# Impact of Interbranch Correlation on Multichannel Spectrum Sensing with SC and SSC Diversity Combining Schemes

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Impact of Interbranch Correlation on Multichannel Spectrum Sensing with SC and SSC Diversity Combining Schemes

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Abstract—Multi antenna receivers are often deployed in cognitive radio systems for accurate spectrum sensing. However, correlation among signals received by multiple antennas in these receivers is often ignored which yield unrealistic results. In this paper, the effect of this correlation is accurately quantified by deriving analytical expressions for the average probability of detection. Alternative simpler expressions are also derived. These are done for Selection Combining (SC) and Switch and Stay (SSC) diversity techniques in dual arbitrarily correlated Nakagami-m fading channels. Then it is repeated for triple exponentially and identically correlated Nakagami-m fading channels with SC diversity technique. Analysis results show that the inter-branch correlation impacts the detector performance significantly, especially in deep fading scenarios. Also, SC outperforms SSC as expected. However, the difference between them becomes very small in low fading and highly correlated scenarios which, indicates that the simpler SSC scheme can as well be deployed in such situations.

Index Terms—Cognitive radio networks, spectrum sensing, inter-branch correlation, diversity combining, selection combining, switch and stay combining.

I. INTRODUCTION

P

ROTECTING the primary users from detrimental interference from the secondary user signals is crucial in cognitive radio systems. Accurate spectrum sensing is essential for this. Simple schemes such as Energy Detectors (ED) are widely used for this purpose that detect weak signals in noisy channels as long as the noise power is known [1]. Accurate spectrum sensing suffers from few issues, multipath fading and shadowing being the leading causes. Multi antenna receivers, with appropriate diversity combining schemes, are designed to overcome these issues. Ideally, wireless channels seen by the multiple antennas shall be independent to obtain the best results from these diversity receivers [2]. However, often this is not the case especially, when antennas are increasingly placed closer to each other as the mobile units get smaller and more demanding. Therefore, ignoring inter-branch correlation yields inaccurate, especially overly optimistic, results. The effects of multipath fading and correlation among antenna branches heavily depend on the type of the diversity combining technique employed. It is well known that Maximal Ratio Combining scheme (MRC) is the optimal scheme which is also the most complex linear diversity scheme. Equal Gain Combining (EGC) diversity technique is a close competitor. Both the MRC and EGC techniques require all or some knowledge of the Channel State Information (CSI) [3]–[5]. Furthermore, in these schemes each diversity branch must be equipped with a single receiver that increases the system complexity. Recently simpler combining schemes such as Switch and Stay Combining (SSC) and Selection Combining (SC) are getting popular due to their simplicity. These are especially useful in cognitive radio networks. With the SC scheme, the receiver simply selects the antenna with the highest received signal power and ignores other antennas. Hence, signal combiners, phase shifters or variable gain controllers are not required [3], [6]. The SSC diversity technique is the least complex system where no real combiner is required. The SSC selects a particular antenna branch until its SNR drops below a predetermined threshold [3]. Both SC and SSC schemes are required to measure only the amplitude on each branch (in order to select the highest one). Hence, they can be employed for both coherent and non-coherent modulation schemes [3]. Different diversity combining techniques have been studied in the literature. In [7], averaging the probability of detection over fading channels with Rayleigh, Nakagami-m and Rician distributions are studied, and closed-form expressions for detector parameters were derived for Nakagami-m channels with integer values of m. In [8]–[9], alternative analytic approaches to that in [1] and [7] were given. Furthermore, in [8], independent and identically distributed (i.i.d) dual and L number of Rayleigh fading branches were considered with SSC and SC diversities. Corresponding average probability of detection expressions \( \overline{P_D} \) were also derived for both techniques. In [10]–[11], closed-form expressions for \( \overline{P_D} \) were derived for i.i.d. diversity branches in Nakagami-m fading channels employing SC technique. In [12], closed-form expressions of \( \overline{P_D} \) for i.i.d dual Nakagami-m fading branches with SSC were derived for real and integer m values. In our previous work [6], we have done an investigation on the probability of detection for SC diversity with correlated Nakagami-m fading branches. A review of prior works reveals that correlated fading branches with SSC diversity is not studied in the literature. However, because of the simplicity, the SSC is particularly valuable for mobile stations that have limited resource and power. This paper aims to fulfill that requirement.

In this paper, we extend our previous investigations by
considering SC and SSC diversity combining techniques with identically correlated branches in Rayleigh and Nakagami-\(m\) fading channels.

Our contribution falls into two folds. First

- We consider SC and SSC schemes with dual arbitrarily correlated branches in Rayleigh and Nakagami-\(m\) fading channels.
- Then, we extend study of SC diversity to triple exponentially correlated branches.
- Corresponding novel expressions for average probability of detection are derived for each case.
- Alternative and more general and simpler expressions are also derived for each case.
- For SSC diversity, we derive an expression which can be solved numerically to calculate the optimal SNR threshold value in order to optimize the detector performance.
- All our derived expressions do converge rapidly.

Secondly, to gain better insight

- We do a performance comparison between the two combining diversity techniques
- Analysis results show that the inter-branch correlation affects the detector performance significantly, especially in deep fading scenarios.
- SC outperforms SSC as expected however; the difference between them becomes very small in low fading scenario with highly correlation among antennas. This indicates that the simpler SSC scheme can be substituted for the SC scheme in these situations.

The rest of the paper is organized as follows. Section II describes the system model. In section III, we study the performance of SC scheme. In section IV, we study the performance of SSC scheme. Section V describes simulation and analysis results. Section VI concludes the paper.

II. SYSTEM MODEL

We follow a binary hypothesis testing on the received signal to declare the presence or absence of the primary user. For this, we employ ED that is widely used in cognitive spectrum sensing. Note that no priori information about the detected signal is needed for ED [13], [14].

Let \(x(t)\) be the observed receptions data

\[
x(t) = h s(t) + n(t),
\]

where, \(h\) is the complex channel gain amplitude coefficient, assumed to be constant during the sensing time, \(s(t)\) is the signal to be detected and, \(n(t)\) is the AWGN noise. This noise is a low-pass Gaussian process with zero mean and variance \(N_0W\) where, \(N_0\) and \(W\) denote Power Spectral Density (PSD) of the Gaussian noise and the signal bandwidth, respectively.

Two hypotheses are defined for the decision statistics. Namely \(H_0\) and \(H_1\), for the absence and the presence of the primary user signal respectively, as follows:

\[
x(t) = \begin{cases} n(t) & \text{under } H_0 \\ h s(t) + n(t) & \text{under } H_1. \end{cases}
\]

The decision statistics is squared and integrated over time \(T\) at the ED. The output is written as

\[
y \triangleq \frac{2}{N_0} \int_0^T |x|^2(t) \, dt.
\]

The Probability Density Function (PDF) of the decision statistics \(y\) is given by [8] and [9]

\[
p_y(y) = \begin{cases} \frac{1}{2\pi} \frac{1}{\sqrt{2\gamma}} e^{-\frac{y}{2\gamma}}, & \text{under } H_0 \\ \frac{1}{2} \frac{1}{\sqrt{2\gamma}} e^{-\frac{y+\psi}{2\gamma}} I_{u-1} \left(\sqrt{2\gamma y}\right), & \text{under } H_1 \end{cases}
\]

where \(\gamma\) denotes the signal-to-noise-ratio, \(\Gamma(.)\) is the Gamma function and, \(I_u(.)\) is the \(u^{th}\) order modified Bessel function of the first kind. The parameter \(u\) depends on the time-bandwidth product. In (4), it is clear that the decision statistics has a central chi-square distribution with \(2u\) degrees of freedom \(\chi_{2u}^2\) in the absence of the primary user signal, i.e. the received samples are noise only. However, it has a non-central chi-square distribution \(\chi_{2u}^2(\psi)\) with \(2u\) degrees of freedom and non-centrality parameter \(\psi = 2\gamma\) in the presence of the primary user signal [8] and [9].

