A Multi-Cell Beamforming Design by Uplink-Downlink Max-Min SINR Duality

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***Abstract*—In this paper, we address the problem of the coordinated beamforming design for multi-cell multiple input single output (MISO) downlink system subject to per-BS power constraints. The objective is taken as the maximization of the minimum signal-to-interference plus noise ratio (SINR), while a complete analysis of the duality between the multi-cell downlink and the virtual uplink optimization problems is provided. A hierarchical iterative scheme is proposed to solve the virtual uplink optimization problem, whose solution is then converted to derive the one of the multi-cell downlink beamforming problem. The proposed algorithm is proved to converge to a stable point. Additional, the complexity of the proposed algorithm is analyzed. Simulation results show that, in contrast to existing multi-cell beamforming schemes, the proposed algorithm achieves better performance in terms of both rate per energy (RPE) and the worst-user rate.**

***Index Terms*—Multi-cell beamforming, Lagrangian duality theory, max-min SINR, min-max SINR, hierarchical iterative algorithm.**

I. Introduction

FREQUENCY reuse has emerged as an attractive strategy to enhance the utilization of spectrum resource in cellular mobile communication system. Meanwhile, frequency reuse can simplify cellular network planning and base station deployment, and has a potential of enhancing the coverage and accommodating more users in cellular network. However, frequency reuse might result in severe interference if treated inappropriately, especially for cell-edge users. To solve this problem, there has been a rapidly growing interest in shifting the design paradigm from single-cell processing to multi-cell cooperative processing.

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The key points of the multi-cell cooperative transmission include cooperating cell clustering and multi-cell beamforming design. Several strategies have been proposed to address the problem of cooperative cell clustering, including network-centric, user-centric method, or the combination of them [1]. By exploiting the idea of dirty paper coding [2], the optimal performance of multi-cell cooperation was investigated. However, this strategy is hard to realize due to the high computational complexity and the requirement of full phase coherence among signals from different base stations (BSs). To reduce the complexity, a centralized multi-cell beamforming scheme was designed based on the idea of block diagonalization precoding [3]. All the above mentioned schemes require traffic data sharing between the cells, causing a heavy burden for the backhaul link. To release this burden, another strategy of multi-cell beamforming which requires no data sharing among cells has been studied recently [4] [5].

Previous works have shown that the uplink-downlink duality theory can be exploited to efficiently design the optimal coordinated multi-cell downlink beamforming. The basic idea is to convert the downlink optimization problem into a virtual uplink optimization problem in which the coupling between different optimization variables in the objective function is considerably released. In this case, the optimization for the multi-cell downlink beamforming can be obtained from the uplink solution. Note that the conventional uplink-downlink duality was first developed in the multiuser context [6]–[8]. It was revealed that the same SINR region can be achieved by both the uplink and the downlink with the same set of beamforming vectors and the same sum power [9]. Concerning a simple sum-power constraint, this duality can be applied to solve the sum power minimization problem subject to SINR constraints, the SINR balancing problem and the capacity computation problem. Furthermore, Yu [10] had extended the duality to solve the sum power minimization problem and the capacity computation problem subject to per-antenna power constraints. In this case, the dual uplink problem suffered from an uncertain noise covariance and therefore it is hard to develop an efficient algorithm. Recently, a more general uplink-downlink duality relationship was disclosed in [11], and a simpler form than that in [10] was obtained. When shifting from the multi-user context to the multi-cell context, Yu [12] had extended the uplink-downlink duality to solve the sum-power minimization problem with SINR constraints. More recently, Huang [13] further applied the duality to achieve a distributed solution to the max-min SINR multi-cell beamforming problem, where the primal problem was addressed by searching for a feasible SINR target for which the sum-power minimization problem was solved to exactly meet the power constraints. Following this strategy, a robust max-min SINR multi-cell coordinated beamforming solution was further proposed to take into account the channel estimation error [14]. It should be noted that the above strategies involved multiple layers of iterations and had a slow convergence when concerning per-BS power constraints.

In this paper, we design the multi-cell beamforming to maximize the minimum SINR, which could provide better user fairness for multi-cell MISO downlink system subject to per-BS power constraints. Different from the existing coordinated beamforming strategies [13]–[15] where the max-min SINR problem was addressed by iteratively solving the sum-power minimization problem to search for a maximum feasible SINR, we target at a more efficient solution by deriving the analytical expression of the max-min SINR duality between the uplink and the downlink. To the best of authors' knowledge, a complete algorithmic analysis for the max-min SINR duality between the uplink and the downlink was only provided in a multiuser context [16] [17] with the nonlinear Perron-Frobenius theory [18]. In the coordinated multi-cell context with per-BS power constraints, the analytical expression of the uplink-downlink duality was only provided for the sum-power minimization problem with given SINR constraints [12]. As shown in [13], though this form of duality can be used to devise an iterative solution to the multi-cell beamforming coordination, it is highly inefficient due to the multi-layer iterations. A complete analytical expression of the uplink-downlink max-min SINR duality has not yet be presented for a multi-cell interference downlink system subject to per-BS power constraints. Here we first present an algorithmic analysis for the uplink-downlink max-min SINR duality in the multi-cell interference downlink system subject to per-BS power constraints, and then achieve an iterative solution to the coordinated multi-cell beamforming problem which has a better convergence behavior. Our main contributions are listed as follows.

1. A complete analytical expression of the uplink-downlink max-min SINR duality for a coordinated multi-cell beamforming system subject to per-BS power constraints is presented, showing that the downlink max-min SINR optimization problem is dual to an uplink min-max problem subject to a sum-sum power constraint plus a weighted-sum virtual noise variance constraint.
2. An iterative algorithm is further developed to solve the virtual uplink min-max SINR optimization problem by revealing its hidden geometric programming (GP) structure. The proposed algorithm is proved to converge to a stable point, and the complexity of the proposed algorithm is also analyzed. Then, the solution to the primal downlink max-min SINR problem is calculated from the uplink solution.

This rest of this paper is organized as follows. The system model is described in Section II. In Section III, we formulate the duality between the downlink optimization problem and the virtual uplink optimization problem. The coordinated beamforming and power allocation algorithm is given in Section IV. The simulation results are shown in Section V. Conclusions are finally given in Section VI.

The following notations are used throughout this paper. Bold lowercase and uppercase letters represent column vectors and matrices, respectively. The function $\lambda_{max}(\boldsymbol{A})$ and $\upsilon_{max}(\boldsymbol{A})$ denote the largest eigenvalue of the matrix $\boldsymbol{A}$ and the eigenvector correspond to the largest eigenvalue of the matrix $\boldsymbol{A}$. The superscript $^{\mathrm{T}}$, $^{\mathrm{H}}$ and † represent the transpose operator, conjugate transpose operator, the Moore Penrose pseudo-inverse of matrix, respectively. Vector $\mathbf{1}_K$ is a $K \times 1$ all one vector.

II. System Model

We consider a multi-cell MISO downlink system with multiple BSs simultaneously transmitting signal to users in its own cell. The channel state information (CSI) needs to be shared between the BSs but no data sharing is required. We assume that the clustering of cooperating BSs and user scheduling have been completed based on a network-centric or user-centric way. We focus on the case where K M-antenna BSs are clustered together and each serves a single-antenna user, while the generalization to multiple users per-cell will be discussed later. An illustration of three-BS cooperating model is shown in Fig. 1. Denoting the i^{th} BS and its served user as BS i and user i, respectively. The received signal of user i is written as

$$y_i = \sqrt{p_i}\boldsymbol{h}_{i,i}^H \boldsymbol{w}_i x_i + \sum_{\substack{k=1 \\ k\neq i}}^{K} \sqrt{p_k}\boldsymbol{h}_{i,k}^H \boldsymbol{w}_k x_k + n_i \tag{1}$$

where p_i denotes the transmit power for user i, $\boldsymbol{h}_{i,k}$ denotes the channel vector from BS k to user i, which incorporates large scale fading, small scale fading and shading fading, $\boldsymbol{w}_i$ denotes the unit-norm beamformer vector for user i, and n_i denotes the additive white Gaussian noise with zero mean and variance σ_i^2, namely, $n_i \sim \mathcal{CN}(0, \sigma_i^2)$. The SINR of user i is

