On the Throughput and Spectrum Sensing Enhancement of Opportunistic Spectrum Access Cognitive Radio Networks

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***Abstract*—Cognitive radio has attracted an increasing amount of interest over the past few years as an effective method of alleviating the spectrum scarcity problem in wireless communications. One of the most promising approaches in cognitive radio is the opportunistic spectrum access, which enables unlicensed users to access licensed frequency bands that are detected to be idle. In this paper, we propose a novel cognitive radio system that exhibits improved throughput and spectrum sensing capabilities compared to the conventional opportunistic spectrum access cognitive radio systems studied so far. More specifically, we study the average achievable throughput of the proposed cognitive radio system under a single high target detection probability constraint, as well as its ergodic throughput under average transmit and interference power constraints, and propose an algorithm that acquires the optimal power allocation strategy and target detection probability, which under the imposed average interference power constraint becomes an additional optimization variable in the ergodic throughput maximization problem. Finally, we provide simulation results, in order to compare the achievable throughput of the proposed cognitive radio system with the respective throughput of the conventional cognitive radio systems and discuss the effects of the optimal power allocation and target detection probability on the ergodic throughput of the proposed cognitive radio system.**

***Index Terms*—Cognitive radio, opportunistic spectrum access, optimal power allocation, spectrum sensing, throughput maximization.**

I. INTRODUCTION

COGNITIVE radio is a new promising technology that aims to alleviate the spectrum scarcity problem in wireless communications by allowing access of unlicensed (secondary) users to frequency bands that are allocated to licensed (primary) users, in a way that does not affect the quality of service (QoS) of the licensed networks [1], [2]. The research in cognitive radio has been encouraged by the measurements of the Federal Communications Commission (FCC), which have revealed that there is a significant amount of licensed spectrum which is largely underutilized in vast temporal and geographic dimensions [3]. The FCC recognizing that there is a significant amount of available spectrum that is currently not being used under the current fixed spectrum allocation policy, has recently allowed the access of unlicensed (secondary) users to the broadcast television spectrum at locations where that spectrum is not being used by licensed services [4]. This unused broadcast television spectrum is often termed as "white spaces" and has been the focus of the IEEE 802.22 WRAN standard that aims to provide broadband wireless internet access to rural areas [5].

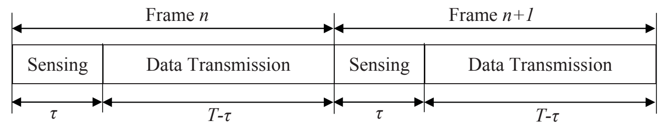

Fig. 1. Frame structure of the conventional opportunistic spectrum access cognitive radio networks.

Two main approaches have been proposed for cognitive radio so far, regarding the way that the cognitive radio users can access the licensed spectrum: (i) through opportunistic spectrum access (OSA) [6], [7], according to which the secondary users are allowed to access a frequency band only when it is detected to be idle, and (ii) through spectrum sharing (SS) [8], [9], according to which the secondary users coexist with the primary users under the condition of protecting the latter from harmful interference. In this paper, we are going to focus on the former approach.

The frame structure of the opportunistic spectrum access cognitive radio systems studied so far consists of a sensing time slot and a data transmission time slot, as depicted in Fig. 1. According to this frame structure, a secondary user ceases transmission at the beginning of each frame and senses for the status of the frequency band (active/idle) for τ units of time, whereas it uses the remaining frame duration $T-\tau$ for data transmission. Therefore, an inherent tradeoff exists in this frame structure between the duration of spectrum sensing and data transmission, hence the throughput of the cognitive radio system. According to the classical detection theory [10], [11], an increase in the sensing time results in a higher detection probability and lower false alarm probability, which in return leads to improved utilization of the available unused spectrum. However, the increase of the sensing time results in a decrease of the data transmission time, hence the achievable throughput

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of the cognitive radio system. This sensing-throughput tradeoff was addressed in [12], where the authors studied the problem of finding the optimal sensing time that maximizes the average achievable throughput of an OSA cognitive radio system under a single high target detection probability constraint for the protection of the QoS of the primary users. In [13], the authors considered the ergodic throughput maximization of an OSA cognitive radio system under an interference power constraint and a single value high target detection probability constraint ($\mathscr{P}_d^{tar} \approx 1$), and proposed an algorithm that obtains the sensing time and power allocation that maximizes the throughput of the cognitive radio system for Rayleigh fading channels.

In this paper, we propose a novel cognitive radio system that overcomes the sensing-throughput tradeoff in opportunistic spectrum access cognitive radio networks by performing spectrum sensing and data transmission at the same time. The way that this is achieved is described in more detail in Section II. Moreover, we compare the average achievable throughput of the proposed cognitive radio system with the respective throughput of the conventional opportunistic spectrum access cognitive radio system in [12], and show that the proposed cognitive radio system exhibits improved throughput under a single high target detection probability constraint imposed for the protection of the primary users. Furthermore, we study the problem of maximizing the achievable ergodic throughput of the proposed cognitive radio system under joint average transmit and interference power constraints, in order to keep the long-term power budget of the secondary users, and effectively protect the primary users from harmful interference for the case that the frequency band is falsely detected to be idle. More specifically, we focus on determining the optimal power allocation strategy for the proposed cognitive radio system, as well as the optimal target detection probability, which under the imposed average interference power constraint becomes an additional optimization variable in the ergodic throughput maximization problem. The effect of the target detection probability on the system's ergodic throughput can be seen more clearly in the simulation results presented in Section V. Finally, we propose an algorithm that acquires the optimal target detection probability and power allocation strategy that maximizes the achievable ergodic throughput of the proposed opportunistic spectrum access cognitive radio system and present simulation results.

Different from [12] and [13], in our work we propose a different receiver structure and spectrum sensing approach for the cognitive radio system, whereas we additionally consider an average transmit power constraint (which was not considered in [13]), in order to keep the long-term power budget of the secondary users, and also an average interference power constraint (which was not considered in [12]), in order to protect the primary users from harmful interference when the frequency band is falsely detected to be idle. Furthermore, we do not consider in our analysis a single high target detection probability constraint ($\mathscr{P}_d \geq \bar{\mathscr{P}}_d$ in [12], and $\mathscr{P}_d^{tar} \approx 1$ in [13]), whereas finally, we propose an algorithm that acquires the optimal power allocation strategy regardless of the channel distribution, and which can be applied even when the frequency band is not underutilized.

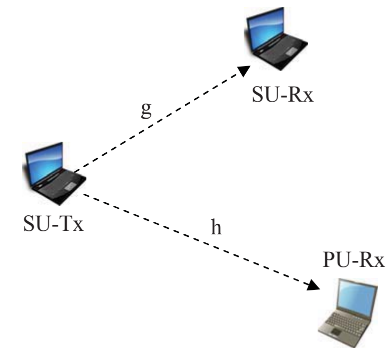

Fig. 2. System model.

The rest of the paper is organized as follows. In Section II, we present the system model and provide an overview of the proposed cognitive radio system. In Section III, we study the average achievable throughput of the proposed cognitive radio system under a high target detection probability constraint. In Section IV, we address the problem of maximizing the ergodic throughput of the proposed cognitive radio system under joint average transmit and interference power constraints, and propose an algorithm that acquires the optimal target detection probability and power allocation strategy for the proposed cognitive radio system. Finally, we present and discuss the simulation results in Section V, whereas the conclusions are drawn in Section VI.

Notations: $\mathbb{E}\{\cdot\}$ denotes the expectation operation, vectors are boldface capital letters, the transpose of the vector $\mathbf{A}$ is denoted by $\mathbf{A}^T$, $[x]^+$ denotes $\max(0, x)$, P denotes power and $\mathscr{P}$ probability.

