Optimal Sensing Time and Power Allocation in Multiband Cognitive Radio Networks

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***Abstract*—Cognitive radio is an emerging technology that aims for efficient spectrum usage by allowing unlicensed (secondary) users to access licensed frequency bands under the condition of protecting the licensed (primary) users from harmful interference. The latter condition constraints the achievable throughput of a cognitive radio network, which should therefore access a wideband spectrum in order to provide reliable and efficient services to its users. In this paper, we study the problem of designing the optimal sensing time and power allocation strategy, in order to maximize the ergodic throughput of a cognitive radio that employs simultaneous multiband detection and operates under two different schemes, namely the wideband sensing-based spectrum sharing (WSSS) and the wideband opportunistic spectrum access (WOSA) scheme. We consider average transmit and interference power constraints for both schemes, in order to effectively protect the primary users from harmful interference, propose two algorithms that acquire the optimal sensing time and power allocation under imperfect spectrum sensing for the two schemes and discuss the effect of the average transmit and interference power constraint on the optimal sensing time. Finally, we provide simulation results to compare the two schemes and validate our theoretical analysis.**

***Index Terms*—Wideband spectrum sensing, throughput maximization, optimal power allocation, optimal sensing time, cognitive radio.**

I. Introduction

THE last decade has witnessed an increasing development and popularity of wireless communications, which has turned the limited spectrum into a scarce resource. Under the current fixed spectrum allocation policy, the frequency bands are exclusively allocated to licensed users, whereas unlicensed users are not allowed to access them even when they are not being used. Moreover, careful studies of the spectrum usage pattern have revealed that the allocated spectrum experiences low utilization. In fact, recent studies by the Federal Communications Commission (FCC) have shown that the utilization of the licensed spectrum varies from 15% to 85%, whereas only 2% of spectrum would be used in the US at any given moment [1].

The realization of the inefficient use of the spectrum under the current fixed spectrum allocation policy, as well as the demand for more and better wireless services has contributed to the reconsideration of the way the spectrum is utilized today and has led very recently to the decision of the Federal Communications Commission (FCC) to allow access of unlicensed users to the broadcast television spectrum at locations where that spectrum is not being used by licensed services [2].

Paper approved by S. Roy, the Editor for Cross-Layer Wireless/Cognitive Radio Networks of the IEEE Communications Society. Manuscript received August 12, 2009; revised March 8, 2010 and June 23, 2010.

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Digital Object Identifier 10.1109/TCOMM.2010.110310.090473

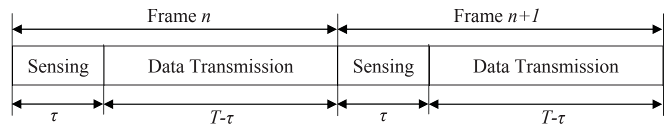

Fig. 1. Frame structure of the cognitive radio network.

The key technology towards efficient spectrum usage is Cognitive Radio (CR), which was first introduced in 1999 by J. Mitola III [3]. Cognitive Radio allows unlicensed (secondary) users to access licensed bands under the condition of protecting the licensed (primary) users from harmful interference [4]. Three approaches have been developed for cognitive radio so far regarding the way a secondary user accesses the licensed spectrum: (i) the opportunistic spectrum access [5], [6], where the secondary users transmit only when a frequency band is detected to be idle, (ii) the spectrum sharing [7], [8], where the secondary users coexist with the primary users and apply an interference constraint to ensure the quality of service (QoS) of the primary network, and (iii) the sensing-based spectrum sharing [9], where the secondary users sense for the status (active/idle) of the channel and adapt their transmit power based on the decision made by spectrum sensing. The latter scheme can be seen as a hybrid approach between the opportunistic spectrum access and the spectrum sharing scheme.

The frame structure of any cognitive radio system that employs spectrum sensing studied so far, consists of a sensing time slot and a data transmission slot, as depicted in Fig. 1. Therefore, an inherent tradeoff exists between the sensing time and the data transmission time, thus the throughput of the cognitive radio network. The problem of the sensing-throughput tradeoff for an opportunistic spectrum access cognitive radio network that employs energy detection for spectrum sensing was addressed in [10] for a single frequency band, where the authors studied the problem of finding the optimal sensing time that maximizes the throughput under a constraint on the probability of detection of the primary users. This work was extended in [11] for a wideband opportunistic spectrum access cognitive radio network, where the problem of finding the optimal sensing time and power allocation scheme that maximizes the average achievable throughput was studied under two different power constraints (an instantaneous and an average transmit power constraint) and it was shown that the average transmit power constraint leads to higher average achievable throughput compared to the instantaneous transmit power constraint.

In this paper, we study the design of the optimal sensing time and power allocation strategy that maximizes the ergodic throughput of a cognitive radio that employs simultaneous multiband detection [13] and operates under two different schemes, namely the wideband sensing-based spectrum sharing (WSSS) and the wideband opportunistic spectrum access (WOSA) scheme. Different from the work in [11], we take into consideration an average interference power constraint in the WOSA scheme (besides the average transmit power constraint), in order to effectively protect the primary users from harmful interference for the realistic scenario of imperfect spectrum sensing, whereas we also demonstrate the effect of this constraint on the optimal sensing time. Moreover, we consider in the optimization process the effect of the quiet sensing time (during which the transmit power is zero) on the average transmit power constraint, namely the fact that an increase of the sensing time leads to reduced data transmission time on the one hand, but also leads to increased transmit power on the other, such that on average the aforementioned transmit power constraint is met but higher throughput is achieved. This can seen in more detail in Section III and IV. In addition, we propose an algorithm that different from the work in [10], [11] can be applied even when not all of the frequency bands are underutilized, in order to achieve the maximum ergodic throughput of the cognitive radio network.

The two latter differences also apply in comparison to the work in [9], where the single-band sensing-based spectrum sharing scheme was studied. Different from the work in [9], by considering all terms in the optimization process, we ensure that the maximum ergodic throughput is achieved on the one hand, and, on the other hand, that the primary users do not suffer from harmful interference, which will be caused by the use of approximations in the average interference power constraint. In addition, we propose an algorithm that can be applied even for the case that not all of the frequency bands are underutilized.

Furthermore, different from the work in [12], where the single-band opportunistic spectrum access scheme was considered under an average interference power constraint for Rayleigh fading channels, we additionally consider an average transmit power constraint, in order to keep the long-term power budget of the secondary users. Moreover, we consider all terms in the ergodic throughput maximization problem, so that to ensure that the maximum throughput is achieved. In addition, we do not consider in our analysis a strict single value constraint regarding the target detection probability ($P_d^{tar} \approx 1$ in [12]) that might not be always possible in practice due to the limitations of the spectrum sensing techniques. Furthermore, we propose an algorithm that acquires the optimal sensing time and power allocation regardless of the channel distribution and which can be applied even when not all of the frequency bands are underutilized.

Finally, we discuss in this paper (i) the effect of the average transmit power constraint and the average interference power constraint on the optimal sensing time, (ii) the effect of selecting a sensing time that is not optimal for all values of maximum average transmit and interference power in terms of throughput loss for the cognitive radio network, (iii) we compare the performance of the two transmission schemes, namely the wideband sensing-based spectrum sharing and the wideband opportunistic spectrum access scheme, and finally (iv) we demonstrate the effect of using the optimal time of one scheme (e.g. WOSA) under the other scheme (e.g. WSSS).

The rest of the paper is organized as follows. In Section II, we present the system model. In Section III, the problem of designing the optimal sensing time and power allocation strategy for a wideband sensing-based spectrum sharing (WSSS) cognitive radio network is studied. The respective problem for a wideband opportunistic spectrum access (WOSA) cognitive radio network is briefly studied in Section IV. The simulation results are presented and discussed in Section V and finally the conclusions are drawn in Section VI.

