Joint Location and Beamforming Optimization for STAR-RIS Aided NOMA-UAV Networks

Yuhua Su, Xiaowei Pang, Weidang Lu, Senior Member, IEEE, Nan Zhao, Senior Member, IEEE, Xianbin Wang, Fellow, IEEE and Arumugam Nallanathan, Fellow, IEEE

Abstract—This work investigates the potential of combining unmanned aerial vehicle (UAV) and simultaneously transmitting and reflecting reconfigurable intelligent surface (STAR-RIS) in wireless networks. In particular, the signals from UAV can be reflected and transmitted through STAR-RIS to the users on both sides of the surface to provide full-space coverage. We formulate a sum-rate maximization problem in STAR-RIS aided non-orthogonal multiple access UAV networks, where the location and power allocation of UAV, and the passive reflection/transmission beamforming of STAR-RIS are jointly optimized, subject to the quality-of-service requirement. Furthermore, we transform the non-convex problem into a convex one and propose an efficient algorithm to obtain the suboptimal solution to the original problem. Simulation results verify the superiority of the proposed scheme compared to other benchmarks.

Index Terms—Non-orthogonal multiple access, reconfigurable intelligent surface, simultaneous transmission and reflection, unmanned aerial vehicle.

I. INTRODUCTION

Reconfigurable intelligent surface (RIS) has been envisioned as a promising technology for future wireless networks [1]. Specifically, each element of RIS, which is made of electromagnetic material and metal patches, can reflect the incident signal by altering its phase shift intelligently [2]. Thus, the wireless propagation environment can be reshaped through collaboratively adjusting the phase shifts of massive reflecting elements. With the advantages of easy deployment, low energy consumption and low hardware cost, RIS has been widely studied in industry and academia to improve the achievable rate, security and reliability [3].

On the other hand, unmanned aerial vehicles (UAVs) have been extensively utilized in wireless networks owing to their low cost, high mobility and line-of-sight (LoS) transmission [4]. In [5], Wang et al. studied a UAV-enabled relay system to maximize the sum harvested energy of users. To efficiently utilize the mobility of UAV, Yuan et al. introduced artificial potential filed to propose a novel optimal trajectory design method in [6]. However, the UAV-ground channel may be blocked in the urban environment, which is a challenging issue. By introducing RIS into UAV communication, additional advantages can be gained to reconfigure the air-ground channels, thereby significantly improving the communication quality [7], [8]. Pang et al. in [9] integrated UAV and RIS to facilitate the security in wireless networks. In [10], Mei et al. maximized the energy efficiency by jointly optimizing the three-dimensional (3D) space of UAV and phase shifts of RIS. However, it is noteworthy that the RIS deployed on the facades of buildings can only reflect signals to the users located on the same side, which limits the coverage [11].

To deal with this challenging issue, the RIS that can simultaneously enable transmission and reflection (STAR-RIS) has emerged [12]. Some promising technologies and successful prototypes have been conducted for the practical implementation of STAR-RIS [13]. Compared to the conventional RIS, the users located in the back side of STAR-RIS can be also served to realize full-space coverage. Meanwhile, by adjusting the phase shifts and power ratio of the reflected and transmitted signals, STAR-RIS can construct favorable propagation environments for the users of both sides. Accordingly, the potential of combining UAV and STAR-RIS to improve the performance is worth investigating. In [14], Liu et al. introduced STAR-RIS into UAV networks to maximize the rate by jointly optimizing the trajectory and phase shifts. The UAV trajectory was jointly optimized by Zhang et al. with power allocation and STAR-RIS’s passive beamforming in [15] to maximize the sum rate.

In STAR-RIS aided UAV networks, the trajectory optimization may become ineffective, due to the delayed and unreliable channel state information (CSI) caused by fading. Motivated by this, in this correspondence, the UAV’s location is jointly optimized with the transmit power and transmission/reflection beamforming based on the hovering characteristic of rotary-wing UAVs, without acquiring offline/online CSI for trajectory planning. We aim at enhancing the sum rate of non-orthogonal multiple access (NOMA) UAV network subject to quality of service (QoS) requirements. An iterative algorithm is developed to solve the joint optimization problem effectively.

