Low-cost blind carrier frequency offset estimator for down-link MIMO multicarrier systems

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Abstract: Under perfect synchronisation conditions, communication systems using multiple transmit and receive antennas, known as the multiple input multiple output (MIMO) technique, along with the multicarrier transmission technique can achieve very high transmission data rates, bandwidth efficiency and robustness to the channel frequency selective fading. In a practical system, however, the presence of a carrier frequency offset (CFO), due to lack of perfect synchronisation, causes a severe degradation in the overall system performance. A low-cost blind CFO estimator is proposed for the downlink of the MIMO multicarrier system, which is based on the null subcarriers concept and Taylor's series expansion. The theoretical mean squared error (MSE) of the proposed CFO estimator is also derived. It is shown in particular, that the computational complexity of the proposed estimator is significantly low, yet its performance is good.

1 Introduction

Multicarrier systems have been broadly used in wireless communications in recent years. By dividing the channel bandwidth into several narrowband subchannels, multicarrier systems can offer high data rate transmission with high bandwidth efficiency. The frequency-selective fading problem is resolved as the channel is converted into a set of parallel narrowband flat-fading subchannels [1, 2]. The intersymbol interference (ISI) can also be eliminated using a time-guard or a cyclic prefix [2]. Orthogonal frequency division multiplexing (OFDM), which is a typical case of a multicarrier system, has been adopted by many standards (e.g. IEEE 802.11a, IEEE 802.11 g and HIPERLAN/2). Multicarrier code division multiple access (MC-CDMA), which is another type of multicarrier system, has also been used in several applications [3].

However, the next generation wireless communication systems require transmission techniques which can reach a data rate of 1 Gbit/s. Multi-input multi-output (MIMO) transmission using multiple antennas is a promising technique to satisfy such a demand [4]. It has been demonstrated that the capacity and the bit error rate are enhanced significantly when the number of antennas are increased [4]. Some other schemes, such as space–time coding (STC) (not used in this work) [5], which can further improve the performance of the system, are often used in MIMO communication systems.

The combination of the two independent techniques, namely the MIMO multicarrier system, offers high data rate, high bandwidth efficiency, robustness to channel frequency-selective fading, etc. [6, 7]. It is considered as a promising technique for future high-speed wireless communication.

Like multicarrier systems, MIMO multicarrier systems are very sensitive to carrier frequency offset (CFO), which destroys the orthogonality between the subcarriers, causes intercarrier interference (ICI) and degrades the BER performance severely [8]. The existing CFO estimators can be classified into two groups; data-aided estimators and non-data-aided or blind estimators.

Data-aided estimators use pilot symbols or training symbols to estimate the CFO [9–12]. Without regard to the performance of data-aided estimators, one of the most obvious disadvantages of these methods is that the pilot and training symbols need additional bandwidth. As a result, blind CFO estimation methods [13–19] have received a lot of attention during the past decade.

In [13], a polynomial cost function is constructed using null subcarriers, and the CFO estimate is the value which minimises this cost function. Then, a truncated Taylor's series expansion is used in [15] to reduce the computational cost of the estimator in [13]. In [13] and [15], the null subcarriers are assumed to be placed at the end of the OFDM block. However, it has been proved in [16] that there may be more than one minimum for the cost function when this null subcarriers placement method is used. In order to overcome this identifiability problem, three new null subcarrier insertion approaches were proposed in [16].

In this paper, we propose a low-cost blind CFO estimator for MIMO multicarrier systems. Compared to the CFO estimator in [17], the proposed algorithm can reduce the computational complexity significantly without sacrificing the performance. Considering the identifiability problem [16], we insert the null subcarriers with distinct spacing so as to ensure that there is a unique minimum to the cost function. Our CFO estimator is based on two practical considerations: (i) In multicarrier systems, CFO is usually divided into an integer part and a residual part; and (ii) The synchronisation is usually done in two-steps. First, the integer part (coarse) of the CFO is detected at the analogue part. Then, in the digital part, only a fine residual CFO is estimated. Thus, we assume that residual CFO $|\omega_0| \ll 1$.

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2 System model

The following notations are used throughout this paper: superscript $(\bullet)^H$ denotes Hermitian; $E[\bullet]$ is the expectation of the random variable within brackets; $\| \bullet \|$ denotes the Euclidean norm; $\Im[\bullet]$ and $\Re[\bullet]$ denote the imaginary and real parts of a complex number respectively; $(\bullet)^*$ is the transpose operator; $\boldsymbol{I}_N$ is an $N \times N$ identity matrix; $\boldsymbol{e}_i$ denotes the $(i+1)$th column of $\boldsymbol{I}_N$; $\boldsymbol{F}_N$ is the $N \times N$ fast Fourier transform (FFT) matrix; the vector $\boldsymbol{f}_N(\omega)$ is defined as $\boldsymbol{f}_N(\omega) := [1, \exp(j\omega), \ldots, \exp(j(N-1)\omega)]^T$; $\boldsymbol{D}_N(h)$ is an $N \times N$ diagonal matrix with $[\boldsymbol{D}_N(\boldsymbol{h})]_{n,n} = [\boldsymbol{h}]_n$ and $\boldsymbol{D}_N(\omega) := \boldsymbol{D}_N(\boldsymbol{f}_N(\omega))$.

Figure 1 shows the general model of a MIMO multicarrier system. This is a simplified down-link model based on the following assumptions:

- All the users' signals, which are transmitted from a specific transmit antenna to another specific receive antenna, are propagated through the same channel.
- All the signals at the user of interest receiver undergo the same carrier frequency offset because they are frequency shifted (up-conversion) using the same oscillator at the transmitter.
- We discuss the signal generation part of the transmitter in two different situations; MIMO-OFDM and MIMO MC-CDMA. The number of transmit and receive antennas are N_t and N_r, respectively. When there is only one user at the transmitter side, the MIMO multicarrier system reduces to MIMO-OFDM. The information of the user is space-division multiplexed to N_t data streams, which pass through serial-to-parallel (S/P) converters and form the vectors $\boldsymbol{s}_\mu(k)$.