Notations: Vectors are boldface, $\mathbb{E}\{\cdot\}$ denotes the expectation operation, $\succeq$ denotes componentwise inequality between vectors, $[x]^+$ denotes $\max(0,x)$, P denotes power and finally $\mathscr{P}$ probability.

II. System Model

We consider a cognitive radio network that can access a wideband spectrum licensed to a primary network, which is divided into M non-overlapping narrowband channels. The system consists of a primary link and a secondary link, as depicted in Fig. 2, where Tx denotes the transmitter and Rx denotes the receiver. In order to access the frequency bands, the secondary user (SU) must first perform spectrum sensing to determine the status (active/idle) of each channel. In this paper, we perform simultaneous spectrum sensing of multiple frequency bands by using the multiband joint detector proposed in [13], which utilizes the energy detection scheme [16], in order to determine the status of the primary user (PU) in each frequency band. The received signal at the secondary user is initially passed through the M down-converters and then to the M individual energy detectors, as shown in Fig. 3. The detection of the status of a frequency band is a binary hypothesis testing problem, i.e. the frequency band k is idle $(H_{0,k})$ or the frequency band k is active $(H_{1,k})$. The primary user's signals are assumed to be complex-valued PSK signals, whereas the noise at the secondary users is assumed to be independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian (CSCG) with zero mean and N_0 variance, namely $\mathcal{CN}(0,N_0)$.

The probability of detection and false alarm for the jth channel under the energy detection scheme is given by

$$\mathscr{P}_{d,j}(\tau,\epsilon_j) = \mathcal{Q}\left(\left(\frac{\epsilon_j}{N_0} - \gamma_j - 1\right)\sqrt{\frac{\tau f_s}{2\gamma_j + 1}}\right) \tag{1}$$

$$\mathscr{P}_{fa,j}(\tau,\epsilon_j) = \mathcal{Q}\left(\left(\frac{\epsilon_j}{N_0} - 1\right)\sqrt{\tau f_s}\right) \tag{2}$$

$$= \mathcal{Q}\left(\sqrt{2\gamma_j + 1}\,\mathcal{Q}^{-1}\left(\mathscr{P}_{d,j}\right) + \sqrt{\tau f_s}\gamma_j\right) \tag{3}$$

respectively [10], where τ represents the sensing time, ϵ_j denotes the decision threshold of the energy detector on channel j, γ_j is the received signal-to-noise ratio (SNR) from the primary user at the secondary detector on channel j and finally f_s represents the sampling frequency.

The frame structure employed in this cognitive radio network is depicted in Fig. 1. A quiet period of duration τ is inserted for spectrum sensing into each transmission frame of duration T. The instantaneous channel power gain of the secondary link, the link between PU-TX and SU-RX, and the link between SU-TX and PU-RX for the jth channel are denoted by $g_{ss,j}$, $g_{ps,j}$ and $g_{sp,j}$ respectively, as shown in Fig. 2. The channels are assumed to be flat fading and the channel power gains are assumed to be ergodic, stationary and known at the secondary users, as in [7],[9], [17],[18].

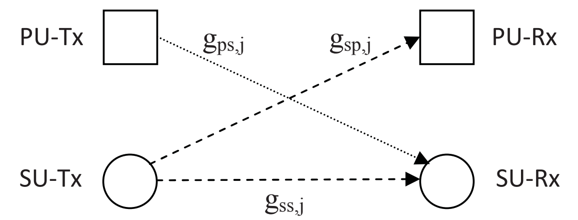

Fig. 2. System model.

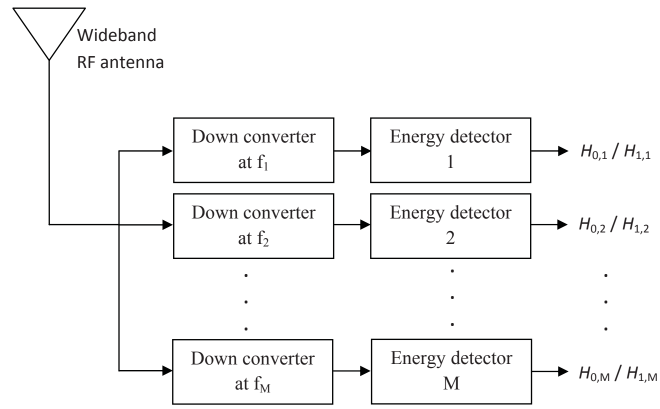

Fig. 3. Block diagram of the secondary user's wideband detector.

III. Wideband Sensing-Based Spectrum Sharing

In the wideband sensing-based spectrum sharing (WSSS) scheme, the secondary user transmits simultaneously on M frequency bands (regardless of the actual status of each frequency band) and adapts its transmit power on each band based on the decision made during the sensing slot at the beginning of each frame (Fig. 1). If the jth frequency band is detected to be idle ($H_{0,j}$), the secondary user transmits using high power $P_{s,j}^{(0)}$ during the data transmission slot, whereas if the jth frequency band is sensed to be active ($H_{1,j}$), then the secondary user transmits using relatively low power $P_{s,j}^{(1)}$, in order to reduce the interference caused to the primary user. This cognitive radio scheme can be seen as a hybrid approach between the opportunistic spectrum access and the spectrum sharing scheme. In this section, we study the problem of optimizing the sensing time and power allocation, in order to maximize the ergodic throughput of the wideband sensing-based spectrum sharing cognitive radio network.

The instantaneous transmission rate of the secondary user on the jth channel, denoted by $r_{0,j}$ for the case of absence of primary user ($H_{0,j}$) and by $r_{1,j}$ for the case of presence of the primary user ($H_{1,j}$), is given by

$$r_{0,j} = \log_2\left(1 + \frac{g_{ss,j}P_{s,j}^{(0)}}{N_0}\right), \tag{4}$$

$$r_{1,j} = \log_2\left(1 + \frac{g_{ss,j}P_{s,j}^{(1)}}{g_{ps,j}P_{p,j} + N_0}\right), \tag{5}$$

respectively, where $P_{p,j}$ denotes the transmit power of the primary user on the jth channel.

However, considering the fact that spectrum sensing is not a perfect function, due to the limitations of the spectrum sensing techniques and the nature of wireless communications that include phenomena such as shadowing and fading, the primary user could either be miss-detected or a false alarm may occur. As a result, four different cases can be distinguished regarding the sensing decision (present or absent) and the actual status of the primary user (active or idle) on each frequency band. Therefore, the following four different instantaneous transmission rates of the secondary user on the jth frequency band occur, where the first index number describes the actual status of the primary user ("0" for idle and "1" for active) and the second index number describes the decision that is made by the secondary users ("0" for absent and "1" for present)

$$r_{00,j} = \log_2\left(1 + \frac{g_{ss,j}P_{s,j}^{(0)}}{N_0}\right), \tag{6}$$

$$r_{01,j} = \log_2\left(1 + \frac{g_{ss,j}P_{s,j}^{(1)}}{N_0}\right), \tag{7}$$

$$r_{10,j} = \log_2\left(1 + \frac{g_{ss,j}P_{s,j}^{(0)}}{g_{ps,j}P_{p,j} + N_0}\right), \tag{8}$$

$$r_{11,j} = \log_2\left(1 + \frac{g_{ss,j}P_{s,j}^{(1)}}{g_{ps,j}P_{p,j} + N_0}\right). \tag{9}$$

Thus, the average throughput of the jth channel for the sensing-based spectrum sharing model (ignoring the sensing time) can be formulated as

$$\begin{aligned} C_j = &\mathscr{P}\left(H_{0,j}\right)\left(1 - \mathscr{P}_{fa,j}\left(\tau, \epsilon_j\right)\right) r_{00,j} \\ &+ \mathscr{P}\left(H_{0,j}\right) \mathscr{P}_{fa,j}\left(\tau, \epsilon_j\right) r_{01,j} \\ &+ \mathscr{P}\left(H_{1,j}\right)\left(1 - \mathscr{P}_{d,j}\left(\tau, \epsilon_j\right)\right) r_{10,j} \\ &+ \mathscr{P}\left(H_{1,j}\right) \mathscr{P}_{d,j}\left(\tau, \epsilon_j\right) r_{11,j} \end{aligned} \tag{10}$$

where $\mathscr{P}\left(H_{0,j}\right)$ denotes the probability that the jth channel is idle and $\mathscr{P}\left(H_{1,j}\right)$ denotes the probability that the jth channel is active.