II. SYSTEM MODEL

Consider a downlink NOMA-UAV network, where a single-antenna UAV connects with two single-antenna users with
the assistance of STAR-RIS. The users are denoted by $U_i$, $i \in \mathcal{I} = \{A, B\}$. As shown in Fig. 1, the incident signal is reconfigured by the STAR-RIS, and then transmitted to the indoor User $A$ and simultaneously reflected to the outdoor User $B$. Assume that the RIS is composed of a uniform planar array (UPA) with $M$ elements. For simplicity, we adopt the same transmission and reflection coefficients for all the STAR-RIS elements. Define $\Phi_A = \sqrt{\beta_A} \text{diag}(e^{j\theta_A^1}, \cdots, e^{j\theta_A^M}) = \sqrt{\beta_A} \Theta_A$ and $\Phi_B = \sqrt{\beta_B} \text{diag}(e^{j\theta_B^1}, \cdots, e^{j\theta_B^h}) = \sqrt{\beta_B} \Theta_B$ as the transmission and reflection beamforming matrices, where $\beta_i$ and $\theta_i^m$ are the transmission/reflection coefficients and phase-shift adjustment of the $m$th element, respectively. Owing to the law of energy conservation, $\beta_A + \beta_B = 1$.

3D Cartesian coordinate system is adopted, where the coordinate of $U_i$ is denoted as $\mathbf{q}_i = [x_i, y_i, z_i]$. The STAR-RIS and UAV are respectively located at $\mathbf{q}_A = [x_A, y_A, z_A]$ and $\mathbf{Q} = [x_U, y_U, H]$, where the UAV is assumed to hover at a fixed height $H$. Denote $\mathbf{h}_{UI} \in \mathbb{C}^{M \times 1}$ and $\mathbf{h}_{hi} \in \mathbb{C}^{1 \times M}$ as the channels from UAV to RIS and from RIS to users, which can be modeled as LoS and expressed as

$$\mathbf{h}_{UI} = \sqrt{\rho_d^{UI}} \mathbf{h}_{UI},$$  
(1)

$$\mathbf{h}_{hi} = \sqrt{\rho_d^{hi}} \mathbf{h}_{hi}, i = A, B,$$  
(2)

where $\rho$ is the channel gain at the reference distance of 1 m, and $d_{UI} = \sqrt{||\mathbf{q}_U - \mathbf{Q}||^2}$ and $d_{hi} = \sqrt{||\mathbf{q}_i - \mathbf{Q}||^2}$ are the distance between UAV and RIS and that between RIS and $U_i$, respectively. $\mathbf{h}_{UI}$ and $\mathbf{h}_{hi}$ are the array responses of STAR-RIS when the signal arrive at and depart from RIS, respectively. Denote $\mathbf{h}_{UB} \in \mathbb{C}^{1 \times 1}$ as the direct link between UAV and $U_B$, which is blocked and modeled by Rayleigh fading as

$$\mathbf{h}_{UB} = \sqrt{\rho_d^{UB}} h_i,$$  
(3)

where $d_{UB} = \sqrt{||\mathbf{Q} - \mathbf{q}_B||^2}$ is the distance between the UAV and $U_B$, $\alpha \geq 2$ is the path-loss exponent and $h \sim \mathcal{CN}(0, 1)$. As the UAV-RIS-user channel is dominated by the LoS component and the UAV is hovering at the fixed location, we assume that the perfect CSI can be obtained with existing channel estimation techniques [17].

The superimposed information is transmitted from the UAV to users via NOMA. Accordingly, the received signals at $U_i$ can be expressed as

$$y_A = (\mathbf{h}_{IA} \Phi_A \mathbf{h}_{UI})(\sqrt{p_A s_A} + \sqrt{p_B s_B}) + n_A,$$  
(4)

$$y_B = (\mathbf{h}_{IB} \Phi_B \mathbf{h}_{UB})(\sqrt{p_A s_A} + \sqrt{p_B s_B}) + n_B,$$  
(5)

where $p_i$ is the transmit power allocated to $U_i$, $s_i$ denotes the transmitted signal for $U_i$, and $n_i \sim \mathcal{CN}(0, \sigma^2)$ denotes the additive white Gaussian noise at $U_i$.