Let us define \(\lambda\) as the decision threshold. Then the probability of false alarm (\(P_F^r\)) and the probability of detection (\(P_D\)) of the ED can be written as

\[
\begin{align*}
P_F &= Pr \left( y > \lambda \mid H_0 \right), \\
P_D &= Pr \left( y > \lambda \mid H_1 \right).
\end{align*}
\]

where \(Pr(.)\) denotes the Cumulative Distribution Function (CDF). Consequently, the probability of false alarm and probability of detection in AWGN channel are given as [8] - [9]

\[
\begin{align*}
P_F &= \frac{\Gamma(u, \frac{\lambda}{2})}{\Gamma(u)}, \\
P_D &= Q_u \left( \sqrt{2\gamma}, \sqrt{\lambda} \right).
\end{align*}
\]

where \(\Gamma(., .)\) and \(Q_u(., .)\) denote the upper incomplete Gamma function and generalized Marcum Q-function, respectively. These detection probabilities are conditioned upon the channel realization. Also, they represent instantaneous probability of detection. Therefore, we need to integrate this instantaneous probability of detection over the SNR’s PDF of the corresponding fading channel \(p_{\gamma, Div}(\gamma)\) to obtain the average probability of detection \(P_{D,Div}\).

\[
P_{D,Div} = \int_0^\infty Q_u \left( \sqrt{2\gamma}, \sqrt{\lambda} \right) p_{\gamma, Div}(\gamma) \, d\gamma.
\]

The expression in (9) will serve as a general expression for the corresponding diversity channel.

Note that the probability of detection expression in (8) is restricted to only integer values of \(u\) since the PDF of the decision statistics in (4) is derived only for even numbers, i.e. \(2u\), as stated in [8]. However, when the alternative Marcum-Q function is employed, \(u\) could be half-odd integer

\footnote{False alarm probability is not a function of SNR as no signal is transmitted, therefore it will remain unchanged as in (7).}
where, \(\Gamma(.)\) denotes the Gamma function, \(\Omega = E[r^2]/m = \frac{\gamma}{m}\) is the mean value of the variable \(r\), and \(m\) (\(m \geq 1/2\)) is the inverse normalized variance of \(r^2\), which describes the fading severity.

We define the instantaneous SNR per symbol per channel \(\gamma_i\) as \(\gamma_i = r_i^2 E[N_0]/E_s\); \(i \in [1, 2, ..., L]\); \(E_s\) is the energy per symbol and, \(N_0\) is the PSD of the Gaussian noise. The average SNR per branch is \(\bar{\gamma}_i = \frac{\gamma_i}{E_s N_0}\) where, \(r_i^2 = E[r_i^2]\) is the expectation of the channel envelop.

### A. SC with Dual arbitrarily Correlated Branches

Using [[17], Eq. (20)] and, by assuming identical diversity branches and by changing variables with some mathematical simplification, the PDF of the output SNR for a dual SC combiner under correlated Nakagami-\(m\) fading channels can be obtained as

\[
p_{\gamma,SC}(\gamma) = \frac{2}{\Gamma(m)} \left(\frac{m}{\gamma}\right)^m \gamma^{m-1} \exp\left(-\frac{m \gamma}{\gamma}\right) \times \left[1 - Q_m\left(\sqrt{2 a \rho \gamma}, \sqrt{2 a \gamma}\right)\right], \quad \gamma \geq 0
\]  

(12)

where, \(\rho\) denote the correlation coefficient between the two fading envelopes, and \(a = \frac{m}{\gamma(1-\rho)}\). Please see Appendix A for detailed derivation.

By substituting (12) into (9), the average probability of detection for dual correlated SC’s diversity branches \(P_{D,SC,2}\) is obtained as

\[
P_{D,SC,2} = \frac{2}{\Gamma(m)} \left(\frac{m}{\gamma}\right)^m \left[I_A - I_B\right],
\]  

(13)

where

\[
I_A = \int_0^{\infty} Q_u\left(\sqrt{2\gamma}, \sqrt{\lambda}\right) \gamma^{m-1} \exp\left(-\frac{m \gamma}{\gamma}\right) d\gamma,
\]  

(14)

and

\[
I_B = \int_0^{\infty} Q_u\left(\sqrt{2\gamma}, \sqrt{\lambda}\right) Q_m\left(\sqrt{2a \rho \gamma}, \sqrt{2a \gamma}\right) \times \gamma^{m-1} e^{-\frac{m \gamma}{\gamma}} d\gamma.
\]  

(15)

Note this lengthy expression consists of two integrals, \(I_A\) and \(I_B\). We solve them separately. Please see Appendix B.

Hence, the average probability of detection for dual SC receiver under correlated identical Nakagami-\(m\) fading branches (restricted to integer \(u\) and \(m\) values) is

\[
P_{D,SC,2} = \frac{2}{\Gamma(m)} \left(\frac{m}{\gamma}\right)^m \left[I_A - I_B\right],
\]  

(16)

where \(c = 1 + a (\rho + 1) + \frac{m}{\gamma}\) and \(G_1\) for integer \(m\) values is

\[
G_1 = \frac{2}{\Gamma(m)} \left(\frac{m}{\gamma}\right)^m \left[I_A - I_B\right],
\]  

(17)

Here \(L_n(.)\) denotes Laguerre polynomial of \(n\)-degree [18], and \(F_1(.,.,.)\) denotes the Confluent Hyergeometric function. This is defined in [[19], Eq. (15.1.1)] as

\[
F_1(a_1, b_1; x) = \frac{\Gamma(b_1)}{\Gamma(a_1)} \sum_{i=0}^{\infty} \frac{\Gamma(a_1 + i) x^i}{i!}.
\]  

(18)

Note (16) reduces to dual correlated Rayleigh fading branches for \(m = 1\). It’s worthwhile to mention that for \(i.i.d\). diversity branches, (16) reduces to [[9], Eq. (7), [8], Eq. (20)] multiplied by 2 (not exceeding unity). The latter expression was derived for the average probability of detection in flat fading. Hence we have improved the detection performance and derived (16) to serve as a proof.

### B. Alternative Expression for \(P_{D,SC,2}\)

Despite the fact that \(Q_u\left(\sqrt{2\gamma}, \sqrt{\lambda}\right)\) portion of the second integral \(I_B\) in (13) is evaluated for \(u\) values not-restricted to integer, (16) is still restricted to integer values. This is because, the first integral \(I_A\) in (13) is only valid for integer \(u\) and \(m\) values. In this section, we derive a more general and simpler
alternative expression for (16) that is not restricted to integer $u$ values. Please see Appendix C for the derivation.

$$\mathcal{P}_{D,SC,2} = 1 - 2 \left( \frac{m}{\gamma} \right)^m e^{-\frac{m}{\gamma}}$$

$$\times \left[ \frac{1}{\Gamma(m)} \sum_{u=0}^{\infty} \left( \frac{\lambda}{2} \right)^n \frac{1}{(n+1)!} F_1 \left( n; 1 + n; \frac{\lambda}{2} \gamma \right) + \frac{1}{\Gamma(m)} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \left( \frac{\lambda}{2} \right)^n \frac{(n+1)!}{\Gamma(n+1)} e^{(n+1)k} \times \Gamma(n+1) \sum_{v=0}^{\infty} \frac{\lambda}{2} e^{-\frac{m}{\gamma} e^{v+1} \gamma^k} \right].$$

(19)

For $m = 1$, (19) reduces to the average probability of detection with dual correlated Rayleigh fading branches and, with $\rho = 0$ to i.i.d dual Rayleigh fading branches given in (8). The error resulting from truncating the infinite series in (19) is upper bounded by the Confluent Hypergeometric function defined in (18). Since this function is monotonically decreasing with $i, k$ and $n$ for given values of $m, \lambda$ and $\gamma$ [20], the number of terms ($N_w$ and $N_h$) that required five digit accuracy could be calculated. These numbers are shown in Table I for different values of $\rho$ and $m$.

It’s worthwhile to mention that several solutions for integrals involving the Marcum Q-function are available in literature [21]–[25]. However, our case of study in (15) differs a solution and more complicated integral which involves a product of two Marcum Q-functions. These solutions are introduced in (57), (69) in Appendices B and C, respectively. To the best of our knowledge, we believe that this solution is new in literature.

C. SC with Triple Correlated Branches

In this section, we consider triple correlated diversity branches. We start from PDF of the fading envelope for trivariate Nakagami-$m$ channel given in (26), Eq. (8). Then, by changing variable and by assuming identical branches ($\gamma_1 = \gamma_2 = \gamma_3$, and the same fading parameter $m$), the PDF of the output SNR for triple SC exponentially correlated Nakagami-$m$ branches can be derived. This is shown below

$$p_{\gamma;SC,3}(\gamma) = \frac{\gamma^{-2m}}{\Gamma(m)} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{|P_{1,2}^{i+j+j}|^2 |P_{2,3}^{i+j}|^2}{P_{1,1}^{i+m} P_{1,2}^{i+j+m+j} P_{2,2}^{i+j+m} P_{2,3}^{j+m}} \times \frac{1}{\Gamma(m+i) \Gamma(m+j) |i+j|!}$$

(20)

where $\Sigma^{-1}$ is the inverse of the correlation matrix, $p_{1,1}^{i_1,1} (i_1, j_1 = 1, 2, 3)$ being its entries and $\Theta_1, \Theta_2$ and $\Theta_3$ are

$$\Theta_1 = \left( \frac{p_{1,1} m}{\gamma} \right)^{i_1+j_1+m} e^{-\frac{p_{1,1} m}{\gamma}} \gamma^{i_1+j_1+m-1}$$

$$\times \gamma \left( i_1+j_1+m, \frac{p_{1,1} m}{\gamma} \right) \gamma \left( j_1+m, \frac{p_{3,3} m}{\gamma} \right) \gamma \left( j_1+m, \frac{p_{3,3} m}{\gamma} \right),$$

(21)

respectively. Here $\gamma(a, x)$ denotes the lower incomplete gamma function with $\gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt$ (118), Eq.(8.350/1)).

