RACH Preamble Repetition in NB-IoT Network

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***Abstract*—NarrowBand-Internet of Things (NB-IoT) is a radio access technology recently standardized by 3GPP. To provide reliable connections with extended coverage, a repetition transmission scheme is applied in both Random Access CHannel (RACH) procedure and data transmission. In this letter, we model RACH in the NB-IoT network taking into account the repeated preamble transmission and collision using stochastic geometry. We derive the exact expression of RACH success probability under time correlated interference, and validate the analysis with different repetition values via independent simulations. Numerical results have shown that the repetition scheme can efficiently improve the RACH success probability in a light traffic scenario, but only slightly improves that performance with very inefficient channel resource utilization in a heavy traffic scenario.**

***Index Terms*—NB-IoT, stochastic geometry, RACH success probability, repetition values.**

I. INTRODUCTION

To support communication of billions of miscellaneous innovative devices, Internet of Things (IoT) has gained unprecedented momentum and commercial interest. In view of this, a new radio access technology was developed by the 3rd generation partnership project (3GPP) named NarrowBand-IoT (NB-IoT), which provide reliable connections among inexpensive IoT devices with extended coverage and low power consumption [1, 2]. NB-IoT is built from existing Long Term evolution (LTE) design, which aims to achieve rapid specification and minimize development effort [3]. Accordingly, it reuses most LTE functionalities to achieve excellent co-existence performance with legacy LTE technologies [4].

However, to meet the requirements of IoT-based applications, such as low data rate, wide coverage, and etc., a number of LTE features are simplified, optimized, or developed [3]. One important key design difference between LTE and NB-IoT is the Physical Random Access CHannel (PRACH) (i.e., the channel used for uplink preamble transmission in the Random Access CHannel procedure, a.k.a. RACH). In more detail, the frequency hopping signal is used to transmit each preamble for time-of-arrival estimation[1], and a repetition scheme is applied to improve the reliability of RACH [1, 2, 4]. In other words, the preamble transmission is repeated for a dedicated number of times according to a repetition value that decided by an evolved NodeB (eNB) [2].

Obviously, increasing the repetition value increases the success opportunities of preamble transmission, but occupies more resources elements, which can bring serious burden for the NB-IoT system leading to reduced resource elements for data transmission. It is still unknown to what extent the repetition scheme improves the RACH success, and how to choose repetition value in different traffic scenarios to balance the RACH success probability and data transmission channel resources. To study the RACH in cellular-based massive IoT network, a general model was provided to analyze preamble transmission based on stochastic geometry [5]. Unfortunately, this model cannot be directly adopted to analyze the RACH in NB-IoT, due to the following three reasons: 1) new preamble format is defined based on frequency hopping; 2) repetition scheme significantly complicate the analysis; 3) collision events are not addressed in [5]. Considering the three issues, this letter provides a mathematical framework to analyze the RACH in NB-IoT, where two independent link outage conditions are focused: 1) the insufficient Signal-to-Interference-plus-Noise Ratio (SINR) of the received preamble; 2) the collision event, which occurs if two or more IoT devices choose the same sub-carrier at the same time.

The rest of the paper is organized as follows. The system model and the analytical methodology are introduced in Section II. The RACH success probability is presented in Section III. Section IV presents the numerical results, and the paper is concluded in Section V.

II. SYSTEM MODEL

In the NB-IoT system, the eNBs and the IoT devices are spatially distributed in $\mathbb{R}^2$ following two independent homogeneous Poisson point process (HPPP), Φ_B and Φ_D, with intensities λ_B and λ_D, respectively. According to [2], each IoT device associates with its geographically nearest eNB. Path-loss attenuation is defined as $r^{-\alpha}$, where r is the propagation distance and α is the path-loss exponent. We consider identically distributed (i.i.d.) Rayleigh fading channel, where the channel power gain h is assumed to be exponentially distributed random variable with unit mean. According to [1, 2], the transmitted power of IoT devices determined by the full path-loss inversion power control, where each IoT device maintains the received signal power in the eNB equalling to a same threshold ρ by compensating its own path-loss.