$$\begin{aligned} \mathrm{SINR}_i^{Down} &= \frac{p_i \boldsymbol{w}_i^H \boldsymbol{h}_{i,i}\boldsymbol{h}_{i,i}^H \boldsymbol{w}_i}{\sum_{k=1,k\neq i}^{K} p_k \boldsymbol{w}_k^H \boldsymbol{h}_{i,k}\boldsymbol{h}_{i,k}^H \boldsymbol{w}_k + \sigma_i^2} \\ &= \frac{p_i \boldsymbol{w}_i^H \boldsymbol{\Omega}_{i,i} \boldsymbol{w}_i}{\sum_{k=1,k\neq i}^{K} p_k \boldsymbol{w}_k^H \boldsymbol{\Omega}_{i,k} \boldsymbol{w}_k + 1} \end{aligned} \tag{2}$$

where $\boldsymbol{\Omega}_{i,k} \triangleq \frac{\boldsymbol{h}_{i,k}\boldsymbol{h}_{i,k}^H}{\sigma_i^2}$, $\forall i,k$. The instantaneous rate of user i can be expressed as

$$R_i = \log_2\left(1 + \mathrm{SINR}_i^{Down}\right) \tag{3}$$

Obviously, the key point of the multi-cell downlink beamforming system above is to jointly design the beamforming vectors $\{\boldsymbol{w}_i\}_{i=1}^K$ and power allocation vector $\boldsymbol{p} = [p_1, \cdots, p_K]^T$. In order to reach a fairness rate optimality, we choose the max-min SINR as the performance metric, and the downlink optimization problem $\mathcal{Q}^{Down}$ is given by

$$\begin{aligned} \mathcal{Q}^{Down}: \quad & \max_{\{\boldsymbol{w}_i, p_i\}_{i=1}^K} \min_i \mathrm{SINR}_i^{Down} \\ s.t. \quad & p_i \geq 0, p_i \leq P_i, ||\boldsymbol{w}_i|| = 1, \forall i \end{aligned} \tag{4}$$

where P_i denotes the individual power constraint for user i.

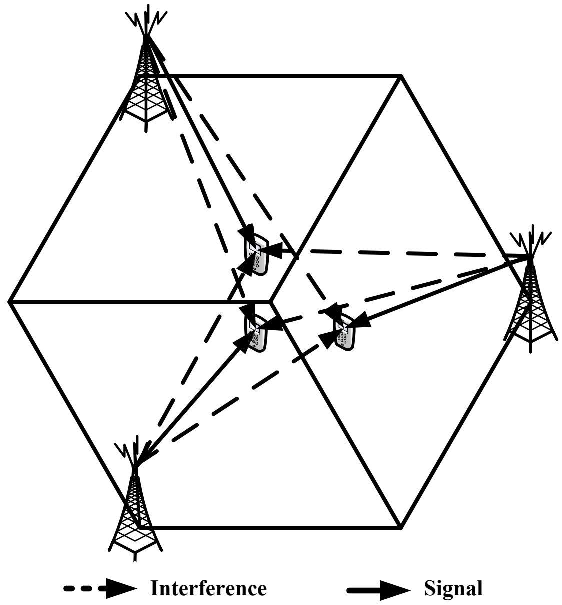

Fig. 1. System model.

The problem is hard to solve directly since the optimization variables are highly coupled. Hence, we aim to develop a solution to the coordinated multi-cell beamforming optimization by analyzing the max-min SINR duality between the uplink and the downlink of the multi-cell MISO downlink system.

III. Uplink-Downlink Max-Min SINR Duality in Multi-cell Systems

The existing uplink-downlink duality theory for multiuser systems says that the uplink and the downlink transmission can achieve the same SINR region with the same sum power and the same set of beamforming vectors [9] [10]. Furthermore, if a simple sum-power constraint is considered, the duality can be used in multi-cell interference downlink system to solve the downlink SINR max-min problem. However, per-BS power constraints should be considered in practical scenario, for which the existing uplink-downlink duality is not applicable. In order to solve the optimization problem (4), we here attempt to provide an algorithmic analysis on the max-min SINR duality between the uplink and the downlink transmission.

Theorem 1. *Considering per-BS transmit power constraints, the multi-cell downlink max-min SINR optimization problem $\mathcal{Q}^{Down}$ is dual to the following virtual uplink min-max SINR optimization problem $\mathcal{Q}^{Up}$:*

$$\begin{aligned} &\mathcal{Q}^{Up}: \min_{\{\upsilon_i\}_{i=1}^K} \max_{\{\boldsymbol{w}_i,\lambda_i\}_{i=1}^K} \min_i \mathit{SINR}_i^{Up} \\ s.t.\quad & \lambda_i \geq 0, \upsilon_i \geq 0, ||\boldsymbol{w}_i|| = 1, \forall i \\ & \sum_{i=1}^K \lambda_i \leq \sum_{i=1}^K P_i, \sum_{i=1}^K \upsilon_i P_i \leq \sum_{i=1}^K P_i \end{aligned} \tag{5}$$

where $\mathit{SINR}_i^{Up} = \frac{\lambda_i \boldsymbol{w}_i^H \boldsymbol{\Omega}_{i,i} \boldsymbol{w}_i}{\sum_{k=1,k\neq i}^K \lambda_k \boldsymbol{w}_i^H \boldsymbol{\Omega}_{k,i} \boldsymbol{w}_i + \upsilon_i}$ is the virtual uplink SINR of user i, the duality variables $\{\lambda_i\}_{i=1}^K$ can be interpreted as the transmit power of each user in the multi-cell virtual uplink, and the duality variables $\{\upsilon_i\}_{i=1}^K$ can be interpreted as the virtual noise variance at each BS.

In the dual virtual uplink problem, the individual power constraint in the downlink problem has been converted into a sum-sum power constraint $\sum_{i=1}^K \lambda_i \leq \sum_{i=1}^K P_i$ plus a weighted-sum constraint of virtual noise variances $\sum_{i=1}^K \upsilon_i P_i \leq \sum_{i=1}^K P_i$. Note that this problem might have multiple solutions with the same worst-user SINR, among which it is easy to understand that the solution with balanced SINR levels consumes the least sum power, i.e., achieves the best balanced energy efficiency. Our aim is to achieve this particular solution.

Remark: Though it looks similar to the BC-MAC duality form which was derived in [11], our proposed duality form is applied to the coordinated multi-cell beamforming systems while the BC-MAC duality is designed for the multiuser systems or the multi-cell joint transmission systems. As shown in [11], the uplink noise variance of the BC-MAC duality is the weighted sum of power constraint matrix, whereas the uplink noise variance here is only an uncertain variable, showing a GP structure.

The proof of this theorem can be found in Appendix A. Based on the above results, we see that the downlink optimization problem $\mathcal{Q}^{Down}$ can be addressed by handling the dual uplink optimization problem $\mathcal{Q}^{Up}$ which is reformulated as

$$\begin{aligned} &\mathcal{Q}^{Up}: \min_{\boldsymbol{\upsilon}} \max_{\boldsymbol{\lambda},\boldsymbol{W},\gamma} \gamma \\ s.t.\quad & \gamma \leq \mathrm{SINR}_i^{Up}, \forall i \\ & \lambda_i \geq 0, \upsilon_i \geq 0, ||\boldsymbol{w}_i|| = 1, \forall i \\ & \sum_{i=1}^K \lambda_i \leq \sum_{i=1}^K P_i, \sum_{i=1}^K \upsilon_i P_i \leq \sum_{i=1}^K P_i \end{aligned} \tag{6}$$

From the proof of Theorem 1, we see that the primal optimization problem and the dual optimization problem have an important property, namely, from the perspective of the minimizing the sum of transmit power, all users can achieve an identical optimal balanced SINR levels when the optimal solution of the max-min SINR optimization problem is obtained. In the following section, we will make use of this property to design a hierarchical iterative optimization algorithm to solve the virtual uplink min-max SINR optimization problem.