II. Overview of the Proposed Cognitive Radio System

We consider the cognitive radio system that is presented in Fig. 2. Let g and h denote the instantaneous channel power gains from the secondary transmitter (SU-Tx) to the secondary receiver (SU-Rx) and the primary receiver (PU-Rx), respectively. The channel power gains g and h are assumed to be ergodic, stationary and known at the secondary users[1] similar to [8], [9], [13], [14], [15], [17], whereas the noise is assumed to be circularly symmetric complex Gaussian (CSCG) with zero mean and variance σ_n^2, namely $\mathcal{CN}(0, \sigma_n^2)$. It should be noted here that knowledge of the precise channel power gain h is very difficult to be obtained in practice and therefore our results serve as upper bounds on the achievable throughput of the cognitive radio system.

A. System overview

The proposed cognitive radio system operates as follows. In the beginning, an initial spectrum sensing is performed, in order to determine the status (active/idle) of the frequency band. When the frequency band is detected to be idle, the secondary transmitter accesses it for the duration of a frame by

[1] In practice, the channel power gain h can be obtained via, e.g., estimating the received signal power from the PU-Rx when it transmits, under the assumptions of the pre-knowledge on the PU-Rx transmit power level and the channel reciprocity [14].

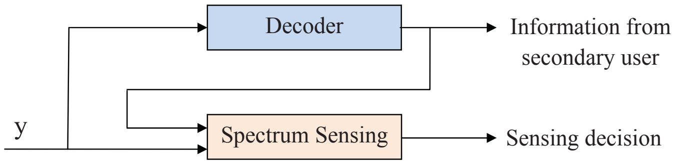

Fig. 3. Receiver structure of the proposed cognitive radio system.

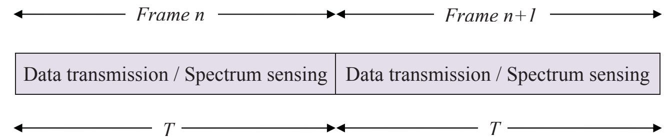

Fig. 4. Frame structure of the proposed cognitive radio system.

transmitting information to the secondary receiver. The latter decodes the signal from the secondary transmitter, strips it away from the received signal, and uses the remaining signal for spectrum sensing, in order to determine the action of the cognitive radio system in the next frame. At the end of the frame, if the presence of primary users is detected, namely if the primary users started transmission after the initial spectrum sensing was performed, data transmission will be ceased, in order to protect the primary users from harmful interference. In the opposite case, the secondary users will access the frequency band again in the next frame. Finally, the process is repeated.

B. Receiver structure

The receiver structure of the proposed cognitive radio system is presented in Fig. 3. The received signal at the secondary receiver is given by

$$y = \theta x_p + x_s + n, \tag{1}$$

where θ denotes the actual status of the frequency band ($\theta = 1$ if the frequency band is active and $\theta = 0$ if it is idle), x_p and x_s represent the received (faded) signal from the primary users and the secondary transmitter, respectively, and finally n denotes the additive noise.

The received signal y is initially passed through the decoder, as depicted in Fig. 3, where the signal from the secondary transmitter is obtained. In the following, the signal from the secondary transmitter is cancelled out from the aggregate received signal y, and the remaining signal

$$\tilde{y} = \theta x_p + n \tag{2}$$

is used to perform spectrum sensing.[2] This is the same signal that the secondary receiver would receive if the secondary transmitter had ceased data transmission, which is the conventional way that was proposed to perform spectrum sensing. Here, instead of using a limited amount of time τ, the whole duration of the frame T can be used for spectrum sensing. This way, we are able to perform spectrum sensing and data transmission at the same time, thus maximizing the duration of both.

C. Frame structure

The frame structure of the proposed cognitive radio system is presented in Fig. 4 and consists of a single slot during which both spectrum sensing and data transmission are performed at the same time, using the receiver structure presented in the previous subsection. The advantage of the proposed frame structure is that the spectrum sensing and data transmission time are simultaneously maximized, whereas, more specifically, they are equal to the frame duration T. The significance of this result is twofold. Firstly, the increased sensing time:

i) enables the detection of very weak signals from the primary users, the detection of which under the frame structure of Fig. 1 would significantly reduce the data transmission time, hence the throughput of the cognitive radio network,

ii) leads to an improved detection probability, thus better protection of the primary users from harmful interference,

iii) results to a decreased false alarm probability, which enables a better use of the available unused spectrum,

iv) facilitates the use of more complex spectrum sensing techniques that exhibit increased sensing capabilities, but require higher sensing time (such as cyclostationary detection [18] or several covariance-based spectrum sensing techniques [19], [20]), which prohibits their application for quick periodical spectrum sensing under the frame structure presented in Fig. 1,

v) the calculation of the optimal sensing time is no longer an issue, since it is maximized and equal to the frame duration T,

vi) continuous spectrum sensing can be achieved under the proposed cognitive radio system, which ensures better protection of the quality of service (QoS) of the primary networks.

The second important aspect is that the sensing time slot τ of the frame structure of Fig. 1 is now used for data transmission, which leads to an increase in the throughput of the cognitive radio network on the one hand, and facilitates the continuity of data transmission on the other.

III. Average Achievable Throughput of the Proposed Cognitive Radio System under a High Target Detection Probability Constraint

In this section, we study the average achievable throughput of the proposed cognitive radio system and compare it with the respective achievable throughput of the cognitive radio system that operates based on the conventional frame structure depicted in Fig. 1. We consider, similar to the work in [12], a single high target detection probability constraint for the protection of the primary users from harmful interference. Considering the fact that the priority of a cognitive radio system is and should be the protection of the quality of service (QoS) of the primary network, a high target detection probability is required, in order to ensure that no harmful interference is caused to the licensed users by the secondary network. For instance, the target probability of detection in the IEEE 802.22 WRAN standard [5] is chosen to be 90% for a

[2] We consider here a similar scenario to spectrum sharing cognitive radio networks, where the secondary users are able to decode the received secondary signal irrespective of the status of the primary users.

signal-to-noise ratio (SNR) as low as -20 dB for the primary user's signal at the secondary detector. We denote this target detection probability in the following by $\bar{\mathscr{P}}_d$.

More specifically, we consider as in [12] the energy detection scheme [21] as a spectrum sensing technique, in order to determine the status (active/idle) of the frequency band. The detection and false alarm probability under the energy detection scheme are given by

$$\mathscr{P}_d = \mathcal{Q}\left(\left(\frac{\epsilon}{\sigma_n^2} - \gamma - 1\right)\sqrt{\frac{\tau f_s}{2\gamma+1}}\right), \tag{3}$$

$$\begin{aligned}\mathscr{P}_{fa} &= \mathcal{Q}\left(\left(\frac{\epsilon}{\sigma_n^2} - 1\right)\sqrt{\tau f_s}\right) = \\ &= \mathcal{Q}\left(\sqrt{2\gamma+1}\mathcal{Q}^{-1}\left(\mathscr{P}_d\right) + \sqrt{\tau f_s}\gamma\right),\end{aligned} \tag{4}$$

respectively [12], where ϵ denotes the decision threshold of the energy detector, γ the received signal-to-noise ratio (SNR) from the primary user at the secondary detector, τ denotes the sensing time and finally f_s represents the sampling frequency. For a given target detection probability $\mathscr{P}_d = \bar{\mathscr{P}}_d$, the decision threshold ϵ is given by

$$\epsilon = \sigma_n^2\left(\sqrt{\frac{2\gamma+1}{\tau f_s}}\mathcal{Q}^{-1}(\bar{\mathscr{P}}_d) + \gamma + 1\right). \tag{5}$$

In the following proposition, we show that the probability of false alarm $\mathscr{P}_{fa}$ of the energy detection given by equation (4) is an increasing and concave function of the probability of detection $\mathscr{P}_d$ for $\mathscr{P}_d \geq 0.5$, two properties that will be discussed further in our analysis.

Proposition 1: The probability of false alarm $\mathscr{P}_{fa}$ under the energy detection scheme given by equation (4) is an increasing function of the probability of detection $\mathscr{P}_d$ and is also a concave function of the probability of detection $\mathscr{P}_d$ for $\mathscr{P}_d \geq 0.5$.

