Security-Aware Cross-Layer Resource Allocation for Heterogeneous Wireless Networks

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Abstract

In this paper, a security-aware energy-efficient resource allocation is modeled as a fractional programming problem for heterogeneous multi-homing networks. The security-aware resource allocation is subject to the average packet delay, the average packet dropping probability, and the total available power consumption. In order to guarantee the packet-level quality of service (QoS), first, the average packet delay and the average packet dropping probability requirement at the link layer for each mobile terminal (MT) are transformed as a minimum secrecy rate constraint at the physical layer. Then, the maximization problem is transformed into a convex problem with an equivalent epigraph form. A security-aware energy-efficient resource allocation algorithm is proposed leveraging dual decomposition method and bi-section search method. Finally, a heuristic security-aware resource allocation algorithm is proposed to serve as a benchmark. Simulation results demonstrate that the proposed security-aware energy-efficient resource allocation algorithm not only improves the secrecy energy efficiency and throughput, but also guarantees the packet-level QoS.

Index Terms

Heterogeneous networks, cross-layer resource allocation, tri-convex programming, packet delay, packet dropping probability.

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I. INTRODUCTION

The future wireless communication networks are expected to aggregate the spectrum resource of heterogeneous wireless networks to support high-quality communication services for mobile terminals (MTs) [1]. In 5G, there exist some promising candidate technologies, e.g., heterogeneous wireless network, non-orthogonal multiple access (NOMA), and massive multiple input and multiple output (MIMO) [2–5]. With the advance of multi-homing technology, each MT is able to connect to available wireless networks via different radio interfaces, and the data stream for each MT is split into multiple sub-streams, which are transmitted simultaneously over different wireless networks [6, 7].

In heterogeneous multi-homing network, each MT has the requirement for secure communication over all available radio interfaces. Therefore, the issue of security becomes critical. Especially, physical layer security (PLS) is an important issue against attacks over heterogeneous wireless medium. Based on information-theoretic approaches, PLS can achieve secrecy via channel coding and advanced signal processing techniques. In comparison with conventional cryptography-based approaches, the advantage of PLS is to achieve perfect secrecy [8]. The performance of PLS can be characterized by the secrecy transmission rate defined as the nonzero transmission rate at which perfectly secure information can be transmitted with arbitrarily small error probability. As a result, PLS is an excellent candidate for heterogeneous multi-homing networks providing both security and quality of experience [9–11].

In order to utilize the radio resource efficiently, the resource allocation algorithms can be divided into three categories. The first category is the resource allocation problem based on single access technology for heterogeneous wireless networks [12–18]. In this scenario, each MT can connect with only one base station (BS). The second category is the resource allocation for heterogeneous wireless networks with multi-homing technology [19–24], which exploits the disparity among different radio interfaces for each MT [18]. The third category is the security-aware resource allocation for wireless network which aims for not only reliable but also secure communications established among the legitimate parties in the presence of eavesdroppers [25–28].

The optimization objectives in the first category include, e.g., spectral efficiency [12, 14], energy efficiency [15, 16], and cross-layer design [13, 17, 18]. In [12], the resource allocation
supporting multiple quality of service (QoS) classes for heterogeneous femtocell networks was investigated with a greedy algorithm. In order to maximize the spectral efficiency, a joint subchannel and power allocation algorithm was proposed for heterogeneous femtocell networks in [14]. For the energy-efficient resource allocation, a power minimization algorithm for interference mitigation was proposed in [15] for heterogeneous femtocell networks. In [16], a joint subcarrier assignment, power allocation and base station selection for heterogeneous femtocell networks was proposed. For the cross-layer scheme, a cross-layer resource allocation algorithm was presented [17], taking both the data-link layer and the physical layer into account. In [18], a joint resource allocation and call admission control problem for heterogeneous femtocell networks was solved by semi-Markov decision process and game theory. A joint resource allocation and call admission control algorithm was proposed for orthogonal frequency division multiple access (OFDMA) femtocell networks in [13].