In addition, Theorem 1 can be easily generalized to the case of multiple users per-cell. Let $x_{i,j}$ denote the transmitted signal for the j^{th} user in the i^{th} cell and $\boldsymbol{w}_{i,j}$ be its associated beamforming vector. The received signal at the j^{th} user in the i^{th} cell, denoted as $y_{i,j}$, is a summation of the intended signal, intracell interference, and intercell interference:

$$y_{i,j} = \sum_{m,n} \sqrt{p_{m,n}} \boldsymbol{h}_{m,i,j}^H \boldsymbol{w}_{m,n} x_{m,n} + n_{i,j} \tag{7}$$

where $p_{i,j}$ denotes the transmit power for the j^{th} user in the i^{th} cell, $\boldsymbol{h}_{m,i,j}$ denotes the channel vector from the BS in the m^{th} cell to the j^{th} user in the i^{th} cell, and $n_{i,j}$ denotes the additive white Gaussian noise with zero mean and variance

$\sigma_{i,j}^2$. The SINR of the j^{th} user in the i^{th} cell is

$$\mathrm{SINR}_{i,j}^{Down} = \frac{p_{i,j}\boldsymbol{w}_{i,j}^H \Omega_{i,i,j}\boldsymbol{w}_{i,j}}{\sum\limits_{(m,n)\neq(i,j)} p_{m,n}\boldsymbol{w}_{m,n}^H \Omega_{m,i,j}\boldsymbol{w}_{m,n} + 1} \tag{8}$$

where, $\boldsymbol{\Omega}_{m,i,j} \triangleq \frac{\boldsymbol{h}_{m,i,j}\boldsymbol{h}_{m,i,j}^H}{\sigma_{i,j}^2}$, $\forall m,i,j$. From a similar procedure of Theorem 1, the following uplink-downlink duality form for the general case can be derived.

Theorem 2. *Considering per-BS power constraints and per-BS serves simultaneously several users in a multi-cell interference downlink, the max-min SINR optimization problem is given by*

$$\begin{aligned} &\max_{\{\boldsymbol{w}_{i,j},p_{i,j}\}\forall i,j} \min_{i,j} \mathit{SINR}_{i,j}^{Down} \\ &s.t. \sum_j p_{i,j} \leq P_i, p_{i,j} > 0, \|\boldsymbol{w}_{i,j}\| = 1, \forall i,j \end{aligned} \tag{9}$$

The above optimization problem is dual to the following virtual uplink min-max SINR optimization problem:

$$\begin{aligned} &\min_{\{\upsilon_i\}\forall i} \max_{\{\lambda_{i,j},\boldsymbol{w}_{i,j}\}\forall i,j} \min_{i,j} \mathit{SINR}_{i,j}^{Up} \\ &s.t. \lambda_{i,j} \geq 0, \upsilon_i \geq 0, \|\boldsymbol{w}_{i,j}\| = 1, \forall i,j \\ &\sum_{i,j}\lambda_{i,j} \leq \sum_i P_i, \sum_i \upsilon_i P_i \leq \sum_i P_i \end{aligned} \tag{10}$$

where $\mathit{SINR}_{i,j}^{Up} = \frac{\lambda_{i,j}\boldsymbol{w}_{i,j}^H\Omega_{i,i,j}\boldsymbol{w}_{i,j}}{\sum_{(m,n)\neq(i,j)}\lambda_{m,n}\boldsymbol{w}_{i,j}^H\Omega_{i,m,n}\boldsymbol{w}_{i,j}+\upsilon_i}$ *is the virtual uplink SINR of user i, the duality variables $\lambda_{i,j}$ can be interpreted as the transmit power of the j^{th} user in the i^{th} cell, the duality variables υ_i can be denotes the virtual noise variance at the BS in the i^{th} cell.*

In the dual virtual uplink problem, the individual power constraint in the downlink problem has been converted into a sum-sum power constraint $\sum_{i,j}\lambda_{i,j} \leq \sum_i P_i$ plus a weighted-sum constraint of virtual noise variances $\sum_i \upsilon_i P_i \leq \sum_i P_i$. Note that this problem might have multiple solutions with the same worst-user SINR, among which it is easy to understand that the solution with balanced SINR levels consumes the least sum power, i.e., achieves the best balanced energy efficiency.

IV. Multi-cell Beamforming Algorithm Design

In order to solve the downlink optimization problem $\mathcal{Q}^{Down}$, here we first design an iterative optimization algorithm to solve the virtual uplink optimization problem $\mathcal{Q}^{Up}$ by using bisection method and GP optimization method, then convert its solution to the downlink. The proposed algorithm employs a two-layer strategy: In the inner layer, bisection method is used to address the optimization with respect to $\{\boldsymbol{w}_i\}_{i=1}^K$, $\{\lambda_i\}_{i=1}^K$ and γ for predefined $\{\upsilon_i\}_{i=1}^K$; While in the outer layer, GP optimization method is used to search for the most suitable values of $\{\upsilon_i\}_{i=1}^K$ for predefined $\{\boldsymbol{w}_i\}_{i=1}^K$ and $\{\lambda_i\}_{i=1}^K$.

A. Optimization of Virtual Uplink Powers and Beamformers

When the values of the optimization variable $\{\upsilon_i\}_{i=1}^K$, $\{\lambda_k\}_{\substack{k=1\\k\neq i}}^K$ and the optimal balanced SINR level γ are predefined, the maximization of the SINR_i^{Up} can be formulated as

$$\max_{\lambda_i}\lambda_i \max_{\boldsymbol{w}_i}\frac{\boldsymbol{w}_i^H\boldsymbol{\Omega}_{i,i}\boldsymbol{w}_i}{\boldsymbol{w}_i^H\boldsymbol{\Xi}_i\boldsymbol{w}_i} = \lambda_{max}\left(\boldsymbol{\Xi}_i^\dagger\boldsymbol{\Omega}_{i,i}\right)\max_{\lambda_i}\lambda_i \tag{11}$$

where $\boldsymbol{\Xi}_i = \sum_{\substack{k=1\\k\neq i}}^K \lambda_k\boldsymbol{\Omega}_{k,i} + \upsilon_i\boldsymbol{I}$. When the worst-user SINR is maximized, all other users can also achieve the optimal balanced SINR levels, therefore, we can get the following update equation of $\lambda_i, \forall i$.

$$\lambda_i^* = \frac{\gamma}{\lambda_{max}\left(\boldsymbol{\Xi}_i^\dagger\boldsymbol{\Omega}_{i,i}\right)} \tag{12}$$

When the values of the optimization variables $\{\upsilon_i\}_{i=1}^K$ and $\{\lambda_i\}_{i=1}^K$ are predefined, the problem $\mathcal{Q}^{Up}$ can be divided into K parallel sub-problems:

$$\begin{aligned} &\max_{\boldsymbol{w}_i}\frac{\lambda_i\boldsymbol{w}_i^H\boldsymbol{\Omega}_{i,i}\boldsymbol{w}_i}{\boldsymbol{w}_i^H\boldsymbol{\Xi}_i\boldsymbol{w}_i} \\ &s.t. \quad ||\boldsymbol{w}_i|| = 1, \forall i \end{aligned} \tag{13}$$

It can be easily known that the solution to (13) is the dominant eigenvector [19], i.e.

$$\boldsymbol{w}_i^* = \nu_{max}\left(\lambda_i\boldsymbol{\Xi}_i^\dagger\boldsymbol{\Omega}_{i,i}\right). \tag{14}$$

B. Optimization of Downlink Powers

Once the optimal beamforming vectors $\{\boldsymbol{w}_i^*\}_{i=1}^K$ and the optimal balanced SINR level γ^* value are obtained, according to the developed uplink-downlink duality and the relation $\mathrm{SINR}_i^{Down} = \gamma, \forall i$, the multi-cell downlink power vector can be calculated as

$$\boldsymbol{p}^* = \boldsymbol{G}^\dagger\boldsymbol{1}_K, \tag{15}$$

where $\boldsymbol{p}^* = [p_1, \cdots, p_K]^T$ and the matrix $\boldsymbol{G}$ is given by

$$[\boldsymbol{G}]_{i,k} = \begin{cases} \frac{\boldsymbol{w}_i^{*H}\boldsymbol{\Omega}_{i,i}\boldsymbol{w}_i^*}{\gamma^*} & i = k \\ -\boldsymbol{w}_k^{*H}\boldsymbol{\Omega}_{i,k}\boldsymbol{w}_k^* & i \neq k. \end{cases} \tag{16}$$