Proof: See Appendix A. ■

We can now focus on the average achievable throughput of the cognitive radio system. The instantaneous transmission rate of the cognitive radio system when the frequency band is actually idle (H_0) is given by

$$r_0 = \log_2\left(1 + \frac{gP}{\sigma_n^2}\right). \tag{6}$$

However, considering the fact that perfect spectrum sensing may not be achievable in practice due to the nature of wireless communications that includes phenomena such as shadowing and fading, we consider the more realistic scenario of imperfect spectrum sensing, where the actual status of the primary users might be falsely detected. Therefore, in this paper, we also consider the case that the frequency band is falsely detected to be idle, when in fact it is active (H_1). Following the approach in [15], [22], the instantaneous transmission rate in this case is given by

$$r_1 = \log_2\left(1 + \frac{gP}{\sigma_n^2 + \sigma_p^2}\right), \tag{7}$$

where σ_p^2 denotes the received power from the primary users.

The average achievable throughput of the cognitive radio system that operates based on the conventional frame structure of Fig. 1 is given by

$$\bar{R}\left(\tau\right) = \bar{R}_0\left(\tau\right) + \bar{R}_1\left(\tau\right), \tag{8}$$

where $\bar{R}_0(\tau)$ and $\bar{R}_1(\tau)$ are given by

$$\bar{R}_0\left(\tau\right) = \frac{T-\tau}{T}\mathscr{P}\left(H_0\right)\left(1 - \mathscr{P}_{fa}\left(\tau\right)\right)r_0, \tag{9}$$

$$\bar{R}_1\left(\tau\right) = \frac{T-\tau}{T}\mathscr{P}\left(H_1\right)\left(1 - \mathscr{P}_{d}\left(\tau\right)\right)r_1, \tag{10}$$

respectively. In the equations above, T represents the frame duration, $\mathscr{P}(H_0)$ the probability that the frequency band is idle, and $\mathscr{P}(H_1)$ the probability that the frequency band is active.

Under the proposed cognitive radio system, spectrum sensing is performed simultaneously with data transmission, whereas the sensing time and data transmission time are equal to the frame duration T, as seen in Fig. 4. Therefore, the average achievable throughput of the proposed cognitive radio system is given by

$$\bar{C} = \bar{C}_0 + \bar{C}_1, \tag{11}$$

where $\bar{C}_0$ and $\bar{C}_1$ denote the average achievable throughput when the frequency band is actually idle and active (but falsely detected to be idle), respectively, and are given by

$$\bar{C}_0 = \mathscr{P}\left(H_0\right)\left(1 - \mathscr{P}_{fa}\left(T\right)\right)r_0, \tag{12}$$

$$\bar{C}_1 = \mathscr{P}\left(H_1\right)\left(1 - \mathscr{P}_{d}\left(T\right)\right)r_1, \tag{13}$$

respectively.

For a target probability of detection $\bar{\mathscr{P}}_d$, we can now show that the proposed cognitive radio system exhibits higher average achievable throughput compared to the cognitive radio system that operates based on the conventional frame structure shown in Fig. 1. Following the FCC requirements in [4], the secondary users should detect a worst-case SNR from the primary users, regardless if the spectrum sensing is performed at the receiver or the transmitter. This worst-case SNR is denoted here by $\bar{\gamma}$. From the classical detection theory [10], [11], it is known that for a target probability of detection $\bar{\mathscr{P}}_d$, the higher the sensing time, the lower the probability of false alarm $\mathscr{P}_{fa}$. Therefore, for a target probability of detection $\mathscr{P}_d = \bar{\mathscr{P}}_d$ and sensing time $0 < \tau \leq T$, it results from the equation (4) that

$$\begin{aligned}\mathscr{P}_{fa}(\tau) =&\mathcal{Q}\left(\sqrt{2\bar{\gamma}+1}\mathcal{Q}^{-1}\left(\bar{\mathscr{P}}_d\right) + \sqrt{\tau f_s}\bar{\gamma}\right)\\ \geq&\mathcal{Q}\left(\sqrt{2\bar{\gamma}+1}\mathcal{Q}^{-1}\left(\bar{\mathscr{P}}_d\right) + \sqrt{T f_s}\bar{\gamma}\right) = \mathscr{P}_{fa}(T),\end{aligned} \tag{14}$$

considering the fact that the complementary cumulative distribution function of the standard Gaussian $\mathcal{Q}\left(x\right)$ is a decreasing function of x. As a result, for a sensing time $0 < \tau \leq T$, it results from the equations (8)-(14) that

$$\begin{aligned}\bar{R}(\tau) =&\bar{R}_0(\tau) + \bar{R}_1(\tau) = \frac{T-\tau}{T}\mathscr{P}(H_0)(1-\mathscr{P}_{fa}(\tau))r_0 + \\ &+\frac{T-\tau}{T}\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)r_1 \\ &< \mathscr{P}(H_0)(1-\mathscr{P}_{fa}(\tau))r_0 + \mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)r_1 \\ &\leq \mathscr{P}(H_0)(1-\mathscr{P}_{fa}(T))r_0 + \mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)r_1 \\ &= \bar{C}_0 + \bar{C}_1 = \bar{C}, \end{aligned} \tag{15}$$

i.e. that the average achievable throughput of the proposed cognitive radio system for a target detection probability $\mathscr{P}_d = \bar{\mathscr{P}}_d$ is higher compared to the respective of the cognitive radio system that employs the frame structure depicted in Fig. 1, namely it results that

$$\bar{C} > \bar{R}(\tau) \tag{16}$$

for a sensing time $0 < \tau \leq T$.

IV. Ergodic Throughput Maximization of the Proposed Cognitive Radio System under Average Transmit and Interference Power Constraints

In this section, we study the problem of determining the optimal power allocation strategy that maximizes the ergodic throughput of the proposed cognitive radio network under joint average transmit and interference power constraints. In order to keep the long-term power budget and effectively protect the primary users from harmful interference, we consider similar to [14], [17], [23] an average (over all different fading states) transmit and interference power constraint that can be formulated as follows

$$\mathbb{E}_{g,h}\left\{\mathscr{P}(H_0)(1-\mathscr{P}_{fa})P + \mathscr{P}(H_1)(1-\mathscr{P}_d)P\right\} \leq P_{av}, \tag{17}$$

$$\mathbb{E}_{g,h}\left\{\mathscr{P}(H_1)(1-\mathscr{P}_d)hP\right\} \leq \Gamma. \tag{18}$$

Here, P_{av} denotes the maximum average transmit power of the secondary users and Γ the maximum average interference power that is tolerable by the primary users. Similar to the previous section, the energy detection scheme is considered here as a method of spectrum sensing, whereas the detection and false alarm probability are now given by

$$\mathscr{P}_d = \mathcal{Q}\left(\left(\frac{\epsilon}{\sigma_n^2} - \gamma - 1\right)\sqrt{\frac{Tf_s}{2\gamma+1}}\right), \tag{19}$$

$$\mathscr{P}_{fa}(\mathscr{P}_d) = \mathcal{Q}\left(\sqrt{2\gamma+1}\mathcal{Q}^{-1}(\mathscr{P}_d) + \sqrt{Tf_s}\gamma\right). \tag{20}$$

As a result, the optimization problem that maximizes the ergodic throughput of the proposed opportunistic spectrum access cognitive radio system under joint average transmit and interference power constraints can be formulated as follows

$$\begin{aligned}\underset{\{P,\mathscr{P}_d\}}{\text{maximize}}\quad & C(P,\mathscr{P}_d) = \mathbb{E}_{g,h}\left\{\mathscr{P}(H_0)(1-\mathscr{P}_{fa}(\mathscr{P}_d))\cdot\right. \\ &\left.\log_2\left(1+\frac{gP}{\sigma_n^2}\right) + \mathscr{P}(H_1)(1-\mathscr{P}_d)\log_2\left(1+\frac{gP}{\sigma_n^2+\sigma_p^2}\right)\right\} \end{aligned} \tag{21}$$

$$\text{subject to}:\ (17),\ (18),\ P \geq 0,\ 0 \leq \mathscr{P}_d \leq 1.$$

By considering an average interference power constraint similar to spectrum sharing cognitive radio networks [8], the probability of detection $\mathscr{P}_d$ becomes an optimization variable in the problem of maximizing the achievable ergodic throughput of the proposed cognitive radio system. The dependance of the ergodic throughput on the probability of detection $\mathscr{P}_d$ can be better observed in the simulation results presented in Section V.