In exponentially correlated model, the diversity antennas are equispaced. Therefore, the correlation matrix can be written as $\Sigma_{i,j} = \rho |i-j| \Sigma_{i,j}$ [27]. Hence, the inverse correlation matrix $\Sigma^{-1}$ is tridiagonal and can be written as

$$\Sigma^{-1} = \frac{1}{\rho^2-1} \begin{bmatrix} -1 & \rho & 0 \\ \rho & -(\rho^2+1) & \rho \\ 0 & \rho & -1 \end{bmatrix},$$

(24)

where $\rho$ denotes the correlation coefficient.

We have made an assumption of identical average SNRs in all three branches above. This assumption is reasonable if the diversity channels are closely spaced and, their gains as well as noise powers are equal [3].

The average probability of detection for triple SC diversity Nakagami-$m$ correlated branches with integer $u$ is derived as below. See Appendix D for details

$$\mathcal{P}_{D,SC,3} = \frac{\Sigma^{-1}}{\Gamma(m)} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{|P_{1,2}^{i+j+j}|^2 |P_{2,3}^{i+j}|^2}{P_{1,1}^{i+m} P_{1,2}^{i+j+m+j} P_{2,2}^{i+j+m} P_{2,3}^{j+m}} \times \frac{1}{\Gamma(m+i) \Gamma(m+j) |i+j|!}$$

(28)

where, $\Xi_1, \Xi_2$ and $\Xi_3$ are as given in (25), (26) and (27) at the top of next page, respectively, and $F_2 (\alpha; \beta; \beta; \gamma_1; x, y)$ denotes the Hypergeometric function of two variables defined in (118), Eq. (9.180.2). Note, for $m = 1$, (28) reduces to triple correlated Rayleigh fading branches.

D. General Expression for Triple Branches

In this section, we will derive a general and simpler alternative expression to (28), where both $u$ and $m$ are not restricted to integer values. See Appendix E for details.
The average probability of detection for triple SC Nakagami-\(m\) correlated branches for not restricted \(u\) or \(m\) integer values is:

\[
\mathcal{P}_{D,SC,3} = \frac{1}{\Gamma(m)} \sum_{n=0}^{\infty} \sum_{j=0}^{\infty} \sum_{i=0}^{\infty} \frac{(\gamma)^n}{(\Gamma(u+n, \frac{\lambda}{\gamma})} \frac{\Gamma(u+n, \frac{\lambda}{\gamma})}{\Gamma(u+n)} \times \frac{p_{1,2}^{2\gamma} p_{2,3}^{2\gamma} \Gamma(2i+2j+3m+n)}{\Gamma(m+i) \Gamma(m+j) \Gamma(m+i+j+n) (\Xi_1 + \Xi_2 + \Xi_3)} \times \frac{p_{1,1} p_{2,2} p_{3,3}}{p_{1,1} + p_{2,2} + p_{3,3} + \frac{\gamma}{m}} (2+2j+3m+n+\gamma). \tag{29}
\]

where \(\Xi_1, \Xi_2\) and \(\Xi_3\) are given in (25), (26) and (27), respectively. As before, for \(m = 1\), (29) reduces to triple correlated Rayleigh fading branches. It’s worthwhile to mention that the Hypergeometric function of two variables \(F_2\left(\alpha; \beta_1, \beta_2; \gamma_1, \gamma_2; x, y\right)\) appears in (28) and (29) converges only for \(|x| + |y| < 1\) [18], where \(|.|\) denotes absolute. Fortunately, this is the case in our above derived equations.

IV. DUAL CORRELATED NAKAGAMI-\(m\) CHANNELS WITH SSC DIVERSITY

The SSC receiver selects a particular diversity branch until its SNR drops below a predetermined threshold value. Hence SSC’s technique is similar to its counterpart SC but. Nevertheless, the SSC receive does not need to continuously monitor the SNR of each branch. Therefore, the SSC is considered as the least complex\(^2\) diversity combining technique [3].

Starting from [[3], p.437, Eq. (9.334)], the SNR’s PDF for a dual and identical correlated Nakagami-\(m\) fading channels with SSC combiner is

\[
p_{\gamma,SSC}(\gamma) = \begin{cases} 
A(\gamma) & \text{if } \gamma \leq \gamma_T \\
A(\gamma) + \left(\frac{m}{\gamma}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)} \exp\left(-\frac{m \gamma}{\gamma}\right) & \text{if } \gamma > \gamma_T, \tag{30}
\end{cases}
\]

where \(\gamma_T\) denotes a predetermined switching threshold and \(A(\gamma)\) is given in [[3], p.437, Eq. (9.335)] as

\[
A(\gamma) = \left(\frac{m}{\gamma}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)} \exp\left(-\frac{m \gamma}{\gamma}\right) \times \left[1 - Q_m\left(\sqrt{2a \rho \gamma}, \sqrt{2a \gamma_T}\right)\right], \tag{31}
\]

where \(a = \frac{m}{\gamma(1-\rho^2)}\) and \(Q_m(., .)\) denotes generalized Marcum Q-function.

The average probability of detection for dual correlated Nakagami-\(m\) fading branches with SSC diversity \((\mathcal{P}_{D,SSC,2})\) is obtained by substituting (30) into (9) and then using the definition \(\int_0^\infty f dx = \int_0^\infty f dx - \int_0^\infty f dx\), which yields

\[
\mathcal{P}_{D,SSC,2} = \frac{1}{\Gamma(m)} \left(\frac{m}{\gamma}\right)^m [I_A - I_B - I_C] \tag{32}
\]

with

\[
I_A = 2 \int_0^\infty Q_a\left(\sqrt{2a \gamma_T}, \sqrt{2a \gamma_T}\right) \gamma^{m-1} \exp\left(-\frac{m \gamma}{\gamma}\right) d\gamma, \tag{33}
\]

\[
I_B = \int_0^\infty Q_a\left(\sqrt{2a \gamma_T}, \sqrt{2a \gamma}\right) Q_m\left(\sqrt{2a \gamma_T}, \sqrt{2a \gamma}\right) \gamma^{m-1} \exp\left(-\frac{m \gamma}{\gamma}\right) d\gamma, \tag{34}
\]

and

\[
I_C = \int_0^\infty Q_a\left(\sqrt{2a \gamma}, \sqrt{2a \gamma}\right) \gamma^{m-1} \exp\left(-\frac{m \gamma}{\gamma}\right) d\gamma. \tag{35}
\]

Before deriving an expression for the probability of detection \(\mathcal{P}_{D,SSC,2}\), it is worthy to investigate (32) for the following two special cases of threshold values.

Case I: \(\gamma_T = 0\)

If \(\gamma_T = 0\), we have \(Q_m\left(\sqrt{2a \rho \gamma}, \sqrt{2a \gamma_T}\right) = 1\) and the third term \(I_C\) vanishes, consequently (32) reduces to single branch detection as

\[
\mathcal{P}_{D,SSC,2} = \frac{1}{\Gamma(m)} \left(\frac{m}{\gamma}\right)^m \int_0^\infty Q_a\left(\sqrt{2a \gamma}, \sqrt{2a \gamma}\right) \gamma^{m-1} \exp\left(-\frac{m \gamma}{\gamma}\right) d\gamma. \tag{36}
\]

Case II: \(\gamma_T \rightarrow \infty\)

If \(\gamma_T \rightarrow \infty\), we have \(Q_m\left(\sqrt{2a \rho \gamma}, \sqrt{2a \gamma_T}\right) = 0\), consequently \(I_B\) vanishes and only \(I_C\) is subtracted from \(I_A\). This results in single branch detection as in (36). Therefore, care must be taken to choose a sensible threshold value. Otherwise, the diversity technique might become useless.

The average probability of detection for dual correlated SSC receiver with Nakagami-\(m\) fading branches where \(u\) and \(m\) are restricted to integer values is given in (37) at the top of next page. Please see Appendix F for detailed derivation.

