

A Joint Resource Allocation Scheme for Multiuser Two-Way Relay Networks

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Abstract—In this letter, we study the problem of resource allocation in *amplify-and-forward* (AF) based multiuser two-way relay network that is operated under orthogonal frequency division multiple access (OFDMA) modulation. We formulate an end-to-end throughput maximization problem subject to limited power constraint at individual user and relay. The optimization targets to find the best sub-carrier allocation to each user, sub-carrier pairing at the relay, as well as the power allocation at all nodes, which turns out to be a mixed integer programming problem. We then derive an asymptotically optimal solution through Lagrange dual decomposition approach and further design a suboptimal algorithm to trade the performance for computational complexity. Finally, simulation results are provided to demonstrate the performance gain of the proposed algorithms.

Index Terms—Two-way relay network, amplify-and-forward, OFDMA, resource allocation, multiuser communications.

I. INTRODUCTION

THE relay networks have gained much interest due to their capability of enhancing the communication reliability and enlarging the transmission range [1], [2]. Meanwhile, multi-carrier transmissions are known to combat the frequency selective fading channels and, when combined with the relay transmission, can provide improved performance through adaptive resource allocation. Hence, various research on multi-carrier aided relay network have been carried out during the past few years, for example, channel estimation [3], precoder design [4], and throughput analysis via resource allocation [5]–[7].

Resource allocation in orthogonal frequency division multiplexing (OFDM) based two-way relay network (TWRN) have

been proposed in [8]–[13]. The authors in [8] studied the throughput maximization problem in a three node network, where two user terminals exchange information with the help of a relay node using OFDM transmission, subject to an individual power constraint at each node. The results showed an enhanced system performance from an optimized power allocation via dual decomposition technique and a greedy sub-carrier pairing scheme. This scheme is further exploited in [9] under a total power constraint where a two step power allocation strategy was proposed. Joint power allocation and sub-carrier assignment problem in multiple-relay scenario, where two terminals exchange information with the help of more than one intermediate relay nodes, was considered in [10]. The problem is solved by a suboptimal algorithm where each resource is optimized by fixing the other. The authors further applied the idea to the orthogonal frequency division multiple access (OFDMA) based multi-user multi-relay systems in [11] and proposed a sub-carrier allocation algorithm for the known power allocation. The work in [12] studied the power and sub-carrier allocation problem in OFDMA multi-user relay network. More recently, relay power allocation problem in a multi-user system, where a number of user pairs exchange information through a single relay station, was considered in [13]. However a unified resource allocation scheme considering tone permutation, power optimization, and sub-carrier allocation all together has not been reported yet, to the best of authors' knowledge.

In this work, we consider a multiuser two-way OFDMA system, where users communicate with each other through a single relay node. The previous reported works have shown the enhanced throughput results in OFDM systems by optimizing either of the following:

- Power allocation over different subcarriers at each transmitting node.
- Subcarrier allocation among different users.
- Subcarrier pairing at relay node, where the signal received at relay over one subcarrier is re-transmitted on a different subcarrier.

This motivates us to find a unified framework where all resources are jointly optimized. Further, the distributed nature of the wireless systems prohibits to impose a total power constraint over all nodes. Thus, we assume that each node has a limited power supply, which makes our consideration closer to practical scenarios. The problem is then formulated as maximizing the end to end system throughput and is solved by dual decomposition technique that yields a nearly optimal solution for OFDMA system when the number of sub-carriers is sufficiently large, regardless of the non-convexity of the original problem [15]. To reduce the complexity, we further

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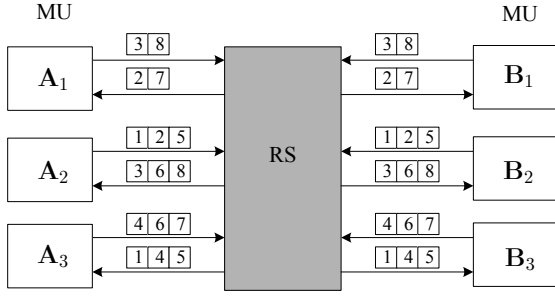


Fig. 1. System model for an OFMDA aided multiuser two-way relay network.

propose a suboptimal method that sacrifices very little on the performance as demonstrated by the numerical examples.

