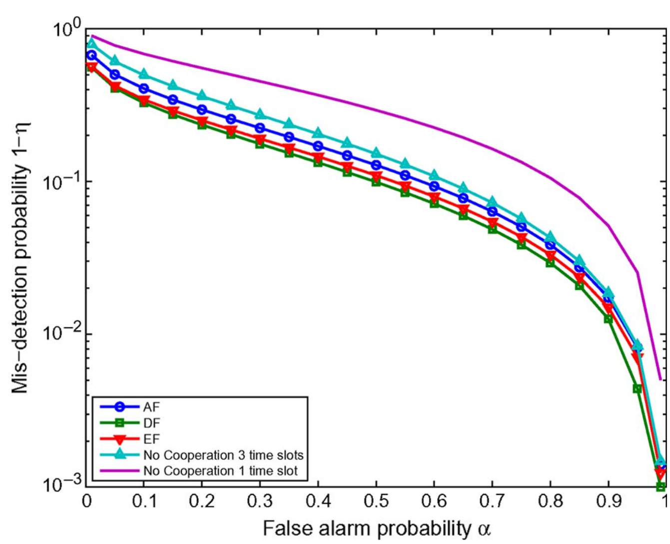


Fig. 8. Performance comparison between different strategies at $\mathbb{U}_2$ under the average power constraint with $\sigma_1^2 = 0.1$, $\sigma_2^2 = 1$ and $P_1 = 5$.

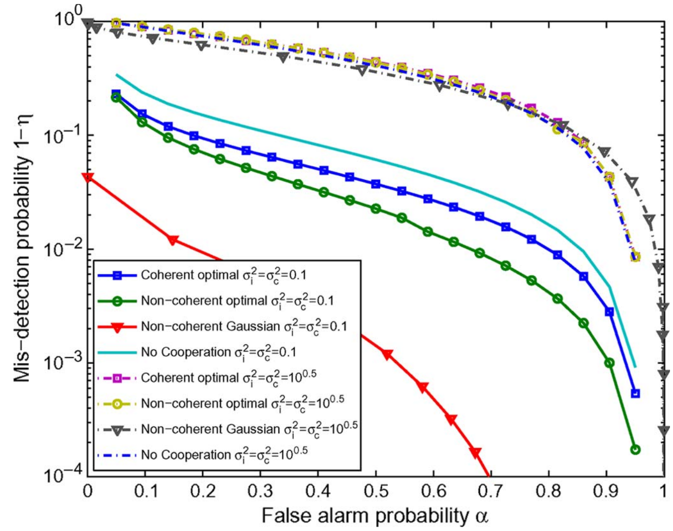


Fig. 9. Performance comparison between coherent and noncoherent global cooperative spectrum sensing under the average power constraint with $P_i = 1$, $\sigma_{hic}^2 = 1\ \forall i = 1, 2, 3$. $N = 3$ SUs are considered.

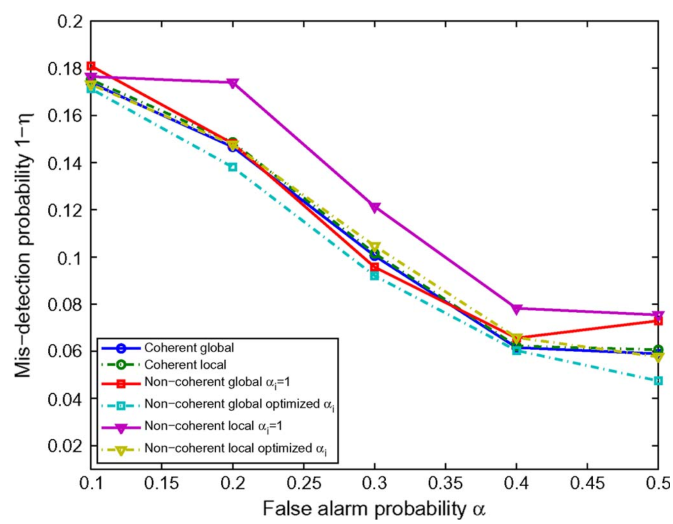


Fig. 10. Performance comparison between coherent and noncoherent global cooperative spectrum sensing over Rayleigh fading channels under the average power constraint with $P_i = 1$, $\sigma_i^2 = 0.1, \forall i = 1, 2, 3\ \forall i = 1, 2, 3$, and $\sigma_c^2 = 0.1$. $N = 3$ SUs are considered.

VI. Conclusion

The development of the IEEE 802.22 WRAN standard [3] aims at using CR techniques to share the unused spectrums that have been allocated to the television broadcast service. In this paper, we have considered cooperative spectrum sensing, which is an important issue in CR. Different from existing works, where each SU transmits its local sensing decision, we have considered SU transmitting a function of its received signal from the PU. We have optimized the relaying function at each SU via functional analysis for both average and peak power constraints. We have discussed optimization of local spectrum sensing with two SUs. The proposed spectrum-sensing algorithms perform significantly better than existing algorithms. It is interesting to investigate how to pair nodes in a large network and what is the best strategy for cooperation among more than two users. In addition, simultaneous consideration of multiple PUs is in place.

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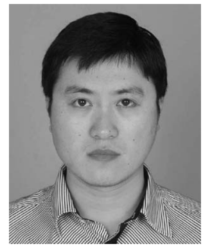

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