In the uplink of NB-IoT, the Narrowband Physical Uplink Shared CHannels (NPUSCHs) are used for data transmission and the Narrowband-PRACHs (NPRACHs) are used for

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[1]The frequency hopping preamble is designed for calculating time-of-arrival to facilitate distance estimation between the IoT device and its associated eNB, which is beyond the scope of this paper [1, 2].

preamble transmission. In more detail, 180 kHz of spectrum is occupied with 3.75 kHz tone spacing (i.e., spans over 48 sub-carriers) or 15 kHz tone spacing (i.e., spans over 12 sub-carriers), where the NPRACH only supports 3.75 kHz tone spacing. To fulfill the coverage requirements of different IoT devices, NB-IoT network can configure up to 3 repetition values from the set {1, 2, 4, 8, 16, 32, 64, 128} in a cell, and allows flexible configuration of NPRACH resources [4]. In this model, we consider single repetition value to provide fundamental insights due to repetition, where the related resources assignment of NPRACHs only takes place in the begin of a transmission time interval (TTI) as shown in Fig. 1.

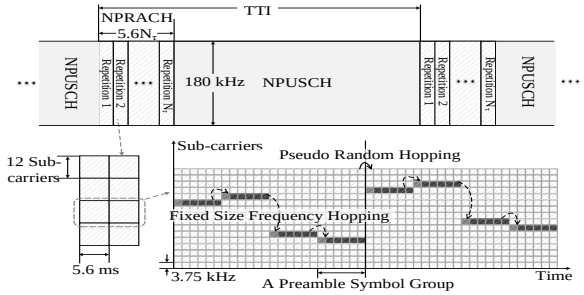


Fig. 1: Structure of NPUSCH and NPRACH.

Due to the single repetition value configuration, each active IoT device will contend on all 48 sub-carriers, and thus each sub-carrier has an equal probability (1/48) to be chosen. As only active IoT devices will try to request for uplink channel resources, we define the active probability of each IoT device $p_a \in [0, 1]$ follows a Bernoulli process. According to the thinning process [6], the density of active IoT devices choosing the same sub-carrier can be derived

$$\lambda_{Da} = p_a \lambda_D / 48. \tag{1}$$

Recalling the repetition scheme, an active IoT device repeats a same preamble N_τ times (i.e., the dedicated repetition value). In each repetition, a preamble consists of 4 symbol groups, where the first preamble symbol group is transmitted via a sub-carrier determined by Pseudo random hopping (i.e., the hopping depends on the current repetition time and the Narrowband physical Cell ID, a.k.a NCellID), and the following 3 preamble symbol groups are transmitted via sub-carriers determined by the fixed size frequency hopping [1]. In other words, if two or more IoT devices chose the same first sub-carrier in a single RACH opportunity, the following sub-carriers (i.e., in the same RACH opportunity) would be same, due to that these two hopping algorithms lead to one-to-one correspondences between the first sub-carrier and the following sub-carriers.

We first formulate the SINR outage condition. In each TTI, a preamble transmission success occurs if any repetition successes, and in a single repetition, a preamble is successfully received at the associated eNB, if all four received SINRs are above the SINR threshold γ_{th}. Overall, the preamble transmission success probability under N_τ repetitions conditioning on n number of intra-cell interfering IoT devices is expressed as

$$p_S(N_\tau, n) = 1 - \prod_{n_\tau=1}^{N_\tau} \Big(1 - \mathbb{P}\{\theta_{n_\tau} | Z_B = n\}\Big), \tag{2}$$

where $Z_B = \|\mathcal{Z}_{\text{intra}}\|$ is the number of active intra-cell interfering IoT devices, and

$$\theta_{n_\tau} \triangleq \left\{ \begin{array}{c} SINR_{n_\tau,1} \geq \gamma_{th}, SINR_{n_\tau,2} \geq \gamma_{th}, \\ SINR_{n_\tau,3} \geq \gamma_{th}, SINR_{n_\tau,4} \geq \gamma_{th} \end{array} \right\}. \tag{3}$$