However, the down-link MIMO MC-CDMA system is a typical MIMO multicarrier system for multiusers. We assume that there are N_u users randomly distributed in a cell site. First, the data bits of the vth user is multiplexed to yield a data stream $\{t_\mu^v(k)\}_{\mu=1}^{N_t}$. Instead of the S/P converters, copiers are used to replicate the data bits $t_\mu^v(k)$. Each data bit of the vth user is multiplexed with a specific spreading sequence $\boldsymbol{c}_\mu^v = [c_\mu^v(0) \;\; c_\mu^v(1) \; \cdots \; c_\mu^v(K-1)]^T$, which is one column of the orthogonal Hadamard matrix. Because the

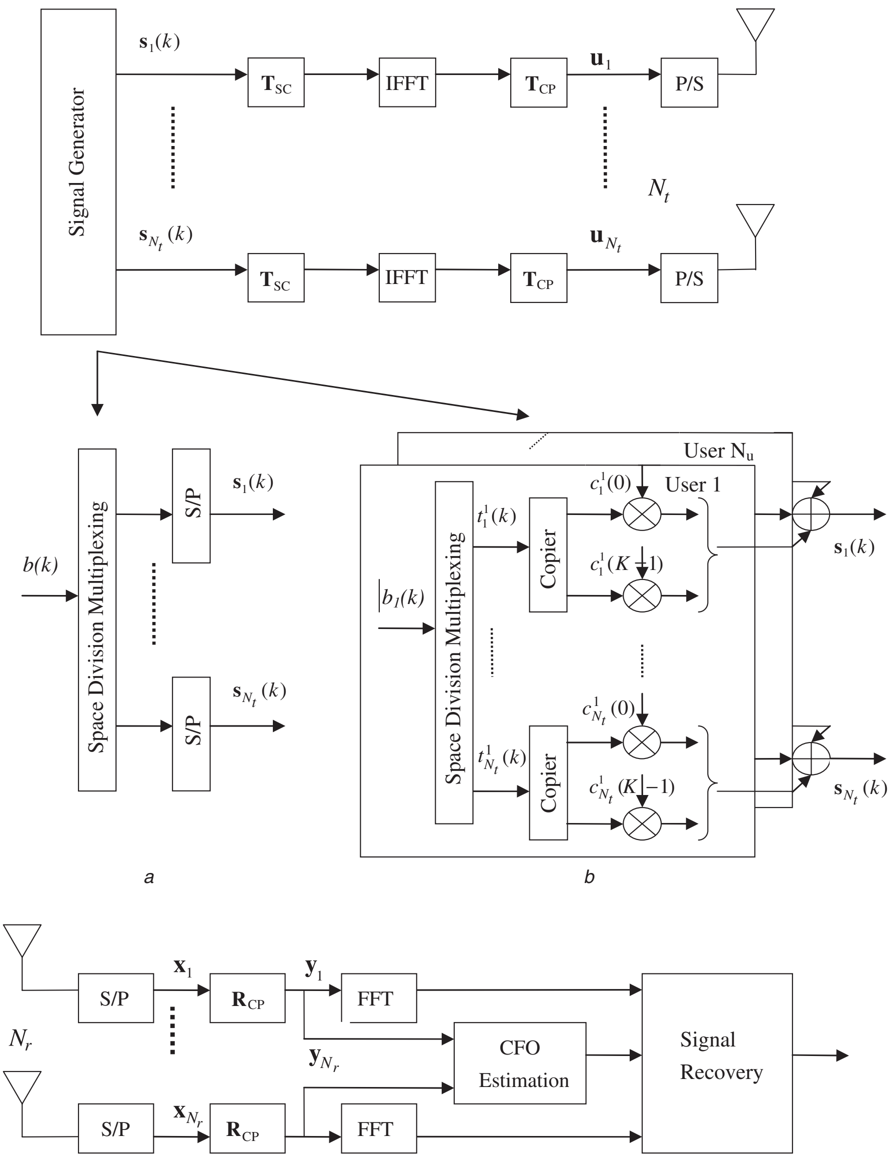

Fig. 1 *General model of MIMO multicarrier system, transmitter and receiver*
a MIMO-OFDM
b MIMO-MC-CMDA

length of the spreading sequence is K, the maximum number of users is determined to be $N_u(\max) = \lfloor K/N_t \rfloor$. All the N_u vectors corresponding to the μth antenna are summed up to form one vector as follows:

$$s_\mu(k) = \sum_{v=1}^{N_u} t_\mu^v(k) c_\mu^v \quad (1)$$

where the symbol vectors $\{s_\mu(k)\}_{\mu=1}^{N_t}$ dimension is $K \times 1$. After null subcarriers insertion, N-point inverse FFT (IFFT) and cyclic prefix (CP) insertion, we obtain the final transmitted block of the μth antenna as [16]

$$u_\mu(k) = T_{CP} F_N^H T_{SC} s_\mu(k) \quad (2)$$

where $T_{SC} := [e_{\bar{i}_1}, \ldots, e_{\bar{i}_K}]$ is the null subcarrier insertion matrix, F_N^H is the $N \times N$ IFFT matrix, and $T_{CP} = [I_{L\times N}^T \; I_N^T]^T$ is the CP insertion matrix. $\bar{i}_1 < \cdots < \bar{i}_K$ are the indices of the information symbols. The channel and CP lengths are both equal to L. In this paper, we choose the number of the null subcarriers as $N-K=L+2$, and the indices of the inserted zeros as $i_k = 2^{k-1}$, $k = 1, 2, \ldots, N-K$, so as to ensure that the cost function has only one minimum [16].

At the transmitter side, the length of the blocks $\{u_\mu(k)\}_{\mu=1}^{N_t}$ is expanded to $P=N+L$. The frequency-selective channels from the μth transmit antenna to the υth receive antenna have discrete-time finite impulse responses $h^{(\upsilon,\mu)}(l)$, $l \in [0, L]$. When the signals arrive at the receiver, they undergo a carrier frequency offset, which could be due to Doppler or mismatch between transmit-receive oscillators, and get corrupted by additive white Gaussian noise. Let us define ω_0 as the normalised carrier frequency offset. The output of the filter of the υth receive antenna can be written as

$$x_\upsilon(i) = e^{j\omega_0 i} \sum_{\mu=1}^{N_t} \sum_{l=0}^{L} h^{(\upsilon,\mu)}(l) u_\mu(i-l) + \eta_\upsilon(i) \quad (3)$$