Assume that the successive interference cancellation (SIC) order is $U_B \rightarrow U_A$. According to NOMA, we first decode the signal of $U_B$ by treating the signal of $U_A$ as noise, and then remove it from the superposed signal. The achievable rate of $U_i$ can be given by

$$R_A = \log_2 (1 + \text{SINR}_A),$$  
(6)

$$R_B = \min \{\log_2 (1 + \text{SINR}^1_B), \log_2 (1 + \text{SINR}^2_B)\}.$$  
(7)

Define the channel gains of $U_A$ and $U_B$ as $\mathbf{H}_A = ||\mathbf{h}_{IA} \Phi_A \mathbf{h}_{UI}||^2$ and $\mathbf{H}_B = ||\mathbf{h}_{IB} \Phi_B \mathbf{h}_{UI} + h_{UB}||^2$, respectively. The signal-to-interference-plus-noise ratio (SINR) of signal $s_A$ at $U_A$ can be expressed as

$$\text{SINR}_A = \frac{p_A \mathbf{H}_A}{\sigma^2},$$  
(8)

and the SINR to decode $s_B$ at $U_B$ and $U_A$ can be respectively expressed as

$$\text{SINR}^1_B = \frac{p_B \mathbf{H}_B}{p_A \mathbf{H}_B + \sigma^2},$$  
(9a)

$$\text{SINR}^2_B = \frac{p_B \mathbf{H}_A}{p_A \mathbf{H}_A + \sigma^2}.$$  
(9b)

III. Problem Formulation

We aim at maximizing the sum rate via jointly optimizing the power allocation, the transmission and reflection beamforming, and the hovering location, subject to QoS requirements. Since $U_B$ owns a much longer distance with $U_A$, leading to a poor quality of the UAV-U$B$ channel, we assume that the channel quality of $U_A$ is better than that of $U_B$, i.e., $\mathbf{H}_A \geq \mathbf{H}_B$. Accordingly, the achievable rate of $U_B$ can be expressed as $R_B = \log_2 (1 + \text{SINR}^2_B)$. Thus, the problem can be formulated as

$$\max_{\Phi_i, p_i, \mathbf{Q}} R_A + R_B$$  
(10a)

s.t.  
$$0 \leq \theta_i^m < 2\pi, \forall i, m,$$  
(10b)

$$\beta_A + \beta_B = 1,$$  
(10c)

$$p_A + p_B = p,$$  
(10d)

$$p_A \geq p_B,$$  
(10e)

$$\text{SINR}_A \geq \gamma_A,$$  
(10f)

$$\text{SINR}^2_B \geq \gamma_B,$$  
(10g)

$$\mathbf{H}_A \geq \mathbf{H}_B,$$  
(10h)

$$\mathbf{Q} \in \mathbb{L},$$  
(10i)

where $p$ is the total transmit power of UAV, $\mathbf{Q}$ denotes the UAV location, and $\mathbb{L}$ is the area where the UAV can hover. It is challenging to solve it due to the non-convex constraints and coupled variables.

IV. Proposed Solutions

In this section, we decompose (10) into two sub-problems based on the block coordinate descent (BCD) method, and an iterative algorithm is proposed to solve them alternatively.

A. Joint Optimization of $\Phi_i$ and $p_i$

With the given UAV location $\mathbf{Q}$, the sub-problem can be formulated as
\[
\max_{\theta_A, \beta, p_i, t} \log_2 \left(1 + \frac{p_A H_A}{\sigma^2} \right) + \log_2 \left(1 + \frac{p_B H_B}{p_A H_B + \sigma^2} \right) \quad (11a)
\]
\[
s.t. \quad (10b) - (10h). \quad (11b)
\]

Note that the STAR-RIS allows for independent optimization of phase shifts of the transmitted and reflected signals. First, we have the following lemma for the closed-form solution to \( \Theta_A \).