Furthermore, in order to keep the long-term power budget of the secondary users, an average transmit power constraint (over all fading states) should be taken into account, as in previous studies of the capacity over fading channels such as

[19]-[21], which can be written as follows

$$\frac{T-\tau}{T}\mathbb{E}\Bigg\{\sum_{j=1}^{M}\Big[\mathscr{P}(H_{0,j})P_{s,j}^{(0)}(1-\mathscr{P}_{fa,j}(\tau,\epsilon_j)) + \mathscr{P}(H_{0,j})P_{s,j}^{(1)}\mathscr{P}_{fa,j}(\tau,\epsilon_j) + \mathscr{P}(H_{1,j})P_{s,j}^{(0)}(1-\mathscr{P}_{d,j}(\tau,\epsilon_j)) + \mathscr{P}(H_{1,j})P_{s,j}^{(1)}\mathscr{P}_{d,j}(\tau,\epsilon_j)\Big]\Bigg\} \le P_{av} \quad (11)$$

where P_{av} denotes the maximum average transmit power of the secondary user.

Since the priority of a cognitive radio network is to protect the quality of service (QoS) of primary users, an interference power constraint should be imposed for the protection of the primary network. In this paper, we will apply an average interference power constraint (averaged over all different fading states), since it was shown in [19] that the average interference power constraint does not only provide higher ergodic throughput for the cognitive radio network compared to the peak interference power constraint, but it also provides better protection of the primary network.

Under the wideband sensing-based spectrum sharing (WSSS) scheme, interference on the jth channel is caused to the primary user in two cases, namely when the primary user is falsely detected to be absent (missed detection) and additionally, when the primary user is detected to be present (correct detection) and therefore a low transmit power $P_{s,j}^{(1)}$ is used. As a result, the average interference power constraint of the wideband sensing-based spectrum access can be formulated as follows

$$\frac{T-\tau}{T}\mathbb{E}\Big\{g_{sp,j}P_{s,j}^{(0)}\mathscr{P}(H_{1,j})(1-\mathscr{P}_{d,j}(\tau,\epsilon_j)) + g_{sp,j}P_{s,j}^{(1)}\mathscr{P}(H_{1,j})\mathscr{P}_{d,j}(\tau,\epsilon_j)\Big\} \le \Gamma, j=1,..,M \quad (12)$$

where the parameter Γ represents the maximum average interference power that is tolerable by the primary users on each frequency band.

Finally, since the main priority of a cognitive radio network is the protection of the primary users, a high detection probability $\mathscr{P}_{d,j}(\tau,\epsilon_j)$ is required. In this paper, we choose the target detection probability to be $\mathscr{P}_{d,j}(\tau,\epsilon_j)=\bar{\mathscr{P}}_{d,j}$ for $j=1,\ldots,M$. Moreover, for a given sensing time $\tau=\bar{\tau}$, we can choose a decision threshold such that the target detection probability $\bar{\mathscr{P}}_{d,j}$ is met. According to (1), this decision threshold is given by

$$\epsilon_j = N_0\left(\sqrt{\frac{2\gamma_j+1}{\bar{\tau}f_s}}\mathcal{Q}^{-1}\left(\bar{\mathscr{P}}_{d,j}\right)+\gamma_j+1\right),\ j=1,\ldots,M. \quad (13)$$

Thus, the optimization problem that maximizes the ergodic throughput of a wideband sensing-based spectrum sharing (WSSS) cognitive radio network, under average transmit and interference power constraints can be formulated as follows

$$\underset{\{\tau,\mathbf{P}_s^{(0)},\mathbf{P}_s^{(1)}\}}{\text{maximize}}\ \mathbb{E}\left\{\sum_{j=1}^{M}\frac{T-\tau}{T}C_j(\tau,\mathbf{P}_s^{(0)},\mathbf{P}_s^{(1)})\right\} \quad (14)$$

$$\text{subject to}\ (11),\ (12),\ P_{s,j}^{(0)}\ge 0,\ P_{s,j}^{(1)}\ge 0,\ j=1,\ldots,M,\ T\ge\tau\ge 0.$$

The optimization problem (14) is convex with respect to the transmit powers $\mathbf{P}_s^{(0)}$ and $\mathbf{P}_s^{(1)}$, but not with respect to the sensing time τ, considering the dependence of the false alarm probability $\mathscr{P}_{fa,j}(\tau)$ on the sensing time τ [10], [23]. Different from the approach in [9], we do not consider any approximation on the objective function or the average interference power constraint, in order to effectively protect the licensed users from harmful interference and achieve the maximum ergodic throughput possible. Due to the non-convexity of the problem (14) with respect to the sensing time τ, the optimal sensing time cannot be obtained using convex optimization techniques. However, considering the fact that the sensing time lies within the interval $(0,T)$, it can be easily acquired using one-dimensional exhaustive search. Therefore, in the following we focus on finding the optimal power allocation strategy that maximizes the ergodic throughput of the wideband sensing-based spectrum sharing (WSSS) cognitive radio network.

The Lagrangian with respect to the transmit powers $\mathbf{P}_s^{(0)}$ and $\mathbf{P}_s^{(1)}$ is given by the following equation

$$L(\mathbf{P}_s^{(0)},\mathbf{P}_s^{(1)},\lambda,\boldsymbol{\mu}) = \mathbb{E}\left\{\sum_{j=1}^{M}\frac{T-\bar{\tau}}{T}(\alpha_{0,j}r_{00,j}+\beta_{0,j}r_{10,j} + \alpha_{1,j}r_{11,j}+\beta_{1,j}r_{01,j})\right\} - \lambda\left[\frac{T-\bar{\tau}}{T}\mathbb{E}\left\{\sum_{j=1}^{M}\Big[(\alpha_{0,j}+\beta_{0,j}) \cdot P_{s,j}^{(0)} + (\alpha_{1,j}+\beta_{1,j})P_{s,j}^{(1)}\Big]\right\} - P_{av}\right] - \sum_{j=1}^{M}\mu_j\left[\frac{T-\bar{\tau}}{T}\cdot \mathbb{E}\Big\{g_{sp,j}\beta_{0,j}P_{s,j}^{(0)}+g_{sp,j}\beta_{1,j}P_{s,j}^{(1)}\Big\}-\Gamma\right], \quad (15)$$

where the parameters $\alpha_{0,j}$, $\beta_{0,j}$, $\alpha_{1,j}$ and $\beta_{1,j}$ are given by

$$\alpha_{0,j} = \mathscr{P}(H_{0,j})(1-\mathscr{P}_{fa,j}(\bar{\tau})), \quad (16)$$

$$\beta_{0,j} = \mathscr{P}(H_{1,j})\left(1-\bar{\mathscr{P}}_{d,j}\right), \quad (17)$$

$$\alpha_{1,j} = \mathscr{P}(H_{0,j})\mathscr{P}_{fa,j}(\bar{\tau}), \quad (18)$$

$$\beta_{1,j} = \mathscr{P}(H_{1,j})\bar{\mathscr{P}}_{d,j}, \quad (19)$$

respectively, for $j=1,\ldots,M$.