For heterogeneous wireless networks with the multi-homing technology, an uplink decentralized joint bandwidth and power allocation algorithm was proposed in [19] utilizing the dual decomposition method. In [20], an uplink optimal resource allocation algorithm with delay-constraint service and best-effort service was proposed. In addition, resource allocation considering max-min fairness and proportional fairness were, respectively, studied in [21, 22]. On the other hand, an optimal joint bandwidth and power allocation algorithm maximizing the energy efficiency was proposed in [23] through a parameter-free approach. In [24], a joint bandwidth and power allocation scheme was also proposed utilizing dual decomposition.

For security-aware resource allocation in wireless network, the security-aware max-min joint subchannel and power allocation was designed for multiuser OFDMA system in [25] with one eavesdropper. For OFDMA network, security-aware resource allocation algorithm with one source and multiple untrusted MTs in the presence of a friendly jammer was proposed in [26]. In [27], the secret key agreement for MTs connected through parallel fading channels was considered, and the security-aware resource allocation with full and partial eavesdropper’s channel state information (CSI) was designed therein. In [28], secure resource allocations for OFDMA two-way relay wireless sensor networks were investigated without and with cooperative jamming is investigated, respectively.

One limitation of the above works for resource allocation in heterogeneous multi-homing networks is that they ignore the impact of the packet delay and the packet dropping probability
on the secrecy transmission. In order to guarantee the security-aware packet-level QoS, the queue buffer occupancy at the link layer should also be analyzed, which affects the resource allocation at the physical layer. In this paper, we investigate the security-aware energy efficiency maximization problem subject to the packet-level QoS for heterogeneous multi-homing networks. We summarize the contributions of this paper as follows: (i) An energy-efficient packet-QoS-aware resource allocation problem is formulated as a fractional programming to jointly allocate subchannels, power and radio interfaces to each MT, subject to the average packet delay constraint, the average packet dropping probability constraint, and the total available power; (ii) we transform the packet-level QoS into the required minimum secrecy transmission rate based on the packet arrival rate; (iii) a resource allocation algorithm is proposed using Lagrangian dual decomposition method, and a heuristic security-aware subchannel and power allocation algorithm is also developed; and (iv) simulation results reveal that the proposed security-aware energy-efficient resource allocation algorithm not only improves the energy efficiency, but also guarantees the packet-level QoS.

The rest of the paper is organized as follows. The system description and optimization formulation are given in Section II. A security-aware cross-layer subchannel and power allocation algorithm is proposed in Section III. A heuristic low-complexity security-aware subchannel and power allocation algorithm is presented in Section IV and Section V provides the computation complexity analysis for the proposed algorithms. Simulation results and conclusions are given in Section VI and Section VII, respectively.

II. SYSTEM MODEL AND OPTIMIZATION PROBLEM FORMULATION

In this section, the system model and the power consumption model are given first. Then, the security-aware energy-efficient resource allocation problem is formulated to ensure the packet-level QoS for heterogeneous multi-homing networks. Finally, the analysis on packet dropping probability and packet delay is presented to simplify the problem.