where $\upsilon \in [1, N_r]$ and $\eta_\upsilon(i)$ is additive white Gaussian noise (AWGN). The received signal is the summation of the signals which come from all N_t transmit antennas. Then, we eliminate the interblock interference (IBI) by left multiplying received blocks ($x_\upsilon(k)$ constructed by stacking (3) into an $L+N$ long column vector) with the CP removing matrix [16]. Hence, the IBI-free block can be written as

$$\begin{aligned} y_\upsilon(k) &= R_{CP} x_\upsilon(k) \\ &= e^{j\omega_0 kP} \sum_{\mu=1}^{N_t} R_{CP} D_P(\omega_0) H^{(\upsilon,\mu)} T_{CP} F_N^H T_{SC} s_\mu(k) \\ &\quad + R_{CP} \eta_\upsilon(k) \end{aligned} \quad (4)$$

where $R_{CP} := [0_{N\times L}\, I_N]$ is the CP removing matrix, $H^{(\upsilon,\mu)}$ is a $P \times P$ lower triangular Toeplitz matrix with first column $[h^{(\upsilon,\mu)}(0), \ldots, h^{(\upsilon,\mu)}(L), 0, \ldots 0]^T$ and the $P \times P$ diagonal matrix $D_P(\omega_0) := diag[1, \exp(j\omega_0), \ldots, \exp(j(P-1)\omega_0)]$, which is caused by CFO ω_0. For simplicity, we rewrite (4) as follows [17]:

$$y_\upsilon(k) = e^{j\omega_0(kP+L)} D_N(\omega_0) \sum_{\mu=1}^{N_t} \tilde{H}^{(\upsilon,\mu)} F_N^H T_{SC} s_\mu(k) + n_\upsilon(k) \quad (5)$$

where $\tilde{H}^{(\upsilon,\mu)} := R_{CP} H^{(\upsilon,\mu)} T_{CP}$, $n_\upsilon(k) := R_{CP}\eta_\upsilon(k)$.

3 Proposed CFO estimator

The cost function of the blind CFO estimator that we construct is based on the received symbols and the orthogonality of the IFFT matrix. Then, Taylor's expansion is used to make an adequate approximation to the cost function. First, we construct a cost function as [16]

$$J(\omega) = \sum_{t=1}^{N-K} f_N^H\left(\frac{2\pi i_t}{N}\right) D_N^{-1}(\omega) R_{yy} D_N(\omega) f_N\left(\frac{2\pi i_t}{N}\right) \quad (6)$$

where R_{yy} is a covariance matrix, which is the summation of the covariance matrices of $\{y_\upsilon(k)\}_{\upsilon=1}^{N_r}$. Let us define $R_{y_\upsilon} := E[y_\upsilon(k) y_\upsilon^H(k)]$. In practice, R_{y_υ} is estimated by averaging across M blocks, i.e.

$$\widehat{R}_{y_\upsilon} = \frac{1}{M} \sum_{k=0}^{M-1} y_\upsilon(k) y_\upsilon^H(k) \quad (7)$$

Hence, the covariance matrix R_{yy} can be written as [17]

$$R_{yy} = \frac{1}{N_r} \sum_{\upsilon=1}^{N_r} R_{y_\upsilon} = \frac{1}{MN_r} \sum_{\upsilon=1}^{N_r} \sum_{k=0}^{M-1} y_\upsilon(k) y_\upsilon^H(k) \quad (8)$$

In (6), ω is the candidate carrier offset. It is proved in [13] that in the absence of noise the cost function $J(\omega_0) = 0$ when $\omega = \omega_0$. As mentioned before, when the number of null subcarriers is not less than $L+2$, and the spacing between any two null subcarriers is distinct, the cost function $J(\omega)$ has a unique minimum, which is the CFO ω_0 [16]. It is easy to get the CFO estimate by searching the candidate carrier offset ω which minimises (6). However, this leads to an algorithm with high computational complexity. In the following, we develop a new method with low computational cost and comparable performance. First, we consider the inverse diagonal matrix $D_N^{-1}(\omega)$, which is defined as

$$D_N^{-1}(\omega) = diag(1, \exp(-j\omega), \exp(-j2\omega), \ldots, \exp(-j(N-1)\omega)) \quad (9)$$

It is known that Taylor's expansion of a complex exponential is

$$e^{jx} = \sum_{n=0}^{+\infty} \frac{(jx)^n}{n!} \quad (10)$$

Substituting (10) into (9), we obtain [15]

$$\begin{aligned} D_N^{-1}(\omega) &= e^{-j\omega((N-1)/2)} diag(e^{j\omega((N-1)/2)}, e^{j\omega((N-3)/2)}, \\ &\qquad \ldots, e^{j\omega((N-N)/2)}) \\ &= e^{-j\omega((N-1)/2)} \times \sum_{n=0}^{+\infty} \frac{(j\omega)^n}{2^n n!} D^n \end{aligned} \quad (11)$$

where $D = diag((N-1), (N-3), \ldots, (1-N))$. As mentioned in Section 1, our method is used to detect the fine fractional CFO, which is usually less than $0.1\,\omega$, where $\omega = 2\pi/N$. In practice, the number of subcarriers N is much larger than 1, so we can assume that the fine fractional CFO $|\omega_0| \ll 1$. In the Taylor's series, we select a suitable integer $Q \ll N$ such that the higher-order terms $(n > Q)$ in (11) can be neglected, and the truncated series can be a good approximation to $D_N^{-1}(\omega)$. The value of Q is chosen such that the absolute value of the ratio of the $(n+1)$th term to the nth term of the Taylor's series of $e^{j\omega(N-1)/2}$ (worst case in (11) as the argument is the largest in absolute value) is smaller than one [15], i.e.