The rest of this letter is organized as follows. In Section II, we present the system model of multi-user two-way relay transmission and formulate the joint resource allocation problem. In Section III, we develop the dual decomposition method as well as the suboptimal method. Simulation results are presented in Section IV and conclusions are made in Section V.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

We consider a two-way multi-user relay network that consists of M pre-assigned pairs of mobile users (MUs) and one fixed relay station (RS), all equipped with only one antenna that cannot transmit and receive simultaneously, as shown in Fig. 1. The multicarrier two-way transmission protocol is divided into two phases: the multiple access (MA) phase and the broadcast (BC) phase. In MA phase, all MUs transmit information to RS simultaneously via non-overlapping carriers. In BC phase RS broadcasts the received signal after certain processing, for example power amplifying and carrier permutation. The two users of the m -th user pair, denoted as A_m and B_m , transmit simultaneously on the same carriers, for example the k th carrier in MA phase,¹ while the received signal will be sent back over the j -th sub-carrier in the BC phase. Assigning which carrier to which user-pair, as well as the pairing strategy (k, j) will be optimized in this letter.

Denote the channel coefficient from A_m to RS as $h_{m,k}$, the one from B_m to RS as $g_{m,k}$, the one from RS to A_m as $\tilde{h}_{m,j}$, and the one from RS to B_m as $\tilde{g}_{m,j}$.² Then the received signal at RS is

$$y_k^{RS} = \sqrt{p_{m,k}^A} h_{m,k} x_{m,k}^A + \sqrt{p_{m,k}^B} g_{m,k} x_{m,k}^B + w_k^{RS}, \quad (1)$$

where $x_{m,k}^A$ and $x_{m,k}^B$ are the information symbols to be exchanged, $p_{m,k}^A$ and $p_{m,k}^B$ are the corresponding powers over the k -th carrier, and w_k^{RS} is the additive white Gaussian noise with variance σ^2 .

If the power allocated at RS over sub-carrier j is represented as p_j^R , then the signals received at the m -th user pair can be

¹That is to say, A_{m_1} and B_{m_2} will not transmit on the same carrier during MA phase.

²By letting $\tilde{h}_{m,j} = h_{m,j}$ and $\tilde{g}_{m,j} = g_{m,j}$, the scenario here reduces to the reciprocal channels.

written as

$$y_{m,j}^A = \sqrt{p_j^R \tilde{h}_{m,j} \rho_j} \sqrt{p_{m,k}^A} h_{m,k} x_{m,k}^A + \sqrt{p_j^R \rho_j \tilde{h}_{m,j}} w_k^{RS} + \sqrt{p_j^R \tilde{h}_{m,j} \rho_j} \sqrt{p_{m,k}^B} g_{m,k} x_{m,k}^B + w_{m,j}^A, \quad (2)$$

$$y_{m,j}^B = \sqrt{p_j^R \rho_j \tilde{g}_{m,j}} \sqrt{p_{m,k}^B} g_{m,k} x_{m,k}^B + \sqrt{p_j^R \rho_j \tilde{g}_{m,j}} w_k^{RS} + \sqrt{p_j^R \tilde{g}_{m,j} \rho_j} \sqrt{p_{m,k}^A} h_{m,k} x_{m,k}^A + w_{m,j}^B, \quad (3)$$

where $\rho_j \triangleq \frac{1}{p_{m,k}^A |h_{m,k}|^2 + p_{m,k}^B |g_{m,k}|^2 + \sigma^2}$ is the scaling factor to keep the power constraint, while $w_{m,j}^A$ and $w_{m,j}^B$ are the received additive white Gaussian noises (AWGN) at A_m and B_m , respectively, both with variance σ^2 . Assuming a perfect self-interference cancellation, the corresponding SNRs can be written as

$$\text{SNR}_{m,j}^A = \frac{p_j^R |\tilde{h}_{m,j}|^2 \rho_j^2 p_{m,k}^B |g_{m,k}|^2}{(p_j^R \rho_j^2 |\tilde{h}_{m,j}|^2 + 1) \sigma^2}, \quad (4)$$

$$\text{SNR}_{m,j}^B = \frac{p_j^R |\tilde{g}_{m,j}|^2 \rho_j^2 p_{m,k}^A |h_{m,k}|^2}{(p_j^R \rho_j^2 |\tilde{g}_{m,j}|^2 + 1) \sigma^2}. \quad (5)$$

B. Problem Formulation

Due to the exclusive sub-carrier pairing constraint, each sub-carrier in MA phase can only be paired with one sub-carrier in BC phase. We then define $\pi_{(k,j)} \in \{0, 1\}$ as the binary variable for the sub-carrier pairing such that $\pi_{(k,j)} = 1$ if the k -th sub-carrier is paired with the j -th sub-carrier, while $\pi_{(k,j)} = 0$ otherwise. Further, we define binary variables $\tau_{m,(k,j)} \in \{0, 1\}$, such that $\tau_{m,(k,j)} = 1$ if sub-carrier pair (k, j) is allocated to the m -th MU pair while $\tau_{m,(k,j)} = 0$ otherwise.