Note that for \(m = 1\), (37) reduces to dual Rayleigh correlated fading branches, and for \(\rho = 0\) it reduces to dual \text{i.i.d.} Nakagami-\(m\) fading branches detection.
\[ P_{D,SSC,2} = \frac{1}{\Gamma(m)} \left( \frac{m}{\gamma} \right)^m e^{-\gamma} \left[ 2 \sum_{j=0}^{\infty} \sum_{k=0}^{j+m-1} \left( \frac{\lambda}{2} \right)^k (j+m-1)! \gamma^{j+m} \right] - \sum_{n=0}^{\infty} \sum_{q=0}^{n} \sum_{i=0}^{n} \sum_{k=0}^{i+m-1} \left( \frac{\lambda}{2} \right)^q n! \gamma^{n+m} \gamma_n^{m+n} \right] \]

\[ P_{D,SSC,2} = \frac{1}{\Gamma(m)} \left( \frac{m}{\gamma} \right)^m \left[ 4 \sum_{j=0}^{\infty} \frac{\Gamma(u+j, \frac{\gamma}{2}) \Gamma(m+j)}{\Gamma(u+j) (1 + \frac{\gamma}{2})} j! m^j \right] - \sum_{n=0}^{\infty} \sum_{i=0}^{n} \sum_{k=0}^{i+m-1} \frac{\Gamma(u+n, \frac{\gamma}{2}) \Gamma(m+n+i)}{\Gamma(u+n) \left( \frac{2m}{\gamma} + \alpha + 1 \right) n!} k! m^j \]

\[ \frac{\partial}{\partial \gamma_T^*} P_{D,SSC,2} = \frac{1}{\Gamma(m)} \left( \frac{m}{\gamma} \right)^m \left[ 2 \sqrt{a} \gamma_T^* e^{-a \gamma_T^*} \sum_{k=0}^{\infty} \frac{a^{m+k-1} \gamma_T^k}{\Gamma(m+k) 2^{m+k+1} k!} \right] - Q_u \left( \sqrt{2 \gamma_T^* \lambda} \right) \gamma_T^{m-1} \exp \left( -\frac{m \gamma_T^*}{\lambda} \right). \]

A. Alternative Solution

The expression \( P_{D,SSC,2} \) in (37) involves many infinite series representations. Some of their upper bounds (number of terms) are dependent on the preceding one. As an example the upper bound of the second sum (\( \sum_{k=0}^{j+m-1} (.) \)) depends on the number of terms (\( N \)) needed for convergence of the previous series. Fortunately, it will not be very difficult to find the number of terms for convergence (with five digit accuracy). However, time for numerical implementation will be rather long. Therefore, we will derive an alternative more general and simpler expression \( P_{D,SSC,2} \) with less number of infinite series representations.

The average probability of detection where \( u \) is not restricted while \( m \geq 1 \) is restricted to integer values is given in (38) at the top of next page. See Appendix G for the derivation.

Note, for \( m = 1 \), (38) reduces to that of a dual SSC receiver with Rayleigh correlated fading branches. For, \( \rho = 0 \) it reduces to the PDF of the dual i.i.d. Nakagami-\( m \) fading branches detection.

Interestingly, the three terms in (38) contain the upper incomplete gamma function in addition to the lower incomplete gamma function in the last term. In fact, we can represent both these functions by the monotonically decreasing confluent hypergeometric function using [[19], Eq. (6.5.12)] and [[28], Eq. (1.6)] for lower and upper incomplete gamma functions, respectively. Consequently the infinite series terms in (38) converges rapidly.

B. Optimal Threshold \( (\gamma_T^*) \)

Optimal threshold \( \gamma_T^* \) is defined as the value of the SNR that maximizes the probability of detection. We maximize the probability of detection by selecting an appropriate SNR for SSC switching. Probability of false alarm is fixed since it's a function of the decision threshold \( \lambda \) and not a function of SNR, as shown in (7). Constant False Alarm Rate (CFAR) is a well-known technique that is often employed in cognitive spectrum sensing. In this technique and using (7), a decision threshold is calculated for fixed probability of false alarm. Then the corresponding probability of detection is calculated using (8) for optimal SNR. We have derived an expression for this optimal threshold given in (39) at the top of this page. This is done by differentiating \( P_{D,SSC,2} \) in (32) with respect to \( \gamma_T \) and solving \( \frac{\partial}{\partial \gamma_T^*} P_{D,SSC,2} = 0 \) for \( \gamma_T^* \). See Appendix H for details. Using Matlab, we can obtain the optimal threshold by evaluating (39) numerically for \( \frac{\partial}{\partial \gamma_T^*} P_{D,SSC,2} = 0 \).

V. Simulation and Analysis Results

The energy detector employed in spectrum sensing is mainly characterized by the probability of false alarm \( P_F \) and probability of detection \( P_D \). In this section we study the impact of the correlation among antenna diversity branches on \( P_D \) (equivalently probability of miss detection \( P_{D_m} = 1 - P_D \) ) as a performance metric using the derived expressions in previous sections. To this end, we produce Complementary Receiver Operating Characteristic (CROC) graphs (\( P_{D_m} \) versus \( P_F \)) for SC and SSC diversity techniques in Nakagami-\( m \) fading channel.

<table>
<thead>
<tr>
<th>Table I: Terms required for five digits accuracy</th>
</tr>
</thead>
</table>
| \( \bar{P}_{D,SSC} \) : \( E[N] \), \( u = 2 \), \( P_F = 0.01 \), \( \bar{\gamma} = 20 \text{ dB} \) | \( \begin{array}{c|c|c|c|c|c}
\rho & m \ y N_2, N_1 \ & 1 & m \ y N_2, N_1 \ & 2 & m \ y N_2, N_1 \ & 3 & m \ y N_2, N_1 \ & 4 \\
0.1 & 15.1 & 15.1 & 15.1 & 15.1 & 15.1 & 15.1 & 15.1 & 15.1 \\
0.2 & 15.3 & 15.3 & 15.3 & 15.3 & 15.3 & 15.3 & 15.3 & 15.3 \\
0.4 & 15.3 & 15.3 & 15.3 & 15.3 & 15.3 & 15.3 & 15.3 & 15.3 \\
0.6 & 15.3 & 15.3 & 15.3 & 15.3 & 15.3 & 15.3 & 15.3 & 15.3 \\
0.8 & 15.4 & 15.4 & 15.4 & 15.4 & 15.4 & 15.4 & 15.4 & 15.4 \\
\end{array} \) |
In this work, we have investigated the impact of correlation among diversity fading branches in multi-antenna cognitive radio spectrum sensing networks. A unified performance analysis was presented for the probability of detection of SC and SSC diversity combining receivers with arbitrary and exponential correlation among fading branches. Exact expressions were derived for the probability of detection for each case. Our result show that the correlation among diversity fading branches causes an adverse impact on the probability of detection, which cannot be ignored especially under severe fading conditions. Consequently, an increase in the interference rate between the primary user and secondary user is observed by three times its rate when independent fading branches is assumed. Our investigations reveal that for low fading environment (large \( m \)-values), correlation effect may be ignored. Furthermore, at low fading and highly correlated environments, SSC which is simpler scheme performs as good as SC which is a more complex scheme.
Figure 2: SC dual correlated Nakagami-$m$ branches with $\bar{\gamma} = 20$ dB for different $\rho$ values.

Figure 3: SC/SSC dual correlated Nakagami-$m$ branches comparison with $\bar{\gamma} = 20$ dB for $\rho = 0$ (solid) and 0.8 (dashed).

**Appendix A:**

**Derivation of (12)**

Using ([17], (20)), the PDF of SC’s output of dual identical correlated Nakagami fading branches is

\[
p_{\gamma, SC}(r) = \frac{4 m^m r^{2m-1}}{\Gamma(m) \Omega^m} \exp \left( -\frac{m r^2}{\Omega} \right) \times \left[ 1 - Q_m \left( \sqrt{2} \rho A r, \sqrt{2} A r \right) \right],
\]

where $A = \sqrt{\frac{m}{\Omega(1-\rho)}}$.

Changing variables using $p_{\gamma}(\gamma) = \frac{p_{r}(\sqrt{\frac{\Omega \gamma}{2(\sqrt{\frac{\Omega}{\gamma}})}})}{2}$ [3] yields

\[
p_{\gamma, SC}(\gamma) = \frac{4 m^m \left( \sqrt{\frac{\Omega \gamma}{2}} \right)^{2m-1}}{\Gamma(m) \Omega^m} \exp \left( -\frac{m \left( \sqrt{\frac{\Omega \gamma}{\bar{\gamma}}} \right)^2}{\Omega} \right) \times \left[ 1 - Q_m \left( \sqrt{2} \rho A \sqrt{\frac{\Omega \gamma}{\bar{\gamma}}}, \sqrt{2} A \sqrt{\frac{\Omega \gamma}{\bar{\gamma}}} \right) \right],
\]
Figure 4: Probability of miss detection versus correlation with $\tilde{\gamma} = 20$ dB, $P_F = 0.01$ and different fading severity for SC and SSC.