**Lemma 1**: With other variables fixed, the optimal phase-shift of STAR-RIS for the transmitted signal can be given by

\[
\theta_A^* = \theta^* - \angle \{W_m\},
\]

where \( W_m \) is the \( m \)-th value of \( W = \text{diag}(h_{IA}) u_{UI} \).

**Proof**: To find the optimal \( \Theta_A \), (11) can be converted to maximize the channel gain of \( U_A \), i.e., \( H_A \). Define 
\[
U_A = [e^{j\theta_1}, \ldots, e^{j\theta_M}].
\]
Then, the channel gain of \( U_A \) can be equivalent as

\[
H_A^* = \beta A |U_A^H \text{diag}(h_{IA}) u_{UI}|^2 \\
= \beta A |U_A^H W|^2 (a) = \beta A \left( \sum_{m=1}^{M} |W_m| \right)^2,
\]

where (a) holds when (13) reaches its maximum, and \( \theta_A^* = \theta^* - \angle \{W_m\} \) with \( \theta^* \) an arbitrary value.

Then, we employ the semi-definite relaxation (SDR) to optimize the passive reflection beamforming of STAR-RIS \( \Phi_B \), which is equal to maximizing \( H_B \). Therefore, let \( v = \sqrt{p_B} [v_1, \ldots, v_M] \) and \( \Phi_B = \text{diag}(h_{IB}) u_{UI} \), where \( v_m = e^{j\theta_m} \) and \( |v_m|^2 = 1 \). As the equation \( x^H \Phi y = \phi^H \text{diag}(x^H) y \), we have \( h_{IB} \Phi_B u_{UI} = v^H L_B v \). To obtain the optimal \( v \), we introduce an auxiliary variable \( u \) and have

\[
\| h_{IB} \Phi_B u_{UI} + u_B \|^2 = \|v^H L_B + u_B \|^2 = v^H X \tilde{v} + \|u_B\|^2,
\]

where

\[
X = \begin{bmatrix}
L_B L_B^H & L_B^H u_B \\
l_B u_B^H & 0
\end{bmatrix}, \tilde{v} = \begin{bmatrix} v \\ u \end{bmatrix}.
\]

Note that \( v^H \tilde{v} = (X \tilde{v})^H \), and we have

\[
H_B^* = \text{tr}(X \tilde{v} \tilde{v}^H) + \|u_B\|^2,
\]

where \( V = \tilde{v}^H \), and \( V \succeq 0 \) and \( \text{rank}(V) = 1 \) should be satisfied.

Accordingly, the simplified achievable rate of \( U_A \) and \( U_B \) can be given by

\[
R_A' = \log_2 \left(1 + p_A \beta A \left( \sum_{m=1}^{M} |W_m| \right)^2 / \sigma^2 \right). \quad (17)
\]

\[
R_B' = \log_2 \left(1 + \frac{p_B H_B^*}{p_A H_B^* + \sigma^2} \right). \quad (18)
\]

Based on the above derivation, (11) can be converted into

\[
\max_{v, \beta, p_i, t} R_A' + R_B' \quad (19a)
\]
\[
s.t. \quad (10c) - (10g), \quad (19b)
\]
\[
H_A^* \geq H_B^*, \quad (19c)
\]
\[
V \succeq 0, \quad (19d)
\]
\[
V_{n,n} = \beta_B, n = 1, \ldots, M, \quad (19e)
\]
\[
V_{M+1,M+1} = 1, \quad (19f)
\]
\[
\text{rank}(V) = 1, \quad (19g)
\]

where (19e) and (19f) enable the equality in (14).