In (3), γ_{th} is the SINR threshold, and $SINR_{n_\tau,1}$ is the received SINR of the $1st$ symbol group in the n_τth repetition. Based on Slivnyak's theorem [6], we formulate this SINR experienced at the typical eNB located at the origin as

$$SINR = \frac{\rho_0 h_0}{\sum\limits_{j \in \mathcal{Z}_{\text{intra}}} \rho_j h_j + \sum\limits_{i \in \mathcal{Z}_{\text{inter}}} P_i \|u_i\|^{-\alpha} h_i + \sigma^2}, \tag{4}$$

where $\|\cdot\|$ is the Euclidean norm, ρ_0 is the full path-loss inversion power control threshold, h_0 is the channel power gain from the typical IoT device to its associated eNB, $\mathcal{Z}_{\text{inter}}$ is set of the active inter-cell interfering IoT devices using the same sub-carrier, σ^2 is the noise power, P_i is the actual transmit power of the ith interfering IoT device, x_i and h_i are distance and channel power gain from the ith interfering IoT device to the typical eNB, respectively.

Then, we formulate the RACH success probability under both SINR outage and collision conditions. Recalling that a collision occurs if an eNB receives multiple preambles from the same set of sub-carriers at the same time. Consequently, a RACH successes when the preamble successfully received in the eNB, as well as no collision occurs, which is presented as

$$P_{N_\tau} = \sum_{n=0}^{\infty} \Bigg(\mathbb{P}\{Z_B = n\} p_{S,0}(N_\tau, n) \prod_{l=1}^{n} \Big(1 - p_{S,l}(N_\tau, n)\Big) \Bigg), \tag{5}$$

where $p_{S,0}(N_\tau, n)$ is the preamble transmission success probability of the typical IoT device, $p_{S,l}(N_\tau, n)$ is the preamble transmission success probability of the lth interfering IoT device located in the typical cell, and $\mathbb{P}\{Z_B{=}n\}$ is the probability of n number of interfering IoT devices located in the typical cell.

For an expected RACH success probability, the repetition value is written as

$$N_\tau = f^{-1}(P_{N_\tau}), \tag{6}$$

where N_τ is the required repetition value.

In a TTI, the repetition value determines the assigned channel resources for NPRACHs, and the least channel resources for data transmission (i.e., NPUSCHs). A proper repetition value can help to improve the channel efficiency, due to that insufficient repetition may results in a low RACH success probability, but redundant repetition may leads to deficiency of NPUSCHs. The time vacancy ratio (i.e., the ratio between the time of NPUSCHs and TTI) is expressed as

$$T_a = (\text{TTI} - 5.6N_\tau)/\text{TTI}. \tag{7}$$

As can be seen from Eq. (7), increasing N_τ decreases the resource elements utilization.

III. RANDOM ACCESS SUCCESS PROBABILITY

Due to that each intra-cell interfering IoT device has the same preamble transmission success probability, the RACH success probability presented in (5) can be simplified as

$$P_{N_\tau} = \sum_{n=0}^{\infty} \mathbb{P}\{Z_B = n\} p_S(N_\tau, n) \Big(1 - p_S(N_\tau, n)\Big)^n, \tag{8}$$

where $\mathbb{P}\{Z_B{=}n\}$ is the probability of the number of interfering IoT devices $Z_B = n$, and $p_S(N_\tau, n)$ is the preamble transmission success probability of an IoT device conditioning on $\{Z_B{=}n\}$. The Probability Mass Function (PMF) of the number of interfering IoT devices Z_B is represented as [5, Eq.(12)]

$$\mathbb{P}\{Z_B = n\} = \frac{c^{(c+1)}\Gamma(n+c+1)(\frac{\lambda_{Da}}{\lambda_B})^n}{\Gamma(c+1)\Gamma(n+1)(\frac{\lambda_{Da}}{\lambda_B}+c)^{n+c+1}}, \quad (9)$$

where λ_{Da} is the intensity of active IoT devices using the same sub-carrier given in (1), λ_B is the intensity of eNBs, $c = 3.575$ is a constant related to the approximate PMF of the PPP Voronoi cell, and $\Gamma\{\cdot\}$ is the gamma function.