$$Q \geq \frac{|\omega|(N-1)}{2} \quad (12)$$

Now, substituting (8) into (6) leads to

$$J(\omega) = \sum_{t=1}^{N-K} \boldsymbol{f}_N^H\left(\frac{2\pi i_t}{N}\right)\boldsymbol{D}_N^{-1}(\omega)\left\{\sum_{\upsilon=1}^{N_r}\boldsymbol{R}_{\boldsymbol{y}_\upsilon}\right\}\boldsymbol{D}_N(\omega)\boldsymbol{f}_N\left(\frac{2\pi i_t}{N}\right)$$
$$\simeq \sum_{t=1}^{N-K}\boldsymbol{f}_N^H\left(\frac{2\pi i_t}{N}\right)\boldsymbol{D}_N^{-1}(\omega) \times\left\{\sum_{\upsilon=1}^{N_r}\left[\frac{1}{M}\sum_{k=0}^{M-1}\boldsymbol{y}_\upsilon(k)\boldsymbol{y}_\upsilon^H(k)\right]\right\}\boldsymbol{D}_N(\omega)\boldsymbol{f}_N\left(\frac{2\pi i_t}{N}\right)$$
$$=\frac{1}{M}\sum_{\upsilon=1}^{N_r}\sum_{k=0}^{M-1}\sum_{t=1}^{N-K}\left\|\boldsymbol{f}_N^H\left(\frac{2\pi i_t}{N}\right)\boldsymbol{D}_N^{-1}(\omega)\boldsymbol{y}_\upsilon(k)\right\|^2 \quad (13)$$

Then, by substituting the Taylor's expansion of the inverse diagonal matrix $\boldsymbol{D}_N^{-1}(\omega)$ into (13), we can rewrite (13) as

$$\boldsymbol{J}(\omega) = \frac{1}{M}\sum_{\upsilon=1}^{N_r}\sum_{k=0}^{M-1}\sum_{t=1}^{N-K}\left\|\boldsymbol{f}_N^H\left(\frac{2\pi i_t}{N}\right)\left[e^{-j\omega\frac{(N-1)}{2}}\sum_{n=0}^{+\infty}\frac{(j\omega)^n}{2^n n!}\boldsymbol{D}^n\right]\boldsymbol{y}_\upsilon(k)\right\|^2$$
$$=\frac{1}{M}\sum_{\upsilon=1}^{N_r}\sum_{k=0}^{M-1}\sum_{t=1}^{N-K}\left\|\sum_{n=0}^{+\infty}\frac{(j\omega)^n}{2^n n!}\boldsymbol{f}_N^H\left(\frac{2\pi i_t}{N}\right)\boldsymbol{D}^n\boldsymbol{y}_\upsilon(k)\right\|^2 \quad (14)$$

In order to simplify this equation, we set

$$a_{i_t,n} = \boldsymbol{f}_N^H\left(\frac{2\pi i_t}{N}\right)\boldsymbol{D}^n\boldsymbol{y}_\upsilon(k) \quad (15)$$

Substituting (15) into (14), and replacing the infinite summations with finite ones, (14) can be rewritten as

$$\boldsymbol{J}_{2Q}(\omega) = \frac{1}{M}\sum_{\upsilon=1}^{N_r}\sum_{k=0}^{M-1}\sum_{t=1}^{N-K}\sum_{n=0}^{Q}\sum_{m=0}^{Q}\frac{(j\omega)^n((j\omega)^m)^*}{2^{n+m}n!m!}a_{i_t,n}a_{i_t,m}^*$$
$$=\frac{1}{M}\sum_{\upsilon=1}^{N_r}\sum_{k=0}^{M-1}\sum_{n=0}^{Q}\sum_{m=0}^{Q}\frac{(-1)^m(j\omega)^{n+m}}{2^{n+m}n!m!}\sum_{t=1}^{N-K}a_{i_t,n}a_{i_t,m}^* \quad (16)$$

It is obvious that $J(\omega) = \lim_{Q\to\infty} J_{2Q}(\omega)$. For simplicity, we can rewrite (16) as follows:

$$\boldsymbol{J}_{2Q}(\omega) = \sum_{l=0}^{2Q}\left(\frac{j}{2}\right)^l\omega^l\sum_{m=0}^{l}\frac{(-1)^m}{(l-m)!m!} \times\sum_{t=1}^{N-K}\left\{\frac{1}{M}\sum_{\upsilon=1}^{N_r}\sum_{k=0}^{M-1}a_{i_t,l-m}a_{i_t,m}^*\right\} = \sum_{l=0}^{2Q}p_l\omega^l \quad (17)$$

where the polynomial coefficients p_l are given by

$$p_l = \left(\frac{j}{2}\right)^l\sum_{m=0}^{l}\frac{(-1)^m}{(l-m)!m!}\sum_{t=1}^{N-K}\left\{\frac{1}{M}\sum_{\upsilon=1}^{N_r}\sum_{k=0}^{M-1}a_{i_t,l-m}a_{i_t,m}^*\right\} \quad (18)$$

keeping in mind that $a_{i,l}(k) = 0$ if $l > Q$.

From (17), we can see that $\boldsymbol{J}_{2Q}(\omega)$ is a polynomial of the real variable ω of degree $2Q$, and the polynomial coefficients p_l can be easily shown to be real. Hence, the value ω which can minimise (17) is the CFO estimate. Because (17) is a polynomial of the variable ω, we can estimate the CFO by computing its derivative with respect to ω and setting it to zero. The derivative is

$$\frac{\partial J_{2Q}(\omega)}{\partial\omega} = \sum_{l=1}^{2Q} l\,p_l\omega^{l-1} = 0 \quad (19)$$

In general, there will be up to $(2Q-1)$ roots. The estimated carrier offset $\hat{\omega}_0$ is the one, which once substituted in (17), gives the minimum of the cost function. Besides the polynomial rooting method, there is another way to compute the CFO estimate as follows. We can evaluate (17) for different values of the angle ω over an interval of interest. The estimated carrier offset $\hat{\omega}_0$ will be the value of ω for which $\boldsymbol{J}_{2Q}(\omega)$ is minimum. This search method is the same as the one we use to find the minimum of (6).

4 Mean squared error (MSE) performance analysis

In this Section, we analyse the perturbation, which is caused by additive white Gaussian noise, to the performance of the proposed estimator. The measure of the performance is the theoretical mean squared error (MSE) of the CFO estimate. A closed-form function is derived to reveal the relationship between the system parameters and the performance of the estimator. Since the MSE of the proposed method varies with the value of Q, we will make use of the cost function (6) to derive a deterministic theoretical MSE in order to benchmark the simulation results of the low-cost estimation algorithm.