However, it is still difficult to optimize the power allocation and transmission/reflection coefficients. To deal with (17), we introduce a slack variable \( t \) which satisfies \( t^2 \leq p_A \beta_A \), resulting in \( \| [2t, p_A - \beta_A] H_B \| \leq p_A + \beta_A \). The lower bound of \( R_A' \) can be approximated by the first-order Taylor series as

\[
R_A' \geq \log_2 \left(1 + 2p_A \beta_A \left( \sum_{m=1}^{M} |W_m| \right)^2 / \sigma^2 \right) \triangleq R_B^{lb}. \quad (20)
\]

For \( R_B' \), it can be expressed with respect to \( p_A \) and \( H_B^* \) as

\[
R_B' = \log_2 (p_B H_B^* + \sigma^2) - \log_2 (p_A H_B^* + \sigma^2) \triangleq R_B^{lb} - R_B^{lb}. \quad (21)
\]

Considering that both \( p_A \) and \( V \) are variables in (21), an auxiliary variable \( r \geq p_A H_B^* \) is introduced. With the given local point \( (p_A, H_B^*) \), its first-order Taylor expansion can be expressed as

\[
r \geq \frac{1}{2} \left[ (p_A + H_B^*)^2 - 2p_A H_B^* + (H_B^*)^2 \right]. \quad (22)
\]

Accordingly, the lower bound of \( R_B \) based on the first-order Taylor expansion can be given by

\[
R_B' \geq R_B^{lb} \left[ - \log_2 (r_0 + \sigma^2) + \log_2 \left( (r_0 + \sigma^2) (r - r_0) \right) \right] \triangleq R_B^{lb}. \quad (23)
\]

To handle the non-convex rank-one constraint (19g), we apply SDR to make it tractable. However, the obtained solution is an upper bound, which may not satisfy \( \text{rank}(V) = 1 \). Hence, an additional step of Gaussian randomization is required to construct a rank-one solution, the details of which are omitted [18]. As a result, the reconfigured problem can be given by

\[
\max_{v, \beta, p_i, t, r} R_A' + R_B^{lb} \quad (24a)
\]
\[
s.t. \quad (10c) - (10g), \quad (24b)
\]
\[
R_A' \geq R_A (\gamma_A), \quad (24c)
\]
\[
R_B^{lb} \geq R_B (\gamma_B), \quad (24d)
\]
\[
\| [2t, p_A - \beta_A] H_B \| \leq p_A + \beta_A, \quad (24e)
\]
\[
(19c) - (19f), (22), \quad (24f)
\]

where the constraints (24c) and (24d) are the QoS requirements of users, which are equivalent to (10f) and (10g), respectively. Then, the problem (24) is convex and can be solved by CVX.

**B. UAV Location Optimization**

With the optimized variables in the last sub-section, the UAV location can be obtained by solving

\[
\max_{\Phi} \log_2 \left(1 + \frac{p_A H_A}{\sigma^2} \right) + \log_2 \left(1 + \frac{p_B H_B}{p_A H_B + \sigma^2} \right) \quad (25a)
\]
\[
s.t. \quad R_A \geq R_A (\gamma_A), \quad (25b)
\]
\[
R_B \geq R_B (\gamma_B), \quad (25c)
\]
\[
H_A \geq H_B, \quad (25d)
\]
\[
Q \in L. \quad (25e)
\]

The objective function is neither convex nor concave. For brevity, we transform the channel gains of \( U_A \) and \( U_B \) into

\[
H_A = \| h_{IA} \Phi_A u_{UI} \|^2 = \frac{\| h_{IA} \Phi_A \tilde{u}_{UI} \|^2}{d_{UI}^2} = \frac{A}{d_{UI}^2}, \quad (26)
\]
where
\[
A = \beta A_0^2M_0^2d_{I_1}^2, \quad \text{(28a)}
\]
\[
B = \rho^2|\mathbf{h}_B| \mathbf{h}_B^H| \mathbf{u}_I|^2, \quad \text{(28b)}
\]
\[
C = \rho|\mathbf{h}_B^H|^2, \quad \text{(28c)}
\]
\[
D = \rho Re\{\mathbf{h}_B^H \mathbf{h}_B \mathbf{u}_I^H \mathbf{h}_B^H\}. \quad \text{(28d)}
\]

Note that the constraints (25b) and (25c) are difficult to handle due to their non-convexity, for which we apply successive convex approximation (SCA) on \(R_A\) and \(R_B\) as follows.