We seek to jointly optimize the sub-carrier allocation, sub-carrier pairing, and the power allocation such that the overall system throughput is maximized under individual power constraints at MUs and RS. Let P_{A_m} , P_R , and P_{B_m} denote the total available powers at A_m , RS, and B_m , respectively. The optimization can be formulated as

$$\begin{aligned} \max_{\pi, \tau, \mathbf{p}^A, \mathbf{p}^R, \mathbf{p}^B} \quad & \sum_{m=1}^M \sum_{k=1}^K \sum_{j=1}^K \pi_{(k,j)} \tau_{m,(k,j)} \left(\frac{1}{2} C(\text{SNR}_{m,j}^A) + \frac{1}{2} C(\text{SNR}_{m,j}^B) \right) \\ \text{s.t.} \quad & \sum_{k=1}^K \pi_{(k,j)} = 1, \forall j, \quad \sum_{j=1}^K \pi_{(k,j)} = 1, \forall k, \\ & \sum_{m=1}^M \tau_{m,(k,j)} = 1, \forall (k, j), \quad \sum_{j=1}^K p_j^R \leq P_R, \\ & \sum_{k=1}^K p_{m,k}^A \leq P_{A_m}, \forall m, \quad \sum_{k=1}^K p_{m,k}^B \leq P_{B_m}, \forall m, \\ & p_{m,k}^A \geq 0, p_j^R \geq 0, p_{m,k}^B \geq 0, \quad \forall m, k, j, \end{aligned} \quad (6)$$

where $C(x) \triangleq \log_2(1 + x)$, and $\tau = \{\tau_{m,(k,j)}\}$, $\pi = \{\pi_{(k,j)}\}$, $\mathbf{p}^A = \{p_{m,k}^A\}$, $\mathbf{p}^B = \{p_{m,k}^B\}$, $\mathbf{p}^R = \{p_j^R\}$ for all $m = \{1, \dots, M\}$, $k = \{1, \dots, K\}$, $j = \{1, \dots, K\}$. The $\frac{1}{2}$ factor appears due to the two time slots used for a complete transmission.

The first and the second constraints are originated from the fact that each sub-carrier in MA phase can be coupled with one and only one sub-carrier in BC phase and vice verse. The third constraint ensures the exclusive allocation of the sub-carrier pair (k, j) to the m -th user pair (A_m, B_m) only. However more than one sub-carrier pairs can be allocated to a particular MU pair. Other constraints represent individual power constraint at each node.

III. RESOURCE ALLOCATION SCHEME

It is easily known that (6) is a mixed integer non-linear programming problem [14], and thus an exhaustive search over all variables is required to find the optimal solution. Thanks to [15], we know that the duality gap between the primal problem and the dual problem in a multi-carrier system approaches to zero for a sufficiently large number of sub-carriers. Thus we can solve the dual problem instead of the original problem. The dual problem associated with the primal problem (6) is defined as [16]

$$\begin{aligned} \min_{\nu, \lambda, \eta} \quad & D(\nu, \lambda, \eta) \\ \text{s.t.} \quad & \nu_m \geq 0, \eta_m \geq 0, \forall m, \lambda \geq 0, \end{aligned} \quad (7)$$

where $D(\nu, \lambda, \eta)$ is the dual function given by

$$\begin{aligned} D(\nu, \lambda, \eta) = \max_{\pi, \tau, \mathbf{p}^A, \mathbf{p}^R, \mathbf{p}^B} \left\{ \sum_{m=1}^M \sum_{k=1}^K \sum_{j=1}^K \pi_{(k,j)} \tau_{m,(k,j)} \left(\frac{1}{2} C(\text{SNR}_{m,j}^A) \right. \right. \\ \left. \left. + \frac{1}{2} C(\text{SNR}_{m,j}^B) \right) + \sum_{m=1}^M \nu_m \left(P_{A_m} - \sum_{k=1}^K p_{m,k}^A \right) \right. \\ \left. + \sum_{m=1}^M \eta_m \left(P_{B_m} - \sum_{k=1}^K p_{m,k}^B \right) + \lambda \left(P_R - \sum_{j=1}^K p_j^R \right) \right. \\ \left. \left| \sum_{k=1}^K \pi_{(k,j)} = 1, \forall j, \sum_{j=1}^K \pi_{(k,j)} = 1, \forall k, \sum_{m=1}^M \tau_{m,(k,j)} = 1, \forall (k,j) \right\}, \quad (8) \end{aligned}$$

and $\nu = [\nu_1, \dots, \nu_M]^T$, $\lambda, \eta = [\eta_1, \dots, \eta_M]^T$ are the associated Lagrange multipliers or the dual variables.

