Doubly Iterative Receiver for Block Transmissions with EM-Based Channel Estimation

The-Hanh Pham, Ying-Chang Liang, *Senior Member, IEEE*, and A. Nallanathan, *Senior Member, IEEE*

***Abstract*—Cyclic-prefix code division multiple access (CP-CDMA), multicarrier CDMA (MC-CDMA) and single carrier cyclic-prefix (SCCP) transmission are some schemes that could support the increasing demand of future high data rate applications. The linear and nonlinear equalizers used to detect the transmitted signal are always far from the Maximum-Likelihood (ML) detection bound. The block iterative generalized decision feedback equalizer (BI-GDFE) is an iterative and effective interference cancelation scheme which could provide near-ML performance yet with very low complexity. In order to deploy this scheme, the channel state information (CSI) must be available at the receiver. In practice, this information has to be estimated by using pilot and data symbols. This paper investigates the problem of channel estimation using the Expectation Maximization (EM) algorithm. The BI-GDFE provides the soft information of the transmitted signals to the EM-based algorithm in the form a combination of hard decision and a coefficient so-called the input-decision correlation (IDC). The resultant receiver becomes a doubly iterative scheme. To evaluate the performance of the proposed estimation algorithm, the Cramér-Rao Lower Bound (CRLB) is also derived. Computer simulations show that the bit error rate (BER) performance of the proposed receiver for joint channel estimation and signal detection can reach the performance of the BI-GDFE with perfect CSI.**

***Index Terms*—Channel estimation, doubly iterative receiver, expectation maximization (EM) algorithm, SCCP, CP-CDMA, MC-CDMA.**

I. INTRODUCTION

THE cyclic-prefix (CP) block-based transmissions have attracted a lot of attention in nowadays wireless systems. Among those systems, cyclic-prefix code division multiple access (CP-CDMA), multicarrier CDMA (MC-CDMA) and single carrier cyclic-prefix (SCCP) transmission are three popular systems.

The CP-CDMA [1] is an advanced version of direct sequence code division multiple access (DS-CDMA). It is the combination of the CP insertion with the DS-CDMA. An alternative to CP-CDMA is multi-carrier CDMA (MC-CDMA) [2] which combines DS-CDMA with orthogonal frequency division multiplexing (OFDM). In MC-CDMA, the data block is spread out in frequency domain using orthogonal spreading sequences and transmitted over many subcarriers. This is different from the CP-CDMA where data block is transmitted directly using a single carrier. MC-CDMA is being extensively investigated for potential deployment in beyond 3G or 4G cellular systems [3].

The SCCP modulation is an alternative to the multicarrier cyclic-prefix (MCCP) modulation such as OFDM in order to suppress intersymbol interference (ISI). The SCCP modulation uses a single carrier for transmission, instead of the many typically used in OFDM, so the peak-to-average power ratio (PAPR) is generally smaller. SCCP has been selected for IEEE 802.16.

In recent years, a lot of efforts have been devoted to the design of nonlinear receivers that have low complexity yet achieve near-ML performance. In [4], one of such receiver, the Block-Iterative GDFE (BI-GDFE) receiver, has been introduced. The BI-GDFE receiver iteratively and simultaneously (but not jointly) detects the transmitted signals by utilizing the decisions from the previous iteration to cancel out the interference. Relying on a statistical reliability factor of hard decisions for each iteration, namely the input-decision correlation (IDC), interference is reduced iteratively, and this improves the bit error rate (BER) performance of the system. The low complexity of the BI-GDFE receiver relies on the fact that the equalizer coefficients can be determined in an off-line manner.

In order to detect the transmitted signals coherently, the channel state information (CSI) must be available at the receiver. The approach that uses not only the information provided by pilots but also the information obtained from the data part is preferable nowadays, where detected symbols are used to refine the CSI estimate. On the other hand, the expectation-maximization (EM) algorithm [5] provides an iterative approach to likelihood-based parameter estimation problems when the direct maximization of likelihood function is computationally prohibitive.

In this paper, we propose an EM-based channel estimation algorithm for three block-based transmission schemes using the BI-GDFE receiver. The hard decisions of the signals as well as the IDC can be used to constitute the soft information of the transmitted signals even in the uncoded system. Therefore, BI-GDFE receiver satisfies the requirement of the expectation of the transmitted signals of the EM-based channel estimation algorithm. To facilitate the evaluation of estimation algorithm, in this paper, we also derive the Cramér-Rao Lower Bound (CRLB). Because of the difficulty in finding a closed-form of the exact CRLB, we rely on the so-called *modified* CRLB [6]. The obtained closed-form modified CRLB agrees with the exact CRLB through simulations; therefore, it is used to evaluate the mean square error (MSE) performance of the estimated parameters.

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The remaining of the paper is organized as follows. In Section II, we describe the system models for three block-based systems, namely CP-CDMA, MC-CDMA and SCCP. The proposed doubly iterative receiver is given in Section III. CRLB derivation is presented in Section IV. In Section V, computer simulations are provided. Finally, conclusions are drawn in Section VI.