The Lagrange dual optimization problem is now given by

$$\underset{\lambda\ge 0,\ \boldsymbol{\mu}\succeq\mathbf{0}}{\text{minimize}}\quad g(\lambda,\boldsymbol{\mu}). \quad (20)$$

In the optimization problem above, the function $g(\lambda,\boldsymbol{\mu})$, which is given by

$$g(\lambda,\boldsymbol{\mu}) = \sup_{\mathbf{P}_s^{(0)},\mathbf{P}_s^{(1)}} L(\mathbf{P}_s^{(0)},\mathbf{P}_s^{(1)},\lambda,\boldsymbol{\mu}) \quad (21)$$

represents the Lagrange dual function. It can be seen from (14) that the primal optimization problem with respect to the transmit powers $\mathbf{P}_s^{(0)}$ and $\mathbf{P}_s^{(1)}$ is convex with affine inequality constraints and that Slater's condition holds [23], i.e. strong duality holds. Therefore, the difference between the optimal value of the objective function of the primal and dual optimization problem (i.e. the optimal duality gap) is zero, which guarantees [22], [23] that the primal optimization problem (14) with respect to the transmit powers $\mathbf{P}_s^{(0)}$ and $\mathbf{P}_s^{(1)}$ can be equivalently solved by the Lagrange dual optimization problem (20). We therefore focus on the Lagrange dual optimization problem (20).

In order to calculate the dual function $g(\lambda, \boldsymbol{\mu})$, we need to find the supremum of the Lagrangian with respect to the transmit powers $\mathbf{P}_s^{(0)}$ and $\mathbf{P}_s^{(1)}$. The joint optimization problem (21) with respect to both transmit powers can be decomposed into two optimization subproblems, namely one for $\mathbf{P}_s^{(0)}$ and one for $\mathbf{P}_s^{(1)}$, as follows

Subproblem 1 (SP1):

$$\begin{aligned}\underset{\mathbf{P}_s^{(0)}\succeq\mathbf{0}}{\text{maximize}}\ &\mathbb{E}\left\{\sum_{j=1}^{M}\frac{T-\bar{\tau}}{T}\left(\alpha_{0,j}r_{00,j}+\beta_{0,j}r_{10,j}\right)\right\}\\ &-\lambda\mathbb{E}\left\{\sum_{j=1}^{M}\frac{T-\bar{\tau}}{T}\left(\alpha_{0,j}+\beta_{0,j}\right)P_{s,j}^{(0)}\right\}\\ &-\sum_{j=1}^{M}\mu_j\frac{T-\bar{\tau}}{T}\mathbb{E}\left\{g_{sp,j}\beta_{0,j}P_{s,j}^{(0)}\right\}\end{aligned}\tag{22}$$

Subproblem 2 (SP2):

$$\begin{aligned}\underset{\mathbf{P}_s^{(1)}\succeq\mathbf{0}}{\text{maximize}}\ &\mathbb{E}\left\{\sum_{j=1}^{M}\frac{T-\bar{\tau}}{T}\left(\alpha_{1,j}r_{11,j}+\beta_{1,j}r_{01,j}\right)\right\}\\ &-\lambda\mathbb{E}\left\{\sum_{j=1}^{M}\frac{T-\bar{\tau}}{T}\left(\alpha_{1,j}+\beta_{1,j}\right)P_{s,j}^{(1)}\right\}\\ &-\sum_{j=1}^{M}\mu_j\frac{T-\bar{\tau}}{T}\mathbb{E}\left\{g_{sp,j}\beta_{1,j}P_{s,j}^{(1)}\right\}\end{aligned}\tag{23}$$

The above subproblems (SP1 and SP2) are convex optimization problems, and by writing their Lagrangian functions and applying the Karush-Kuhn-Tucker (KKT) conditions, the optimal power when the primary user is detected to be idle on the jth frequency band ($H_{0,j}$) can be obtained as

$$P_{s,j}^{(0)}=\left[\frac{A_{0,j}+\sqrt{\Delta_{0,j}}}{2}\right]^{+}\tag{24}$$

where

$$A_{0,j}=\frac{\log_2(e)(\alpha_{0,j}+\beta_{0,j})}{\lambda(\alpha_{0,j}+\beta_{0,j})+\mu_j\beta_{0,j}h}-\frac{2N_0+\delta_j}{g_{ss,j}}\tag{25}$$

$$\Delta_{0,j}=A_{0,j}^2-\frac{4}{g_{ss,j}}\cdot\left\{\frac{N_0^2+\delta_jN_0}{g_{ss,j}}-\frac{\log_2(e)[\alpha_{0,j}(N_0+\delta_j)+\beta_{0,j}N_0]}{\lambda(\alpha_{0,j}+\beta_{0,j})+\mu_j\beta_{0,j}h}\right\}\tag{26}$$

$$\delta_j=g_{ps,j}P_{p,j}\tag{27}$$

whereas the optimal power when the primary user is detected to be active on the jth frequency band ($H_{1,j}$) is given by the following equation

$$P_{s,j}^{(1)}=\left[\frac{A_{1,j}+\sqrt{\Delta_{1,j}}}{2}\right]^{+}\tag{28}$$

where

$$A_{1,j}=\frac{\log_2(e)(\alpha_{1,j}+\beta_{1,j})}{\lambda(\alpha_{1,j}+\beta_{1,j})+\mu_j\beta_{1,j}h}-\frac{2N_0+\delta_j}{g_{ss,j}}\tag{29}$$

$$\Delta_{1,j}=A_{1,j}^2-\frac{4}{g_{ss,j}}\cdot\left\{\frac{N_0^2+\delta_jN_0}{g_{ss,j}}-\frac{\log_2(e)[\alpha_{1,j}(N_0+\delta_j)+\beta_{1,j}N_0]}{\lambda(\alpha_{1,j}+\beta_{1,j})+\mu_j\beta_{1,j}h}\right\}\tag{30}$$

$$\delta_j=g_{ps,j}P_{p,j}\tag{31}$$

and $[x]^+$ denotes $\max(x,0)$.

In order to find the optimal power allocation strategy for the wideband sensing-based spectrum sharing (WSSS) cognitive radio network, the optimal values of λ and $\boldsymbol{\mu}$ that minimize the dual function $g(\lambda,\boldsymbol{\mu})$ need to be found. The ellipsoid method [25] is used here to find the optimal solution, which requires the subgradient of the dual function $g(\lambda,\boldsymbol{\mu})$. The latter is given by the following proposition.

Proposition 1: The subgradient of the dual function $g(\lambda,\boldsymbol{\mu})$ is $\left[D,\mathbf{E}^T\right]$, where D is given by $D=P_{av}-\frac{T-\bar{\tau}}{T}\mathbb{E}\left\{\sum_{j=1}^{M}\left[\left(\alpha_{0,j}+\beta_{0,j}\right)P_{s,j}^{(0)}+\left(\alpha_{1,j}+\beta_{1,j}\right)P_{s,j}^{(1)}\right]\right\}$ and $\mathbf{E}$ is a vector with jth element $E_j=\Gamma-\frac{T-\bar{\tau}}{T}\mathbb{E}\left\{g_{sp,j}\beta_{0,j}P_{s,j}^{(0)}+g_{sp,j}\beta_{1,j}P_{s,j}^{(1)}\right\}$, $j=1,\ldots,M$, $\lambda\geq 0$, $\boldsymbol{\mu}\succeq\mathbf{0}$, whereas $\mathbf{P}_s^{(0)}$ and $\mathbf{P}_s^{(1)}$ denote the optimal power allocation in (21) for fixed λ, $\boldsymbol{\mu}$.