A. Lagrange Dual Decomposition: Solving the Dual Function

To proceed with the dual problem (7), we need to first find the dual function (8) for given initial λ, ν , and η . The dual function can be re-expressed as

$$\begin{aligned} D(\nu, \lambda, \eta) = \max_{\pi, \tau, \mathbf{p}^A, \mathbf{p}^R, \mathbf{p}^B} \left\{ \sum_{m=1}^M \sum_{k=1}^K \sum_{j=1}^K \pi_{(k,j)} \tau_{m,(k,j)} \left(\frac{1}{2} C(\text{SNR}_{m,j}^A) \right. \right. \\ \left. \left. + \frac{1}{2} C(\text{SNR}_{m,j}^B) - \nu_m p_{m,k}^A - \lambda p_j^R - \eta_m p_{m,k}^B \right) \right. \\ \left. + \sum_{m=1}^M \nu_m P_{A_m} + \lambda P_R + \sum_{m=1}^M \eta_m P_{B_m} \right| \end{aligned}$$

$$\begin{aligned} \sum_{k=1}^K \pi_{(k,j)} = 1, \forall j, \sum_{j=1}^K \pi_{(k,j)} = 1, \forall k, \\ \sum_{n=1}^N \sum_{m=1}^M \tau_{m,(k,j)} = 1, \forall (k,j) \}. \quad (9) \end{aligned}$$

Clearly, for given π, τ , the optimal $\mathbf{p}^A, \mathbf{p}^R$, and \mathbf{p}^B could be found from the following sub-problems:

$$\begin{aligned} \max_{p_{m,k}^A, p_j^R, p_{m,k}^B} \quad & \frac{1}{2} C(\text{SNR}_{m,j}^A) + \frac{1}{2} C(\text{SNR}_{m,j}^B) - \nu_m p_{m,k}^A \\ & - \lambda p_j^R - \eta_m p_{m,k}^B, \\ \text{s.t.} \quad & p_{m,k}^A \geq 0, p_j^R \geq 0, p_{m,k}^B \geq 0. \end{aligned} \quad (10)$$

We solve (10) for all m, k, j , thus there are total MK^2 sub-problems. The power allocation problem in (10) is non-convex and finding the closed form solution is not trivial. Nevertheless, the optimal solution $(\hat{p}_{m,k}^A, \hat{p}_j^R, \hat{p}_{m,k}^B)$ can be obtained through searching over $p_{m,k}^A, p_j^R$, and $p_{m,k}^B$, assuming that each takes discrete values [8], [9]. This approach requires $O(Z^3)$ computational complexity where Z is the number of power levels that can be taken by each of $p_{m,k}^A, p_j^R$, and $p_{m,k}^B$. Therefore the total complexity of solving power allocation for all $m, (k, j)$ is $O(MK^2 Z^3)$.

Substituting optimal power values $\hat{p}_{m,k}^A, \hat{p}_j^R$, and $\hat{p}_{m,k}^B$ into (9), we obtain

$$\begin{aligned} D(\nu, \lambda, \eta) \\ = \max_{\pi, \tau} \left\{ \sum_{m=1}^M \sum_{k=1}^K \sum_{j=1}^K \pi_{(k,j)} \tau_{m,(k,j)} F_{m,(k,j)} + \sum_{m=1}^M \nu_m P_{A_m} \right. \\ \left. + \lambda P_R + \sum_{m=1}^M \eta_m P_{B_m} \right| \sum_{k=1}^K \pi_{(k,j)} = 1, \forall j, \\ \sum_{j=1}^K \pi_{(k,j)} = 1, \forall k, \sum_{m=1}^M \tau_{m,(k,j)} = 1, \forall (k,j) \}, \quad (11) \end{aligned}$$

where $F_{m,(k,j)}$ is obtained by substituting $\hat{p}_{m,k}^A, \hat{p}_j^R$, and $\hat{p}_{m,k}^B$ into the objective of (10).