C. Optimization of Virtual Uplink Noise Powers

For given values of the optimization variables $\{\lambda_i\}_{i=1}^K$ and the optimization beamforming vectors $\{\boldsymbol{w}_i\}_{i=1}^K$, the optimization problem $\hat{\mathcal{Q}}$ can be formulated as

$$\begin{aligned} \hat{\mathcal{Q}}: &\min_{\{\upsilon_i\}_{i=1}^K}\max_i \mathrm{SINR}_i^{Up} \\ &s.t. \quad \upsilon_i \geq 0, \forall i, \sum_{i=1}^K \upsilon_i P_i \leq \sum_{i=1}^K P_i. \end{aligned} \tag{17}$$

Similar to the method used in [14], we introduce slack variable t. Let $y_i = \sum_{\substack{k=1\\k\neq i}}^K \lambda_k\boldsymbol{w}_i^H\boldsymbol{\Omega}_{k,i}\boldsymbol{w}_i + \upsilon_i$, then, $\upsilon_i = y_i - \sum_{\substack{k=1\\k\neq i}}^K \lambda_k\boldsymbol{w}_i^H\boldsymbol{\Omega}_{k,i}\boldsymbol{w}_i$. The above optimization problem can be rewritten as

$$\begin{aligned} \check{\mathcal{Q}}: &\min_{\{\upsilon_i\}_{i=1}^K} \quad t \\ &s.t. \quad (\lambda_i\boldsymbol{w}_i^H\boldsymbol{\Omega}_{i,i}\boldsymbol{w}_i)t^{-1}y_i^{-1} \leq 1, \forall i \\ &\left(\sum_{k=1,k\neq i}^K \lambda_k\boldsymbol{w}_i^H\boldsymbol{\Omega}_{k,i}\boldsymbol{w}_i\right)y_i^{-1} \leq 1, \forall i \\ &\sum_{i=1}^K \frac{P_i}{\sum\limits_{j=1}^K P_j\left(\sum\limits_{\substack{k=1\\k\neq j}}^K \lambda_k\boldsymbol{w}_j^H\boldsymbol{\Omega}_{k,j}\boldsymbol{w}_j + 1\right)}y_i \leq 1 \end{aligned} \tag{18}$$

It is easily seen that the optimization problem (17) is transformed into standard GP problem form [20] and hence can be solved using standard optimization packages.

D. Hierarchical Iterative Algorithm

Based on the above analysis, we propose the following hierarchical iterative algorithm to solve the optimization problem $\mathcal{Q}^{Up}$.

0) Let $\upsilon_i^{(0)} = \epsilon$, $t^{(0)} = 0, n = 0$.
1) Let $\lambda_i^{(0)} = 0, p_i^{(0)} = 0, \forall i, m = 0, n = n + 1$.
2) Let $m = m + 1$, $\gamma_{min} = 0$, $\gamma_{max} = \frac{\sum_{i=1}^{K} P_i}{\sum_{i=1}^{K} \frac{1}{\lambda_{max}\left(\boldsymbol{\Xi}_i^{(m)\dagger}\boldsymbol{\Omega}_{i,i}\right)}}$, where $\boldsymbol{\Xi}_i^{(m)} = \sum_{k=1,k\neq i}^{K} \lambda_k^{(m-1)}\boldsymbol{\Omega}_{k,i} + \upsilon_i^{(n-1)}\boldsymbol{I}$.
3) Let $\gamma = \frac{\gamma_{min}+\gamma_{max}}{2}$, then computing $\{\lambda_i^{(*)}\}_{i=1}^K$ with (12), $\{\lambda_i^{(m-1)}\}_{i=1}^K$ and γ.
4) Compute $\{\boldsymbol{w}_i^{(*)}\}_{i=1}^K$ with (14) and $\{\lambda_i^{(*)}\}_{i=1}^K$, computing $\{p_i^{(*)}\}_{i=1}^K$ with (15), $\{\boldsymbol{w}_i^{(*)}\}_{i=1}^K$ and γ. If $0 \leq p_i^{(*)} \leq P_i, \forall i$, let $\gamma_{min} = \gamma$, $\lambda_i^{(m)} = \lambda_i^{(*)}$, $\boldsymbol{w}_i^{(m)} = \boldsymbol{w}_i^{(*)}$, $p_i^{(m)} = p_i^{(*)}$, $\forall i$, otherwise, let $\gamma_{max} = \gamma$.
5) If $|\gamma_{max} - \gamma_{min}| \leq \zeta$, then go to step 6, otherwise go to step 3.
6) If $\sum_{i=1}^K |\lambda_i^{(m)} - \lambda_i^{(m-1)}| \leq \delta$, then go to step 7, otherwise go to step 2.
7) Solve (18) with $\{\lambda_i^{(m)}, \boldsymbol{w}_i^{(m)}\}_{i=1}^K$, then get $\{\upsilon_i^{(n)}\}_{i=1}^K$ and $t^{(n)}$. If $|t^{(n)} - t^{(n-1)}| < \xi$, then output $\boldsymbol{w}_i^{(m)}$ and $p_i^{(m)}$, $\forall i$, otherwise go to step 1.

where ϵ is an arbitrarily small positive number, while ζ, δ and ξ are predefined thresholds. The convergence of this two-layer iterative algorithm is proved in Appendix B.

We proceed to analyze the computational complexity of the above algorithm. For each inner iteration, its main complexity involves the element computation of $\boldsymbol{\Xi}_i$, the inversion of $\boldsymbol{\Xi}_i$ and the eigenvalue decomposition of $\boldsymbol{\Xi}_i^\dagger \boldsymbol{\Omega}_{i,i}$. Defining a flop as real floating-point operation, the element computation of $\boldsymbol{\Xi}_i$ needs about $8KM^2$ flops. As matrix $\boldsymbol{\Xi}_i$ is positive definition, its inversion requires about $\frac{8}{3}M^3 - \frac{3}{2}M^2 + \frac{7}{6}M$ flops [21], and the eigenvalue decomposition of $\boldsymbol{\Xi}_i^\dagger \boldsymbol{\Omega}_{i,i}$ requires about $126M^3$ flops. Therefore, Step 3 involves about $\mathcal{F}_3 = 129M^3 + 8KM^2$ flops, while Step 4 involves about $\mathcal{F}_4 = 132M^3 + 8KM^2$ flops. As a result, the whole inner iteration loop approximately needs $N_6 K\left(\mathcal{F}_3 + N_4\mathcal{F}_4\right)$ flops, where N_4 and N_6 are the number of Step 4 and Step 6 involved in each inner iteration loop, respectively. Then, the whole computational complexity of the proposed iterative algorithm involves about $N_{out}N_6 K\left(\mathcal{F}_3 + N_4\mathcal{F}_4\right)$ flops, where N_{out} denotes the iteration number of the outer iteration loop.

V. Simulation Results

In this section, the performance of the proposed iterative multi-cell beamforming scheme is investigated via numerical examples. We consider a cell cluster with inter-BS distance of 1km. We also assume that all users are located in the cell edge such that each user will receive significant intercell interference in the same cluster. More specifically, we consider that each user has a distance $d_1 = 400$m from its serving BS and a distance $d_2 = 600$m from other BSs in the cluster. The channel coefficient $\boldsymbol{h}_{k,j}$ between BS j and user k is generated based on the scenario of urban macro with non-line-of-sight (NLoS) so that $\boldsymbol{h}_{k,j} \triangleq \gamma_{k,j}\widetilde{\boldsymbol{h}}_{k,j}$, where $\widetilde{\boldsymbol{h}}_{k,j}$ denotes the small scale fading channel coefficient and is assumed to be zero-meam Gaussian distributed with the covariance matrix of $\boldsymbol{I}$, and $\gamma_{k,j} = \frac{\beta\chi_{k,j}}{d_{k,j}^{\alpha}}$ denotes the large scale fading factor, β is a scale factor, α denotes the path loss exponent (typically, $\alpha > 2$), $d_{k,j}$ represents the distance between the BS j and the user k, and $\chi_{k,j}$ denotes the lognormal shadowing. In particular, we choose $\beta = 10^{-3.45}$, $\alpha = 3.8$, then the large scale fading is given $10\log_{10}(\gamma_{k,j}) = -38\log_{10}(d_{k,j}) - 34.5 + \eta_{k,j}$ in decibel, where $\eta_{k,j}$ represents the shadow fading in decibel and follows the distribution $\mathcal{N}(0, 8\text{dB})$ [22]. Each BS is equipped with $M = 4$ transmit antennas, and has the same transmit power constraint P, or the multi-cell MISO downlink system has a sum-power constraint KP if only a total-power limit is considered, the noise figure at each user terminal is 9dB, and in the simulation results we use the transmit power constraint in dBm over 10MHz bandwidth to indicate the average SNR level. The stop thresholds of the proposed iterative algorithm are respectively given by $|\gamma_{max} - \gamma_{min}| \leq 10^{-5}$, $\sum_{i=1}^K |\Delta\lambda_i| \leq 10^{-5}$ and $|\Delta t| \leq 10^{-5}$. The RPE performance merit [23] is defined as