Now returning to the optimization problem (21), it can be seen that the problem (21) is convex with respect to the transmit power P, but not with respect to the probability of detection $\mathscr{P}_d$ due to the dependance of the probability of false alarm $\mathscr{P}_{fa}$ on the probability of detection $\mathscr{P}_d$ [24]. Therefore, the optimal target detection probability can not be obtained using convex optimization techniques, but by taking into consideration that the detection probability lies in the interval $[0,1]$, it can be easily obtained using one-dimensional exhaustive search. As a result, we will focus in the following on determining the optimal power allocation strategy that maximizes the ergodic throughput of the proposed opportunistic spectrum access cognitive radio system for a target probability of detection $\mathscr{P}_d = \bar{\mathscr{P}}_d$, whereas the optimal value of the latter will be found using one-dimensional exhaustive search in the interval $[0,1]$, as seen in Algorithm 1 in the following.

The Lagrangian with respect to the transmit power P for a target detection probability $\mathscr{P}_d = \bar{\mathscr{P}}_d$ is given by

$$\begin{aligned} L(P,\lambda,\mu) =& \mathbb{E}_{g,h}\left\{\mathscr{P}(H_0)(1-\mathscr{P}_{fa}(\bar{\mathscr{P}}_d))\log_2\left(1+\frac{gP}{\sigma_n^2}\right)\right. \\ &\left.+\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)\log_2\left(1+\frac{gP}{\sigma_n^2+\sigma_p^2}\right)\right\} - \lambda\left[\mathbb{E}_{g,h}\left\{P\cdot\right.\right. \\ &\left.\left.\cdot\mathscr{P}(H_0)\left(1-\mathscr{P}_{fa}\left(\bar{\mathscr{P}}_d\right)\right) + \mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)P\right\} - P_{av}\right] \\ &-\mu\left[\mathbb{E}_{g,h}\left\{\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)hP\right\} - \Gamma\right]. \end{aligned} \tag{22}$$

The Lagrange dual optimization problem is now given by

$$\underset{\lambda\geq 0,\ \mu\geq 0}{\text{minimize}}\quad g(\lambda,\mu), \tag{23}$$

where $g(\lambda,\mu)$ denotes the Lagrange dual function that is given by the following equation

$$g(\lambda,\mu) = \sup_{P} L(P,\lambda,\mu). \tag{24}$$

It can be seen from (21) that the primal optimization problem with respect to the transmit power P is convex with linear inequality constraints and that Slater's condition holds [24]. Therefore, the difference between the optimal value of the objective function of the primal and dual optimization problem (namely the optimal duality gap) is zero, which guarantees [24] that the primal optimization problem (21) with respect to the transmit power P can be equivalently solved by the dual optimization problem (23). We therefore focus in the following on solving the Lagrange dual optimization problem (23).

In order to calculate the Lagrange dual function $g(\lambda,\mu)$, we need to find the supremum of the Lagrangian $L(P,\lambda,\mu)$ with respect to the transmit power P, as seen from the equation (24). By applying the Karush-Kuhn-Tucker (KKT) conditions

$$A = \frac{\log_2(e)\left[\mathscr{P}(H_0)\left(1-\mathscr{P}_{fa}\left(\bar{\mathscr{P}}_d\right)\right)+\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)\right]}{\lambda\left[\mathscr{P}(H_0)\left(1-\mathscr{P}_{fa}\left(\bar{\mathscr{P}}_d\right)\right)+\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)\right]+\mu\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)h} - \frac{2\sigma_n^2+\sigma_p^2}{g}, \tag{26}$$

$$\Delta = A^2 + \frac{4}{g}\left\{\frac{\log_2(e)\left[\mathscr{P}(H_0)\left(1-\mathscr{P}_{fa}\left(\bar{\mathscr{P}}_d\right)\right)\left(\sigma_n^2+\sigma_p^2\right)+\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)\sigma_n^2\right]}{\lambda\left[\mathscr{P}(H_0)\left(1-\mathscr{P}_{fa}\left(\bar{\mathscr{P}}_d\right)\right)+\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)\right]+\mu\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)h} - \frac{\sigma_n^2\left(\sigma_n^2+\sigma_p^2\right)}{g}\right\}, \tag{27}$$

$$\hat{P} = \left\{\frac{\log_2(e)\left[\mathscr{P}(H_0)\left(1-\mathscr{P}_{fa}(\hat{\mathscr{P}}_d)\right)+\mathscr{P}(H_1)\left(1-\hat{\mathscr{P}}_d\right)\right]}{\lambda\left[\mathscr{P}(H_0)\left(1-\mathscr{P}_{fa}(\hat{\mathscr{P}}_d)\right)+\mathscr{P}(H_1)\left(1-\hat{\mathscr{P}}_d\right)\right]+\mu\mathscr{P}(H_1)\left(1-\hat{\mathscr{P}}_d\right)h} - \frac{\sigma^2}{g}\right\}^+. \tag{28}$$

[24], the optimal power allocation P for given Lagrange multipliers λ and μ can be obtained by

$$P = \left[\frac{A+\sqrt{\Delta}}{2}\right]^+, \tag{25}$$

where the parameters A and Δ can be found at the top of this page and $[x]^+$ denotes $\max(0,x)$.

In order to determine the optimal power allocation strategy for the proposed cognitive radio system, the optimal values of the Lagrangian multipliers λ and μ that minimize the dual function $g(\lambda,\mu)$ need to be found. The ellipsoid method [25] is used here to find the optimal solution, which requires the subgradient [26] of the dual function $g(\lambda,\mu)$. The latter is given by the following proposition.

Proposition 2: The subgradient of the dual function $g(\lambda,\mu)$ is $[D,E]$, where D is given by $D = P_{av} - \mathbb{E}_{g,h}\left\{\mathscr{P}(H_0)\left(1-\mathscr{P}_{fa}\left(\bar{\mathscr{P}}_d\right)\right)P+\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)P\right\}$, and E is given by $E = \Gamma - \mathbb{E}_{g,h}\left\{\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)hP\right\}$, where $\lambda \geq 0$, $\mu \geq 0$, and P denotes the optimal power allocation for fixed λ, μ.

Proof: See Appendix B. ■

The algorithm that acquires the optimal target detection probability and power allocation strategy of the proposed opportunistic spectrum access cognitive radio system is presented in the following table.

Algorithm 1: Optimal detection probability and power allocation for the proposed opportunistic spectrum access cognitive radio system.

► For $\bar{\mathscr{P}}_d = 0:1$
1) Initialize λ, μ.
2) Repeat:
- calculate P using (25)-(27);
- update λ, μ using the ellipsoid method;
3) Until λ, μ converge.
► End.
► Optimal detection probability and power allocation:
$\bar{\mathscr{P}}_d^{opt} = \arg\max C\left(\bar{\mathscr{P}}_d, P\right)$ and $P_{opt} = \{P\}_{\bar{\mathscr{P}}_d = \bar{\mathscr{P}}_d^{opt}}$