Proof: See Appendix A. ■

Finally, the algorithm that obtains the optimal sensing time and power allocation strategy of the wideband sensing-based spectrum sharing (WSSS) cognitive radio network is presented in the following table.

Algorithm 1: Optimal sensing time and power allocation for wideband sensing-based spectrum sharing (WSSS) cognitive radio networks.

► For $\bar{\tau}=0:T$
   1) Initialize λ, $\boldsymbol{\mu}$.
   2) Repeat:
      - calculate $\mathbf{P}_s^{(0)}$, $\mathbf{P}_s^{(1)}$ using (24)-(31);
      - update λ, $\boldsymbol{\mu}$ using the ellipsoid method;
   3) Until λ, $\boldsymbol{\mu}$ converge.

► End.

► Optimal sensing time and power allocation:

$$\bar{\tau}_{opt}=\arg\max R\left(\bar{\tau},\mathbf{P}_s^{(0)},\mathbf{P}_s^{(1)}\right),$$

$$\left[\mathbf{P}_{s,opt}^{(0)},\mathbf{P}_{s,opt}^{(1)}\right]=\left[\mathbf{P}_s^{(0)},\mathbf{P}_s^{(1)}\right]_{\bar{\tau}=\bar{\tau}_{opt}}.$$

IV. Wideband Opportunistic Spectrum Access

In the wideband opportunistic spectrum access (WOSA) scheme, the secondary users simultaneously sense all frequency bands and access only those that are detected to be idle. Therefore, the role of spectrum sensing is of utmost importance and a high target detection probability is required, in order to avoid the degradation of the quality of service (QoS) of the primary network. However, as mentioned in the previous section, due to the limitations of spectrum sensing techniques and the nature of wireless communications, it is almost inevitable that there will be a probability of missed detection. In this case, the secondary user will detect that a frequency band is idle (when in fact it is not) and it will access it. As a result, the average throughput of the jth channel (ignoring the sensing time) for the wideband opportunistic spectrum access scheme is given by

$$\begin{aligned}\mathscr{C}_j =& \mathscr{P}(H_{0,j})(1-\mathscr{P}_{fa,j}(\tau,\epsilon_j))\, r_{0,j}\\ &+ \mathscr{P}(H_{1,j})(1-\mathscr{P}_{d,j}(\tau,\epsilon_j)) r_{1,j} \end{aligned} \tag{32}$$

where $r_{0,j}$ and $r_{1,j}$ are given by (4) and (5), respectively.

When the secondary user falsely detects the status of a frequency band, it is going cause harmful interference to the primary user. Since the priority of cognitive radio networks is to protect the quality of service (QoS) of primary users, an interference power constraint should be imposed for the protection of the primary networks. In this paper, we will apply in addition to the following average transmit power constraint (averaged over all different fading states)

$$\begin{aligned}\frac{T-\tau}{T}\mathbb{E}\Bigg\{\sum_{j=1}^{M}&\big[\mathscr{P}(H_{0,j})(1-\mathscr{P}_{fa,j}(\tau))P_{s,j}\\ &+\mathscr{P}(H_{1,j})(1-\bar{\mathscr{P}}_{d,j})P_{s,j}\big]\Bigg\} \le P_{av}, \end{aligned} \tag{33}$$

an average interference power constraint, which can be written as follows

$$\frac{T-\tau}{T}\,\mathbb{E}\left\{g_{sp,j}\mathscr{P}(H_{1,j})(1-\mathscr{P}_{d,j}(\tau,\epsilon_j))P_{s,j}\right\} \le \Gamma, \tag{34}$$

where Γ denotes the average interference power threshold.

Finally, we can formulate the optimization problem that maximizes the ergodic throughput of a wideband opportunistic spectrum access (WOSA) cognitive radio network for a target probability of detection $\mathscr{P}_{d,j}(\tau,\epsilon_j) = \bar{\mathscr{P}}_{d,j}$ for $j = 1,\dots,M$, as follows

$$\begin{aligned}&\underset{\{\tau,\mathbf{P}_s\}}{\text{maximize}}\ \ \mathbb{E}\left\{\sum_{j=1}^{M}\frac{T-\tau}{T}\mathscr{C}_j(\tau,\mathbf{P}_s)\right\}\\ &\text{subject to}\ \ (33),(34),\ P_{s,j}\ge 0,\ j=1,\dots,M,\\ &\qquad\qquad\ T\ge\tau\ge 0.\end{aligned} \tag{35}$$

The optimization problem (35) is convex with respect to the transmit power $\mathbf{P}_s$, but not with respect to the sensing time τ, considering the dependence of the false alarm probability $\mathscr{P}_{fa,j}(\tau)$ on the sensing time. Due to the non-convexity of the problem (35) with respect to the sensing time, we consider (similar to the wideband sensing-based spectrum sharing scheme) one-dimensional exhaustive search. Therefore, in the following we focus on finding the optimal power allocation that maximizes the ergodic throughput of the wideband opportunistic spectrum access (WOSA) cognitive radio network for a given sensing time $\tau = \bar{\tau}$.

By writing the Lagrangian $L(\mathbf{P}_s,\lambda,\boldsymbol{\mu})$ and applying the Karush-Kuhn-Tucker (KKT) conditions, the optimal transmit power can be obtained as

$$P_{s,j} = \left[\frac{A_j+\sqrt{\Delta_j}}{2}\right]^+ \tag{36}$$

where the parameters A_j and Δ_j are given at the bottom of this page and $[x]^+$ denotes $\max(x,0)$.

In order to find the optimal power allocation, the optimal values of λ and $\boldsymbol{\mu}$ that minimize the dual function

$$h(\lambda,\boldsymbol{\mu}) = \sup_{\mathbf{P}_s} L(\mathbf{P}_s,\lambda,\boldsymbol{\mu}) \tag{37}$$

need to be found. The ellipsoid method is considered here, in order to determine the optimal values of λ and $\boldsymbol{\mu}$, whereas the required subgradient [24] of the dual function $h(\lambda,\boldsymbol{\mu})$ is given by the following proposition.

Proposition 2: The subgradient of the dual function $h(\lambda,\boldsymbol{\mu})$ is $[F,\mathbf{G}^T]$, where F is given by $F = P_{av} - \frac{T-\bar{\tau}}{T}\ \mathbb{E}\left\{\sum_{j=1}^{M}\left[\mathscr{P}(H_{0,j})(1-\mathscr{P}_{fa,j}(\bar{\tau}))P_{s,j} + \mathscr{P}(H_{1,j})(1-\bar{\mathscr{P}}_{d,j})P_{s,j}\right]\right\}$, $\mathbf{G}$ is a vector with $G_j = \Gamma - \mathbb{E}\left\{\frac{T-\bar{\tau}}{T}g_{sp,j}\mathscr{P}(H_{1,j})(1-\bar{\mathscr{P}}_{d,j})P_{s,j}\right\}$ $j=1,\dots,M$, $\lambda\ge 0$, $\boldsymbol{\mu}\succeq\mathbf{0}$, and $\mathbf{P}_s$ is the corresponding optimal power allocation in (37) for fixed λ, $\boldsymbol{\mu}$.