To find the optimum sub-carrier allocation under a given sub-carrier pairing, (11) becomes

$$\begin{aligned} \max_{\tau} \left\{ \sum_{m=1}^M \sum_{k=1}^K \sum_{j=1}^K \tau_{m,(k,j)} F_{m,(k,j)} + \sum_{m=1}^M \nu_m P_{A_m} + \lambda P_R \right. \\ \left. + \sum_{m=1}^M \eta_m P_{B_m} \right| \sum_{m=1}^M \tau_{m,(k,j)} = 1, \forall (k,j) \}. \quad (12) \end{aligned}$$

The optimal solution of (12) is obtained by choosing an MU pair that maximizes $F_{m,(k,j)}$, i.e.,

$$\hat{\tau}_{m,(k,j)} = \begin{cases} 1, & \text{for } m = \arg \max_m F_{m,(k,j)}, \forall (k,j), \\ 0, & \text{otherwise.} \end{cases} \quad (13)$$

For a given $\pi_{(k,j)}$, each maximization operation in (13) has the complexity of $O(M)$ and the total complexity of solving sub-carrier allocation problem thus is $O(MK^2)$.

It remains to find the optimal sub-carrier pairing $\hat{\pi}$. Substituting (13) into (11), we obtain

$$D(\nu, \lambda, \eta) = \max_{\pi} \left\{ \sum_{k=1}^K \sum_{j=1}^K \pi(k, j) F_{m^*, (k, j)} + \sum_{m=1}^M \nu_m P_{A_m} + \lambda P_R + \sum_{m=1}^M \eta_m P_{B_m} \middle| \sum_{k=1}^K \pi(k, j) = 1, \forall j, \sum_{j=1}^K \pi(k, j) = 1, \forall k \right\}, \quad (14)$$

where $F_{m^*, (k, j)} = \max_m F_{m, (k, j)}, \forall (k, j)$. Let \mathbf{F} be a $K \times K$ matrix such that

$$\mathbf{F} = \begin{bmatrix} F_{m^*, (1, 1)} & F_{m^*, (1, 2)} & \cdots & F_{m^*, (1, K)} \\ F_{m^*, (2, 1)} & F_{m^*, (2, 2)} & \cdots & F_{m^*, (2, K)} \\ \vdots & \vdots & \ddots & \vdots \\ F_{m^*, (K-1, 1)} & F_{m^*, (K-1, 2)} & \cdots & F_{m^*, (K-1, K)} \\ F_{m^*, (K, 1)} & F_{m^*, (K, 2)} & \cdots & F_{m^*, (K, K)} \end{bmatrix}. \quad (15)$$

The matrix \mathbf{F} can be considered as a profit matrix with row indices being different operators and column indices being different machines to be operated, i.e., a total of K different machines to be operated by K different operators. The value of each entry can be treated as the profit from operating a particular machine by a particular operator. Problem (14) is equivalent to maximizing the sum profit by choosing the best strategy where each operator (k) can operate only one machine (j). Such kind of linear assignment problem can be solved efficiently from the standard Hungarian algorithm with the complexity $O(K^3)$ [17]. The steps of Hungarian algorithm are briefly described as follows:

- 1) Subtract the values in each row from the maximum number in the row. and subtract the minimum number in each column from the entire column.
- 2) Cover all zeroes in the matrix with as few lines as possible.
- 3) If the number of lines equals to the size of the matrix, find the solution. Otherwise, find the minimum number that is uncovered. Subtract this minimum number from all uncovered values and add it to values at the intersections of lines, and go to step 2.

Interested readers are referred to [17] for more details.

Finally, the dual function can be obtained by substituting $\hat{\pi}$, $\hat{\tau}$, \hat{p}^A , \hat{p}^R , and \hat{p}^B into (8).

B. Solving the Dual Problem with Sub-gradient Method

Next we solve the dual problem (7) to find the optimal values of dual variables. From the sub-gradient method [18], we could pick up initial dual variables $\lambda^{(0)}$, $\nu^{(0)}$, and $\eta^{(0)}$ to find the power allocation in (10). Then with the obtained \hat{p}^A , \hat{p}^R , and \hat{p}^B , the dual variables at $(i+1)$ -th iteration should

be updated as

$$\nu_m^{(i+1)} = \left[\nu_m^{(i)} - \delta^{(i)} \left(P_{A_m} - \sum_{k=1}^K \sum_{j=1}^K \hat{\pi}(k, j) \hat{\tau}_{m, (k, j)} \hat{p}_{m, k}^A \right) \right]^+, \quad (16)$$