Furthermore, we additionally study in this section the problem where the received power from the primary users at the secondary receiver (σ_p^2) is considered to be unknown at the secondary transmitter. By considering a single noise power σ^2, the ergodic throughput maximization problem under this scenario can be formulated as follows

$$\underset{\{\hat{P},\hat{\mathscr{P}}_d\}}{\text{maximize}}\ \mathscr{C}(\hat{P},\hat{\mathscr{P}}_d) = \mathbb{E}_{g,h}\Bigg\{\mathscr{P}(H_0)\left(1-\mathscr{P}_{fa}(\hat{\mathscr{P}}_d)\right)\cdot \cdot\log_2\left(1+\frac{g\hat{P}}{\sigma^2}\right)+\mathscr{P}(H_1)\left(1-\hat{\mathscr{P}}_d\right)\log_2\left(1+\frac{g\hat{P}}{\sigma^2}\right)\Bigg\} \tag{29}$$

$$\text{subject to: } \mathbb{E}_{g,h}\left\{\mathscr{P}(H_1)\left(1-\hat{\mathscr{P}}_d\right)h\hat{P}\right\} \leq \Gamma,$$
$$\mathbb{E}_{g,h}\Bigg\{\mathscr{P}(H_0)\left(1-\mathscr{P}_{fa}(\hat{\mathscr{P}}_d)\right)\hat{P}+ +\mathscr{P}(H_1)(1-\hat{\mathscr{P}}_d)\hat{P}\Bigg\} \leq P_{av},$$
$$\hat{P} \geq 0,\ 0 \leq \hat{\mathscr{P}}_d \leq 1.$$

By using a similar analysis to the one considered for the case when the received power from the primary users is considered to be known, the optimal power allocation $\hat{P}$ under fixed Lagrange multipliers λ and μ for this scenario can be obtained from the equation (28). The latter can be found at the top of this page. As a result, the optimal target detection probability and power allocation strategy that maximizes the ergodic throughput for the scenario, where the received power from the primary users at the secondary receiver is consider to be unknown, can be obtained from Algorithm 1, by using the equation (28) in the place of the equations (25)-(27).

V. Simulation Results

In this section, we present the simulation results for the proposed opportunistic spectrum access cognitive radio system using the energy detection scheme as a spectrum sensing technique. The frame duration is set to $T = 100$ ms, the probability that the frequency band is idle is considered to be $\mathscr{P}(H_0) = 0.6$, whereas the sampling frequency f_s is assumed to be 6 MHz. The channels g and h are assumed to follow the Rayleigh fading model and more specifically, they are the squared norms of independent CSCG random variables that are distributed as $\mathcal{CN}(0,1)$ and $\mathcal{CN}(0,10)$, respectively. The average tolerable interference power at the primary receiver is considered to be $\Gamma = 1$ and the received SNR from the primary user is considered to be $\gamma = -20$ dB. As in [14], an additional channel power gain attenuation is considered here for the channel h between the secondary transmitter and the primary receiver, where an attenuation of 10 dB for example, means that $\mathbb{E}\{h\} = 1$.

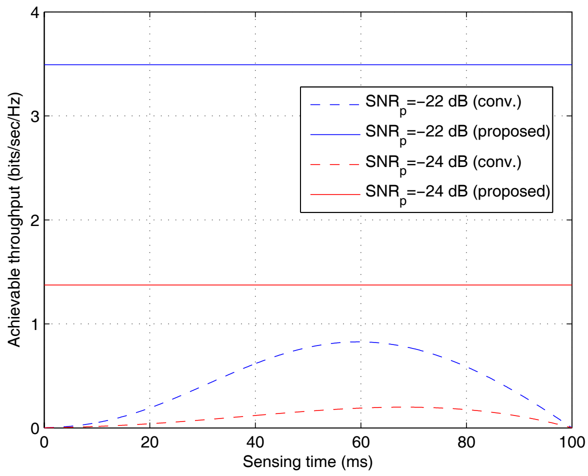

Fig. 5. Average achievable throughput of the proposed and conventional opportunistic spectrum access cognitive radio system versus the sensing time τ, for various values of the target detection SNR from the primary user ($\mathrm{SNR_p}$) and for a target detection probability $\bar{\mathscr{P}}_d = 99.99\%$.

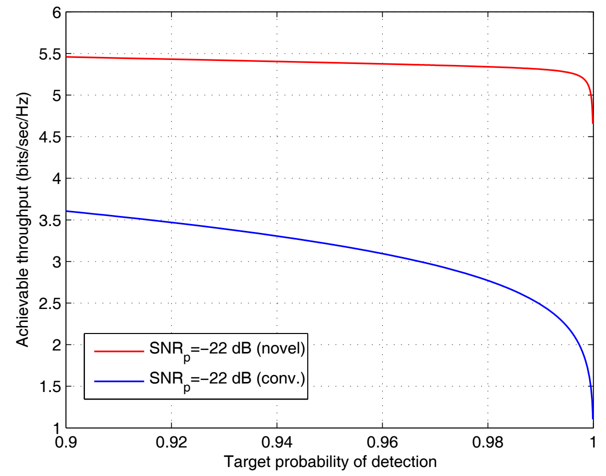

Fig. 6. Average achievable throughput of the proposed and conventional opportunistic spectrum access cognitive radio system versus the target probability of detection $\bar{\mathscr{P}}_d$ for various values of the target detection SNR from the primary user ($\mathrm{SNR_p}$).

In Fig. 5, the average achievable throughput versus the sensing time τ is presented for the proposed cognitive radio system (solid line) and the cognitive radio system that employs the conventional frame structure of Fig. 1 (dashed line), for the case of a single high target detection probability constraint that was studied in Section III. The received signal-to-noise ratio (SNR) from the secondary transmitter at the secondary receiver is considered to be $SNR_s = 20$ dB as in [12], the target probability of detection is set to $\bar{\mathscr{P}}_d = 99.99\%$, in order to effectively protect the primary users from harmful interference, whereas different values of the target detection signal-to-noise ratio from the primary user (denoted by $\mathrm{SNR_p}$) are presented. One can clearly see that the average achievable throughput of the proposed cognitive radio system (solid line) is significantly higher compared to the respective achievable throughput of the cognitive radio system that employs the conventional frame structure of Fig. 1 (dashed line). This throughput improvement can be explained by the fact that the whole duration of the frame T is used for data transmission, as opposed to the conventional frame structure of Fig. 1, where only a part of the frame is used for data transmission (i.e. $T - \tau$). Moreover, the improved sensing capabilities of the proposed cognitive radio system also contribute to the throughput improvement of the cognitive radio system by enabling a more efficient usage of the available unused spectrum. More specifically, it can be seen from Fig. 5 and the equation (4) that for the same target probability of detection $\bar{\mathscr{P}}_d$, the probability of false alarm $\mathscr{P}_{fa}$ for the optimal sensing time under the conventional frame structure is higher compared to the respective false alarm probability of the proposed cognitive radio system. The latter remark can be explained by the fact that the whole duration of the frame T is used for spectrum sensing in the proposed system, as opposed to merely a part of the frame under the conventional frame structure of Fig. 1.

In Fig. 6, the average achievable throughput is presented versus the target probability of detection $\bar{\mathscr{P}}_d$, for a target detection signal-to-noise ratio from the primary user equal to $\mathrm{SNR_p} = -22$ dB. It can be clearly seen from Fig. 6 that the average achievable throughput under the proposed cognitive radio system is significantly higher compared to the respective achievable throughput of the system that employs the frame structure presented in Fig. 1, whereas the decrease in the average achievable throughput as the target probability of detection $\bar{\mathscr{P}}_d$ receives higher values is small, especially compared to the respective of the secondary users that employ the conventional frame structure of Fig. 1. This means that the proposed cognitive radio system can provide better protection for the primary users on the one hand, while achieving an increased throughput for its users on the other, even for very high values of target detection probability and very weak signals from the primary users. This can be further seen from Fig. 7, where the average achievable throughput is presented versus the target detection signal-to-noise ratio

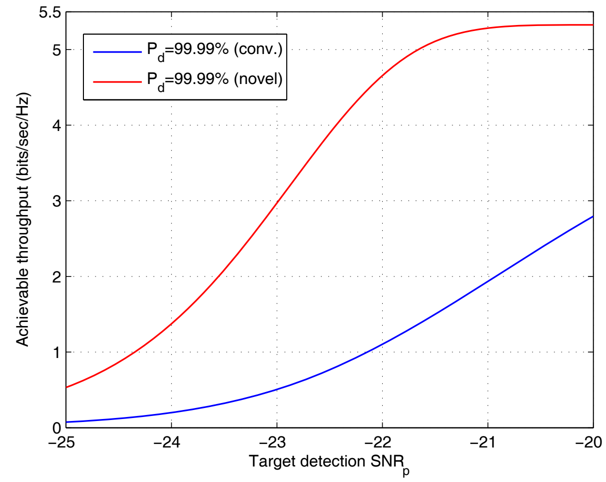

Fig. 7. Average achievable throughput of the proposed and conventional opportunistic spectrum access cognitive radio system versus the target detection SNR from the primary user ($\mathrm{SNR_p}$) for a target detection probability $\bar{\mathscr{P}}_d = 99.99\%$.