Notations: the transpose, conjugate and Hermitian conjugate of a vector/matrix are denoted by $(\cdot)^{\mathcal{T}}$, $(\cdot)^*$ and $(\cdot)^{\mathcal{H}}$, respectively. The (m,n) element of a matrix $\boldsymbol{A}$ is denoted by $(\boldsymbol{A})_{m,n}$; the n^{th} element of a vector $\boldsymbol{a}$ is denoted as $(\boldsymbol{a})_n$. $\mathrm{tr}\{\boldsymbol{A}\}$ is the trace of a matrix $\boldsymbol{A}$. $\mathrm{diag}\{\boldsymbol{A}\}$ is a diagonal matrix which takes the diagonal elements from a matrix $\boldsymbol{A}$; $\mathrm{diag}\{\boldsymbol{a}\}$ is the diagonal matrix whose diagonal elements are from a vector $\boldsymbol{a}$. $\boldsymbol{W}$ is the N-point discrete Fourier transform matrix. $\boldsymbol{F}_L$ is constructed by the first L columns of the $\sqrt{N}\boldsymbol{W}$. $\boldsymbol{I}_N$ denotes the identity matrix of size N.

II. System Models

In this section, the systems models for various CP-based systems are presented.

A. CP-based CDMA systems

We assume that the two CP-based CDMA systems (CP-CDMA and MC-CDMA) have T users in a single cell environment under synchronous downlink transmission scenario. The signal block consisting of a CP of length P and the data of length N. The symbols are transmitted over a frequency-selective fading channel which is characterized by the channel vector of length L, $\boldsymbol{h} = [h_0\ h_1\ \cdots\ h_{L-1}]^{\mathcal{T}}$. We assume that the channel is static over the entire transmission of a frame. The CP length satisfies $P \geq L$.

1) MC-CDMA [2]: Let Q and G be the number of symbols that one user can transmit in one block and the processing gain for every user, respectively. Then $N = QG$ is the total number of sub-carriers. The i^{th} user has the short code of $\boldsymbol{c}_i = [c_i(0)\ \cdots\ c_i(G-1)]^{\mathcal{T}}$ where $\boldsymbol{c}_i^{\mathcal{H}}\boldsymbol{c}_j = \delta(i-j)$. The long scrambling code at the t^{th} block is $\boldsymbol{D}(t) = \mathrm{diag}\{d(t;0)\ \cdots\ d(t;N-1)\}$ with $|d(t;k)| = 1$. At the receiver, the received signal block is written as

$$\boldsymbol{y}(t) = \boldsymbol{\Lambda\Pi D}(t)\boldsymbol{Cs}(t) + \boldsymbol{n}(t),\ t = 1,2,\ldots,T, \quad (1)$$

where $\boldsymbol{\Pi}$ is the interleaver matrix used to spread the transmitted chip sequence from the same symbol to possibly non-consecutive sub-carriers and other quantities are defined as $\boldsymbol{y}(t) = [y(t;0)\ \cdots\ y(t;N-1)]^{\mathcal{T}}$, $\boldsymbol{C} = \mathrm{diag}\{\bar{\boldsymbol{C}}\ \cdots\ \bar{\boldsymbol{C}}\}$, $\boldsymbol{s}(t) = [\bar{\boldsymbol{s}}_0^{\mathcal{T}}(t)\ \cdots\ \bar{\boldsymbol{s}}_{Q-1}^{\mathcal{T}}]^{\mathcal{T}}$ with $\bar{\boldsymbol{C}} = [\boldsymbol{c}_0\ \cdots\ \boldsymbol{c}_{T-1}]$ and $\bar{\boldsymbol{s}}_k(t) = [s_0(t;k)\ \cdots\ s_{T-1}(t;k)]^{\mathcal{T}}$. In (1), $\boldsymbol{\Lambda}$ is a diagonal matrix in which the diagonal elements correspond to the frequency responses of channel vector $\boldsymbol{h}$. The noise vector $\boldsymbol{n}(t)$ is a realization of the zero-mean complex Gaussian random vector with covariance matrix $\sigma_n^2\boldsymbol{I}_N$.

2) CP-CDMA [1]: For this system, we have the following model

$$\boldsymbol{y}(t) = \boldsymbol{\Lambda W D}(t)\boldsymbol{Cs}(t) + \boldsymbol{n}(t),\ t = 1,2,\ldots,T, \quad (2)$$

where $\boldsymbol{y}(t)$, $\boldsymbol{C}$, $\boldsymbol{s}(t)$ and $\boldsymbol{n}(t)$ are defined in the MC-CDMA model system.