$$\eta_m^{(i+1)} = \left[\eta_m^{(i)} - \delta^{(i)} \left(P_{B_m} - \sum_{k=1}^K \sum_{j=1}^K \hat{\pi}(k, j) \hat{\tau}_{m, (k, j)} \hat{p}_{m, k}^B \right) \right]^+, \quad (17)$$

for all m , and

$$\lambda^{(i+1)} = \left[\lambda^{(i)} - \delta^{(i)} \left(P_R - \sum_{j=1}^K \hat{p}_j^R \right) \right]^+, \quad (18)$$

where $[x]^+ \triangleq \max(0, x)$, and $\delta^{(i)}$ is an appropriate step size of the i th iteration. Note that, for each iteration, all the variables $\hat{\pi}(k, j)$, $\hat{\tau}_{m, (k, j)}$, $\hat{p}_{m, k}^A$, $\hat{p}_{m, k}^B$ and \hat{p}_j^R should be re-computed under $\lambda^{(i)}$, $\nu_m^{(i)}$, and $\eta_m^{(i)}$. The iteration will be stopped once certain criterion is fulfilled. Then, we normalize \hat{p}^A , \hat{p}^R , and \hat{p}^B so that the power constraint at each node is satisfied.

If the dual objective function $D(\nu, \lambda, \eta)$ is minimized within N iterations, the total computational complexity of our proposed scheme becomes $O(NK^2(M(Z^3 + 1) + K))$ which is much less than that of solving problem by exhaustive search, i.e., $O(NMK^1Z^3)$.

C. Suboptimal Algorithm

The algorithm derived in previous subsections provides a near optimal solution for the large number of sub-carriers. However the computational efficiency decreases with the increasing of K and M . In this subsection we propose a suboptimal algorithm which trades the performance for lower complexity. We solve the optimization (6) following a step-wise approach where each resource is optimized while fixing the others. The algorithm is outlined as:

1) *Sub-carrier Allocation for Given Power Allocation:* Initially, we fix the power allocation by equally distributing the available powers at RS and each MU to the K sub-carriers, i.e., $p_k^R = \frac{P_R}{K}, \forall k$, $p_{m, k}^A = \frac{P_{A_m}}{K}, \forall m, k$, and $p_{m, k}^B = \frac{P_{B_m}}{K}, \forall m, k$. Then each sub-carrier k is assigned to an m -th user pair, denoted as m_k^* , such that

$$m_k^* = \arg \max_m (\text{SNR}_{m, k}^A + \text{SNR}_{m, k}^B), \forall k, \quad (19)$$

where $\text{SNR}_{m, k}^A = \frac{p_k^R |\tilde{h}_{m, k}|^2 \rho_k^2 p_{m, k}^B |g_{m, k}|^2}{(p_k^R \rho_k^2 |\tilde{h}_{m, k}|^2 + 1) \sigma^2}$, $\text{SNR}_{m, k}^B = \frac{p_k^R |\tilde{g}_{m, k}|^2 \rho_k^2 p_{m, k}^A |h_{m, k}|^2}{(p_k^R \rho_k^2 |\tilde{g}_{m, k}|^2 + 1) \sigma^2}$. In this process a set of K_m number of sub-carriers, denoted as Ω_m , is assigned to m -th MU pair such that $0 \leq K_m \leq K$, and $\sum_{m=1}^M K_m = K$.

For a given k , obtaining the optimum m_k^* in (19) requires a complexity of $O(M)$, and thus the total computational complexity of sub-carrier allocation becomes $O(MK)$ which is NK times less than that from (13).

2) *Sub-carrier Pairing for Given Power Allocation and Sub-carrier Allocation*: To find the sub-carrier pairing, we first re-distribute the power at each of the MU such that $p_{m,k}^A = \frac{P_m^A}{K_m}$ and $p_{m,k}^B = \frac{P_m^B}{K_m}$, $\forall m, k \in \Omega_m$. For the m -th user pair, we choose a carrier k^* in MA phase such that

$$k^* = \arg \max_{k \in \Omega_m} p_{m,k}^A |h_{m,k}|^2 + p_{m,k}^B |g_{m,k}|^2, \quad (20)$$

and pair it with sub-carrier j^* in the BC phase, where

$$j^* = \arg \max_{j \in \Omega_m} (\text{SNR}_{m,j,k^*}^A + \text{SNR}_{m,j,k^*}^B), \quad (21)$$