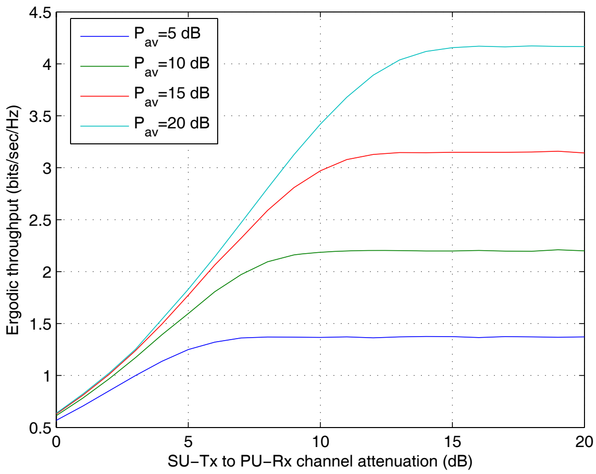

Fig. 8. Ergodic throughput of the proposed cognitive radio system versus the additional channel power gain attenuation for different values of average transmit power P_{av} and target detection probability $\bar{\mathscr{P}}_d = 90\%$.

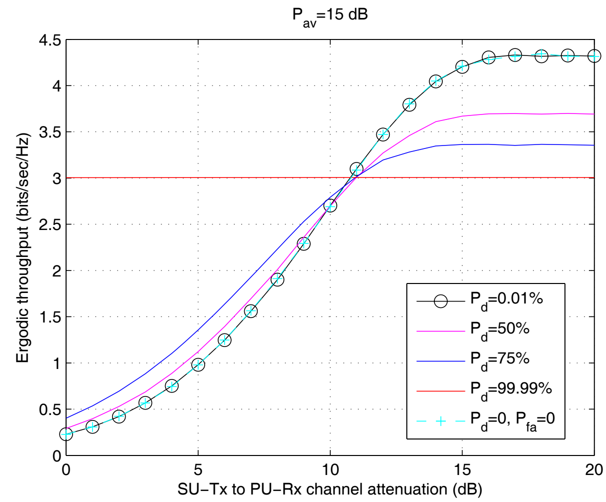

Fig. 9. Ergodic throughput of the proposed cognitive radio system versus the additional channel power gain attenuation for different values of target detection probability $\bar{\mathscr{P}}_d$ and average transmit power $P_{av} = 15$ dB.

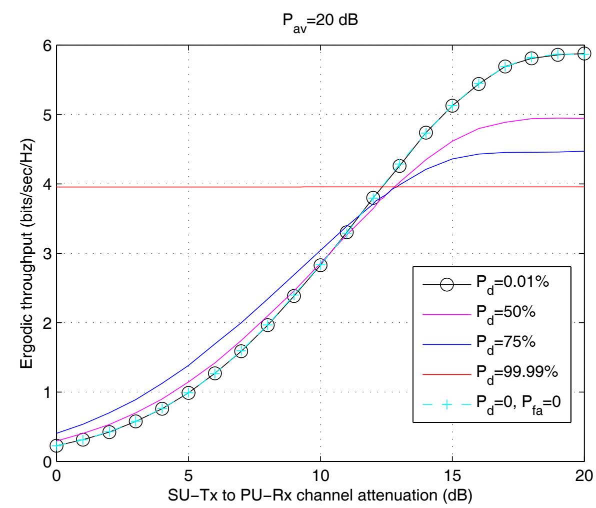

Fig. 10. Ergodic throughput of the proposed cognitive radio system versus the additional channel power gain attenuation for different values of target detection probability $\bar{\mathscr{P}}_d$ and average transmit power $P_{av} = 20$ dB.

from the primary users ($\mathrm{SNR_p}$), for a target probability of detection equal to $\bar{\mathscr{P}}_d = 99.99\%$.

In Fig. 8, the ergodic throughput of the proposed cognitive radio system is presented versus the additional channel power gain attenuation between the secondary transmitter (SU-Tx) and the primary receiver (PU-Rx) for different values of the average transmit power P_{av} of the secondary user and for a target detection probability $\bar{\mathscr{P}}_d = 90\%$. It can be clearly seen from Fig. 8 that for all values of the average transmit power P_{av}, the achievable ergodic throughput increases as the channel power gain attenuation between the secondary transmitter and the primary receiver obtains higher values. This can be easily explained by the fact that as the channel power gain attenuation increases, the average interference power constraint allows the use of higher transmit power P, which leads to an increased achievable ergodic throughput for the cognitive radio system. Furthermore, it can be seen in Fig. 8 that the achievable ergodic throughput reaches a maximum for all values of the average transmit power constraint P_{av}, whereas the point that this maximum is achieved, depends on the value of the average transmit power P_{av}. As the average transmit power P_{av} increases, this point is reached for higher values of channel power gain attenuation, which is due to the increased transmit power P that is available to the cognitive radio users, in combination with the imposed interference power constraint.

In Fig. 9, the ergodic throughput of the proposed cognitive radio system is presented versus the additional channel power gain attenuation between the secondary transmitter and the primary receiver for different values of target detection probability $\bar{\mathscr{P}}_d$ and for an average transmit power of the secondary user equal to 15 dB. In this figure, several interesting results can be observed regarding the achievable ergodic throughput of the cognitive radio system. Firstly, the optimal detection probability up to a certain point (i.e. around 10 dB) is approximately equal to 1, which shows that according to the optimal power allocation formulas, no knowledge of the channel power gain h is required to obtain the maximum throughput. Secondly, beyond that point, an imposed high detection probability $\bar{\mathscr{P}}_d = 99.99\%$ not only does *not* provide better protection for the primary users, but it also has a negative effect on the achievable ergodic throughput of the secondary system.[3] It can be clearly seen from Fig. 9 that lower values of target detection probability lead to higher achievable ergodic throughput, whereas the optimal value of the target detection probability appears to be zero. This interesting result can be explained by the fact that after a certain value of channel power gain attenuation, the average transmit power constraint becomes the dominant constraint on the optimal power allocation process, as opposed to the average interference power constraint that was dominant before. As a result, a lower target detection probability while

[3]The proposed cognitive radio system can allocate the power without any knowledge of the channel h (as seen by the red solid line in Fig. 9), however, it will not achieve the maximum throughput beyond the value of around 10 dB (as seen by the cyan dashed line in Fig. 9).

satisfying the average interference power constraint on the one hand, leads to a lower false alarm probability on the other. This in return leads to higher allocated transmit power P for the secondary users (as seen from the inequality (17)) and therefore to higher ergodic throughput for the cognitive radio system. Based on the latter remark that after a certain value of channel power gain attenuation, the optimal target detection probability turns out to be zero, the next logical step is to consider shutting down the spectrum sensing unit, namely consider $\mathscr{P}_d = 0$ and $\mathscr{P}_{fa} = 0$. As seen from Fig. 9, it turns out that after a certain value of channel power gain attenuation, shutting down the spectrum sensing unit leads to the highest achievable ergodic throughput and therefore becomes the optimal spectrum access mode for the cognitive radio system. Viewing this from a different perspective, it indicates that beyond a certain point, the opportunistic spectrum access scheme is actually suboptimal compared to the spectrum sharing scheme (i.e. when $\mathscr{P}_d = 0$ and $\mathscr{P}_{fa} = 0$).