For \(R_A\), the first-order Taylor expansion at the local point \(Q^k\) in the \(k\)th iteration can be given by
\[
R_A = \log_2 \left( 1 + \frac{p_A A}{|q_I - Q_k|^2 / 2\sigma^2} \right) \geq \hat{A}^k - \hat{A}^k \left( |q_I - Q_k|^2 - |q_I - Q^k|^2 \right) = R_A^k,
\]
where
\[
\hat{A}^k = \log_2 \left( 1 + \frac{p_A A}{|q_I - Q^k|^2 / 2\sigma^2} \right), \quad \text{(30)}
\]
\[
\hat{A}^k = \frac{p_A A \log_2 e}{(p_A A + 2\sigma^2 |q_I - Q^k|^2) |q_I - Q^k|^2}. \quad \text{(31)}
\]

In order to tackle the non-convexity of \(R_B\), we introduce two auxiliary variables \(z\) and \(s\) satisfying
\[
z \leq \frac{p_B}{p_A + \sigma^2}, \quad \text{(32)}
\]
\[
s \leq \frac{B}{d_{U_I}^2} + \frac{C}{d_{U_B}^2} + \frac{2D}{d_{U_I}d_{U_B}^2} = \tilde{B} + \tilde{C} + \tilde{D}. \quad \text{(33)}
\]

Accordingly, the achievable rate of \(U_B\) can be expressed as
\[
R_B = \log_2 \left( 1 + \frac{p_B}{p_A + \sigma^2 |N_B|} \right) \geq \log_2 \left( 1 + \frac{p_B}{p_A + \sigma^2 / s} \right) \geq \log_2 (1 + z) = R_B^k. \quad \text{(34)}
\]

However, (32) and (33) are non-convex. By the Taylor expansion at the given point \(z_0\), (32) can be rewritten as
\[
\sigma^2 / s \leq p_B (1 / z_0 - (z - z_0) / z_0^2) - p_A, \quad \text{(35)}
\]

To cope with (33), the three terms on the right side of the inequality are approximated separately. Since both \(B\) and \(C\) are positive, \(B\) and \(C\) are convex with \(d_{U_I}\) and \(d_{U_B}\). Their lower bound can be expressed as the corresponding first-order Taylor series as
\[
B = Bd_{U_I}^2 \geq B\{3d_{U_I} - 2d_{U_I}d_{U_B}^3\} = B_1, \quad \text{(36)}
\]
\[
C = Cd_{U_B}^2 \geq C\{(1 + \alpha)d_{U_B} - \alpha d_{U_B}d_{U_B}^3\} = C_1. \quad \text{(37)}
\]

where \(d_{U_I} = \sqrt{|q_I - Q^k|^2}\) and \(d_{U_B} = \sqrt{|q_B - Q^k|^2}\) denote the given local points in the \(k\)th iteration.

Nevertheless, it is not guaranteed that \(D\) is positive or negative due to different channel qualities. When \(D > 0\), the lower bound of \(\tilde{D}\) can be expressed as
\[
\tilde{D} = 2D_0 d_{U_I}^{-\alpha / 2} d_{U_B}^{-\alpha / 2} \geq 2D \left( \frac{2 + \alpha / 2}{d_{U_B}^{-\alpha / 2} d_{U_B}^{-\alpha / 2}} \right) \geq D_1, \quad \text{(38)}
\]

When \(D < 0\), we introduce two auxiliary variables \(m \leq d_{U_I}\) and \(n \leq d_{U_B}\), which can be written as \(m^2 \leq |q_I - Q_k|^2\) and \(n^2 \leq |q_B - Q_k|^2\), respectively. Considering that \(|q_I - Q_k|^2\) is convex with the UAV location \(Q\), its lower bound approximated by the Taylor expansion can be given by
\[
|q_I - Q_k|^2 \geq |q_I - Q^k|^2 - 2(q_I - Q^k)^T(Q - Q^k). \quad \text{(39)}
\]