$$RPE = \frac{1}{K}\sum_{i=1}^{K}\frac{R_i}{p_i} \tag{19}$$

where, p_i is the effective transmit power for user i and R_i is the rate of user i. The effective transmit power save proportion merit is defined as

$$Pro = \sum_{i=1}^{K}\frac{p_i^{Total} - p_i^{Per}}{p_i^{Total}} \times 100\% \tag{20}$$

where p_i^{Total} is the effective transmit power for user i with total power constraint and p_i^{Per} is the effective transmit power for user i with per-BS power constraints. The performance merit of the worst-user rate among all users is defined as $R = \min_i R_i$.

The legends in the figures are defined as: the term "Algorithm 1" denotes the proposed algorithm with per-BS power constraints, "Algorithm 2" denotes the special case with a sum-power constraint [13], "SLNR" denotes SLNR beamforming algorithm with per-BS power constraints [19] and "MRT" represents the maximum ratio transmission (MRT) beamforming algorithm with per-BS power constraints.

Fig.2 illustrates the RPE performance of the above four schemes for two-BS and three-BS cooperating scenarios, respectively. The simulation results show that Algorithm 1 has an advantage over other multi-cell coordinated beamforming algorithms in terms of RPE, and the gain increases with the transmit power. In particular, Algorithm 1 achieves almost the same RPE performance in two-BS and three-BS cooperating scenarios. While the RPE performance of other three algorithms in three-BS scenario exhibits some performance loss compared to that in two-BS scenario. The minimum user rate performance of these four algorithms is shown in Fig.3. It is seen that Algorithm 1 shows a better performance than the

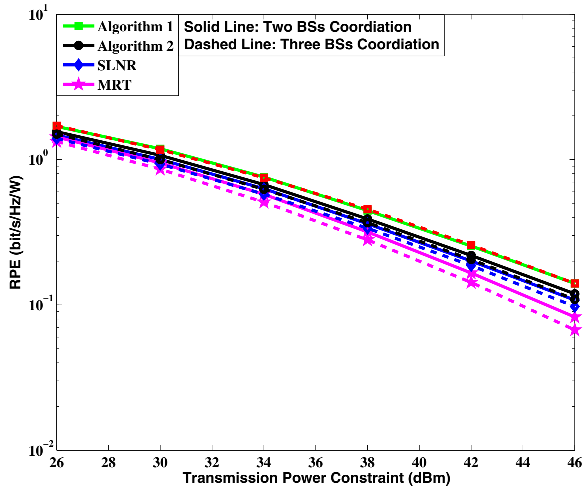

Fig. 2. The RPE Performance in the proposed multi-cell cooperative systems $K = 2$ and $K = 3$.

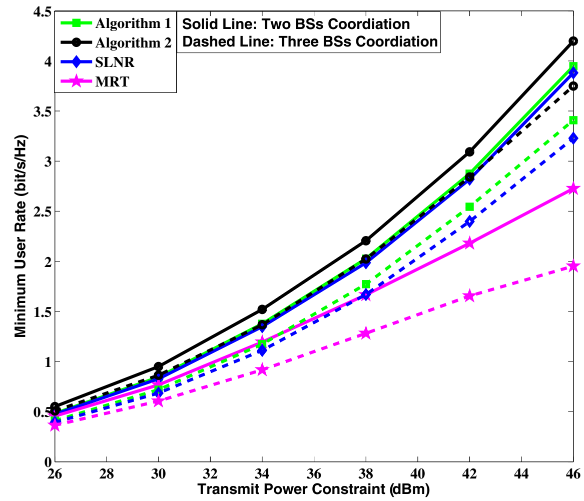

Fig. 3. The Minimum User Rate Performance in the proposed multi-cell cooperative systems $K = 2$ and $K = 3$.

SLNR and MRT algorithms. As the number of cooperating BSs increases, the performance gap also increases. Compared with Algorithm 2, Algorithm 1 has a performance loss due to the imposing of more stringent per-BS constraints. Furthermore, The RPE of these algorithms relative to the worst-user rate are simulated and shown in Fig.4. The results demonstrate that Algorithm 1 and Algorithm 2 achieve almost the same RPE performance and both outperform the SLNR and MRT algorithms. The effective consumption powers at each BS of Algorithm 1 and Algorithm 2 for three-BS cooperation are given in Fig.5. Results show that in contrast to Algorithm 2, Algorithm 1 can save a considerable portion of transmit power at each BS. It is further seen that transmit power saving ratio reduces with the increase of the transmit power.

Since the channel state information at the BSs are usually obtained through a finite-rate feedback channel, it is useful to evaluate the impact of limited feedback on the algorithm performance. Fig.6 illustrates the RPE performance of the multi-cell beamforming algorithms with limited feedback CSI based on Grassmannian codebook [24]. It is shown that the quantization of CSI results in a performance loss both in terms of the RPE performance and the worst-user rate. However, the loss becomes smaller with the increasing of codebook size.

Finally, the convergence behavior of the proposed multi-cell beamforming algorithm is illustrated in Fig.7 for a random channel realization. The results show that the two key steps of the proposed algorithm only need around 3 iterations to reach the balance point, revealing that the proposed hierarchical algorithm has a fast convergence and hence has a low computational complexity.

VI. Conclusions

The duality between the per-BS power constrained multi-cell MISO downlink max-min SINR optimization problem and the virtual uplink min-max SINR optimization problem is first revealed using the Lagrangian duality theory. A hierarchical iterative optimization algorithm is developed to solve the virtual uplink min-max SINR optimization problem using the bisection method and the GP optimization method jointly. The uplink solution is then converted to achieve the solution to the multi-cell MISO downlink beamforming problem. The convergence of the proposed algorithm is also proved. Compared to the existing algorithms on inter-cell beamforming and power allocation, the proposed algorithm can achieve better performance in terms of rate per energy and user fairness.

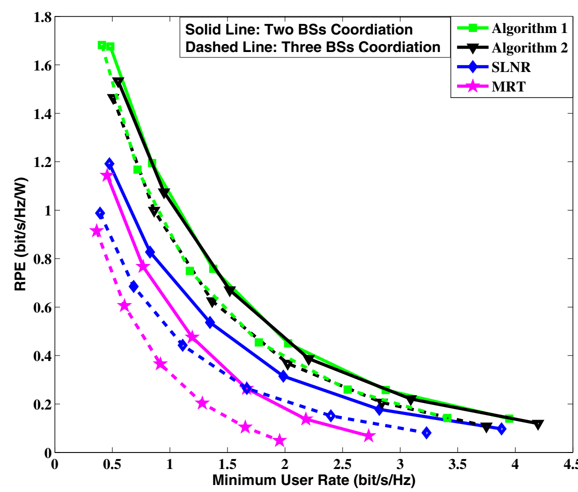

Fig. 4. The Energy Efficient Performance of the worst cell in the proposed multi-cell cooperative systems $K = 2$ and $K = 3$.