Proof: The proof is similar to the one of Proposition 1 and is therefore omitted. ■

Finally, the algorithm that optimizes the sensing time and power allocation strategy of a wideband opportunistic spectrum access (WOSA) cognitive radio network with an average transmit and interference power constraint is presented in the following table.

$$A_j = \frac{\log_2(e)\left[\mathscr{P}(H_{0,j})(1-\mathscr{P}_{fa,j}(\bar{\tau})) + \mathscr{P}(H_{1,j})\left(1-\bar{\mathscr{P}}_{d,j}\right)\right]}{\lambda\left[\mathscr{P}(H_{0,j})(1-\mathscr{P}_{fa,j}(\bar{\tau})) + \mathscr{P}(H_{1,j})\left(1-\bar{\mathscr{P}}_{d,j}\right)\right] + \mu_j\mathscr{P}(H_{1,j})\left(1-\bar{\mathscr{P}}_{d,j}\right)g_{sp,j}} - \frac{2N_0+g_{ps,j}P_{p,j}}{g_{ss,j}},$$

$$\Delta_j = A_j^2 - \frac{4}{g_{ss,j}}\left[\frac{N_0+g_{ps,j}P_{p,j}}{g_{ss,j}N_0^{-1}} - \frac{\lambda^{-1}\log_2(e)\left[\mathscr{P}(H_{0,j})(1-\mathscr{P}_{fa,j}(\bar{\tau}))(N_0+g_{ps,j}P_{p,j}) + \mathscr{P}(H_{1,j})\left(1-\bar{\mathscr{P}}_{d,j}\right)N_0\right]}{\left[\mathscr{P}(H_{0,j})(1-\mathscr{P}_{fa,j}(\bar{\tau})) + \mathscr{P}(H_{1,j})\left(1-\bar{\mathscr{P}}_{d,j}\right)\right] + \mu_j\mathscr{P}(H_{1,j})\left(1-\bar{\mathscr{P}}_{d,j}\right)g_{sp,j}}\right].$$

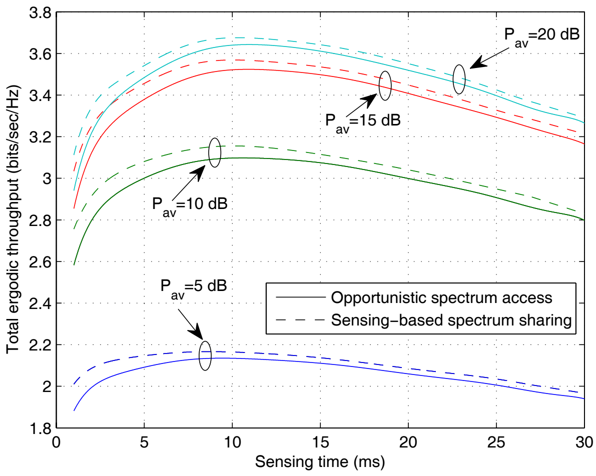

Fig. 4. Total ergodic throughput versus sensing time under $\mathscr{P}(H_{0,j}) = 0.6$ and $\Gamma = -10$ dB for different values of total transmit power P_{av} for the wideband opportunistic spectrum access and sensing-based spectrum sharing cognitive radio network.

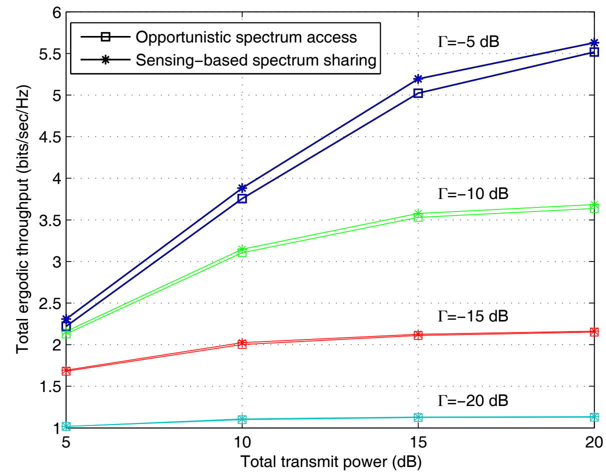

Fig. 5. Total ergodic throughput versus total transmit power P_{av} under $\mathscr{P}(H_{0,j}) = 0.6$ for different values of average interference power Γ for the wideband opportunistic spectrum access and sensing-based spectrum sharing cognitive radio network.

Algorithm 2: Optimal sensing time and power allocation for wideband opportunistic spectrum access (WOSA) cognitive radio networks.

- ▶ For $\bar{\tau} = 0 : T$
  1) Initialize λ, $\boldsymbol{\mu}$.
  2) Repeat:
     - calculate $\mathbf{P}_s$ using (36);
     - update λ, $\boldsymbol{\mu}$ using the ellipsoid method;
  3) Until λ, $\boldsymbol{\mu}$ converge.
- ▶ End.
- ▶ Optimal sensing time and power allocation:
  $\bar{\tau}_{opt} = \arg\max R(\bar{\tau}, \mathbf{P}_s)$ and
  $\mathbf{P}_{s,opt} = [\mathbf{P}_s]_{\bar{\tau}=\bar{\tau}_{opt}}$.

V. Simulation Results

In this section, we present and discuss the simulation results for two cognitive radio networks: one that operates under the wideband sensing-based spectrum sharing scheme (WSSS) and one that employs the wideband opportunistic spectrum access scheme (WOSA). Three narrowband frequency channels are considered here, each of 6 MHz bandwidth. The channels are assumed to be block faded and their power gains ergodic, stationary and exponentially distributed with unit mean. The frame duration of the secondary networks is fixed and set to $T = 100$ ms and the sampling frequency to 6 MHz. The target detection probability for all channels is set to $\bar{\mathscr{P}}_{d,j} = 90\%$, whereas the worst-case received SNR from the primary user at the secondary detector on each of the three channels is considered to be $\gamma_1 = -12$ dB, $\gamma_2 = -15$ dB and $\gamma_3 = -20$ dB, respectively. Finally, the transmit power of the primary user on all channels is assumed to be $P_{p,j} = 10$ dB, whereas the noise variance equal to $N_0 = 1$.

In Fig. 4, the total ergodic throughput versus the sensing time τ is presented for the WSSS and the WOSA cognitive radio network for several values of the total average transmit power P_{av} of the secondary user. The maximum average interference power is set to $\Gamma = -10$ dB, whereas the probability that the frequency band j is idle is assumed to be $\mathscr{P}(H_{0,j}) = 0.6$, $j = 1, 2, 3$. It can be clearly seen from Fig. 4 that the total ergodic throughput of both secondary networks is a convex function of the sensing time τ, which yields that an optimal sensing time exists for the WSSS and the WOSA cognitive radio network. Furthermore, as seen from Fig. 4, the optimal sensing time $\bar{\tau}_{opt}$ for both secondary networks is around 10 ms, which is slightly increased compared to the results presented in [11], where (under the same scenario) the optimal sensing time was found to be around 6 ms. In addition, it results from Fig. 4 that the total ergodic throughput of the WSSS is higher compared to the respective of the WOSA for all considered values of the total average transmit power P_{av}, which is due to the fact that the WSSS scheme allows data transmission (using spectrum sharing) even when the primary user is detected to be active.