Without loss of generality, we assume each IoT device remains spatially static during a TTI (i.e., we assume that the preamble format 0 is used, where a preamble repetition only takes 5.6 ms [1]), and thus the mutual interference among IoT devices is temporally correlated [7]. This temporal correlation complicate the derivation of the preamble transmission success probability, which is the main challenge of RACH success analysis. The preamble transmission success probability with N_τ repetitions $p_S(N_\tau, n)$ is derived in the following Theorem.

Theorem 1. *The preamble transmission success probability of a randomly chosen IoT device with N_τ preamble repetitions $p_S(N_\tau, n)$ is expressed as*

$$p_S(N_\tau, n) = 1 - \prod_{n_\tau=1}^{N_\tau}\Big(1 - \mathbb{P}\{\theta_{n_\tau}\big|Z_B = n\}\Big)$$

$$= \sum_{n_\tau=1}^{N_\tau}(-1)^{n_\tau+1}\binom{N_\tau}{n_\tau}\mathbb{P}\{\theta_1, \cdots, \theta_{n_\tau}\big|Z_B = n\}, \quad (10)$$

where $\binom{N_\tau}{n_\tau} = \frac{N_\tau!}{n_\tau!(N_\tau - n_\tau)!}$ is the binomial coefficient, and $\mathbb{P}\{\theta_1, \cdots, \theta_{n_\tau}\big|Z_B = n\}$ is the probability that all of $4 \times n_\tau$ (i.e., a preamble consists of 4 symbol groups) time-correlated preamble symbol groups are successfully received in the eNB. For ease of presentation, we assume $m = 4 \times n_\tau$, and $\mathbb{P}\{\theta_1, \cdots, \theta_{n_\tau}\big|Z_B = n\}$ is expressed as

$$\mathbb{P}\{\theta_1, \cdots, \theta_{n_\tau}\big|Z_B = n\} =$$

$$\frac{\exp\Big(-\frac{m\gamma_{th}\sigma^2}{\rho} - 2(\gamma_{th})^{\frac{2}{\alpha}}\frac{\lambda_{Da}}{\lambda_B}\int_{(\gamma_{th})^{-\frac{1}{\alpha}}}^{\infty}\Big[1 - (\frac{1}{1+y^{-\alpha}})^m\Big]ydy\Big)}{(1+\gamma_{th})^{mn}}, \quad (11)$$

where α is the path-loss parameter, γ_{th} is the received SINR threshold, σ^2 is the noise, ρ is the full path-loss inversion power control threshold.

Proof. Based on the Binomial theorem, the preamble transmission success probability of N_τ repetitions $\mathbb{P}\{\theta_1, \cdots, \theta_{n_\tau}\big|Z_B = n\}$ is expressed as

$$\mathbb{P}\{SINR_1 \geq \gamma_{th}, \cdots, SINR_m \geq \gamma_{th}\big|Z_B = n\} =$$

$$\exp(-\frac{m\gamma_{th}\sigma^2}{\rho})E[e^{-s\sum_{k=1}^{m}\mathcal{I}_k^{\text{intra}}}\big|Z_B = n]E[e^{-s\sum_{k=1}^{m}\mathcal{I}_k^{\text{inter}}}], \quad (12)$$

where $s = \gamma_{th}/\rho$, $\mathcal{I}_k^{\text{intra}}$ and $\mathcal{I}_k^{\text{inter}}$ are the aggregate intra-cell interference and the aggregate inter-cell interference when the kth preamble symbol group is transmitted, respectively.

In (12), the Laplace Transform of the aggregate interference from the intra-cell interfering IoT devices received at the typical eNB conditioning on $Z_B = n$ is obtained as

$$E[e^{-s\sum_{k=1}^{m}\mathcal{I}_k^{\text{intra}}}\big|Z_B = n] = E\Big[\exp\big(-s\sum_{j\in\mathcal{Z}_{\text{intra}}}\rho\sum_{k=1}^{m}h_j^k\big)\Big]$$

$$= E\Big[\prod_{j=1}^{n}\exp\big(-s\rho\sum_{k=1}^{m}h_j^k\big)\Big] \overset{(a)}{=} (\frac{1}{1+s\rho})^{mn}, \quad (13)$$

where (a) follows by taking the average with respect to h_j^1, h_j^2, $\cdots$, h_j^m.