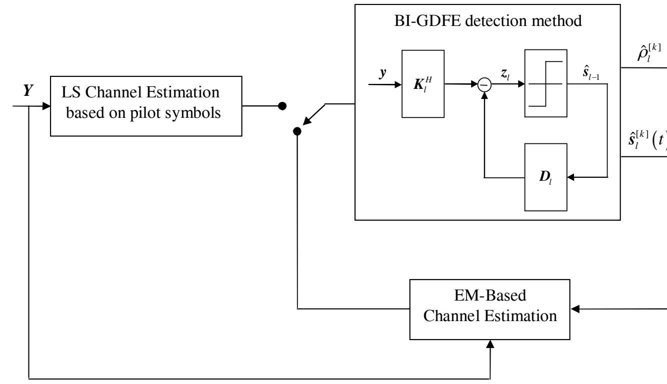

Fig. 1. The block diagram of the EM-based channel estimation for the BI-GDFE receiver.

B. SCCP System [7]

The input-output model for SCCP is given by

$$\boldsymbol{y}(t) = \boldsymbol{\Lambda W s}(t) + \boldsymbol{n}(t), \quad t = 1,2,\ldots,T. \quad (3)$$

C. General Model

From the previous subsections, we observe that MC-CDMA, CP-CDMA and SCCP models share some similarities. In this section, we generalize the above three models for the purpose of signal detection and channel estimation.

- *Signal detection*: Equations (1), (2) and (3) can be presented by a unique MIMO model as:

$$\boldsymbol{y}(t) = \boldsymbol{H}(t)\boldsymbol{s}(t) + \boldsymbol{n}(t), \quad t = 1,2,\ldots,T, \quad (4)$$

  where $\boldsymbol{H}(t)$ is $\boldsymbol{\Lambda\Pi D}(t)\boldsymbol{C}$, $\boldsymbol{\Lambda W D}(t)\boldsymbol{C}$ and $\boldsymbol{\Lambda W}$ for MC-CDMA, CP-CDMA and SCCP, respectively.
- *Channel estimation*: Equations (1), (2) and (3) can be written as:

$$\boldsymbol{y}(t) = \boldsymbol{\mathcal{D}}(t)\boldsymbol{F}_L\boldsymbol{h} + \boldsymbol{n}(t), \quad t = 1,2,\ldots,T, \quad (5)$$

  where $\boldsymbol{\mathcal{D}}(t)$ is $\mathrm{diag}\{\boldsymbol{\Pi D}(t)\boldsymbol{Cs}(t)\}$, $\mathrm{diag}\{\boldsymbol{WD}(t)\boldsymbol{Cs}(t)\}$ and $\mathrm{diag}\{\boldsymbol{Ws}(t)\}$ for MC-CDMA, CP-CDMA and SCCP, respectively.

The general model of (4) will be used in the BI-GDFE algorithm and the general model of (5) will assist us in the EM-based channel estimation which will be proposed in Section III.

III. Proposed Doubly Iterative Receiver

In this section, the proposed receiver is presented. The complexity of the receiver is also considered.

A. Proposed Receiver

The EM algorithm can provide the ML estimation in an iterative manner as long as the initial value of the interested parameter is good enough. In our proposed doubly iterative receiver, the initial estimate of the channel coefficients can be obtained by using pilot symbols. After that, the EM algorithm uses not only the initial estimates but also the information provided by the received signals during data transmission to get a better channel coefficient estimation.

The combination of the BI-GDFE detection method and the EM-based channel estimation is illustrated in Fig. 1. In this doubly iterative structure we have two loops, the first one is the BI-GDFE detection method which is referred as the inner loop and the other is the loop of the EM-based algorithm which is called as the outer loop. After the l^{th} iteration of the BI-GDFE which uses the CSI from the $(k-1)^{th}$ iteration of the EM-based algorithm, we obtain the IDC and the hard decisions of the transmitted signal blocks for the whole frame. These information would be used by the EM-based algorithm to update the CSI. Below is the details of the k^{th} iteration of the EM-based algorithm.

We start the EM-based channel estimation algorithm by considering the following model from (5)

$$\boldsymbol{y}(t) = \boldsymbol{\mathcal{D}}(t)\boldsymbol{F}_L\boldsymbol{h} + \boldsymbol{n}(t),\ t = 1, 2, \ldots, T. \tag{6}$$

According to the EM terminology, the set of $\{\boldsymbol{y}(t)\}_{t=1}^{T}$ is the *incomplete data space*. The parameter to be estimated is $\boldsymbol{h}$. We define a *complete data space*, $\boldsymbol{X} = \big(\{\boldsymbol{y}(t)\}_{t=1}^{T}, \{\boldsymbol{s}(t)\}_{t=1}^{T}\big)$, for the parameter we want to estimate. The probability density function of $\boldsymbol{X}$ as a function of $\boldsymbol{h}$ is

$$\begin{aligned} f\big(\boldsymbol{X}\big|\boldsymbol{h}\big) &= f\big(\{\boldsymbol{y}(t)\}_{t=1}^{T}, \{\boldsymbol{s}(t)\}_{t=1}^{T}\big|\boldsymbol{h}\big) \\ &= f\big(\{\boldsymbol{y}(t)\}_{t=1}^{T}\big|\{\boldsymbol{s}(t)\}_{t=1}^{T}, \boldsymbol{h}\big) f\big(\{\boldsymbol{s}(t)\}_{t=1}^{T}\big|\boldsymbol{h}\big). \end{aligned} \tag{7}$$