In Fig. 5 and 6, the total ergodic throughput versus the average transmit power constraint P_{av} of the secondary user is presented for various values of the maximum average interference power Γ and for probabilities that the frequency bands are idle equal to $\mathscr{P}(H_{0,j}) = 0.6$ and $\mathscr{P}(H_{0,j}) = 0.8$, respectively. It is rather interesting to notice in Fig. 5 and 6 that under low values of the maximum average interference power Γ, the performance of the two networks is almost the same, something that indicates that the transmit power in the WSSS is mainly allocated at the periods when the frequency band is detected to be idle, whereas the contribution of the spectrum sharing function of the WSSS (i.e. when the primary user is detected to be active) is only significant when the maximum average interference power Γ receives higher values. Viewing this from a different angle, it indicates that as the distance between the primary receiver and the secondary transmitter increases, more transmit power is allocated during the spectrum sharing periods and the WSSS scheme can achieve higher total ergodic capacity compared to the WOSA

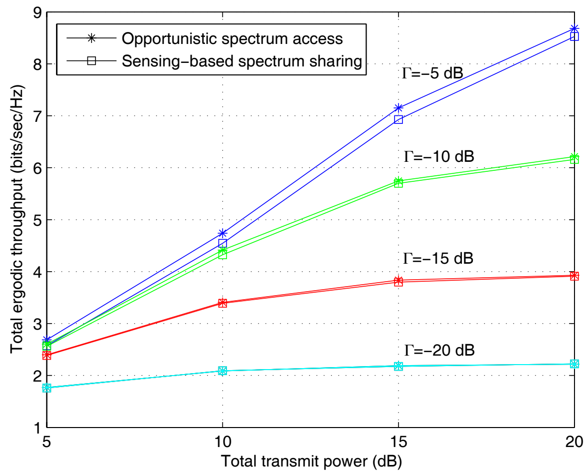

Fig. 6. Total ergodic throughput versus total transmit power P_{av} under $\mathscr{P}(H_{0,j}) = 0.8$ for different values of average interference power Γ for the wideband opportunistic spectrum access and sensing-based spectrum sharing cognitive radio network.

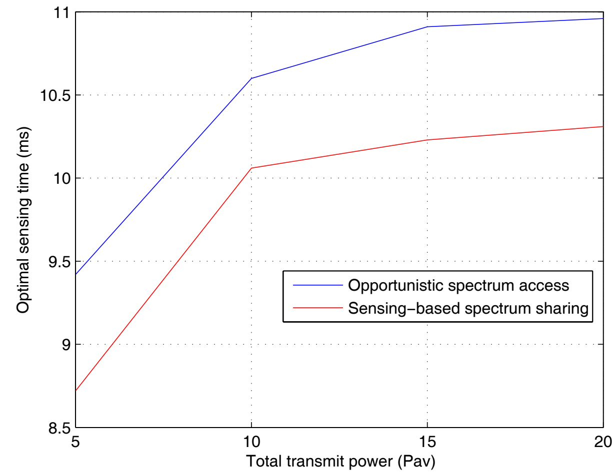

Fig. 7. Optimal sensing time versus total transmit power P_{av} under $\mathscr{P}(H_{0,j}) = 0.6$ and $\Gamma = -10$ dB for the wideband opportunistic spectrum access and sensing-based spectrum sharing cognitive radio network.

Table I: Optimal sensing time for WSSS and $P_{av} = 20$ dB

Total transmit power	10 dB	15 dB	20 dB
WSSS throughput loss	0.003%	0.004%	0%
WOSA throughput loss	0.004%	0.02%	0.03%

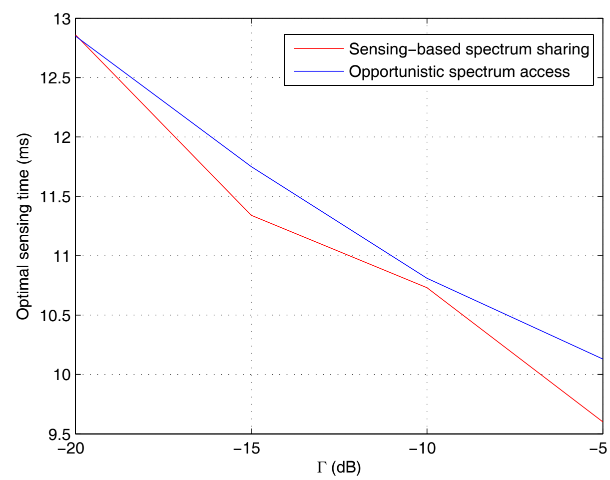

Fig. 8. Optimal sensing time versus maximum average interference power Γ under $\mathscr{P}(H_{0,j}) = 0.6$ and $P_{av} = 20$ dB for the wideband opportunistic spectrum access and sensing-based spectrum sharing cognitive radio network.

scheme.

In Fig. 7, the optimal sensing time versus the total transmit power is presented for the WSSS and the WOSA cognitive radio network under $\mathscr{P}(H_{0,j}) = 0.6$ and $\Gamma = -10$ dB. It can be observed that the optimal sensing time for both schemes (WSSS and WOSA) increases as the total transmit power receives higher values, whereas the optimal sensing time for the WOSA is slightly higher compared to the respective of the WSSS. However, as discussed in [11], it is better in practice to fix the sensing time for a good protocol design. It can be seen from Fig. 4 that the total ergodic throughput varies slightly around the peak value for both schemes (WSSS and WOSA) and therefore by fixing the sensing time, the resulting loss in the throughput is not significant. For instant, if we fix the sensing time to be optimal for the highest total transmit power $P_{av} = 20$ dB, the respective throughput losses for lower values of total transmit power for the WSSS and the WOSA cognitive radio network are presented in Table I and II, respectively. It can be observed that the resulting losses are actually very small and a rather interesting result that can be deduced from Tables I and II is that by fixing the sensing time, the total throughput losses are insignificant regardless of the scheme that the cognitive radio network employs. This means that the cognitive radio network could change the transmission scheme from wideband opportunistic spectrum access (WOSA) to wideband sensing-based spectrum sharing (WSSS) without having to change the sensing time of its (secondary) users, hence being able to increasing its total throughput by employing cognitive behavior and adapting its transmission scheme to its environment.

Finally, the optimal sensing time versus the maximum average interference power Γ is presented in Fig. 8 for the WSSS and the WOSA cognitive radio network under $\mathscr{P}(H_{0,j}) = 0.6$ and $P_{av} = 20$ dB. It can be seen from Fig. 8 that the optimal sensing time for the wideband opportunistic spectrum access (WOSA) system is slightly higher compared to the respective of the wideband sensing-based spectrum sharing (WSSS) system, whereas the optimal sensing time for both schemes decreases as the maximum average interference power Γ receives higher values. Again, if we fix the sensing time for instance to the optimal solution for the lowest value of the maximum average interference power $\Gamma = -20$ dB, the resulting losses in the total ergodic throughput of the cognitive radio network for higher values of maximum average interference power Γ, as well as for the other transmission scheme in each case, are small, something that can be seen in Tables III and IV, where the throughput losses for the

Table II: Optimal sensing time for WOSA and $P_{av} = 20$ dB

Total transmit power	10 dB	15 dB	20 dB
WOSA throughput loss	0.006%	$2 \cdot 10^{-4}$%	0%
WSSS throughput loss	0.05%	0.04%	0.02%

Table III: Optimal sensing time for WSSS and $\Gamma = -20$ dB

Maximum interference power	-10 dB	-15 dB	-20 dB
WSSS throughput loss	0.46%	0.32%	0%
WOSA throughput loss	0.32%	0.02%	10^{-7}%

WSSS and the WOSA cognitive radio system are presented, respectively.