Accordingly, we have
\[
m^2 \leq |q_I - Q^k|^2 - 2(q_I - Q^k)^T(Q - Q^k), \quad \text{(40)}
\]
\[
n^2 \leq |q_B - Q^k|^2 - 2(q_B - Q^k)^T(Q - Q^k). \quad \text{(41)}
\]

The lower bound of \(\tilde{D}\) when \(D \leq 0\) can be expressed as
\[
\tilde{D} = 2D_0 d_{U_I}^{-\alpha / 2} d_{U_B}^{-\alpha / 2} \geq D_1 \left( \frac{2 + \alpha / 2}{d_{U_B}^{-\alpha / 2} d_{U_B}^{-\alpha / 2}} \right) \geq D_1 \left( \frac{2 + \alpha / 2}{d_{U_B}^{-\alpha / 2} d_{U_B}^{-\alpha / 2}} \right) \geq D_1 \left( \frac{2 + \alpha / 2}{d_{U_B}^{-\alpha / 2} d_{U_B}^{-\alpha / 2}} \right) \geq D_1. \quad \text{(42)}
\]

Define a binary variable \(flag\). When \(D > 0\), \(flag = 1\); otherwise, \(flag = 0\). The constraint (33) can be replaced by
\[
s \leq B_1 + C_1 + flagD_1 + (1 - flag)D_1. \quad \text{(43)}
\]

It is noted that we should approximate the constraint (25d) to make it convex, which can be expressed as
\[
Ad_{U_I}^2 \geq Bd_{U_I}^2 + Cd_{U_B}^2 + 2Dd_{U_I}d_{U_B}^2. \quad \text{(44)}
\]

Similar to the processing of \(\tilde{B}\), we have
\[
Ad_{U_I}^2 \geq A|3d_{U_I} - 2d_{U_I}d_{U_B}| = A_1. \quad \text{(45)}
\]

Considering the constraint (33), (44) can be rewritten as
\[
A_1 \geq s. \quad \text{(46)}
\]

As a result, the sub-problem of UAV’s location can be transformed into a convex one as
\[
\max_{Q_s, z, m, n} R_A^k + R_B^k \quad \text{s.t. } Q \in L, \quad \text{(35), (40), (41), (43), (46)}, \quad \text{(47c)}
\]

which can be solved by CVX.

C. Algorithm Design

Based on the above derivations, we propose an iterative algorithm, which is summarized in Algorithm 1. In particular, (11) and (25) are solved iteratively until convergence. Define \(R_{I_k}(p_i^{(k-1)}, \beta_i^{(k-1)}, \Theta_i^{(k-1)}, Q_i^{(k-1)})\) as the objective value of (10). After Step 4, we have \(R_{I_k}(p_i^{(k-1)}, \beta_i^{(k-1)}, \Theta_i^{(k-1)}, Q_i^{(k-1)}) \leq R_{I_k}(p_i^{(k)}, \beta_i^{(k)}, \Theta_i^{(k)}, Q_i^{(k-1)})\), where the objective value of (24) is a lower bound of (11) and the approximation is tight at the optimal points. Similarly, we obtain \(R_{I_k}(p_i^{(k)}, \beta_i^{(k)}, \Theta_i^{(k)}, Q_i^{(k-1)}) \leq R_{I_k}(p_i^{(k)}, \beta_i^{(k)}, \Theta_i^{(k)}, Q_i^{(k)})\) by Step 6. Therefore, the objective value of (10) is non-decreasing over iterations. In addition, the optimal value of
Algorithm 1 Iterative Algorithm for Problem (10)

1. **Initialization**: Initialize $p_i^0, \beta_j^0, V_0, Q_0, t_0, r_0, s_0$ and $z_0$.
   Set the iteration number $k = 0$;

2. **repeat**
   3. With given $Q_k$, calculate the optimal $\Theta_A$ with (12).
   4. With given $Q_k$ and $\Theta_A$, solve the problem (24) and update $p_i^k, \beta_j^k, V_k, t_k$ and $r_k$;
   5. Find $\Theta_k^B$ with Gaussian randomization and obtain $\Phi_k^B$;
   6. With given $\Phi_k^B$ and $p_i^k$, update $Q_k$ by solving the problem (47);
   7. Calculate the sum rate by utilizing (6) and (7);
   8. Set $k = k + 1$;

3. **until** : Convergence.