In (12), the Laplace Transform of the aggregate interference from the inter-cell interfering IoT devices received at the typical eNB is obtained as

$$E[e^{-s\sum_{k=1}^{m}\mathcal{I}_k^{\text{inter}}}] = E\Big[\exp\Big(-s\sum_{i\in\mathcal{Z}_{\text{inter}}}P_i(\sum_{k=1}^{m}h_i^k)\|u_i\|^{-\alpha}\Big)\Big]$$

$$\overset{(a)}{=} E\Big[\prod_{i\in\mathcal{Z}_{\text{inter}}}(\frac{1}{1+sP_i{x_i}^{-\alpha}})^m\Big]$$

$$\overset{(b)}{=} \exp\Big(-2\pi\lambda_{Da}\int_{(\frac{P}{\rho})^{\frac{1}{\alpha}}}^{\infty}E_P\Big[1 - (\frac{1}{1+sPx^{-\alpha}})^m\Big]xdx\Big)$$

$$\overset{(c)}{=} \exp\Big(-2\pi\lambda_{Da}s^{\frac{2}{\alpha}}E[P^{\frac{2}{\alpha}}]\int_{(s\rho)^{-\frac{1}{\alpha}}}^{\infty}\Big[1 - (\frac{1}{1+y^{-\alpha}})^m\Big]ydy\Big), \quad (14)$$

where (a) obtained by taking the average with respect to h_i^1, h_i^2, $\cdots$, h_i^m, (b) follows from the probability generation functional (PGFL) of the HPPP, (c) follows by changing the variables $y = x/(sP)^{\frac{1}{\alpha}}$, and $E[P^{\frac{2}{\alpha}}] = \rho^{\frac{2}{\alpha}}/\pi\lambda_B$ is the moments of the transmit power [5, Eq.(A.2,A.3)]. Substituting (13) and (14) into (12), we derive the probability that all of m preamble symbol groups are successfully transmitted. □

For simplicity, we present a special case of RACH with two preamble repetitions, and the preamble transmission success probability $p_S(2, n)$ is expressed as

$$p_S(2, n) = 1 - \Big(1 - \mathbb{P}\{\theta_1\big|Z_B = n\}\Big)\Big(1 - \mathbb{P}\{\theta_2\big|Z_B = n\}\Big)$$

$$\overset{(a)}{=} 2\mathbb{P}\{\theta_1\big|Z_B = n\} - \mathbb{P}\{\theta_1, \theta_2\big|Z_B = n\}, \quad (15)$$

where (a) follows from $\mathbb{P}\{\theta_1\big|Z_B{=}n\}{=}\mathbb{P}\{\theta_2\big|Z_B{=}n\}$. Substituting (15) and (9) into (8), we obtain the RACH success probability when $N_\tau = 2$.

IV. NUMERICAL RESULTS

In this section, the derived analytical results are validated via Monte Carlo simulations. The eNBs and IoT devices are deployed in a 10^4 km^2 circle area. We set λ_B = 0.1 eNBs/km^2, γ_{th} = 0 dB, α = 4, ρ = $-$130 dBm, the bandwidth of a sub-carrier is $BW = 3.75$ kHz, and thus the noise is $\sigma^2 = -174 + 10log_{10}(BW) = -138.3$ dBm. Different from LTE with TTI = 40 ms [8], we set TTI in NB-IoT network as 640 ms following

the defined Narrowband Physical Broadcast Channel (NPBCH) TTI [2]. The packet arrival periods of IoT devices are from a few minutes to several days [9], hence we assume two traffic scenarios, where the light traffic scenario is 1 packet/hour of each IoT device, and the heavy traffic scenario is 1 packet/10 minutes of each IoT device. Therefore, the active probabilities of each IoT device during 640 ms are $p_{a_l} = 640/3600000 = 0.00018$ and $p_{a_h} = 640/600000 = 0.0011$, respectively.