First, we write the received block (5) as $\boldsymbol{y}_\upsilon(k) = \tilde{\boldsymbol{y}}_\upsilon(k) + \boldsymbol{n}_\upsilon(k)$, where $\tilde{\boldsymbol{y}}_\upsilon(k)$ represents the information part, and $\boldsymbol{n}_\upsilon(k)$ represents the perturbation part, which is assumed to be independent and identically distributed (i.i.d.) circular complex-valued additive noise of zero-mean. Based on perturbation to the CFO estimates and the linearisation of the cost function using Newton's method, we can write [20]

$$\Delta\omega = -\frac{\left.\dfrac{\partial J(\omega)}{\partial\omega}\right|_{\omega=\omega_0}}{\left.\dfrac{\partial^2 J(\omega)}{\partial\omega^2}\right|_{\omega=\omega_0}} \quad (20)$$

where $\Delta\omega = \hat{\omega}_0 - \omega_0$. The numerator of (20) is the first-order derivative of the cost function, which can be expressed as [20]

$$\left.\frac{\partial J(\omega)}{\partial\omega}\right|_{\omega=\omega_0} = \frac{2}{M}\Im\left\{\sum_{\upsilon=1}^{N_r}\sum_{k=0}^{M-1}\sum_{t=1}^{N-K}\left(\boldsymbol{f}_N^H\left(\frac{2\pi i_t}{N}\right)\boldsymbol{D}_N^{-1}(\omega_0) \times\tilde{\boldsymbol{D}}_N\tilde{\boldsymbol{y}}_\upsilon(k)\,\boldsymbol{n}_\upsilon^H(k)\boldsymbol{D}_N(\omega_0)\boldsymbol{f}_N\left(\frac{2\pi i_t}{N}\right)\right)\right\} \quad (21)$$

where $\tilde{\boldsymbol{D}}_N = diag(0, 1, \ldots, N-1)$. The denominator can be obtained by differentiating (21) with respect to ω

$$\left.\frac{\partial^2 J(\omega)}{\partial\omega^2}\right|_{\omega=\omega_0} = \frac{2}{M}\left\{\sum_{\upsilon=1}^{N_r}\sum_{k=0}^{M-1}\sum_{t=1}^{N-K}\left(\boldsymbol{f}_N^H\left(\frac{2\pi i_t}{N}\right)\boldsymbol{D}_N^{-1}(\omega_0) \times\tilde{\boldsymbol{D}}_N\tilde{\boldsymbol{y}}_\upsilon(k)\tilde{\boldsymbol{y}}_\upsilon^H(k)\tilde{\boldsymbol{D}}_N\boldsymbol{D}_N(\omega_0)\boldsymbol{f}_N\left(\frac{2\pi i_t}{N}\right)\right)\right\}$$
$$-\frac{2}{M}\Re\left\{\sum_{\upsilon=1}^{N_r}\sum_{k=0}^{M-1}\sum_{t=1}^{N-K}\left(\boldsymbol{f}_N^H\left(\frac{2\pi i_t}{N}\right)\boldsymbol{D}_N^{-1}(\omega_0) \times\tilde{\boldsymbol{D}}_N^2\tilde{\boldsymbol{y}}_\upsilon(k)\boldsymbol{n}_\upsilon^H(k)\boldsymbol{D}_N(\omega_0)\boldsymbol{f}_N\left(\frac{2\pi i_t}{N}\right)\right)\right\} \quad (22)$$

At high SNR, $\|\boldsymbol{n}_\upsilon\| \ll \|\tilde{\boldsymbol{y}}_\upsilon\|$, the second term of (22), which is due to the perturbation, is much smaller than the first term and can be neglected. So, the perturbation can be simplified as

$$\Delta\omega = \frac{\Im\left\{\sum_{\upsilon=1}^{N_r}\sum_{k=0}^{M-1}\sum_{t=1}^{N-K}\left(\boldsymbol{f}_N^H\left(\frac{2\pi i_t}{N}\right)\boldsymbol{D}_N^{-1}(\omega_0)\tilde{\boldsymbol{D}}_N\tilde{\boldsymbol{y}}_\upsilon(k)\boldsymbol{n}_\upsilon^H(k)\boldsymbol{D}_N(\omega_0)\boldsymbol{f}_N\left(\frac{2\pi i_t}{N}\right)\right)\right\}}{\sum_{\upsilon=1}^{N_r}\sum_{k=0}^{M-1}\sum_{t=1}^{N-K}\left(\boldsymbol{f}_N^H\left(\frac{2\pi i_t}{N}\right)\boldsymbol{D}_N^{-1}(\omega_0)\tilde{\boldsymbol{D}}_N\tilde{\boldsymbol{y}}_\upsilon(k)\tilde{\boldsymbol{y}}_\upsilon^H(k)\tilde{\boldsymbol{D}}_N\boldsymbol{D}_N(\omega_0)\boldsymbol{f}_N\left(\frac{2\pi i_t}{N}\right)\right)} \tag{23}$$

Then, we compute the expectation of the error $(\Delta\omega)^2$, which is the theoretical MSE of the estimator. For simplicity, let us set

$$\alpha_{t,k}^\upsilon = \boldsymbol{f}_N^H\left(\frac{2\pi i_t}{N}\right)\boldsymbol{D}_N^{-1}(\omega_0)\tilde{\boldsymbol{D}}_N\tilde{\boldsymbol{y}}_\upsilon(k) \tag{24}$$

Then, the theoretical MSE is given by [20]

$$E[(\Delta\omega)^2] = E\left\{\frac{\sum_{\upsilon,\bar{\upsilon}=1}^{N_r}\sum_{k,\bar{k}=0}^{M-1}\sum_{t,\bar{t}=1}^{N-K}\left(\Im\left(\alpha_{t,k}^\upsilon\boldsymbol{n}_\upsilon^H(k)\boldsymbol{D}_N^{-1}(\omega_0)\boldsymbol{f}_N\left(\frac{2\pi i_t}{N}\right)\right)\times\Im\left(\boldsymbol{f}_N^H\left(\frac{2\pi i_{\bar{t}}}{N}\right)\boldsymbol{D}_N(\omega_0)\boldsymbol{n}_{\bar{\upsilon}}(k)(\alpha_{\bar{t},\bar{k}}^{\bar{\upsilon}})^*\right)\right)}{2\sum_{\upsilon,\bar{\upsilon}=1}^{N_r}\sum_{k,\bar{k}=0}^{M-1}\sum_{t,\bar{t}=1}^{N-K}(\alpha_{t,k}^\upsilon(\alpha_{t,k}^\upsilon)^*\alpha_{\bar{t},\bar{k}}^{\bar{\upsilon}}(\alpha_{\bar{t},\bar{k}}^{\bar{\upsilon}})^*)}\right\} \tag{25}$$