D. Multi-user Scenario

The proposed scheme can be extended to a multi-user case after incorporating proper kind of multiple access and can be solved similarly. Specifically, the users can be divided into $J$ groups, each of which consists of two users on both sides of STAR-RIS, served via NOMA. In this case, the closed-form solution to the transmission phase shifts is not applicable and the sum transmit power in each pair should be optimized. To handle this problem, we divide it into three subproblems and optimize the variables $\{\Phi_A, \Phi_B, p_{i,j}, Q\}$ alternately until convergence, where $p_{i,j}$ is the transmit power for the $i$-th user in the $j$-th group, $j = \{1, 2, \cdots, J\}$. Specifically, we first optimize the transmmision beamforming by using SDR similarly to the reflection beamforming. Then, for the power allocation problem, the objective function can be rewritten similarly to (17) and (21), and its lower bound can be obtained based on the first-order Taylor expansion. Finally, the UAV location optimization in the multi-user case can be solved by referring to (47).

V. SIMULATION RESULTS

Simulation results are presented to demonstrate the effectiveness of the proposed scheme. We set $\rho = -20$ dB, $\sigma^2 = -90$ dBm and $\alpha = 3.8$. The total transmit power, the number of STAR-RIS elements and the QoS requirement of users are set as 10 mW, 36 and 2 bit/s/Hz, if not specified [20]. The QoS requirements of $U_A$ and $U_B$ are assumed to be equal. Assume that the UAV hovers at $H = 100$ m in the area $L$. $U_A$, $U_B$ and STAR-RIS are assumed to be located at $q_A = [10, -5, 30]$ m, $q_B = [50, 50, 0]$ m and $q_I = [20, 0, 30]$ m, respectively.

Fig. 2 presents the sum rate versus the number of STAR-RIS reflecting elements $M$. For performance comparison, several benchmarks are considered. The benchmark “Without STAR-RIS optimization” shows the sum rate when applying random phase shifts. “Without location optimization” performs the power allocation and phase shifts design with the fixed UAV location. “RIS, only transmit” indicates the performance when the elements of RIS only transmit to $U_A$ and $U_B$ in the reflection side can only receive the signal from the direct link. It can be observed that the achievable rate increases with $M$ for all the schemes. This is because with a larger RIS size, better passive beamforming gain can be achieved for users. Due to the optimization limitations, different schemes have different effects on the sum rate. The performance of the proposed scheme is much better than that of the benchmarks, due to the joint optimization. Their gaps indicate the advantages of the optimization of UAV location, the full coverage and the beamforming of STAR-RIS, respectively.

In Fig. 3, we present the sum rate and the reflection coefficient $\beta_B$ of $U_B$ versus the QoS requirement. As expected, the sum rate/reflection coefficient increases/decreases with the total transmit power $P_{\text{sum}}$, as higher signal-to-noise ratio (SNR) can be achieved. In addition, it can be observed that the sum rate decreases with $\gamma$ and the reflection coefficient increases with $\gamma$. The reason is that with lower QoS requirement, more degree of freedom can be obtained for the power allocation and transmission/reflection coefficient design, which leads to
In this correspondence, we have investigated the sum-rate maximization in STAR-RIS assisted NOMA-UAV networks, where the location of UAV, the transmit power and the transmission/reflection beamforming of STAR-RIS are jointly optimized. The problem is decomposed into two sub-problems, and the non-convex constraints are transformed into convex via SCA. Then, an iterative algorithm is developed to optimize the two subproblems alternatively. Simulation results show the effectiveness of the proposed iterative algorithm.

VI. CONCLUSION

In this correspondence, we have investigated the sum-rate maximization in STAR-RIS assisted NOMA-UAV networks, where the location of UAV, the transmit power and the