Since $\boldsymbol{y}(t), t = 1, 2, \ldots, T$, are independent, we can write $f\big(\{\boldsymbol{y}(t)\}_{t=1}^{T}\big|\{\boldsymbol{s}(t)\}_{t=1}^{T}, \boldsymbol{h}\big)$ as

$$\begin{aligned} &f\big(\{\boldsymbol{y}(t)\}_{t=1}^{T}\big|\{\boldsymbol{s}(t)\}_{t=1}^{T}, \boldsymbol{h}\big) \\ &= \frac{1}{(\pi\sigma_n^2)^{NT}} \exp\Big\{-\frac{1}{\sigma_n^2}\sum_{t=1}^{T}\big\|\boldsymbol{y}(t) - \boldsymbol{\mathcal{D}}(t)\boldsymbol{F}_L\boldsymbol{h}\big\|^2\Big\}. \end{aligned} \tag{8}$$

One iteration of the EM-based algorithm consists of two steps: the first one is the E-step and the other is the M-step. Suppose we have the estimate $\hat{\boldsymbol{h}}^{[k]}$ after the $(k-1)^{th}$ iteration of EM. Then, the k^{th} iteration of the EM algorithm comprises the following two steps.

1) E-step: In this step, we calculate

$$Q\big(\boldsymbol{h}\big|\hat{\boldsymbol{h}}^{[k]}\big) = E\big\{\log f\big(\boldsymbol{X}\big|\boldsymbol{h}\big)\big|\{\boldsymbol{y}(t)\}_{t=1}^{T}, \hat{\boldsymbol{h}}^{[k]}\big\}. \tag{9}$$

Because of the independence between the signal vectors $\{\boldsymbol{s}(t)\}_{t=1}^{T}$ and the channel vector $\boldsymbol{h}$, the probability density function $f\big(\{\boldsymbol{s}(t)\}_{t=1}^{T}\big|\boldsymbol{h}\big)$ is not a function of $\boldsymbol{h}$. Hence, it is bypassed when we consider (9). Replacing (8) into (9), after dropping some constants, (9) can be written as:

$$\begin{aligned} Q\big(\boldsymbol{h}\big|\hat{\boldsymbol{h}}^{[k]}\big) &= C_1 + \boldsymbol{h}^{\mathcal{H}}\boldsymbol{F}_L^{\mathcal{H}}\Big(\sum_{t=1}^{T} E\big\{\boldsymbol{\mathcal{D}}^{\mathcal{H}}(t)\big|\boldsymbol{y}(t), \hat{\boldsymbol{h}}^{[k]}\big\}\boldsymbol{y}(t)\Big) \\ &+ \Big(\sum_{t=1}^{T}\boldsymbol{y}^{\mathcal{H}}(t) E\big\{\boldsymbol{\mathcal{D}}(t)\big|\boldsymbol{y}(t), \hat{\boldsymbol{h}}^{[k]}\big\}\Big)\boldsymbol{F}_L\boldsymbol{h} \\ &- \boldsymbol{h}^{\mathcal{H}}\boldsymbol{F}_L^{\mathcal{H}}\Big(\sum_{t=1}^{T} E\big\{\boldsymbol{\mathcal{D}}^{\mathcal{H}}(t)\boldsymbol{\mathcal{D}}(t)\big|\boldsymbol{y}(t), \hat{\boldsymbol{h}}^{[k]}\big\}\Big)\boldsymbol{F}_L\boldsymbol{h}, \end{aligned} \tag{10}$$

where C_1 is a constant independent of $\boldsymbol{h}$.

2) M-step: M-step is to find the $\boldsymbol{h}$ that maximizes (10) and this value of $\boldsymbol{h}$ is denoted by $\hat{\boldsymbol{h}}^{[k+1]}$. It is obtained by

$$\hat{\boldsymbol{h}}^{[k+1]} = \arg\max_{\boldsymbol{h}}\ Q\big(\boldsymbol{h}\big|\hat{\boldsymbol{h}}^{[k]}\big). \tag{11}$$

Differentiating (10) with respect to $\boldsymbol{h}$ [8], we obtain

$$\begin{aligned} \frac{\partial Q(\boldsymbol{h}|\hat{\boldsymbol{h}}^{[k]})}{\partial \boldsymbol{h}} &= -\Big(\boldsymbol{F}_L^{\mathcal{H}}\Big(\sum_{t=1}^{T} E\big\{\boldsymbol{\mathcal{D}}^{\mathcal{H}}(t)|\boldsymbol{y}(t), \hat{\boldsymbol{h}}^{[k]}\big\}\boldsymbol{y}(t)\Big)\Big)^* \\ &+ \Big(\boldsymbol{F}_L^{\mathcal{H}}\Big(\sum_{t=1}^{T} E\big\{\boldsymbol{\mathcal{D}}^{\mathcal{H}}(t)\boldsymbol{\mathcal{D}}(t)|\boldsymbol{y}(t), \hat{\boldsymbol{h}}^{[k]}\big\}\Big)\boldsymbol{F}_L\boldsymbol{h}\Big)^*. \end{aligned} \tag{12}$$