We compute the numerator and the denominator of (25) separately. The numerator of (25) is

$$\begin{aligned}E[Num] &= \frac{1}{2}E\left\{\sum_{\upsilon,\bar{\upsilon}=1}^{N_r}\sum_{k,\bar{k}=0}^{M-1}\sum_{t,\bar{t}=1}^{N-K}\left(\alpha_{t,k}^\upsilon\begin{pmatrix}\boldsymbol{n}_\upsilon^H(k)\boldsymbol{D}_N^{-1}(\omega_0)\boldsymbol{f}_N\left(\frac{2\pi i_t}{N}\right)\\ \cdot\boldsymbol{f}_N^H\left(\frac{2\pi i_{\bar{t}}}{N}\right)\boldsymbol{D}_N(\omega_0)\boldsymbol{n}_{\bar{\upsilon}}(k)\end{pmatrix}\times(\alpha_{\bar{t},\bar{k}}^{\bar{\upsilon}})^*\right)\right\}\\ &= \frac{\sigma_{\boldsymbol{n}}^2}{2}\left\{\sum_{\upsilon=1}^{N_r}\sum_{k=0}^{M-1}\sum_{t=1}^{N-K}\left(\alpha_{t,k}^\upsilon(\alpha_{t,k}^\upsilon)^*\right)\right\}\end{aligned} \tag{26}$$

and the denominator of (25), which is independent from noise, can be expressed as

$$\begin{aligned}Den &= 2\sum_{\upsilon,\bar{\upsilon}=1}^{N_r}\sum_{k,\bar{k}=0}^{M-1}\sum_{t,\bar{t}=1}^{N-K}(\alpha_{t,k}^\upsilon(\alpha_{t,k}^\upsilon)^*\alpha_{\bar{t},\bar{k}}^{\bar{\upsilon}}(\alpha_{\bar{t},\bar{k}}^{\bar{\upsilon}})^*)\\ &= \left(\sum_{\upsilon=1}^{N_r}\sum_{k=0}^{M-1}\sum_{t=1}^{N-K}\alpha_{t,k}^\upsilon(\alpha_{t,k}^\upsilon)^*\right)^2\end{aligned} \tag{27}$$

Then (25) is simplified as

$$E[(\Delta\omega)^2] = \frac{\sigma_{\boldsymbol{n}}^2}{2\sum_{\upsilon=1}^{N_r}\sum_{k=0}^{M-1}\sum_{t=1}^{N-K}|\alpha_{t,k}^\upsilon|^2} \tag{28}$$

Then, substituting the expression of the received block in (28), leads to

$$E[(\Delta\omega)^2] = \frac{\sigma_{\boldsymbol{n}}^2}{2\sum_{\upsilon=1}^{N_r}\sum_{k=0}^{M-1}\sum_{t=1}^{N-K}\left|\boldsymbol{f}_N^H\left(\frac{2\pi i_t}{N}\right)\tilde{\boldsymbol{D}}_N\boldsymbol{g}_\upsilon(k)\right|^2} \tag{29}$$

where $\boldsymbol{g}_\upsilon(k) = \sum_{\mu=1}^{N_t}\tilde{\boldsymbol{H}}^{(\upsilon,\mu)}\boldsymbol{F}_N^H\boldsymbol{T}_{SC}\boldsymbol{s}_\mu(k)$.

5 Computational complexity

In this section, we show how the proposed method can achieve a significant reduction in complexity as compared to the method in [17]. In our calculations, we took into account that some matrices are diagonal and that the cost of multiplying matrices and vectors together can be made small if the multiplication is done following a certain order. Let us first discuss the computational cost when we use (6) to estimate the CFO. From (8), we can see that the computational complexity of the correlation matrix $\widehat{\boldsymbol{R}}_{\boldsymbol{yy}}$ is $O(N^2MN_r)$. The computation of (6) requires $O(N^2(N-K))$ complex multiplications for each value of ω. Since $N-K \ll N$, the complexity of this part becomes only $O(N^2)$. We use a search method to find the minimum of the derivative of (6). Assuming that there are S samples of ω, then the search process requires a complexity of about $O(N^2S)$ complex multiplications. So the total computational complexity of the first method is $O(N^2MN_r)+O(N^2S)$, which is quite high. In a typical application, S can be of the order of a few thousands depending on whether a coarse or fine (more accurate estimation) search is required.

For the proposed estimator, we compute the polynomial coefficients using (15), (18) and (19). The computational complexity is $O(NMN_r(N-K)Q^2) \approx O(NMN_r)$ since $N-K \ll N$, $N \gg Q$ and the index $l \leq Q$ in (18) as $a_{i,l}(k)=0$ for $l > Q$. However, the complexity of the rooting method depends only on the polynomial degree, i.e. $O(Q^2)$ [22] real multiplications, which can be neglected since Q is less or equal to four in our work. However, if we also use a search method instead, then the complexity will be $O((2Q+1)S) \approx O(S)$. Therefore, the computational complexity of the proposed method is about $O(NMN_r)+O(S)$ when a search is used, or only $O(NMN_r)$ when a polynomial rooting method is used.

In the original method, the computation of the correlation matrix $\widehat{\boldsymbol{R}}_{\boldsymbol{yy}}$ is inevitable, whereas for the proposed method, this cost has been included in the computation of the polynomial coefficients. Compared with the original estimator, the complexity of the proposed estimator is at least one order lower than the original one. For easy reference, all the results are summarised in Table 1.

6 Simulation results

In this Section, we present the simulation results of the proposed CFO estimator and compare it to the performance of a high-cost method from the literature under a

Table 1: Computational cost of the different methods

Method	Original cost function (6)	Proposed method: search	Proposed method: polynomial rooting
Total cost	$O(N^2MN_r)+O(N^2S)$ complex multiplications	$O(NMN_r)$ compex multiplications $+O(S)$ real multiplications	$O(NMN_r)$ complex multiplications

Rayleigh-fading channel. In all simulations, we use QPSK modulation. The MSE is defined as

$$MSE = \frac{1}{N_S}\sum_{i=1}^{N_S}\left(\frac{\hat{\omega}_0 - \omega_0}{\varpi}\right)^2 \quad (30)$$

where $N_s = 100$ represents the total number of Monte Carlo trials, $\varpi = 2\pi/N$ is the subcarrier spacing and ω_0 is the true CFO. We assume that the channel order is $L = 3$ and choose $N = 32$ (the number of symbols in each transmitted block after null subcarrier insertion). The null subcarriers are inserted into the blocks with distinct spacing. The estimated correlation matrix $\widehat{\boldsymbol{R}}_{yy}$ is averaged across $M = 320$ blocks.