Appendix A: The Proof of the Theorem 1

Proof: Similar to the method [14], we introduce a slack variable γ for the optimization problem Q^{Down}. Then, the

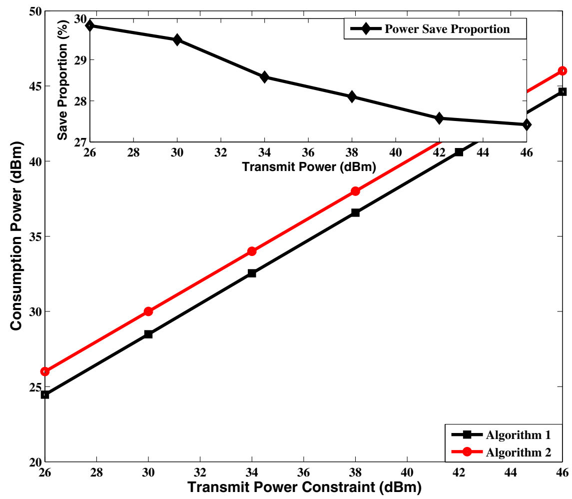

Fig. 5. The Power Save Proportion Performance in the proposed multi-cell cooperative systems $K = 3$.

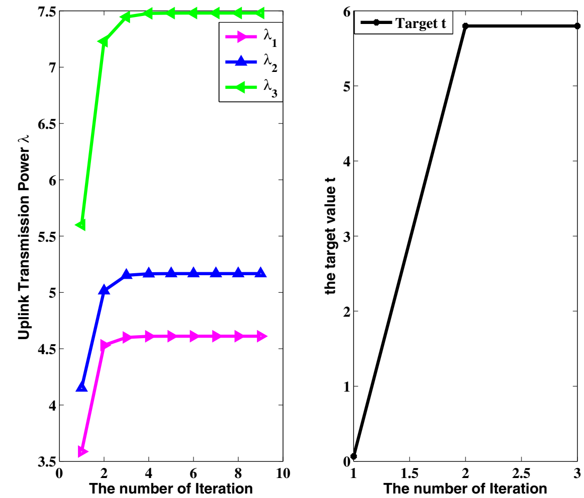

Fig. 7. The Convergence Speed of the Two Key Steps of the Proposed Algorithm, $K = 3$, $P = 38dBm$.

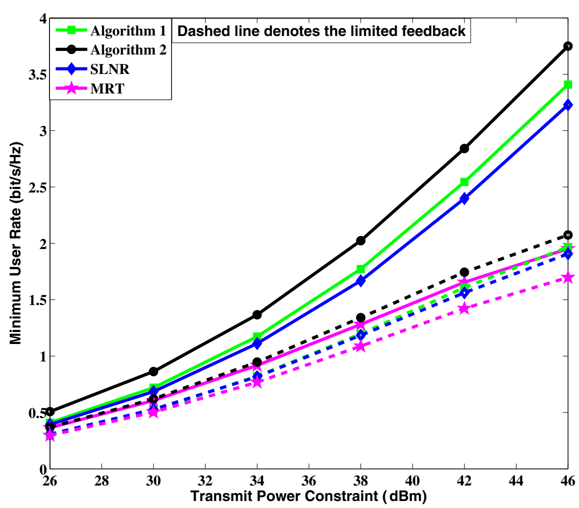

Fig. 6. The Minimum User Rate of Performance of Limited Feedback in the proposed multi-cell cooperative systems $K = 3$ and the size of the codebook is 64.

problem can be rewritten as:

$$\begin{aligned} \tilde{\mathcal{Q}}: \quad & \max_{\{\boldsymbol{w}_i, p_i\}_{i=1}^K, \gamma} \gamma \\ s.t. \quad & \text{SINR}_i^{Down} \geq \gamma, \forall i \\ & p_i \geq 0, p_i \leq P_i, ||\boldsymbol{w}_i|| = 1, \forall i. \end{aligned} \tag{21}$$

Substituting the expression of SINR of (2) into (21), we get the following form:

$$\begin{aligned} \tilde{\mathcal{Q}}: \quad & \max_{\{\boldsymbol{w}_i, p_i\}_{i=1}^K, \gamma} \gamma \\ s.t. \quad & \sum_{k=1, k\neq i}^{K} p_k \boldsymbol{w}_k^H \boldsymbol{\Omega}_{i,k} \boldsymbol{w}_k + 1 \leq \frac{p_i \boldsymbol{w}_i^H \boldsymbol{\Omega}_{i,i} \boldsymbol{w}_i}{\gamma}, \forall i \\ & p_i \geq 0, p_i \leq P_i, ||\boldsymbol{w}_i|| = 1, \forall i. \end{aligned} \tag{22}$$

The Lagrangian function of this optimization problem can be written as (23), shown at the top of the next page, where, $\boldsymbol{\phi} = \{\phi_i \geq 0\}_{i=1}^K$ are the Lagrange multipliers associated with the SINR constraints, $\boldsymbol{\mu} = \{\mu_i \geq 0\}_{i=1}^K$ are the Lagrange multiplies associated with the nonnegativity of the BS transmit power constraints, $\boldsymbol{\varphi} = \{\varphi_i \geq 0\}_{i=1}^K$ are the Lagrange multipliers associated with the per-BS transmit power constraints, $\boldsymbol{W} = \{\boldsymbol{w}_i\}_{i=1}^K$ is the collection of the transmit beamforming vectors at the BSs, $\boldsymbol{p} = \{p_i\}_{i=1}^K$ is the collection of the transmit power at the BSs. Then, the corresponding duality objective function can be expressed as

$$\max_{\gamma, \boldsymbol{W}, \boldsymbol{p}} \boldsymbol{L}(\gamma, \boldsymbol{W}, \boldsymbol{p}, \boldsymbol{\phi}, \boldsymbol{\mu}, \boldsymbol{\varphi}). \tag{24}$$

It is known from the extreme value principle that the optimum transmit power point fulfills $\frac{\partial \boldsymbol{L}}{\partial p_i} = 0$. Combining with the condition $\mu_i \geq 0$, it yields

$$\gamma \geq \frac{\phi_i \boldsymbol{w}_i^H \boldsymbol{\Omega}_{i,i} \boldsymbol{w}_i}{\sum_{k=1, k\neq i}^K \phi_k \boldsymbol{w}_i^H \boldsymbol{\Omega}_{k,i} \boldsymbol{w}_i + \varphi_i}. \tag{25}$$

Thus, the corresponding Lagrangian duality optimization problem can be represented as

$$\begin{aligned} \tilde{\mathcal{Q}}: \quad & \min_{\boldsymbol{\phi}, \boldsymbol{\varphi}} \max_{\boldsymbol{W}, \gamma} \left(\gamma - \sum_{i=1}^{k} \phi_i + \sum_{i=1}^{K} \varphi_i P_i \right) \\ s.t. \quad & \gamma \geq \frac{\phi_i \boldsymbol{w}_i^H \boldsymbol{\Omega}_{i,i} \boldsymbol{w}_i}{\sum_{k=1, k\neq i}^K \phi_k \boldsymbol{w}_i^H \boldsymbol{\Omega}_{k,i} \boldsymbol{w}_i + \varphi_i}, \forall i \\ & \phi_i \geq 0, \varphi_i \geq 0, ||\boldsymbol{w}_i|| = 1, \forall i \end{aligned} \tag{26}$$

$$\begin{aligned} \boldsymbol{L}(\gamma, \boldsymbol{W}, \boldsymbol{p}, \boldsymbol{\phi}, \boldsymbol{\mu}, \boldsymbol{\varphi}) &= \gamma - \sum_{i=1}^{K} \phi_i \left(\sum_{k=1, k\neq i}^{K} p_k \boldsymbol{w}_k^H \boldsymbol{\Omega}_{i,k} \boldsymbol{w}_k + 1 - \frac{p_i \boldsymbol{w}_i^H \boldsymbol{\Omega}_{i,i} \boldsymbol{w}_i}{\gamma} \right) + \sum_{i=1}^{K} \mu_i p_i - \sum_{i=1}^{K} \varphi_i (p_i - P_i) \\ &= \gamma - \sum_{i=1}^{K} \phi_i + \sum_{i=1}^{K} \varphi_i P_i - \sum_{i=1}^{K} p_i \left(\sum_{k=1, k\neq i}^{K} \phi_k \boldsymbol{w}_i^H \boldsymbol{\Omega}_{k,i} \boldsymbol{w}_i + \varphi_i - \frac{\phi_i \boldsymbol{w}_i^H \boldsymbol{\Omega}_{i,i} \boldsymbol{w}_i}{\gamma} - \mu_i \right) \end{aligned} \tag{23}$$