Equating (12) to $\mathbf{0}$, we have $\hat{\boldsymbol{h}}^{[k+1]}$ as follows:

$$\begin{aligned} \hat{\boldsymbol{h}}^{[k+1]} &= \Big(\boldsymbol{F}_L^{\mathcal{H}}\Big(\sum_{t=1}^{T} E\big\{\boldsymbol{\mathcal{D}}^{\mathcal{H}}(t)\boldsymbol{\mathcal{D}}(t)|\boldsymbol{y}(t), \hat{\boldsymbol{h}}^{[k]}\big\}\Big)\boldsymbol{F}_L\Big)^{-1} \\ &\times\Big(\boldsymbol{F}_L^{\mathcal{H}}\Big(\sum_{t=1}^{T} E\big\{\boldsymbol{\mathcal{D}}^{\mathcal{H}}(t)|\boldsymbol{y}(t), \hat{\boldsymbol{h}}^{[k]}\big\}\boldsymbol{y}(t)\Big)\Big). \end{aligned} \tag{13}$$

The general equation of (13) is calculated for three systems as follows:

$$\Big(E\big\{\boldsymbol{\mathcal{D}}^{\mathcal{H}}(t)|\boldsymbol{y}(t), \hat{\boldsymbol{h}}^{[k]}\big\}\Big)_{i,i} = \begin{cases} \Big(\boldsymbol{\Pi}\boldsymbol{D}(t)\boldsymbol{C}\hat{\rho}_l^{[k]}\hat{\boldsymbol{s}}_l^{[k]}(t)\Big)_i & \text{for MC-CDMA} \\ \Big(\boldsymbol{W}\boldsymbol{D}(t)\boldsymbol{C}\hat{\rho}_l^{[k]}\hat{\boldsymbol{s}}_l^{[k]}(t)\Big)_i & \text{for CP-CDMA} \\ \Big(\boldsymbol{W}\hat{\rho}_l^{[k]}\hat{\boldsymbol{s}}_l^{[k]}(t)\Big)_i & \text{for SCCP} \end{cases}, \tag{14}$$

and

$$\Big(E\big\{\boldsymbol{\mathcal{D}}^{\mathcal{H}}(t)\boldsymbol{\mathcal{D}}(t)|\boldsymbol{y}(t), \hat{\boldsymbol{h}}^{[k]}\big\}\Big)_{i,i} = \Big|\Big(E\big\{\boldsymbol{\mathcal{D}}^{\mathcal{H}}(t)|\boldsymbol{y}(t), \hat{\boldsymbol{h}}^{[k]}\big\}\Big)_{i,i}\Big|^2 \tag{15}$$

where $\hat{\rho}_l^{[k]}$ and $\hat{\boldsymbol{s}}_l^{[k]}(t),\ t = 1, 2, \ldots, T$, are the IDC and the hard decisions after the l^{th} iterations of the BI-GDFE method using the channel estimate $\hat{\boldsymbol{h}}^{[k]}$, respectively.

B. Complexity

The proposed receiver has two loops. The inner loop (the BI-GDFE detection method) complexity has been investigate in [4]. If the number of iteration of the BI-GEFE method is I_{bi} then the number of complex multiplications (CMs) for a frame is $I_{bi}(\mu_1+1)N^3+2I_{bi}TN^2$ where μ_1 is a scaling factor depending on the matrix inversion algorithm.

The outer loop is used to calculate (13). In order to determine that, we first have to calculate (14) which requires TN^2, $T(N^2 + \frac{N}{2}\log_2 N)$ and $T\frac{N}{2}\log_2 N$ CMs for MC-CDMA, CP-CDMA and SCCP, respectively. Here multiplications by matrix $\boldsymbol{W}$ can be efficiently performed by means of DFT units and need $\frac{N}{2}\log_2 N$ CMs. The computation of the (13) then requires another $\mu_1 N^3+2N^2L+NL+L^2$ CMs. Table I (on the top of next page) shows the complexity comparison between the inner loop and the outer loop with $T = 10$, $L = 17$, $N = 64$ and assuming $\mu_1 = 1$. From the Table I we observe that the complexity of the outer loop is much smaller than that of the inner loop. The required calculation can be easily performed by the available hardware technology such as the Ti DSP TMS320C6416 or the Xilinx FPGA XCV2V6000.