6.1 Single-user MIMO multicarrier system (MIMO-OFDM)

In the simulations, we choose $K = 27$ to be the number of information symbols in each transmitted block. Figure 2 shows the results for a different number of antennas with the proposed method for $Q = 2$. At low SNR, the MSE decays rapidly. However, at high SNR, there is an error floor which is due to the fact that the truncation error is dominant as compared to the additive white Gaussian noise and, therefore, even if we increase the SNR, the performance of the estimator does not improve. To overcome this, we increase the order in Taylor's series as this leads to a smaller truncation error and hence a better performance for the estimator (see Fig. 3 where SNR goes all the way to 40 dB).

Figure 3 compares the performance of the high-cost method with the proposed one when the number of antennas $N_t = N_r = 3$. We can notice that for $Q > 2$, the proposed method gives comparable results to the one in [17].

Figure 4 shows the bit error rate (BER) of MIMO-OFDM systems when $N_t = N_r = 1$. As compared to the high-cost estimator [17], the proposed method gives comparable results when $Q = 2$.

6.2 Multi-user MIMO multicarrier system (MIMO MC-CDMA)

In the simulation, we select $K = 24$, because K is also the order of the Hadamard matrix, which must be an integer multiple of two. We also insert $L + 2$ null subcarriers to the information blocks and the remaining will be inserted at the end of the blocks. When we construct the cost function, only the $L + 2$ distinct-spacing null subcarriers are used.

Figure 5 shows the performance of the proposed method when the true carrier frequency offset $\omega_0 \in [-0.5\varpi\ 0.5\varpi]$. We can notice that when $Q = 1$, the low-cost method does not estimate the CFO accurately when ω_0 is relatively large.

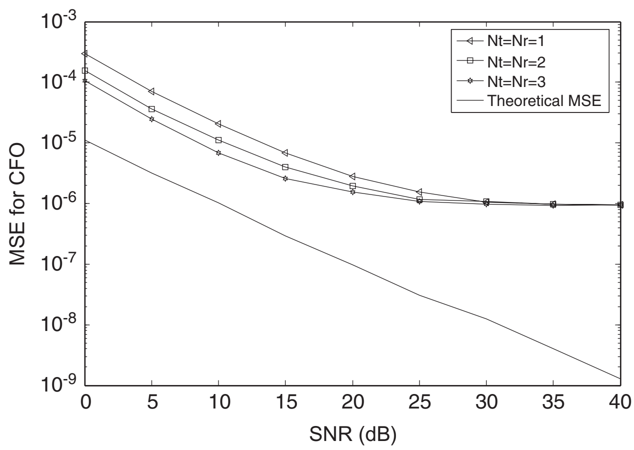

Fig. 2 *MSE of CFO estimation for MIMO-OFDM system using the proposed method for $Q = 2$, $\omega_0 = 0.1\varpi$ and number of transmit/receive antennas $N_t = N_r = 1, 2, 3$*

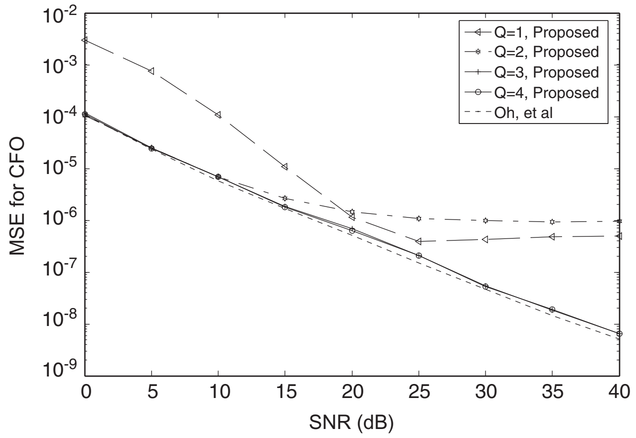

Fig. 3 *MSE of CFO estimation for MIMO-OFDM system using both the proposed and Oh et al. [17] methods, $N_t = N_r = 3$ and $\omega_0 = 0.1\varpi$*

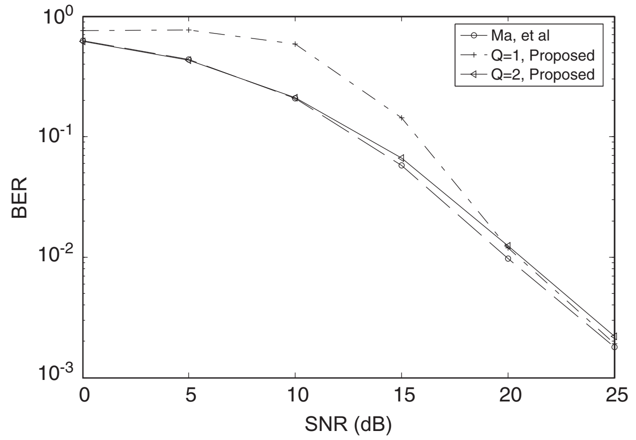

Fig. 4 *BER for MIMO-OFDM system using both the proposed and Oh et al. [17] methods, $N_t = N_r = 1$ and $\omega_0 = 0.1\varpi$*

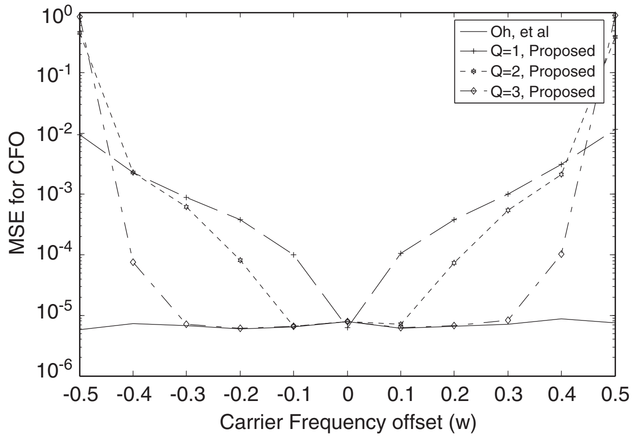

Fig. 5 *MSE of CFO estimation for MIMO MC-CDMA system using both the proposed and Oh et al. [17] methods, SNR = 10, $N_t = N_r = 3$, $N_u = 8$ and $\omega_0 \in [-0.5\varpi\ 0.5\varpi]$*

If $\omega_0 \in [-0.1\varpi\ 0.1\varpi]$, then the performance of the proposed method with $Q=2$ is as good as the high-cost one [17]. It is clear that the larger the CFO, the more terms of the Taylor's series are required for estimation.