Finally, in Fig. 10, the ergodic throughput of the proposed cognitive radio system is presented versus the additional channel power gain attenuation for different values of target detection probability $\bar{\mathscr{P}}_d$, and for a higher (compared to Fig. 9) average transmit power equal to $P_{av} = 20$ dB. Comparing Fig. 9 and Fig. 10, an interesting observation can be made: the value of the channel power gain attenuation after which the average transmit power constraint becomes dominant is larger as the average transmit power increases. This can be explained by the fact that for higher values of average transmit power P_{av}, the average transmit power constraint becomes dominant in higher values of channel power gain attenuation due to the increased transmit power that is available to the cognitive radio users, but which is restricted for the protection of the primary users by the average interference power constraint. This is also in accordance with Fig. 8, which illuminates this result from a different angle.

VI. Conclusions

In this paper, we proposed a novel cognitive radio system that significantly improves the achievable throughput of opportunistic spectrum access cognitive radio systems by performing data transmission and spectrum sensing at the same time. More specifically, we studied the average achievable throughput of the proposed cognitive radio system under a single high target detection probability constraint and showed that it can achieve significantly improved throughput compared to the respective conventional cognitive radio systems. In addition, we studied the problem of maximizing the ergodic throughput under joint average transmit and interference power constraints, and proposed an algorithm that acquires the optimal target detection probability and power allocation strategy that maximizes the ergodic throughput of the proposed cognitive radio system. Furthermore, we provided simulation results, which revealed that for low values of channel power gain attenuation between the secondary transmitter and the primary receiver, a high target detection probability ($\bar{\mathscr{P}}_d \simeq 1$) leads to the maximum achievable ergodic throughput, whereas for higher values of channel power gain attenuation, spectrum sensing not only does *not* provide better protection for the primary users, but it also has a negative effect on the achievable ergodic throughput of the cognitive radio system and should therefore be avoided. Finally, in our future research we plan to extend this work for the case of imperfect secondary signal subtraction at the cognitive radio receiver.

Appendix A
Proof of Proposition 1

By setting $\alpha = \sqrt{2\gamma + 1}$ and $\beta = \sqrt{\tau f_s}\gamma$ in equation (4), the false alarm probability $\mathscr{P}_{fa}$ is now given by

$$\mathscr{P}_{fa}(\mathscr{P}_d) = \mathcal{Q}\left(\alpha \mathcal{Q}^{-1}(\mathscr{P}_d) + \beta\right).$$

In order to prove that the probability of false alarm $\mathscr{P}_{fa}$ is an increasing function of the probability of detection $\mathscr{P}_d$, we take the derivative of the probability of false alarm with respect to the probability of detection. The latter is given by

$$\begin{aligned}\frac{d\mathscr{P}_{fa}}{d\mathscr{P}_d} =& \frac{d}{d\mathscr{P}_d}\Big[\mathcal{Q}\left(\alpha\mathcal{Q}^{-1}(\mathscr{P}_d)+\beta\right)\Big] = \\ =& -\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{\left[\alpha\mathcal{Q}^{-1}(\mathscr{P}_d)+\beta\right]^2}{2}\right\}\cdot \\ &\cdot\frac{d}{d\mathscr{P}_d}\Big[\alpha\mathcal{Q}^{-1}(\mathscr{P}_d)+\beta\Big] = \\ =& -\frac{\alpha}{\sqrt{2\pi}}\exp\left\{-\frac{\left[\alpha\mathcal{Q}^{-1}(\mathscr{P}_d)+\beta\right]^2}{2}\right\}\cdot \\ &\cdot\frac{d\mathcal{Q}^{-1}(\mathscr{P}_d)}{d\mathscr{P}_d}.\end{aligned} \tag{30}$$

Considering that

$$\mathcal{Q}^{-1}(\mathscr{P}_d) = \sqrt{2}\mathrm{erf}^{-1}(1-2\mathscr{P}_d),$$

we have

$$\begin{aligned}\frac{d\mathcal{Q}^{-1}(\mathscr{P}_d)}{d\mathscr{P}_d} &= \sqrt{2}\cdot\frac{d\left[\mathrm{erf}^{-1}(1-2\mathscr{P}_d)\right]}{d\mathscr{P}_d} = \\ &= -\sqrt{2\pi}\exp\left\{\left[\mathrm{erf}^{-1}(1-2\mathscr{P}_d)\right]^2\right\}.\end{aligned} \tag{31}$$

Therefore, from the equations (30) and (31), it results that

$$\begin{aligned}\frac{d\mathscr{P}_{fa}}{d\mathscr{P}_d} = \alpha\cdot\exp\Bigg\{&\left[\mathrm{erf}^{-1}(1-2\mathscr{P}_d)\right]^2 - \\ &-\frac{1}{2}\cdot\left[\alpha\mathcal{Q}^{-1}(\mathscr{P}_d)+\beta\right]^2\Bigg\}.\end{aligned} \tag{32}$$

Since $\alpha = \sqrt{2\gamma+1} > 0$, it results from (32) that

$$\frac{d\mathscr{P}_{fa}}{d\mathscr{P}_d} \geq 0$$

and therefore the probability of false alarm $\mathscr{P}_{fa}(\mathscr{P}_d)$ is an increasing function of the probability of detection $\mathscr{P}_d$.

Now, by taking the second derivative of the false alarm probability $\mathscr{P}_{fa}$ with respect to the detection probability $\mathscr{P}_d$, we have

$$\begin{aligned}\frac{d^2\mathscr{P}_{fa}}{d\mathscr{P}_d^2} =& \frac{\alpha^2[\alpha\mathcal{Q}^{-1}(\mathscr{P}_d)+\beta]}{\sqrt{2\pi}}\left[\frac{d\mathcal{Q}^{-1}(\mathscr{P}_d)}{d\mathscr{P}_d}\right]^2\cdot \\ &\cdot\exp\left\{-\frac{[\alpha\mathcal{Q}^{-1}(\mathscr{P}_d)+\beta]^2}{2}\right\} - \frac{d^2\mathcal{Q}^{-1}(\mathscr{P}_d)}{d\mathscr{P}_d^2}\cdot \\ &\cdot\frac{\alpha}{\sqrt{2\pi}}\exp\left\{-\frac{\left[\alpha\mathcal{Q}^{-1}(\mathscr{P}_d)+\beta\right]^2}{2}\right\},\end{aligned} \tag{33}$$

where

$$\begin{aligned}\frac{d^2\mathcal{Q}^{-1}(\mathscr{P}_d)}{d\mathscr{P}_d^2} &= \frac{d}{d\mathscr{P}_d}\left(-\sqrt{2\pi}\exp\left\{\left[\operatorname{erf}^{-1}(1-2\mathscr{P}_d)\right]^2\right\}\right)\\ &= -\sqrt{2\pi}\exp\left\{\left[\operatorname{erf}^{-1}(1-2\mathscr{P}_d)\right]^2\right\}\cdot\\ &\cdot\frac{d}{d\mathscr{P}_d}\left(\left[\operatorname{erf}^{-1}(1-2\mathscr{P}_d)\right]^2\right) =\\ &= -2\sqrt{2\pi}\left[\operatorname{erf}^{-1}(1-2\mathscr{P}_d)\right]\exp\left\{\left[\operatorname{erf}^{-1}(1-2\mathscr{P}_d)\right]^2\right\}\cdot\\ &\cdot\frac{d}{d\mathscr{P}_d}\left(\operatorname{erf}^{-1}(1-2\mathscr{P}_d)\right) =\\ &= 2\sqrt{2}\pi\left[\operatorname{erf}^{-1}(1-2\mathscr{P}_d)\right]\exp\left\{2\left[\operatorname{erf}^{-1}(1-2\mathscr{P}_d)\right]^2\right\}.\end{aligned} \tag{34}$$