TABLE I
COMPLEXITY COMPARISON FOR INNER ANR OUTER LOOP

Loops	Complex Multiplications	$I_{bi}=2$	$I_{bi}=5$
Inner Loop	$I_{bi}(\mu_1+1)N^3+2I_{bi}TN^2$	1.2×10^6	3×10^6
Outer loop	$(\mu_1 N^3+2N^2L+NL+L^2)+TN^2$ for MC-CDMA	4.4×10^5	4.4×10^5
	$(\mu_1 N^3+2N^2L+NL+L^2)+T(N^2+\frac{N}{2}\log_2 N)$ for CP-CDMA	4.5×10^5	4.5×10^5
	$(\mu_1 N^3+2N^2L+NL+L^2)+T\frac{N}{2}\log_2 N$ for SCCP	4×10^5	4×10^5

IV. CRAMÉR-RAO LOWER BOUND (CRLB)

The CRLB for the channel vector $\boldsymbol{h}$ in the general model of (6) is given by

$$\mathrm{CRLB}(\boldsymbol{h}) = \mathrm{tr}\{\boldsymbol{I}^{-1}(\boldsymbol{h})\}, \tag{16}$$

where $\boldsymbol{I}(\boldsymbol{h})$ is the Fisher information matrix [8]

$$\boldsymbol{I}(\boldsymbol{h}) = E\Big\{\frac{\partial}{\partial \boldsymbol{h}}\log f\big(\{\boldsymbol{y}(t)\}_{t=1}^{T}\big|\boldsymbol{h}\big) \Big(\frac{\partial}{\partial \boldsymbol{h}}\log f\big(\{\boldsymbol{y}(t)\}_{t=1}^{T}\big|\boldsymbol{h}\big)\Big)^{\mathcal{H}}\Big\}. \tag{17}$$

From (6), we have the conditional probability density function of $\{\boldsymbol{y}(t)\}_{t=1}^{T}$ given $\boldsymbol{h}$:

$$f\big(\{\boldsymbol{y}(t)\}_{t=1}^{T}\big|\boldsymbol{h}\big) = \frac{1}{(\pi\sigma_n^2)^{NT}}\exp\Big\{-\frac{1}{\sigma_n^2}\sum_{t=1}^{T}\big\|\boldsymbol{y}(t)-\boldsymbol{\mathcal{D}}(t)\boldsymbol{F}_L\boldsymbol{h}\big\|^2\Big\}, \tag{18}$$

where we assume that the signal is known (in the form of the matrix $\boldsymbol{\mathcal{D}}(t),\ t=1,2,\dots,T$). Therefore, the probability density function is not conditioned on $\boldsymbol{\mathcal{D}}(t),\ t=1,2,\dots,T$. After differentiating the logarithm of (18) with respect to $\boldsymbol{h}$ and substituting the result to (17), we obtain

$$\boldsymbol{I}(\boldsymbol{h}) = \frac{1}{\sigma_n^2}\boldsymbol{F}_L^{\mathcal{T}}\Big(\sum_{t=1}^{T}\boldsymbol{\mathcal{D}}^{\mathcal{T}}(t)\boldsymbol{\mathcal{D}}^{*}(t)\Big)\boldsymbol{F}_L^{*}. \tag{19}$$

It is clear from (19) that the CRLB changes from a frame of T blocks to another because of different signal blocks transmitted. Hence, we have the average CRLB [8], denoted by $\mathrm{aCRLB}(\boldsymbol{h})$, as follows

$$\mathrm{aCRLB}(\boldsymbol{h}) = E\left\{\mathrm{CRLB}(\boldsymbol{h})\right\}, \tag{20}$$

where the expectation is performed with respect to the transmitted signal blocks in the frame of length T. Finding $\mathrm{CRLB}(\boldsymbol{h})$ or $\mathrm{aCRLB}(\boldsymbol{h})$ is not an easy task because it is not straightforward to obtain an explicit expression for the inverse of $\boldsymbol{I}(\boldsymbol{h})$.

Another CRLB is called the modified CRLB [6], which is denoted by mCRLB. It is defined as:

$$\mathrm{mCRLB}(\boldsymbol{h}) = \sum_{i=0}^{L-1}\frac{1}{E\Big\{\big(\boldsymbol{I}(\boldsymbol{h})\big)_{i,i}\Big\}} = \frac{L\sigma_n^2}{T\sum_{n=0}^{N-1}E\Big\{\big|\big(\boldsymbol{\mathcal{D}}(t)\big)_{n,n}\big|^2\Big\}}. \tag{21}$$

Equation (21) is applied to three block-based transmission schemes and (after some simple derivation) produces the same results for the three systems as follows

$$\mathrm{mCRLB}(\boldsymbol{h}) = \frac{L\sigma_n^2}{TN\sigma_s^2}. \tag{22}$$

Note that the mCRLBs do not depend on the probabilistic model of channel vector $\boldsymbol{h}$; it depends on the length of $\boldsymbol{h}$. Hence, it can be applied to any multipath fading channel. Reference [6] shows that the $\mathrm{aCRLB}(\boldsymbol{h})$ is always tighter than the $\mathrm{mCRLB}(\boldsymbol{h})$. However, as shown in Section V, the $\mathrm{aCRLB}(\boldsymbol{h})$ and $\mathrm{mCRLB}(\boldsymbol{h})$ curves agree to each other and this justifies the use of $\mathrm{mCRLB}(\boldsymbol{h})$ as a performance measure for unbiased channel estimation algorithms in the interested systems.