Simplifying, (41) becomes

$$p_{\gamma SC}(\gamma) = \left(\frac{\sqrt{\Omega}}{\gamma}\right) \frac{2^{m^2} \gamma^{2m-1}}{\Gamma(m) \Gamma(m)} \exp\left(-\frac{m \gamma}{\tilde{\gamma}}\right) \times \left[1 - Q_m \left(\sqrt{2\rho \frac{m \gamma}{\tilde{\gamma}} \sqrt{2A \frac{\Omega \gamma}{\tilde{\gamma}}}}\right)\right].$$

Substituting $A = \sqrt{\frac{m}{1 - \rho}}$ and simplifying, yields

$$p_{\gamma SC}(\gamma) = \frac{2^{m^2} \gamma^{2m-1}}{\sqrt{\Gamma(m) \Gamma(m)}} \exp\left(-\frac{m \gamma}{\tilde{\gamma}}\right) \times \left[1 - Q_m \left(\sqrt{2\rho \frac{m \gamma}{\tilde{\gamma}} \sqrt{2A \frac{\Omega \gamma}{\tilde{\gamma}}}}\right)\right].$$

Simplifying

$$p_{\gamma SC}(\gamma) = \frac{2^{m^2} \gamma^{2m-1}}{\sqrt{\Gamma(m) \Gamma(m)}} \exp\left(-\frac{m \gamma}{\tilde{\gamma}}\right) \times \left[1 - Q_m \left(\sqrt{2\rho \frac{m \gamma}{\tilde{\gamma}} \sqrt{2A \frac{\Omega \gamma}{\tilde{\gamma}}}}\right)\right].$$

Simplifying and rearranging, this concludes the derivation.

**APPENDIX B**

**EXPRESSION FOR DUAL SC**

In this appendix, we derive the expression in (16).

1) Evaluating $I_A$ in (14): Introducing changing variable $x = \sqrt{2\gamma}$, we can derive

$$I_A = \frac{1}{2m-1} \int_0^\infty Q_u(x, \sqrt{\lambda}) x^{2m-1} \exp\left(-\frac{m x^2}{2 \lambda}\right) \, dx.$$  

Using [[29], Eq. (29)], we write

$$\int_0^\infty Q_u(\alpha x, \beta) x^q e^{-\frac{x^2}{2}} \, dx \equiv G_u$$

$$= \frac{\Gamma\left(\frac{2m+1}{2}\right)}{2 (u-1)!} \sqrt{\frac{\beta^2}{2}} e^{-\frac{\beta^2}{2}}$$

$$\times 1 F_1\left(\frac{q + 1}{2}; u; \frac{\beta^2}{2} + \frac{\lambda}{2}\right), \quad q > -1,$$

we can solve $I$ by evaluating $G_u$ recursively for $q > -1$ and restricted $u$ integer values as

$$G_u = \frac{G_{u-1} + A_{u-1} F_u}{G_{u-2} + A_{u-2} F_{u-2} + A_{u-1} F_{u-1}}$$

$$\vdots$$

$$= G_1 + \sum_{n=1}^{u-1} A_n F_{n+1}.$$  

where $A_n$ and $F_n$ are given as

$$A_n = \frac{1}{2 (n!) \left(\frac{\beta^2 + \lambda}{2}\right)^{\frac{n+1}{2}}} \Gamma\left(\frac{q + 1}{2}\right) \left(\frac{\beta^2}{2}\right)^n e^{-\frac{\beta^2}{2}}.$$  

$$F_n = \frac{A_{n+1}}{A_n}.$$
Substituting (50) and (57) into (13), this concludes the derivation.

APPENDIX C

ALTERNATIVE EXPRESSION FOR DUAL SC

In this Appendix, we derive (19). Using the alternative expression for Marcum Q-function given in [3], Eq.(4.63), where \( u \) is not restricted to integer values, we can write

\[
Q_u \left( \sqrt{2\gamma}, \sqrt{\lambda} \right) = 1 - e^{-\frac{\gamma^2 + \lambda}{2}} \sum_{n=u}^{\infty} \left( \frac{\sqrt{\lambda}}{\sqrt{2\gamma}} \right)^n I_n \left( \sqrt{2\gamma} \right).
\]

Then substituting (12) in (9) and using the definition of the PDF as

\[
\int_{\gamma}^{\infty} p_\gamma (\gamma) \, d\gamma = 1,
\]

with simplification, we can derive

\[
\overline{P}_{D,SC,2} = 1 - [I_A - I_B],
\]

where

\[
I_A = \frac{2}{\Gamma(m)} \left( \frac{m}{\gamma} \right)^m \int_0^\infty \gamma^{m-1} e^{-\frac{\gamma^2 + \lambda}{2}} \sum_{n=u}^{\infty} \left( \frac{\sqrt{\lambda}}{\sqrt{2\gamma}} \right)^n I_n \left( \sqrt{2\gamma} \right) \, d\gamma,
\]

and

\[
I_B = \frac{2}{\Gamma(m)} \left( \frac{m}{\gamma} \right)^m \int_0^\infty \gamma^{m-1} e^{-\frac{\gamma^2 + \lambda}{2}} \sum_{n=u}^{\infty} \left( \frac{\sqrt{\lambda}}{\sqrt{2\gamma}} \right)^n I_n \left( \sqrt{2\gamma} \right) \, d\gamma, \quad \gamma \geq 0.
\]

1) Evaluating \( I_A \) in (61): Simplifying and rearranging (61), we derive

\[
I_A = \frac{2}{\Gamma(m)} \left( \frac{m}{\gamma} \right)^m e^{-\frac{\gamma^2 + \lambda}{2}} \sum_{n=u}^{\infty} \left( \frac{\lambda}{2} \right)^n \int_0^\infty \gamma^{m-1} e^{-\gamma^2} \, \, d\gamma.
\]

Using [18], Eq. (6.643/2) given as

\[
\int_0^\infty x^\nu - \frac{1}{2} e^{-\alpha x} I_{2\nu} (2\beta \sqrt{x}) \, dx = \frac{\Gamma(\mu + \nu + \frac{1}{2})}{\Gamma(2\nu + 1)} \beta e^{\frac{\alpha^2}{2}} \alpha^{-\mu} M_{\mu,\nu} \left( \frac{\beta^2}{\alpha} \right),
\]

satisfying the condition therein,

\[
0^\infty x^\nu - \frac{1}{2} e^{-\alpha x} \, dx = \frac{\Gamma(\mu + \nu + \frac{1}{2})}{\Gamma(2\nu + 1)} \beta e^{\frac{\alpha^2}{2}} \alpha^{-\mu} M_{\mu,\nu} \left( \frac{\beta^2}{\alpha} \right),
\]

\[\text{Re} \left( \mu + \nu + \frac{1}{2} \right) > 0, \quad (64)\]

where \( M_{\mu,\nu}(z) \) denotes Whittaker function given by [18]

\[
M_{\mu,\nu}(z) = z^{\nu + \frac{1}{2}} e^{-\frac{z}{2}} F_1 \left( \nu + \mu + \frac{1}{2}; 1 + 2 \nu; z \right), \quad (65)
\]
with some simplification and rearranging, the solution of (63) can be derived as
\[ I_A = \frac{2}{d^m} \left( \frac{m}{\gamma} \right)^m e^{-\frac{a}{2}} \sum_{n=0}^{\infty} \frac{\left( \frac{\lambda}{2} \right)^n}{n!} \frac{1}{\Gamma(n+1)} \text{ for } d = \frac{2+m}{\gamma}. \]