and $\text{SNR}_{m,j,k^*}^A = \frac{p_j^R |\tilde{h}_{m,j}|^2 \rho_j^2 p_{m,k^*}^B |g_{m,k^*}|^2}{(p_j^R \rho_j^2 |\tilde{h}_{m,j}|^2 + 1) \sigma^2}$, $\text{SNR}_{m,j,k^*}^B = \frac{p_j^R |\tilde{g}_{m,j}|^2 \rho_j^2 p_{m,k^*}^A |h_{m,k^*}|^2}{(p_j^R \rho_j^2 |\tilde{g}_{m,j}|^2 + 1) \sigma^2}$. Each of the maximization in (20) and (21) have the complexity of $O(K_m)$, and hence the sub-carrier pairing for all M users require the complexity of $O(\sum_{m=1}^M 2K_m) = O(2K)$.

3) *Power Allocation for Given Sub-carrier Allocation and Sub-carrier Pairing*: For the obtained sub-carrier allocation and sub-carrier pairing, we recalculate the power allocation using the dual decomposition approach, where the dual function can be decomposed into K sub-problems, each being similar to (10). The dual variables are found from sub-gradient method in subsection III-B. The solution will converge after N' updates of (16), (17), and (18).

The power allocation requires a complexity of $O(N'KZ^3)$, and thus the total complexity of the algorithm from step 1 to step 3 is $O(K(M + N'Z^3 + 2))$. Without loss of generality, let $N' = N$. The overall complexity of the proposed suboptimal algorithm is much less than $O(NK^2(M(Z^3 + 1) + K))$, the complexity of the joint resource allocation scheme.

IV. SIMULATION RESULTS

In this section, we provide simulation results to evaluate the performance of our proposed algorithms. We consider 6-tap channels taken from i.i.d. Gaussian random variables for all links, while the total number of sub-carriers is set as 32. Without loss of generality, we assume equal power at each node. The figure of the merit is taken as the per tone rate, i.e., sum rate divided by K .

We compare the following algorithms:

- JntOpt: The joint optimal solution proposed in subsection III-A and subsection III-B.
- SubOpt: The suboptimal solution presented in subsection III-C.
- SolWOP: A solution where power allocation and sub-carrier allocation is optimized but sub-carrier pairing is not considered. Algorithm follows the steps for joint optimization algorithm in subsection III-A and III-B with $k = j$ and omits the sub-carrier pairing step (14) and (15).
- Static: A fixed resource allocation solution where each user is randomly assigned an amount of sub-carriers and then the available power is distributed evenly among the allocated sub-carriers. The tone permutation is not considered.

The complexity involved in each algorithms is summarized in Table I, where N'' denotes the number of iterations required for subgradient convergence in SolWOP algorithm. Further,

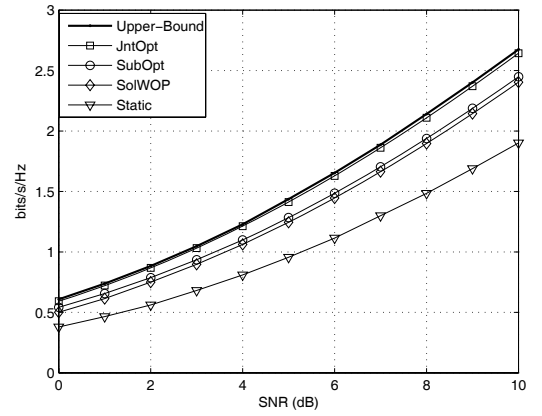


Fig. 2. Throughput versus SNR for $M = 10$.

the running time of different schemes for different number of users at SNR= 10 are also displayed. In the first example, we show the throughput performance of different algorithms versus SNR for $M = 10$ in Fig. 2. The objective of the dual problem (Upper-Bound) is also displayed in the same figure. We observe that the gap between the primal objective and the dual objective, i.e., the duality gap is close to zero for all SNR region, which validates the optimality of the proposed scheme. Moreover it can be seen that JntOpt yields the best performance over all SNR values. We notice a performance gain of 2.4 dB over the Static solution at rate equal to 1 bits/sec/Hz, and it increases to 2.85 dB at rate equal to 1.8 bits/sec/Hz. In comparison the rate losses of the suboptimal algorithm is 0.6 dB and 1 dB, respectively. We observe that the SolWOP exhibits a slightly lower performance to the SubOpt over all SNR region but with a much higher complexity.