Figure 6 shows the performance when the CFO ω_0 is varied uniformly over the interval $[-0.125\varpi\ 0.125\varpi]$. Similarly to Fig. 3, better performance is obtained when $Q > 1$.

Figure 7 gives the performance of the proposed method for a different number of antennas. It shows that the performance of the estimator only depends on the number of receive antennas irrespective of the number of transmit antennas.

Figure 8 shows the MSE performance of the proposed method against the number of antennas. We can find that the performance for $Q=3$ is not significantly better than $Q=2$, which means that when $\omega_0 \in [-0.1\varpi\ 0.1\varpi]$, $Q=2$ is adequate.

From the simulation results, we can notice that the performance of MIMO multi-carrier systems is better than SISO systems. This is because in an SISO system, $\boldsymbol{R}_{yy}$ is estimated by

$$\widehat{\boldsymbol{R}}_{yy} = \frac{1}{M}\sum_{k=0}^{M-1} \boldsymbol{y}(k)\boldsymbol{y}^H(k)$$

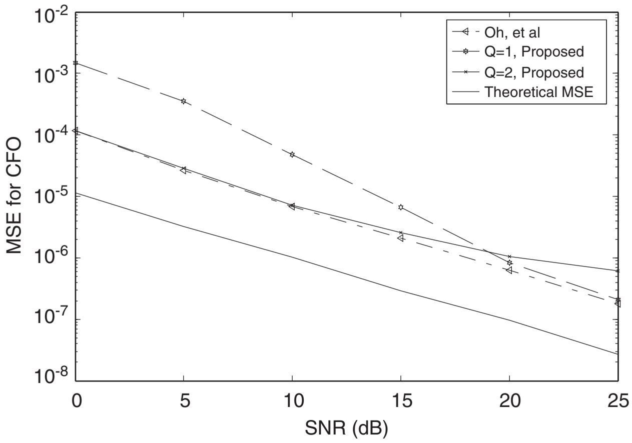

Fig. 6 *MSE of CFO estimation for MIMO MC-CDMA system using both the proposed and Oh et al. [17] methods, $N_t = N_r = 3$ and $\omega_0 \in [-0.125\varpi\ 0.125\varpi]$*

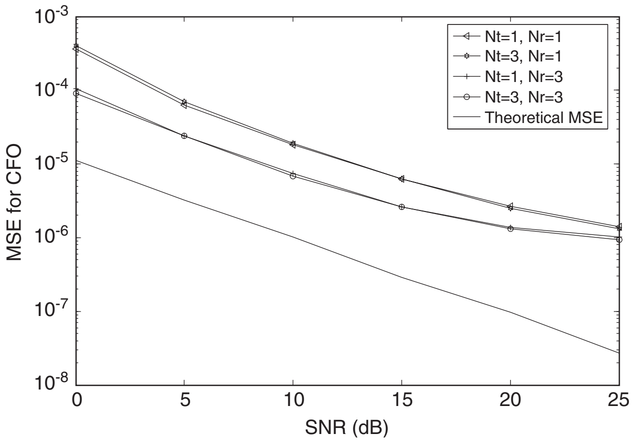

Fig. 7 *MSE of CFO estimation for MIMO MC-CDMA system using the proposed method for $Q=2$, $\omega_0 = 0.1\varpi$, and different numbers of transmit and receive antennas*

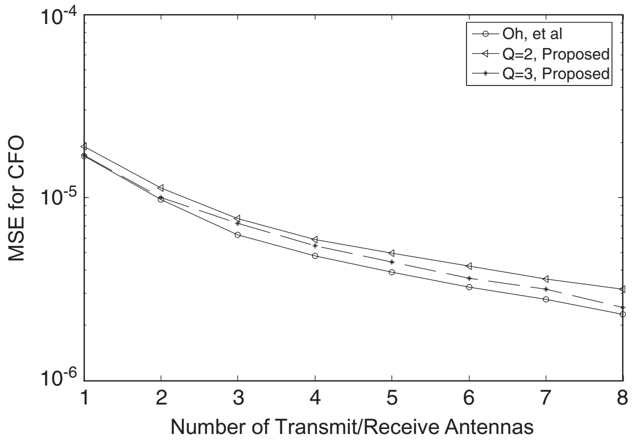

Fig. 8 *MSE of CFO estimation for MIMO MC-CDMA system using both the proposed and Oh et al. [17] methods, $\omega_0 = 0.1\varpi$ and SNR = 10*

while in MIMO systems, $\boldsymbol{R}_{yy}$ is estimated as

$$\widehat{\boldsymbol{R}}_{yy} = \frac{1}{MN_r}\sum_{\upsilon=1}^{N_r}\sum_{k=0}^{M-1} \boldsymbol{y}_\upsilon(k)\boldsymbol{y}_\upsilon^H(k)$$

which is equal to the averaging across N_rM blocks. Therefore, the performance of the system improves when the number of antennas increases.

7 Conclusions

In this paper, we have proposed a low-cost blind method to estimate residual carrier frequency offset for down-link MIMO multicarrier systems. Based on the null subcarriers concept and Taylor's series expansion, we built a cost function whose minimisation leads to the CFO estimate at a relatively low computational complexity cost. In the simulation section, we presented two cases of a MIMO multicarrier system, namely MIMO-OFDM and MIMO MC-CDMA, with different numbers of users and antennas, which show that the performance of the proposed low-cost estimator is comparable with a high-cost method from the literature [17]. We also derived the theoretical MSE of the proposed estimator and compared it with the results obtained through simulations.

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