2) Evaluating \( I_B \) in (62): Simplifying and rearranging (62), we derive
\[ I_B = \frac{2}{\Gamma(m)} \left( \frac{m}{\gamma} \right)^m e^{-\frac{a}{2}} \sum_{n=0}^{\infty} \frac{\left( \frac{\lambda}{2} \right)^n}{n!} \frac{1}{\Gamma(n+1)} \int_0^\infty \gamma^{-\frac{a}{2}+i+k-1} e^{-\gamma} d\gamma, \]
where \( c = 1 + a (\rho + 1) + \frac{m}{\gamma} \). Similarly, implementing same procedures as (66), the solution of (66) can be given as
\[ I_B = \frac{2}{\Gamma(m)} \left( \frac{m}{\gamma} \right)^m e^{-\frac{a}{2}} \sum_{n=0}^{\infty} \frac{\left( \frac{\lambda}{2} \right)^n}{n!} \frac{1}{\Gamma(n+1)} \Gamma(m+i+k) \Gamma(m+i+k+1+n) \left( \frac{\lambda}{2} \right)^k \frac{1}{\Gamma(k+1)} \times \int_0^\infty \gamma^{-\frac{a}{2}+i+k-1} e^{-\gamma} d\gamma, \]
Using (54) with simplification and rearrangement, we write (67) as
\[ I_B = \frac{2}{\Gamma(m)} \left( \frac{m}{\gamma} \right)^m e^{-\frac{a}{2}} \sum_{n=0}^{\infty} \frac{\left( \frac{\lambda}{2} \right)^n}{n!} \frac{1}{\Gamma(n+1)} \Gamma(m+i+k) \Gamma(m+i+k+1+n) \left( \frac{\lambda}{2} \right)^k \frac{1}{\Gamma(k+1)} \times \int_0^\infty \gamma^{-\frac{a}{2}+i+k-1} e^{-\gamma} d\gamma, \]
where \( \eta = \frac{1+1}{m+1} \). Similarly, implementing same procedures as (66), the solution of (66) can be given as
\[ I_B = \frac{2}{\Gamma(m)} \left( \frac{m}{\gamma} \right)^m e^{-\frac{a}{2}} \sum_{n=0}^{\infty} \frac{\left( \frac{\lambda}{2} \right)^n}{n!} \frac{1}{\Gamma(n+1)} \Gamma(m+i+k) \Gamma(m+i+k+1+n) \left( \frac{\lambda}{2} \right)^k \frac{1}{\Gamma(k+1)} \times \int_0^\infty \gamma^{-\frac{a}{2}+i+k-1} e^{-\gamma} d\gamma, \]
Substituting (66) and (69) into (60), this concludes the derivation.

APPENDIX D

EXPRESSION FOR TRIPLE SC

In this section, we derive the expression in (28). Using (53) and substituting (20) into (9), we derive the average probability of detection as
\[ P_{D,SC,3} = \frac{1}{\Gamma(m)} \sum_{n=0}^{\infty} \frac{1}{\Gamma(n+1)} \int_0^\gamma \gamma^{-\frac{a}{2}} e^{-\gamma} \left[ \Theta_1 + \Theta_2 + \Theta_3 \right] d\gamma. \]

Following same procedures in (71)-(76), then substituting (76) into (77), this concludes the derivations.

APPENDIX E

GENERAL EXPRESSION FOR TRIPLE SC

In this section, we derive the expression in (29). Using (52) and substituting (20) into (9) we derive
\[ P_{D,SC,3} = \frac{1}{\Gamma(m)} \sum_{n=0}^{\infty} \frac{1}{\Gamma(n+1)} \int_0^\gamma \gamma^{-\frac{a}{2}} e^{-\gamma} \left[ \Theta_1 + \Theta_2 + \Theta_3 \right] d\gamma. \]

Appendix F

EXPRESSION FOR DUAL SSC

In this section, we will derive the expression in (37) by evaluating \( P_{D,SC,2} \) in (32) as follows.

1) Integral \( I_A \) in (33): Using Marcum Q-function alternative representation (53), we rewrite (33) as
\[ I_A = 2 e^{-\frac{a}{2}} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{\left( \frac{\lambda}{2} \right)^k}{j!k!} \int_0^\infty \gamma^{j+m-1} e^{-\gamma} d\gamma. \]
Using (56) and satisfying the condition therein, we solve (78) as
\[ I_A = 2 e^{-\frac{z^2}{4}} \sum_{j=0}^{\infty} \left( \frac{\lambda}{2} \right)^j \frac{1}{j!} \left( \frac{\bar{\gamma}}{\gamma + m} + 1 \right)^{j+m} . \] 
(79)

2) Integral $I_B$ in (34): Following the same procedures as in (78), we rewrite (34) as
\[ I_B = e^{-z^2} \sum_{n=0}^{\infty} \sum_{q=0}^{\infty} \sum_{i=0}^{\infty} \sum_{k=0}^{\infty} \left( \frac{\lambda}{2} \right)^q \frac{a^{i+k} \rho^i}{i! k! q!} e^{-a \gamma T} \gamma^k_T \cdot \left( \frac{\bar{\gamma}}{\gamma + m} + 1 \right)^{n+m+i} \exp \left\{ -\gamma \left( a \rho + \frac{m}{\bar{\gamma}} + 1 \right) \right\} d\gamma. \] 
(80)

Similarly as we did in (79), we solve (80) as
\[ I_B = e^{-z^2} \sum_{n=0}^{\infty} \sum_{q=0}^{\infty} \sum_{i=0}^{\infty} \sum_{k=0}^{\infty} \left( \frac{\lambda}{2} \right)^q \frac{a^{i+k} \rho^i}{i! k! q!} \left( \frac{m + n + i - 1}{m + n + i + 1} \right) \cdot \left( a \rho + \frac{m}{\bar{\gamma}} + 1 \right) \exp \left\{ -\gamma \left( a \rho + \frac{m}{\bar{\gamma}} + 1 \right) \right\} d\gamma. \] 
(81)

3) Integral $I_C$ in (35): Using (53), we rewrite (35) as
\[ I_C = e^{-z^2} \sum_{n=0}^{\infty} \sum_{q=0}^{\infty} \sum_{i=0}^{\infty} \sum_{k=0}^{\infty} \left( \frac{\lambda}{2} \right)^q \frac{1}{n! q!} e^{-a \gamma T} \gamma^k_T \cdot \left( \frac{\bar{\gamma}}{\gamma + m} + 1 \right)^{n+m+i} \exp \left\{ -\gamma \left( a \rho + \frac{m}{\bar{\gamma}} + 1 \right) \right\} d\gamma. \] 
(82)

Using [[18], Eq. (3.51.1)], where
\[ \int_0^z x^n e^{-\mu x} dx = \frac{n!}{\mu^{n+1}} - e^{-\mu z} \sum_{k=0}^{n} \frac{n!}{k!} \frac{z^k}{\mu^{n-k+1}} \] 
(83)

\[ \left\{ z > 0, \Re \mu > 0, n = 0, 1, 2, \ldots \right\}, \]

we derive (82) as
\[ I_C = e^{-\frac{z^2}{4}} \sum_{n=0}^{\infty} \sum_{q=0}^{\infty} \frac{\lambda^q}{n! q!} \frac{1}{\left( \frac{\bar{\gamma}}{\gamma + m} + 1 \right)} \cdot \exp \left\{ -\gamma \left( a \rho + \frac{m}{\bar{\gamma}} + 1 \right) \right\} \] 
(84)

Substituting (79), (80) and (84) into (32), this concludes the derivation.

### Appendix G

**Alternative Expression for Dual SSC**

In this section, we will derive the expression in (38) by evaluating $P_{D, SSC, 2}$ in (32) for alternative expression as follows.

1) Integral $I_A$ in (33): Let $x = \sqrt{2\gamma}$, we rewrite (33) as
\[ I_A = \frac{4}{2^m} \int_0^{\infty} Q_u \left( x, \sqrt{\lambda} \right) x^{m-1} \exp \left\{ -\frac{m x^2}{2 \gamma} \right\} dx. \] 
(85)

Using [[31], Eq. (8)], we solve (85) as
\[ I_A = 4 \sum_{j=0}^{\infty} \frac{\Gamma(m + j) \Gamma(u + j, \frac{\lambda}{2})}{\Gamma(u + j)} \frac{1}{\Gamma \left( u + j + 1, m + j + 1 \right)} \] 
(86)

2) Integral $I_B$ in (34): Using (52) and (53) for $Q_u \left( \sqrt{2u}, \sqrt{\lambda} \right)$ and $Q_m \left( \sqrt{2a \rho \gamma}, \sqrt{2 \alpha \gamma T} \right)$, we rewrite (34) as
\[ I_B = \sum_{n=0}^{\infty} \sum_{i=0}^{\infty} \sum_{k=0}^{\infty} \frac{\Gamma(u + n + \frac{1}{2}) \Gamma(u + n, \frac{1}{2})}{\Gamma(u + n + i) n! k!} \gamma^k_T e^{-a \gamma T} \times \int_0^{\infty} \frac{\Gamma \left( \frac{m}{\bar{\gamma}} + a \rho + 1 \right) \Gamma \left( m + n + i \right) n! k!}{\Gamma(u + n + i) n! k!} x^{m+n+i-1} e^{-\gamma \left( \frac{m}{\bar{\gamma}} + a \rho + 1 \right)} d\gamma. \] 
(87)