VI. Conclusions

In this paper, we studied the problem of designing the optimal sensing time and power allocation strategy that maximizes the ergodic throughput of a wideband sensing-based spectrum sharing (WSSS) cognitive radio network and a wideband opportunistic spectrum access (WOSA) cognitive radio network under both average transmit and interference power constraints. We proposed two algorithms that acquire the optimal sensing time and power allocation strategy under imperfect spectrum sensing and discussed the effects of the total average transmit power and the average tolerable interference power on the optimal sensing time. Numerical results indicate that the wideband sensing-based spectrum sharing (WSSS) scheme exhibits higher ergodic throughput compared to the wideband opportunistic spectrum access (WOSA) scheme as the average tolerable interference power receives higher values or as the distance between the secondary transmitter and the primary receiver increases, whereas the optimal sensing time was shown to be vary slightly with respect to the total transmit power, the average tolerable interference power and the secondary transmission scheme used (WSSS/WOSA).

Acknowledgement

The authors would like to thank the anonymous reviewers for their critical comments that greatly improved the quality of this paper.

Appendix A
Proof of Proposition 1

Let $\hat{\lambda}$ and $\hat{\boldsymbol{\mu}}$ be any feasible values of the dual function $g(\lambda, \boldsymbol{\mu})$. If we prove that $g(\hat{\lambda}, \hat{\boldsymbol{\mu}}) \geq g(\tilde{\lambda}, \tilde{\boldsymbol{\mu}}) + \left([\hat{\lambda}, \hat{\boldsymbol{\mu}}] - [\tilde{\lambda}, \tilde{\boldsymbol{\mu}}]\right) \mathbf{S}^T$ holds for any $\hat{\lambda}$, $\hat{\boldsymbol{\mu}}$, then $\mathbf{S}$ must be a subgradient of $g(\tilde{\lambda}, \tilde{\boldsymbol{\mu}})$ at $\tilde{\lambda}$, $\tilde{\boldsymbol{\mu}}$. We have

$$\begin{aligned}
g(\hat{\lambda}, \hat{\boldsymbol{\mu}}) &= \sup_{\mathbf{P}_s^{(0)}, \mathbf{P}_s^{(1)}} L(\mathbf{P}_s^{(0)}, \mathbf{P}_s^{(1)}, \hat{\lambda}, \hat{\boldsymbol{\mu}}) \\
&= \mathbb{E}\left\{\sum_{j=1}^{M} \frac{T-\bar{\tau}}{T} \left(\alpha_{0,j}\hat{r}_{00,j} + \beta_{0,j}\hat{r}_{10,j} + \alpha_{1,j}\hat{r}_{11,j} \right.\right. \\
&\quad \left.\left. + \beta_{1,j}\hat{r}_{01,j}\right)\right\} - \hat{\lambda}\left[\frac{T-\bar{\tau}}{T}\mathbb{E}\left\{\sum_{j=1}^{M}\Big[\left(\alpha_{0,j}+\beta_{0,j}\right)\hat{P}_{s,j}^{(0)}\right.\right. \\
&\quad \left.\left. + \left(\alpha_{1,j}+\beta_{1,j}\right)\hat{P}_{s,j}^{(1)}\Big]\right\} - P_{av}\right] - \sum_{j=1}^{M}\hat{\mu}_j\left[\frac{T-\bar{\tau}}{T} \cdot \right. \\
&\quad \left. \mathbb{E}\left\{g_{sp,j}\beta_{0,j}\hat{P}_{s,j}^{(0)} + g_{sp,j}\beta_{1,j}\hat{P}_{s,j}^{(1)}\right\} - \Gamma\right] \\
&\geq \mathbb{E}\left\{\sum_{j=1}^{M} \frac{T-\bar{\tau}}{T} \left(\alpha_{0,j}\tilde{r}_{00,j} + \beta_{0,j}\tilde{r}_{10,j} + \alpha_{1,j}\tilde{r}_{11,j} \right.\right. \\
&\quad \left.\left. + \beta_{1,j}\tilde{r}_{01,j}\right)\right\} - \hat{\lambda}\left[\frac{T-\bar{\tau}}{T}\mathbb{E}\left\{\sum_{j=1}^{M}\Big[\left(\alpha_{0,j}+\beta_{0,j}\right)\tilde{P}_{s,j}^{(0)}\right.\right. \\
&\quad \left.\left. + \left(\alpha_{1,j}+\beta_{1,j}\right)\tilde{P}_{s,j}^{(1)}\Big]\right\} - P_{av}\right] - \sum_{j=1}^{M}\hat{\mu}_j\left[\frac{T-\bar{\tau}}{T} \cdot \right. \\
&\quad \left. \mathbb{E}\left\{g_{sp,j}\beta_{0,j}\tilde{P}_{s,j}^{(0)} + g_{sp,j}\beta_{1,j}\tilde{P}_{s,j}^{(1)}\right\} - \Gamma\right] \\
&= g(\tilde{\lambda}, \tilde{\boldsymbol{\mu}}) + (\hat{\lambda} - \tilde{\lambda})\left(P_{av} - \frac{T-\bar{\tau}}{T}\mathbb{E}\left\{\sum_{j=1}^{M}\left[\left(\alpha_{0,j}+\beta_{0,j}\right) \cdot \right.\right.\right. \\
&\quad \left.\left.\left. \tilde{P}_{s,j}^{(0)} + \left(\alpha_{1,j}+\beta_{1,j}\right)\tilde{P}_{s,j}^{(1)}\right]\right\}\right) + \sum_{j=1}^{M}\left(\hat{\mu}_j - \tilde{\mu}_j\right)\left(\Gamma - \frac{T-\bar{\tau}}{T}\right. \\
&\quad \left. \cdot\, \mathbb{E}\left\{g_{sp,j}\beta_{0,j}\tilde{P}_{s,j}^{(0)} + g_{sp,j}\beta_{1,j}\tilde{P}_{s,j}^{(1)}\right\}\right),
\end{aligned}$$

where $\hat{\mathbf{P}}_s^{(0)}$ and $\hat{\mathbf{P}}_s^{(1)}$ denote the optimal solutions when $\lambda = \hat{\lambda}$ and $\boldsymbol{\mu} = \hat{\boldsymbol{\mu}}$, whereas $\tilde{\mathbf{P}}_s^{(0)}$ and $\tilde{\mathbf{P}}_s^{(1)}$ represent the optimal solutions when $\lambda = \tilde{\lambda}$ and $\boldsymbol{\mu} = \tilde{\boldsymbol{\mu}}$. The inequality above results from the fact that $\hat{\mathbf{P}}_s^{(0)}$ and $\hat{\mathbf{P}}_s^{(1)}$ are the optimal solutions for $\lambda = \hat{\lambda}$ and $\boldsymbol{\mu} = \hat{\boldsymbol{\mu}}$. Therefore, the subgradient $\mathbf{S}^T$ of the dual function $g(\lambda, \boldsymbol{\mu})$ is given by $\left[D, \mathbf{E}^T\right]$, where D is given by

$$\begin{aligned}
D = P_{av} - \frac{T-\bar{\tau}}{T}\mathbb{E}\left\{\sum_{j=1}^{M}\Big[\left(\alpha_{0,j}+\beta_{0,j}\right)P_{s,j}^{(0)}\right. \\
\left. + \left(\alpha_{1,j}+\beta_{1,j}\right)P_{s,j}^{(1)}\Big]\right\}
\end{aligned}$$

and $\mathbf{E}$ is a vector with jth element $(j = 1, \ldots, M)$ that is equal to

$$E_j = \Gamma - \frac{T-\bar{\tau}}{T}\mathbb{E}\left\{g_{sp,j}\beta_{0,j}P_{s,j}^{(0)} + g_{sp,j}\beta_{1,j}P_{s,j}^{(1)}\right\}. \qquad \blacksquare$$

Table IV: Optimal sensing time for WOSA and $\Gamma = -20$ dB

Maximum interference power	-10 dB	-15 dB	-20 dB
WOSA throughput loss	0.32%	0.02%	0%
WSSS throughput loss	0.4%	0.1%	10^{-4}%

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