Introducing a variable χ, the above optimization problem can be reformulated as

$$\begin{aligned} \tilde{\mathcal{Q}}: \quad & \max_{\chi} \min_{\boldsymbol{\phi}, \boldsymbol{\varphi}} \max_{\boldsymbol{W}, \gamma} \left(\gamma - \chi + \sum_{i=1}^{K} \varphi_i P_i \right) \\ s.t. \quad & \gamma \geq \frac{\phi_i \boldsymbol{w}_i^H \boldsymbol{\Omega}_{i,i} \boldsymbol{w}_i}{\sum_{k=1, k\neq i}^{K} \phi_k \boldsymbol{w}_i^H \boldsymbol{\Omega}_{k,i} \boldsymbol{w}_i + \varphi_i}, \forall i \\ & \phi_i \geq 0, \varphi_i \geq 0, ||\boldsymbol{w}_i|| = 1, \forall i, \sum_{i=1}^{K} \phi_i \leq \chi. \end{aligned} \tag{27}$$

Similar to [25], letting $\phi_i = \chi' \lambda_i$, $\varphi_i = \chi' \upsilon_i, \forall i$, $\chi = \chi' \sum_{i=1}^{K} P_i, \chi' \geq 0$, the optimization problem (27) can be rewritten as

$$\begin{aligned} \tilde{\mathcal{Q}}: \quad & \max_{\chi'} \min_{\boldsymbol{\lambda}, \boldsymbol{\upsilon}} \max_{\boldsymbol{W}, \gamma} \left(\gamma + \chi' \left(\sum_{i=1}^{K} \upsilon_i P_i - \sum_{i=1}^{K} P_i \right) \right) \\ s.t. \quad & \gamma \geq \frac{\lambda_i \boldsymbol{w}_i^H \boldsymbol{\Omega}_{i,i} \boldsymbol{w}_i}{\sum_{k=1, k\neq i}^{K} \lambda_k \boldsymbol{w}_i^H \boldsymbol{\Omega}_{k,i} \boldsymbol{w}_i + \upsilon_i}, \lambda_i \geq 0, \forall i \\ & \upsilon_i \geq 0, ||\boldsymbol{w}_i|| = 1, \forall i, \sum_{i=1}^{K} \lambda_i \leq \sum_{i=1}^{K} P_i. \end{aligned} \tag{28}$$

If we regard χ' as a duality variable for the minimization over $\boldsymbol{\upsilon}$ with the constraint $\sum_{i=1}^{K} \upsilon_i P_i \leq \sum_{i=1}^{K} P_i$, and for other fixed variable optimization over $\boldsymbol{\upsilon}$ is a convex problem (the proofs are given in Appendix B) that guarantees the strong duality, the optimization problem (28) is equivalent to

$$\begin{aligned} \tilde{\mathcal{Q}}: \quad & \min_{\boldsymbol{\lambda}, \boldsymbol{\upsilon}} \max_{\boldsymbol{W}, \gamma} \quad \gamma \\ s.t. \quad & \gamma \geq \frac{\lambda_i \boldsymbol{w}_i^H \boldsymbol{\Omega}_{i,i} \boldsymbol{w}_i}{\sum_{k=1, k\neq i}^{K} \lambda_k \boldsymbol{w}_i^H \boldsymbol{\Omega}_{k,i} \boldsymbol{w}_i + \upsilon_i}, \forall i \\ & \lambda_i \geq 0, \upsilon_i \geq 0, ||\boldsymbol{w}_i|| = 1, \forall i \\ & \sum_{i=1}^{K} \lambda_i \leq \sum_{i=1}^{K} P_i, \sum_{i=1}^{K} \upsilon_i P_i \leq \sum_{i=1}^{K} P_i. \end{aligned} \tag{29}$$

From the perspective of minimizing the sum of transmit power, we can know that all user can achieve same optimal balanced SINR levels when the worst-user SINR is maximized. This implies that it will not affect the solution if we reformulate the optimization problem by reversing the inequalities of SINR constraints and reversing the minimization over λ to maximization, with the equivalent form of (29) given as follow [25].

$$\begin{aligned} \tilde{\mathcal{Q}}: \quad & \min_{\boldsymbol{\upsilon}} \max_{\boldsymbol{\lambda}, \boldsymbol{W}, \gamma} \quad \gamma \\ s.t. \quad & \gamma \leq \frac{\lambda_i \boldsymbol{w}_i^H \boldsymbol{\Omega}_{i,i} \boldsymbol{w}_i}{\sum_{k=1, k\neq i}^{K} \lambda_k \boldsymbol{w}_i^H \boldsymbol{\Omega}_{k,i} \boldsymbol{w}_i + \upsilon_i}, \forall i \\ & \lambda_i \geq 0, \upsilon_i \geq 0, ||\boldsymbol{w}_i|| = 1, \forall i \\ & \sum_{i=1}^{K} \lambda_i \leq \sum_{i=1}^{K} P_i, \sum_{i=1}^{K} \upsilon_i P_i \leq \sum_{i=1}^{K} P_i. \end{aligned} \tag{30}$$

By replacing the objective γ with the right term of SINR constraint inequality, we finally obtain the dual problem formulation in theorem 1. ■

Appendix B: The Convergence of the Proposed Algorithm

Proof: In order to demonstrate the convergence of the outer layer of the proposed iterative algorithm for the optimization problem $\mathcal{Q}^{Up}$, we prove that the optimization problem $\mathcal{Q}^{Up}$ is a convex problem over $\boldsymbol{\upsilon}$ when the other optimization variables are fixed. Based on the expression of the optimization problem $\mathcal{Q}^{Up}$, we only need to prove that the feasible set $\mathcal{D}$ of the variables $\boldsymbol{\upsilon}$ is a convex set. Let $\forall \boldsymbol{\upsilon^1}, \boldsymbol{\upsilon^2} \in \mathcal{D}$ and $0 \leq \alpha \leq 1$. Without loss of generality, let $\lambda_i \boldsymbol{w}_i^H \boldsymbol{\Omega}_{i,i} \boldsymbol{w}_i > 0, \forall i$ and $\gamma > 0$, the SINR inequality constraints in (6) can be rewritten as

$$\begin{aligned} \frac{\lambda_i \boldsymbol{w}_i^H \boldsymbol{\Omega}_{i,i} \boldsymbol{w}_i}{\gamma} - \sum_{k=1, k\neq i}^{K} \lambda_k \boldsymbol{w}_i^H \boldsymbol{\Omega}_{k,i} \boldsymbol{w}_i \geq \upsilon_i^1, \forall i \\ \frac{\lambda_i \boldsymbol{w}_i^H \boldsymbol{\Omega}_{i,i} \boldsymbol{w}_i}{\gamma} - \sum_{k=1, k\neq i}^{K} \lambda_k \boldsymbol{w}_i^H \boldsymbol{\Omega}_{k,i} \boldsymbol{w}_i \geq \upsilon_i^2, \forall i \end{aligned} \tag{31}$$

After some basic operations, we can obtain the following inequality

$$\gamma \leq \frac{\lambda_i \boldsymbol{w}_i^H \boldsymbol{\Omega}_{i,i} \boldsymbol{w}_i}{\sum_{\substack{k=1 \\ k\neq i}}^{K} \lambda_k \boldsymbol{w}_i^H \boldsymbol{\Omega}_{k,i} \boldsymbol{w}_i + \alpha \upsilon_i^1 + (1-\alpha) \upsilon_i^2}, \forall i \tag{32}$$

Furthermore, we have the following inequality

$$\begin{aligned} & \sum_{i=1}^{K} (\alpha \upsilon_i^1 + (1-\alpha) \upsilon_i^2) P_i = \\ & \sum_{i=1}^{K} \alpha \upsilon_i^1 P_i + \sum_{i=1}^{K} (1-\alpha) \upsilon_i^2 P_i \leq \\ & (\alpha + (1-\alpha)) \sum_{i=1}^{K} P_i = \sum_{i=1}^{K} P_i \end{aligned} \tag{33}$$

We can draw a conclusion that $\alpha\boldsymbol{v}^1 + (1-\alpha)\boldsymbol{v}^2 \in \mathcal{D}$, so the feasible set $\mathcal{D}$ of the variable $\boldsymbol{v}$ is a convex set and the convergence of the outer iterative of the proposed algorithm can be guaranteed. Based on the results in [14] and the monotonic boundary sequence theory, we know that the convergence of the inner layer of the proposed iterative algorithm can also be guaranteed. As a result, the proposed algorithm is also guaranteed to converge to a fixed point. ■

Appendix C: The Uplink-Downlink Max-Min SINR Duality With Sum-Power Constraint

When a simple sum-power constraint is considered, the uplink-downlink max-min SINR duality of the Theorem 1 can be simplified, which is similar to our previous work [13].