Using (56), we solve (87) as
\[ I_B = \sum_{n=0}^{\infty} \sum_{i=0}^{\infty} \sum_{k=0}^{\infty} \frac{\Gamma \left( u + n, \frac{1}{2} \right) \Gamma \left( m + n + i \right) \Gamma \left( \frac{m}{\bar{\gamma}} + a \rho + 1 \right) \Gamma \left( m + n + i \right) \Gamma \left( \frac{m}{\bar{\gamma}} + a \rho + 1 \right)}{\Gamma(u + n) n! k!} x^{m+n+i-1} e^{-\gamma \left( \frac{m}{\bar{\gamma}} + a \rho + 1 \right)} d\gamma. \] 
(88)

3) Integral $I_C$ in (35): Using (52), we rewrite (35) as
\[ I_C = \sum_{p=0}^{\infty} \frac{\Gamma \left( u + p, \frac{1}{2} \right) \Gamma \left( \frac{m}{\bar{\gamma}} + m \right) \Gamma \left( m + n + i \right) \Gamma \left( \frac{m}{\bar{\gamma}} + m \right)}{\Gamma(u + p) p!} \int_0^{\infty} \frac{x^{m+n+i-1} e^{-\gamma \left( \frac{m}{\bar{\gamma}} + a \rho + 1 \right)}}{\Gamma(u + p) p!} \frac{\Gamma \left( \frac{m}{\bar{\gamma}} + a \rho + 1 \right) \Gamma \left( m + n + i \right) \Gamma \left( \frac{m}{\bar{\gamma}} + a \rho + 1 \right)}{\Gamma(u + n) n! k!} \frac{x^{m+n+i-1} e^{-\gamma \left( \frac{m}{\bar{\gamma}} + a \rho + 1 \right)}}{\Gamma(u + p) p!} \frac{\Gamma \left( \frac{m}{\bar{\gamma}} + a \rho + 1 \right) \Gamma \left( m + n + i \right) \Gamma \left( \frac{m}{\bar{\gamma}} + a \rho + 1 \right)}{\Gamma(u + n) n! k!} \] 
(90)

Substituting (86), (88) and (90) into (32), this concludes the derivation.

### Appendix H

**Expression for Optimal Threshold**

In this section, we will derive the expression in (39).

Employing Leibniz’s rule [[19], Eq. (3.3.7)] with the aid of following identity given in [[29], Eq. (9)] as
\[ \frac{\partial}{\partial \beta} Q_u (\alpha, \beta) = -\beta \left( \frac{\beta}{\alpha} \right)^{u-1} \exp \left\{ -\frac{\alpha^2 + \beta^2}{2} \right\} I_{u-1} (\alpha \beta), \] 
(91)

we rewrite (32) as
\[ \frac{\partial}{\partial \beta} P_{D, SSC, 2} = \frac{1}{\Gamma(m)} \left( \frac{m}{\gamma} \right)^m \left( \frac{1}{\rho} \right)^{m-1} \Gamma(m+1)(2a) e^{-a \gamma \frac{\gamma}{\gamma}} \times \int_0^{\infty} Q_u \left( 2 \sqrt{\gamma}, \sqrt{\lambda} \right) \gamma^{m-1} e^{-\gamma T} I_{m-1} \left( 2a \sqrt{\rho T \gamma} \right) d\gamma \] 
(92)

To solve the integral $I$ in (92), we perform changing variable along with the aid of the series expansion of the modified Bessel function given in [[18], Eq. (8.445)] as
\[ I_u(z) = \sum_{k=0}^{\infty} \frac{1}{\Gamma(n + k + 1, k!)} \left( \frac{z}{2} \right)^{\nu+2k}. \] 
(93)
Then, we drive (92) as
\[
\frac{\partial}{\partial \gamma_i^*} \mathcal{P}_{D, \text{SSC}_2} = \frac{1}{\Gamma(m)} \left( \frac{m}{\gamma} \right)^m \left[ \frac{1-m}{\rho^T} \sqrt{2\mu_y^T} e^{-\gamma_i^*} \right] \times \sum_{k=0}^{\infty} \frac{1}{\Gamma(m+k)2^{m+k}k!} (\sqrt{\rho})^{m+2k-1} \gamma_i^* \times \int_0^{\infty} Q_u(x, \sqrt{\lambda}) x^{2(m+k)-1} e^{-\frac{1}{2}x^2} dx \tag{94}
\]

Using [(29), Eq. (29)] by following same procedures as in (50), we can solve the integral \( I_A \) in (94) as
\[
I_A = G'_1 + \frac{1}{2} \sum_{n=1}^{\infty} \left( \frac{\lambda}{2} \right)^n \Gamma(m+k) \left( \frac{a+1}{2} \right)^{m+k} n!
\times \frac{1}{2} \left( \frac{\lambda}{a+1} \right) \tag{95}
\]
where \( G'_1 \) can be obtained by evaluating the following integral containing the first order of Marcum Q-function \( Q(.,.) \) for integer \( m \) values as
\[
G'_1 = \int_0^{\infty} Q(x, \sqrt{\lambda}) x^{2(m+k)-1} e^{-\frac{1}{2}x^2} dx \tag{96}
\]
Using [(29), Eq. (25)], we evaluate \( G'_1 \) for integer \( m \) values as
\[
G'_1 = \frac{2^{m+k-1}}{a^{2(m+k)}} \left[ \frac{1}{a+1} \right] e^{-\frac{1}{2}a^2} \times \left[ (1+a) \left( \frac{a}{a+1} \right)^{m+k-1} \right] \tag{97}
\]
where \( L_n(\cdot) \) denotes Laguerre polynomial of \( n \)-degree [18]. Substituting (95) into (94), this concludes the derivation.

REFERENCES


Dear Respected Editor and Reviewers,

Thanks for taking time to carefully review the paper. Your constructive comments were very helpful for us to revise and significantly improve the paper. Please see our responses below. If you still find any shortcoming, it may be an oversight.

Reviewer: 1

<b>ADDITIONAL COMMENTS TO THE AUTHOR</b>: The revised paper has been significantly improved but I still have some concerns:

1. The authors addressed several of my previous comments but, unfortunately, not all to my full satisfaction.

In particular, I am not satisfied with the response to my first comment. While I agree that several of the integrals involving the Marcum Q-function that are solved by the authors are different from the ones in the five references that I cited, there are some overlaps.

- For example, it appears that the integral in (14) and (33) of the revised paper are similar to the ones in [R2] and [R5] in my previous review.
- As another example, note that (66) is same as [R2, eq. (9)].

- Response:
Thanks for your constructive comments. First, please note that (14) & (33) given below are the same equations except the multiplying factor 2. Therefore, we continue to discuss (14) only.

\[
I_A = 2 \int_0^\infty Q_u \left( \sqrt{2\gamma}, \sqrt{\lambda} \right) \gamma^{m-1} \exp \left( -\frac{m\gamma}{\gamma} \right) d\gamma, \quad (33)
\]

\[
I_A = \int_0^\infty Q_u \left( \sqrt{2\gamma}, \sqrt{\lambda} \right) \gamma^{m-1} \exp \left( -\frac{m\gamma}{\gamma} \right) d\gamma, \quad (14)
\]

Next, please note we derived two solutions for (14). First is (50), which is much less mathematically complicated than the solution suggested in [R2] and [R5] without
embedded infinite series. Our second, alternative solution to (14) as shown in the Appendix C is given below. This (66) is similar to [R2, eq. (9)].

\[
I_A = \frac{2}{d^m} \left( \frac{m}{\gamma} \right)^m e^{-\frac{\lambda}{2}} \sum_{n=0}^{\infty} \left( \frac{\lambda}{2} \right)^n \frac{1}{\Gamma(n+1)} \times _1 F_1 \left( m; 1 + n; \frac{\lambda}{2d} \right),
\]

In fact, both [R2] and [R5] gave a solution for (14) as in [R2, eq. (9)] and [R5, eq. (7)], respectively, almost using same method. However, [R5]'s solution was more general, consequently, more mathematically complicated.

Furthermore, please note that (16) and (19) are the solutions for (13) given below.

\[
P_{D,SC,2} = \frac{2}{\Gamma(m)} \left( \frac{m}{\gamma} \right)^m \left[ I_A - I_B \right], \quad (13)
\]

Here, we had to deal with two integrals \( I_A \) and \( I_B \), which represents integrals involving combination of a product of two Marcum Q-functions: one with single and the other with double variables. Therefore, our study represents solving a different and more complicated integral than that in the mentioned references.