V. SIMULATION RESULTS

Computer simulations are carried out to evaluate the performance of the doubly iterative receiver for the three systems: MC-CDMA , CP-CDMA and SCCP. For all systems, QPSK modulation is used. The channel is assumed to be frequency-selective fading with L chip-delayed (for the two CP-based CDMA systems)/symbol-delay (for the SCCP system) taps with uniform power delay profile. The ML bound of the three systems cannot be presented because of the large signal size. Instead, the single user matched filter bound (SU-MFB) [9] is used as the lowest bound for evaluating the performance of the proposed receiver. First of all, we compare the aCRLB and mCRLB derived in Section IV.

A. Comparison of aCRLB and mCRLB

Fig. 2 illustrates the aCRLB and mCRLB for the three systems. The channel length is $L = 17$. We observe from the figure that the aCRLB's for the three systems are the same and the mCRLB agrees with them well. This figure justifies our usage of mCRLB in the evaluation of the MSE performance of $\boldsymbol{h}$.

B. MC-CDMA

A fully loaded MC-CDMA system with long spreading sequences is simulated with $N = G = 64$. In this section, we consider the frequency-selective fading channel with channel length $L = 17$. We assume that the channel is unchanged over a frame of $T = 10$ signal blocks. The first signal block is devoted to pilot symbols and the remaining blocks are left for data symbols. Fig. 3 provides the BER performance when the number of inner loops (i.e., the number of iterations of the

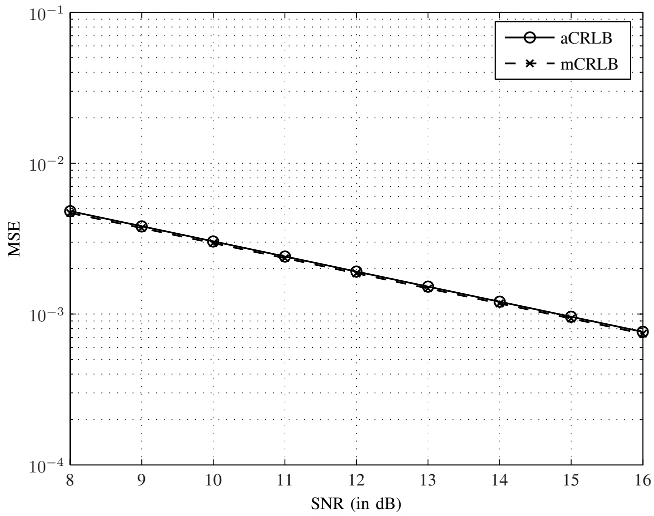

Fig. 2. Comparison of average CRLB with modified CRLB.

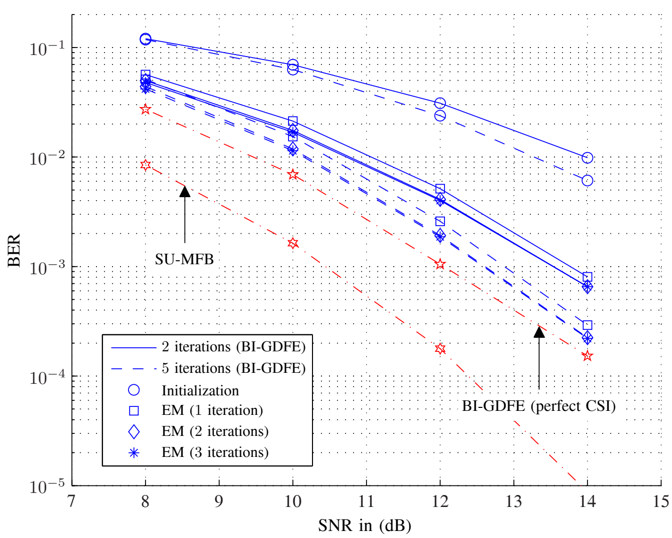

Fig. 3. BER v.s. SNR for different iterations of BI-GDFE and EM for MC-CDMA.