Thus, it results from the equations (31), (33) and (34), that the second derivative of the false alarm probability $\mathscr{P}_{fa}$ with respect to the detection probability $\mathscr{P}_d$ is finally given by

$$\begin{aligned}\frac{d^2\mathscr{P}_{fa}}{d\mathscr{P}_d^2} &= \left\{\alpha\left[\alpha\mathcal{Q}^{-1}(\mathscr{P}_d)+\beta\right]-\sqrt{2}\operatorname{erf}^{-1}(1-2\mathscr{P}_d)\right\}\cdot\\ &\alpha\sqrt{2\pi}\exp\left\{\frac{4\left[\operatorname{erf}^{-1}(1-2\mathscr{P}_d)\right]^2-\left[\alpha\mathcal{Q}^{-1}(\mathscr{P}_d)+\beta\right]^2}{2}\right\}\\ &= \alpha\sqrt{2\pi}\left[\left(\alpha^2-1\right)\mathcal{Q}^{-1}(\mathscr{P}_d)+\alpha\beta\right]\cdot\\ &\cdot\exp\left\{\frac{4\left[\operatorname{erf}^{-1}(1-2\mathscr{P}_d)\right]^2-\left[\alpha\mathcal{Q}^{-1}(\mathscr{P}_d)+\beta\right]^2}{2}\right\}.\end{aligned} \tag{35}$$

For a target detection probability

$$\mathscr{P}_d \geq \mathcal{Q}\left(-\frac{\alpha\beta}{\alpha^2-1}\right) \geq 0.5,$$

the second derivative of the false alarm probability $\mathscr{P}_{fa}$ with respect to the detection probability $\mathscr{P}_d$ from (35) turns out to be

$$\frac{d^2\mathscr{P}_{fa}}{d\mathscr{P}_d^2} \leq 0.$$

Thus, the probability of false alarm $\mathscr{P}_{fa}(\mathscr{P}_d)$ is a concave function of the detection probability $\mathscr{P}_d$ for $\mathscr{P}_d \geq 0.5$. ■

APPENDIX B
PROOF OF PROPOSITION 2

Let $\hat{\lambda}$ and $\hat{\mu}$ be any feasible values of the Lagrange dual function $g(\lambda,\mu)$. If we prove that

$$g(\hat{\lambda},\hat{\mu}) \geq g(\tilde{\lambda},\tilde{\mu}) + \left([\hat{\lambda},\hat{\mu}]-[\tilde{\lambda},\tilde{\mu}]\right)\mathbf{S}^T$$

holds for any $\hat{\lambda}$, $\hat{\mu}$, then $\mathbf{S}$ must be a subgradient of the Lagrange dual function $g(\tilde{\lambda},\tilde{\mu})$ at $\tilde{\lambda}$, $\tilde{\mu}$. We have

$$\begin{aligned}g(\hat{\lambda},\hat{\mu}) =& \sup_P L(P,\hat{\lambda},\hat{\mu}) =\\ =&\mathbb{E}_{g,h}\left\{\mathscr{P}(H_0)\left(1-\mathscr{P}_{fa}\left(\bar{\mathscr{P}}_d\right)\right)\log_2\left(1+\frac{g\hat{P}}{\sigma_n^2}\right)+\right.\\ &\left.+\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)\log_2\left(1+\frac{g\hat{P}}{\sigma_n^2+\sigma_p^2}\right)\right\}\\ &-\hat{\lambda}\left[\mathbb{E}_{g,h}\left\{\mathscr{P}(H_0)\left(1-\mathscr{P}_{fa}\left(\bar{\mathscr{P}}_d\right)\right)\hat{P}+\right.\right.\\ &\left.\left.+\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)\hat{P}\right\}-P_{av}\right]\\ &-\hat{\mu}\left[\mathbb{E}_{g,h}\left\{\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)h\hat{P}\right\}-\Gamma\right]\\ \geq&\mathbb{E}_{g,h}\left\{\mathscr{P}(H_0)\left(1-\mathscr{P}_{fa}\left(\bar{\mathscr{P}}_d\right)\right)\log_2\left(1+\frac{g\tilde{P}}{\sigma_n^2}\right)+\right.\\ &\left.+\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)\log_2\left(1+\frac{g\tilde{P}}{\sigma_n^2+\sigma_p^2}\right)\right\}\\ &-\hat{\lambda}\left[\mathbb{E}_{g,h}\left\{\mathscr{P}(H_0)\left(1-\mathscr{P}_{fa}\left(\bar{\mathscr{P}}_d\right)\right)\tilde{P}+\right.\right.\\ &\left.\left.+\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)\tilde{P}\right\}-P_{av}\right]\\ &-\hat{\mu}\left[\mathbb{E}_{g,h}\left\{\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)h\tilde{P}\right\}-\Gamma\right]\\ =&\mathbb{E}_{g,h}\left\{\mathscr{P}(H_0)\left(1-\mathscr{P}_{fa}\left(\bar{\mathscr{P}}_d\right)\right)\log_2\left(1+\frac{g\tilde{P}}{\sigma_n^2}\right)+\right.\\ &\left.+\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)\log_2\left(1+\frac{g\tilde{P}}{\sigma_n^2+\sigma_p^2}\right)\right\}\\ &-\tilde{\lambda}\left[\mathbb{E}_{g,h}\left\{\mathscr{P}(H_0)\left(1-\mathscr{P}_{fa}\left(\bar{\mathscr{P}}_d\right)\right)\tilde{P}+\right.\right.\\ &\left.\left.+\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)\tilde{P}\right\}-P_{av}\right]\\ &-\tilde{\mu}\left[\mathbb{E}_{g,h}\left\{\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)h\tilde{P}\right\}-\Gamma\right]\\ &+\hat{\lambda}\left[P_{av}-\mathbb{E}_{g,h}\left\{\mathscr{P}(H_0)\left(1-\mathscr{P}_{fa}\left(\bar{\mathscr{P}}_d\right)\right)\tilde{P}+\right.\right.\\ &\left.\left.+\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)\tilde{P}\right\}\right]\\ &-\tilde{\lambda}\left[P_{av}-\mathbb{E}_{g,h}\left\{\mathscr{P}(H_0)\left(1-\mathscr{P}_{fa}\left(\bar{\mathscr{P}}_d\right)\right)\tilde{P}+\right.\right.\\ &\left.\left.+\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)\tilde{P}\right\}\right]\\ &+\hat{\mu}\left[\Gamma-\mathbb{E}_{g,h}\left\{\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)h\tilde{P}\right\}\right]\\ &-\tilde{\mu}\left[\Gamma-\mathbb{E}_{g,h}\left\{\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)h\tilde{P}\right\}\right]\\ =&g(\tilde{\lambda},\tilde{\mu})+(\hat{\lambda}-\tilde{\lambda})\left(P_{av}-\mathbb{E}_{g,h}\left\{\mathscr{P}(H_0)\cdot\right.\right.\\ &\left.\left.\cdot\left(1-\mathscr{P}_{fa}\left(\bar{\mathscr{P}}_d\right)\right)\tilde{P}+\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)\tilde{P}\right\}\right)+\\ &+(\hat{\mu}-\tilde{\mu})\left(\Gamma-\mathbb{E}_{g,h}\left\{\mathscr{P}(H_1)\left(1-\bar{\mathscr{P}}_d\right)h\tilde{P}\right\}\right),\end{aligned}$$

where $\hat{P}$ denotes the optimal solution when $\lambda=\hat{\lambda}$ and $\mu=\hat{\mu}$, whereas $\tilde{P}$ represents the optimal solution when $\lambda=\tilde{\lambda}$ and $\mu=\tilde{\mu}$. The inequality above results from the fact that $\hat{P}$ is the optimal solution for $\lambda = \hat{\lambda}$ and $\mu = \hat{\mu}$.

Therefore, the subgradient $\mathbf{S}^T$ of the Lagrange dual function $g(\lambda, \mu)$ is given by $[D, E]$, where D is given by

$$D = P_{av} - \mathbb{E}_{g,h}\Big\{ \mathscr{P}(H_0)\left(1 - \mathscr{P}_{fa}\left(\bar{\mathscr{P}}_d\right)\right) P + \\ + \mathscr{P}(H_1)\left(1 - \bar{\mathscr{P}}_d\right) P \Big\},$$

and E is given by

$$E = \Gamma - \mathbb{E}_{g,h}\Big\{ \mathscr{P}(H_1)\left(1 - \bar{\mathscr{P}}_d\right) hP \Big\}.$$

■

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