- Therefore, I still would like the authors to make a general remark in the paper pointing out the fact that several integrals involving the Marcum Q-function are available in the literature (citing some of the references) and stating how the integrals in (14), (15), (33)-(35) are more general, better, different, etc.

- **Response:**
  Thanks for your constructive comments. Yes, we made that in p4, col1, 3\textsuperscript{rd} paragraph, the following statement:

  “It’s worthwhile to mention that several solutions for integrals involving the Marcum Q-function are available in literature [21]–[25]. However, our case of study in (15) solves a different and more complicated integral which involves a product of two Marcum Q-functions. These solutions are introduced in (57), (69) in Appendices B and C, respectively. To the best of knowledge, we believe that this solution is new in literature.”
2. I think that Appendix A is not necessary and should be removed. In fact, (12) can be obtained directly from [5, eq.(3)] of reference [17] of the revised manuscript.

- **Response:**
  Thanks for your constructive comments. In fact, reviewer 2 suggested having this derivation, so we did that in Appendix A.

3. There are still some typos and I ask the authors read the manuscript carefully; e.g.,
   - page 1, line 18, column 2: "MRC and EGC techniques..."
   - page 1, line 34, column 2: "schemes are required"
   - page 1, line 59, column 1: "complex linear diversity scheme"
   - page 1, line 60, column 1: "technique is a close"
   - page 2, in equations (2) and (4), "if" may be replaced with "under"
   - page 2, first line of equation (4): the exponential term is missing a -ve sign.
   - page 2, please define the pdf in equation (9).

- **Response:**
  Thanks for your constructive comments. Corrections are all done.

**Reviewer: 2**

**ADDITIONAL COMMENTS TO THE AUTHOR:**
The authors addressed the raised comments.

- Thanks.

**Reviewer: 3**

**ADDITIONAL COMMENTS TO THE AUTHOR:**
Important comment.

Major part of the manuscript is utilized for calculating complicated expressions for pd. The core contribution claimed in the abstract and conclusion is the impact of inter-branch correlation. This is demonstrated through simulations or perhaps through graphs that plot some functions. The expressions are complicated and hence the impact is not clearly visible from them. The authors may want to add in, if possible, simpler approximation in terms of $\rho$ (much more simpler than (38)). Nevertheless, the problem is relevant and requires attention and authors have highlighted that. The authors still need to address the following issues (some are minor).
1. The grammatical errors and typos have reduced significantly but they still exist in different parts of a paper. Proof-reading is still required.

- **Response:**
  Thanks for your constructive comments. These are done.

2. Kindly highlight your contributions in the introduction better. A point wise list may be a good option against the background provided.

- **Response:**
  Thanks for your constructive comments. We did as in p2, col1, 2\textsuperscript{nd} paragraph.

3. In the abstract “Alternative, simpler expressions are also derived for comparison“ of ... and ... ?

- **Response:**
  Thanks for your constructive comments. Yes, corrected: “Alternative, simpler expressions are also derived.”

4. The reviewer suggests that even if you are referring to [8] and [9] you should reproduce the basic sampling model here for convenience. As such from the author’s side it seems that (3) is what has chi-squared distribution but is it not clear how? How is (3) is related sum of Gaussian Squares?

- **Response:**
  Thanks for your constructive comments. The derivations as stated in [8] and [9] are considered for Gaussian channel. Therefore, (3) represents the sum of Gaussian Squares. This is well explained in [8] and [9]. Therefore and for the sake of brevity, we didn’t reproduce that basic sampling model in our paper.

5. Page 5 column 2 line 51 – what is |x| and |y| here? This should be shown in the manuscript itself.

- **Response:**
  Thanks for your constructive comments. We added: “where |. | denotes absolute x”

6. Complexity is an issue which should be highlighted in any case. The reviewer wishes the author to provide the number of computations in terms of number of samples required for each technique. Following this the authors can claim their statement on the complexity.
• **Response:**
  Thanks for your constructive comments. In fact, we discussed this issue in p4, col 1, 3rd paragraph and p6, col 1, 2nd paragraph and we produced Table 1 in p6. However, we didn’t attempt to quantify the complexity as it’s not our main focus in this work. Our main focus is to investigate the impact of the correlation on spectrum sensing and how to reduce this impact in order to maximize the probability of detection. Our work in fact produces simpler expressions the ones previously reported (such as in [R2 and R5]). If necessary, investigating complexity issue could be done in future work.

7. “Optimal threshold $\gamma^*$ is defined as the value of the SNR that maximizes the probability of detection. Therefore corresponding optimal threshold would also be different.”
   Firstly, kindly put the footnote in line else there is a disconnect between the two statements.
   Secondly, it is still not clear whether the authors are maximizing probability of detection by selecting an appropriate SNR while the probability of false alarm is fixed. If agreed that for rest of the parameters fixed, there is a one-to-one mapping between pf and pd, then maximizing pd will naturally result in maximizing pf. Is this always desirable? Precise statements are missing that use (39) to explain this.

• **Response:**
  Thanks for your constructive comments. We have done the first suggestion.

Answer to the second suggestion is ‘yes’. We maximize the probability of detection by selecting an appropriate SNR for fixed probability of false alarm. We have explained that in p6, col 1 & 2, Sec B as following:

We maximize the probability of detection by selecting an appropriate SNR for SSC switching. Probability of false alarm is fixed since it’s a function of the decision threshold $\lambda$ and not a function of SNR, as shown in (7). Constant False Alarm Rate (CFAR) is a well-known measure that is often employed in cognitive spectrum sensing. In this technique and using (7), a decision threshold is calculated for fixed probability of false alarm. Then the corresponding probability of detection is calculated using (8) for optimal SNR.

8. “We have derived the expression of this optimal threshold that is given in (39) at the top of this page.”
   Also (39) does not provide the expression but is apparently the solution to the RHS of (39) being equated to zero. What inference is obtained about the nature of optimal threshold from (39)?

• **Response:**
  Thanks for your constructive comments. In fact (39) is derived, as shown in appendix H, by differentiating (32) which consists of three integrals ($I_A, I_B$ & $I_C$) given in (33), (34) & (35) respectively. Then, we simplified the expression by solving the integral $I$
to obtain (39). We agree with the reviewer that (39) doesn’t give clear and quick clue about the impact of the optimal threshold on the probability of detection. However, by equating (39) to zero and then solving numerically, we can calculate the optimal threshold $\gamma_0^*$. Unfortunately it has to be solved numerically due to the presence of three complicated integrals that cannot be simplified further. However, we did investigate and explained the impact of the threshold SNR on the probability of detection in p5, col1 & col2 in Sec IV when we discussed the two special cases $\gamma_0^* = 0$ & $\gamma_0^* = \infty$ in (32). We concluded there that both very low and very high SNR turns dual SSC into just single branch diversity, thus reducing the probability of detection. Therefore the optimal threshold should be set carefully using (39).

9. “The energy detector employed in spectrum sensing is mainly characterized by the probability of false alarm PF and probability of detection PD. Therefore,” This part in Numerical result section is unnecessary. Again what is numerically obtained here that this section is called numerical results.

- **Response:**
  Thanks for your constructive comments. We removed the word “Therefore”. We called this section “Simulation and Analysis Results”


- **Response:**
  Thanks for your constructive comments. All these are corrected.

11. The paper is mathematically motivated and results can be presented in proposition-lemma-theorem-corollary format. This will make the results more visible and make relegating the proofs/derivations to the appendix.

**Response:**
Thanks for your constructive comments. Actually, we moved most of the derivations to the appendices so as the reader not get confused by derivations details.

Dear Mr. Al-Juboori:

The review of your paper


has been completed.

Below please find comments from the reviewers.

Based on the reviewers' comments, publication of the paper in its present form is not recommended. It is recommended that you resubmit your manuscript, WITHIN TWO MONTHS from the date of this email, as revised in accordance with the comments. If you fail to do so, your paper will be considered withdrawn from the review process.

Please note that this recommendation of resubmission should not be treated as an implication of eventual acceptance of your paper. It only refers to a deferral of the editorial decision to see whether the reviewers' comments could be addressed. In the event that your revision is accepted, the manuscript must be in a format that is in accordance with IEEE final submission requirements. For detailed final submission requirements, kindly refer to the information for IEEE Authors guidelines contained on the website http://www.ieee.org/pubs/authors.html.

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Also, please submit your responses, detailing the changes you have made, either in the
"Respond to these comments" box in Step 1 or by attaching a PDF version of your response in Step 6, of the revision submission process. If you choose to attach a PDF file, please indicate so in the "Respond to these comments" box in Step 1.

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Sincerely yours,

Dr Xin Wang
Editor, IEEE Transactions on Vehicular Technology

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