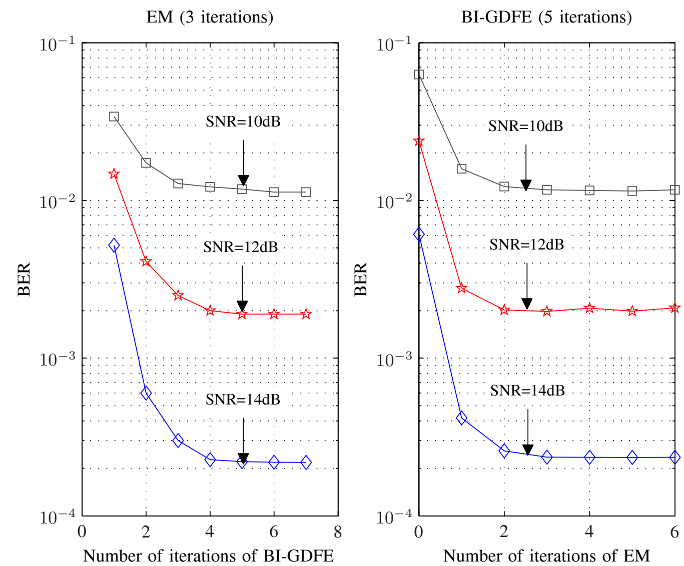

Fig. 4. BER v.s. number of iterations of BI-GDFE and EM for MC-CDMA.

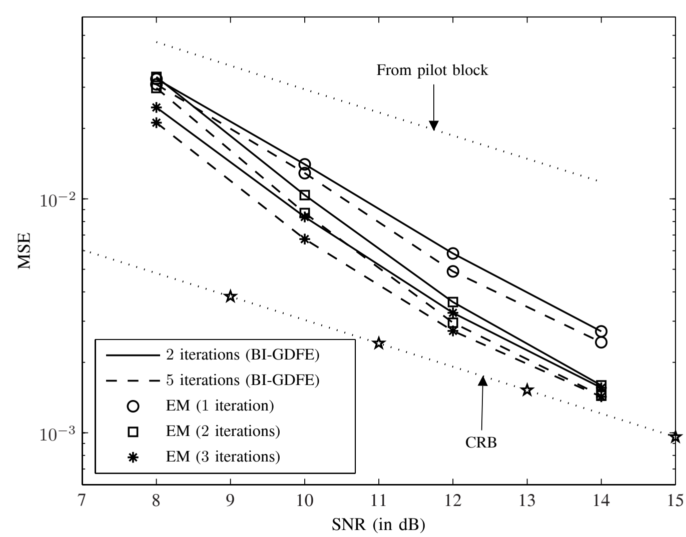

Fig. 5. MSE performance of MC-CDMA.

BI-GDFE method) is 2 and 5; and the number of outer loops (i.e., the number of iterations of the EM-based algorithm) is 3. We can see that after the first outer loop, the BER decreases greatly; after the second outer loop, the BER improvement is marginal. The gap between the proposed receiver (using 5 inner loops and 3 outer loops) and the BI-GDFE using perfect CSI is around 0.5dB. We can also observe that the BER improvement is more significant when the number of inner loops is larger.

In order to see the relationship between the BER performance and the number of the inner/outer loops deeply, we examine Fig. 4 in which the BER as a function of the inner/outer loops is presented for the above system set-up. In the sub-figure on the left hand side, the BER as a function of number of inner loops is provided given 3 outer loops. We can see that the BER does not change if we use 5 or 6 inner loops. Comparing different SNR values, BER drops faster for the larger SNR. Moving to the right hand side where BER as a function of the outer loop is given, we can see that after 3 iterations of the outer loop, BER is stable. In this sub-figure, the results corresponding to the 0^{th} outer loop is the performance of the BI-GDFE using the CSI obtained from the pilot block. These are coincident with the initialization curve in Fig. 3. A great performance gain can be observed with the EM-based channel estimation. Here, we once again see that the improvement is more significant for larger SNR.

To examine the MSE performance of the channel estimation, Fig. 5 provides the MSE performance of $\boldsymbol{h}$ for different scenarios. We can see that the MSE given only by the pilot block is far away from the CRLB. The MSE decreases dramatically with the use of the proposed channel estimation. We could not see a big gap in the MSE performance when we use different numbers of inner loops as compared with BER performance. The proposed channel estimation algorithm provides a near CRLB performance and the gap becomes smaller with an increase of SNR.

C. CP-CDMA and SCCP

The proposed receiver can apply to CP-CDMA and SCCP systems and obtain the same simulation results as in MC-CDMA system.

VI. CONCLUSIONS

Based on the EM-based algorithm for channel estimation and the BI-GDFE receiver for signal detection, a doubly iterative receiver is proposed for three block-based transmission schemes: MC-CDMA, CP-CDMA and SCCP. The inner loop is the BI-GDFE detection algorithm, which is a near-ML detection with low complexity. The BI-GDFE provides not only the hard decision of transmitted signals but also the IDC coefficient, which constitute the soft information of transmitted signals. This soft information is used for the EM-based channel estimation algorithm even when uncoded systems are considered. Simulations have shown that our proposed receiver has a small gap to the BI-GDFE with perfect CSI. To facilitate the evaluation of the channel estimation, the CRLB is also derived in this paper. Due to the difficulty in deriving the exact CRLB, the modified CRLB is obtained which gives the closed-form solution. The modified CRLB agrees with the exact CRLB and it is used to evaluate the performance of the proposed channel estimation. Simulations have shown that our obtained MSE performance is close to the theoretical CRLB.

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