Next we examine the performance of the end-to-end rate versus the number of MUs. The corresponding curves at SNR= 10 dB are shown in Fig. 3. It can be seen that JntOpt always yields the best performance, and a significant gain over Static is observed when the number of the users increases. This is because Static does not exploit multi-user diversity and the optimization becomes more significant while increasing the number of users. The performance of SubOpt and SolWOP also increases with the number of users due to the similar reasons and both exhibits much better gain over Static. On the other hand, we observe a significant increase in running time of JntOpt and SolWOP in table I, when the number of users increases. The running times of both SubOpt and Static is much less than that of both JntOpt and SolWOP, and do not increase much with the increasing of the number of users.

To get a more insight into the performance gain achieved from sub-carrier pairing, in the next example we compare JntOpt and SubOpt with SolWOP under the case when $|h_{m,1}| > |h_{m,2}| \dots > |h_{m,K}|$, $|\tilde{h}_{m,1}| < |\tilde{h}_{m,2}| \dots < |\tilde{h}_{m,K}|$, $|g_{m,1}| > |g_{m,2}| \dots > |g_{m,K}|$, $|\tilde{g}_{m,1}| < |\tilde{g}_{m,2}| \dots < |\tilde{g}_{m,K}|$, $\forall m$. The throughput curves versus SNR for $M = 2$ and $M = 10$ are shown in Fig. 4. It can be seen that JntOpt and SubOpt yield good performance. However, SolWOP exhibits much worse performance as compared to that in Fig. 2 because the good channel in MA phase is always paired with the bad channel in BC phase when no pairing strategy is adopted.

TABLE I
COMPLEXITY COMPARISON OF DIFFERENT ALGORITHMS

Algorithm	Complexity	Running Time (Seconds)				
		M=2	M=5	M=10	M=15	M=20
JntOpt	$O(NK^2(M(Z^3 + 1) + K))$	113.2310	297.6780	598.6954	868.5970	1177.769
SubOpt	$O(K(M + N'Z^3 + 2))$	1.7810	1.8281	1.8910	1.8984	1.9010
SolWOP	$O(MN''K^2(Z^3 + 1))$	3.6250	9.1240	19.0121	27.405	35.951
Static	$O(M)$	0.0155	0.060	0.060	0.060	0.0601

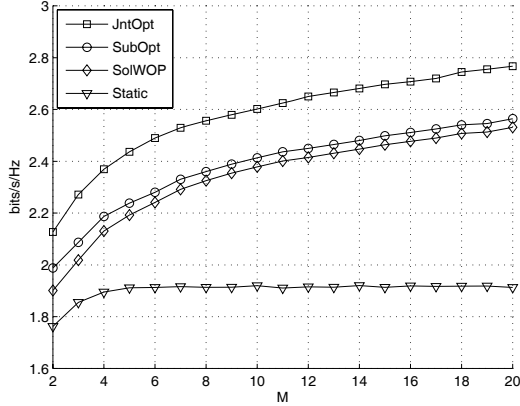


Fig. 3. Throughput versus the number of users at SNR = 10 dB.

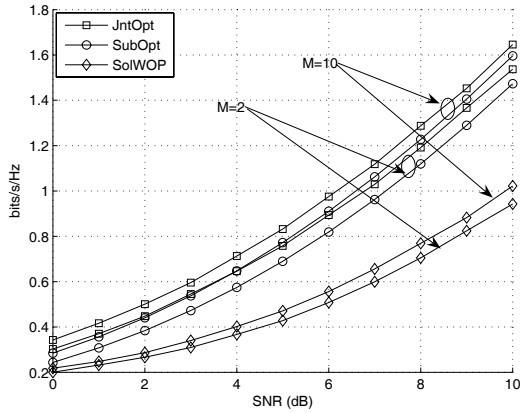


Fig. 4. Throughput versus SNR under anti-symmetric channels for $M = 2$ and $M = 10$, respectively.

V. CONCLUSION

In this letter, we studied the problem of joint resource allocation for OFDMA assisted two-way relay system. The objective function is to maximize the sum-rate through joint sub-carrier allocation, sub-carrier pairing, and power allocation, under the individual power constraints at each transmitting node. The problem is solved from the dual decomposition technique and an asymptotically optimal solution is found, thanks to the previous result that the duality gap approaches

zero when the number of the sub-carriers is large. To reduce the complexity of the algorithm, we further proposed a sub-optimal algorithm which showed its comparable performance via simulation results. Numerical examples demonstrated that the proposed algorithms significantly outperform other candidates.

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