Corollary 1. *Considering a sum-power constraint in a multi-cell interference downlink system, the max-min dowlink SINR optimization problem is given by*

$$\begin{aligned} \overleftarrow{\mathcal{Q}}: &\quad \max_{\{\boldsymbol{w}_i, p_i\}_{i=1}^K} \min_i \mathrm{SINR}_i \\ s.t. &\quad p_i \geq 0, ||\boldsymbol{w}_i|| = 1, \forall i, \sum_{i=1}^{k} p_i \leq P, \end{aligned} \tag{34}$$

The above optimization problem is dual to the following virtual uplink optimization problem,

$$\begin{aligned} \overrightarrow{\mathcal{Q}}: &\quad \max_{\{\boldsymbol{w}_i, \lambda_i\}_{i=1}^K} \min_i \frac{\lambda_i \boldsymbol{w}_i^H \boldsymbol{\Omega}_{i,i} \boldsymbol{w}_i}{\sum_{k=1,k\neq i}^K \lambda_k \boldsymbol{w}_i^H \boldsymbol{\Omega}_{k,i} \boldsymbol{w}_i + 1} \\ s.t. &\quad \lambda_i \geq 0, ||\boldsymbol{w}_i|| = 1, \forall i, \sum_{i=1}^{K} \lambda_i = P \end{aligned} \tag{35}$$

where $\{\lambda_i\}_{i=1}^K$ denotes the transmit power of each user in the virtual uplink, $\sum_{i=1}^K \lambda_i = P$ represents the sum-power constraint in the virtual uplink and is identical to the downlink. When the optimal solution was obtained, all users achieve a same optimal balanced SINR level.

References

[1] 3GPP R1-090140, "Clustering for CoMP transmission," Nortel.
[2] H. Zhang and H. Dai, "Cochannel interferecence mitigation and cooperative processing in dwonlink multicell multiuser MIMO networks," *EURASIP J. Wireless Commun. Netw.,* vol. 2, pp. 222–235, 2004.
[3] J. Zhang, R. Chen, J. Andrews, A. Ghosh, and R. Heath Jr., "Networked MIMO with clustered linear precoding," *IEEE Trans. Wireless Commun.*, vol. 8, no. 4, pp. 1910–1921, Apr. 2009.
[4] J. Zhang and J. Andrews, "Adaptive spatial intercell interference cancellation in multicell wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 28, no. 9, pp. 1455–1468, Dec. 2010.
[5] D. Gesbert, S. Hanly, H. Huang, S. Shitz, O. Simeone, and W. Yu, "Multi-cell MIMO cooperative networks: a new look at interference," *IEEE J. Sel. Areas Commun.*, vol. 28, no. 9, pp. 1380–1408, Dec. 2010.
[6] F. Rashid-Farrokhi, K. Liu, and L. Tassiulas, "Transmit beamforming and power control for cellular wireless systems," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1437–1450, Oct. 1998.
[7] E. Visotsky and U. Madhow, "Optimum beamforming using transmit antenna arrays," in *Proc. 1999 IEEE. Veh. Tech. Conf.*, vol. 1, pp. 851–856.
[8] F. Rashid-Farrokhi, L. Tassiulas, and K. Liu, "Joint optimal power control and beamforming in wireless networks using antenna arrays," *IEEE J. Sel. Areas Commun.*, vol. 46, no. 10, pp. 1313–1324, Oct. 1998.
[9] M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," *IEEE Trans. Veh. Technol.*, vol. 53, no. 1, pp. 18–28, Jan. 2004.
[10] W. Yu and T. Lan, "Transmitter optimization for the multi-antenna downlink with per-antenna power constraints," *IEEE Trans. Signal Process.*, vol. 55, no. 6, pp. 2646–2660, June 2007.
[11] L. Zhang, R. Zhang, Y. Liang, V. Xin, and H. Poor, "On Gaussian MIMO BC-MAC duality with multiple transmit covariance constraints," in *Proc. 2009 ISIT*, pp. 2502–2506.
[12] H. Dahrouj and W. Yu, "Coordinated beamforming for the multicell multi-antenna wireless system," *IEEE Trans. Wireless Commun.*, vol. 9, no. 5, pp. 1748–1759, May 2010.
[13] Y. Huang, G. Zheng, M. Bengtsson, K. Wong, L. Yang, and B. Ottersten, "Distributed multi-cell beamforming design with limited intercell coordination," *IEEE Trans. Signal Process.*, vol. 59, no. 2, pp. 728–738, Feb. 2011.
[14] A. Tajer, N. Prasad, and X. Wang, "Robust linear precoder design for multi-cell downlink transmission," *IEEE Trans. Signal Process.*, vol. 59, no. 1, pp. 235–251, Jan. 2011.
[15] A. Wiesel, Y Eldar, and S. Shamai, "Linear precoding via conic optimization for fixed MIMO receivers," *IEEE Trans. Signal Process.*, vol. 54, no. 1, pp. 161–176, Jan. 2006.
[16] D. Cai, T. Quek, and C. Tan, "A unified analysis of max-min weighted SINR for MIMO downlink system," *IEEE Trans. Signal Process.*, vol. 59, no. 8, pp. 3850–3862, Aug. 2011.
[17] C. Tan, M. Chiang, and R. Srikant, "Maximizing sum rate and minimizing MSE on multiuser downlink: optimality, fast algorithms and equivalence via max-min SINR," *IEEE Trans. Signal Process.*, vol. 59, no. 12, pp. 6127–6143, Dec. 2011.
[18] S. Pillai, T. Suel, and S. Cha, "The Perron-Frobenius theorem: some of its applications," *IEEE Signal Process. Mag.*, vol. 22, no. 2, pp. 62–75, Mar. 2005.
[19] M. Sadek, A. Tarighat, and A. Sayed, "A leakage-based precoding scheme for downlink multi-user MIMO channels," *IEEE Trans. Wireless Commun.* vol. 6, no. 5, pp. 1711–1721, May 2007.
[20] S. Boyd, S. Kim, L. Vandenberghe, and A. Hassibi, "A tutorial on geometric programming," *Optim. Eng. (2007)* vol. 8, pp. 67–127, DOI 10.1007/s11081-007-9001-7.
[21] X. Zhang and J. Lee, "Low complexity MIMO scheduling with channel decomposition using capacity upperbound," *IEEE Trans. Commun.*, vol. 56, no. 6, pp. 871–876, June 2008.
[22] 3GPP TR25.996 v.8.0.0 Release 8, "Spatial channel model for multiple input multiple output (MIMO) simulations," 3GPP, Tech. Rep., Jan. 2009.
[23] R. Madan, N. Mehta, A. Molisch, and J. Zhang, "Energy-efficient cooperative relaying over fading channels with simple relay selection," *IEEE Trans. Wireless Commun.*, vol. 7, no. 8, pp. 3013–3025, Aug. 2008.
[24] D. Love, R. Heath, Jr., and T. Strohmer, "Grassmannian beamforming for multiple-input multiple-output wireless systems," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2735–2747, Oct. 2003.
[25] J. Yang and D. Kim, "Multi-cell uplink-downlink beamforming throughput duality based on Lagrangian duality with per-base station power constraints," *IEEE Commun. Lett.*, vol. 12, no. 12 pp. 277–279, Apr. 2008.

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