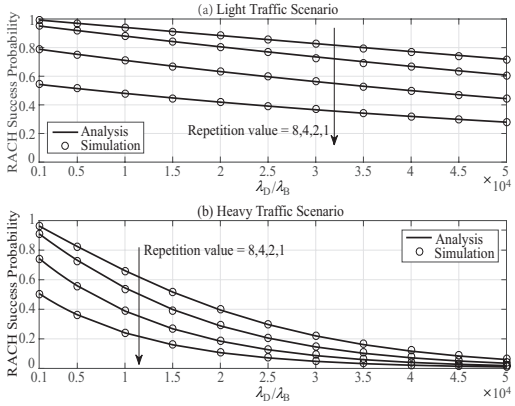


Fig. 2: RACH success probability.

Fig. 2(a) and Fig. 2(b) plot the RACH success probabilities of a randomly chosen IoT device in the light traffic scenario and the heavy traffic scenario, respectively. The analytical curves of the RACH success probability are plotted using (8), which closely match with simulation points that validates the accuracy of developed mathematical framework. We first observe that for all curves, the RACH success probability decreases with the increase of the density ratio between IoT devices and eNBs (λ_D/λ_B), which is due to the following two reasons: 1) increasing the number of IoT devices generating more interference leads to lower received SINR at the eNB; 2) increasing the number of IoT devices results in higher probability of collision. In both two sub-figures, increasing repetition value increases the RACH success probability, which is due to that it offers more opportunities to re-transmit a preamble with the time and frequency diversity.

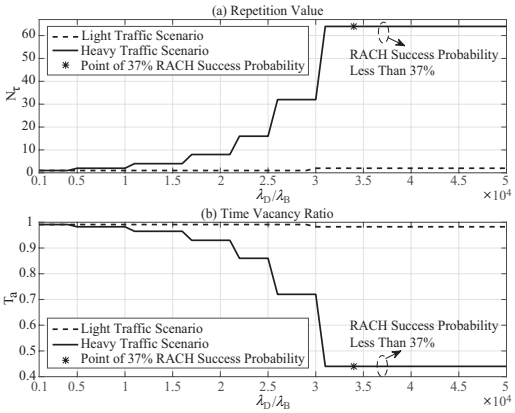


Fig. 3: Required repetition values and related time vacancy ratio.

Access success probability of a IoT device is expected to reach at least 99% within *10 RACH opportunities* in the cellular-based IoT network [10]. To meet this lower bound, the RACH success probability for one single RACH opportunity should achieve at least $1 - \sqrt[10]{1-99\%} \approx 37\%$. Fig. 3(a) plots the minimum repetition value (N_τ) to achieve a RACH success probability that is greater than or equal to 37%, and Fig. 3(b) plots the time vacancy ratio (T_a) under the condition of required repetition value. We assume N_τ is configured from set $\{1, 2, 4, 8, 16, 32, 64\}$, and when neither a value can help IoT devices to reach 37% RACH success probability, the eNB will choose maximum repetition value 64. We first observe that N_τ increases with λ_D/λ_B in Fig. 3(a), which leads to an opposite down trend of T_a in Fig. 3(b). In the light traffic scenario, RACH success probability is always larger than 37% with relatively small repetition values, which contributes to a relatively high time vacancy ratio. However, in the heavy traffic scenario, the required repetition value increased rapidly with λ_D/λ_B, and the performance cannot meet the requirement of 37% RACH success probability after a certain point despite that the maximum repetition value is set.

V. CONCLUSION

In this letter, we developed a stochastic geometry framework to analyze the RACH under the repetition scheme in the NB-IoT system. To evaluate how the repetition mechanism fulfills the requirement of improving RACH reliability, we derived the exact expression of the RACH success probability under the time correlated interference. Different from existing works, we considered both SINR outage and collision from the network point of view. The numerical results shed light on that the RACH success probability increases with the repetition value, and also revealed that very high repetition value will lead to a low channel resources utilization in the heavy traffic scenario.

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