A. System Description

Consider $\mathcal{N} = \{1, 2, \cdots, N\}$ of wireless networks, which are operated by different wireless technologies and different operators. In each network, there is a set of $\mathcal{S}_n = \{1, 2, \cdots, S_n\}$ BSs for network $n$ in the geographical region. At the physical layer, orthogonal frequency
division multiplexing (OFDM) technology is adopted, and the bandwidth is divided into many orthogonal subchannels. The subchannel set in network $n$ for BS $s$ is $K_{ns} = \{1, 2, \cdots, K_{ns}\}$. $ho_{nsm}^k$ is the subchannel allocation indicator for network $n$ BS $s$ MT $m$ over the $k$th subchannel. If $\rho_{nsm}^k = 1$, the subchannel $k$ at network $n$ BS $s$ is allocated to MT $m$; otherwise, $\rho_{nsm}^k = 0$. In the same network, there are $M = \{1, 2, \cdots, M\}$ MTs in the geographical region and $M_{ns} = \{1, 2, \cdots, M_{ns}\} \in M$ is a subset of MTs, which lie in the coverage area of BS $s$ in network $n$. As shown in Fig. 1, we consider secrecy downlink transmission between the BSs and the associated MTs in their respective coverage region in the presence of one external eavesdropper. The eavesdropper attempts to intercept the legitimate transmission across all the subchannels. Using multiple radio interfaces, each MT can communicate with multiple BSs simultaneously. In this work, we focus on the resource allocation for multi-homing heterogeneous wireless networks. Consequently, we assume that orthogonal spectrum resources are used at different BSs and no interference occurs among different BSs. Additionally, this assumption follows the existing references for heterogeneous wireless networks, e.g., [6, 24]. Consider real-time traffic application, the packet-level QoS for the real-time traffic is packet delay and packet dropping probability. Each MT has real-time traffic flow to transmit via near BSs. In the control center, the incoming packets for each MT are queued in its own buffer [29]. Time is partitioned into time slots, $\mathcal{T} = \{1, 2, \cdots\}$, with equal duration $T_0$.

The eavesdropper aims for wiretapping the transmitted signal within all the data-bearing subchannels. The eavesdropper's channel is assumed to be known by the controller\(^1\). Although we consider only one eavesdropper in this work, the proposed security-aware energy-efficient resource allocation scheme can be applied to the scenario with multiple eavesdroppers with some minor changes. Multi-Eves scenario corresponds to a different achievable secrecy rate formula, which could be referred to the paper [32, 33]. Hence our proposed algorithms should change in what kind of way, which is not significantly different from the current results, but needs to be carefully changed. The reason we considered the one-Eve case is because this is the most simple model in which our formulated problem has not been well understood. Additionally, more general case with multiple Eves will be left for the future work.

\(^1\)Despite of being passive, the eavesdropper, whose channel is still possible to be detected and then known by the controller [30, 31].
A flat-fading model is assumed where each subchannel remains constant within its bandwidth and varies from subchannel to subchannel. $h_{nsm}^k$ is the channel gain of the $k$th subchannel from network $n$ BS $s$ to MT $m$, and $g_{nsk}$ is the channel gain of the $k$th subchannel from network $n$ BS $s$ to the eavesdropper $\forall k \in K_n, \forall n \in N, \forall s \in S_n$, and $\forall m \in M_{ns}$. We assume $h_{nsm}^k$ is perfectly known for resource allocation. At the beginning of each time slot, a centralized controller collects all the required information including CSI and queuing state information (QSI), and then conduct the resource allocation algorithm to select some packets in the queue such that the packet-level QoS is satisfied.

The secrecy transmission rate at the $k$th subchannel for network $n$ BS $s$ to communicate with MT $m$ is

$$R_{nsm}^k = \begin{cases} R_{se}, & \text{if } h_{nsm}^k > g_{nsk} \\ 0, & \text{if } h_{nsm}^k \leq g_{nsk} \end{cases}$$

(1)

and

$$R_{se} = B_{ns} \log_2 \left( 1 + \frac{p_{nsm}^k h_{nsm}^k}{B_{ns} n_0} \right) - B_{ns} \log_2 \left( 1 + \frac{p_{nsm}^k g_{nsk}}{B_{ns} n_0} \right)$$

$$= B_{ns} \log_2 \left( \frac{B_{ns} n_0 + p_{nsm}^k h_{nsm}^k}{B_{ns} n_0 + p_{nsm}^k g_{nsk}} \right)$$

(2)
where $P_{nsm}^k$ is the power allocated to the $k$th subchannel for BS $s$ to communicate with MT $m$ at network $n$, $B_{ns}$ is the bandwidth of each subchannel for network $n$ BS $s$, and $n_0$ is one-sided noise power spectral density.

**B. Power Consumption Model**

The circuit power consumption for each BS consists of two components [34]: the first component is fixed power consumption for each BS, where $P_{RF}$ is the radio frequency power consumption and $P_{BB}$ is the baseband unit power consumption; and the second component is dynamic power consumption, where $\xi$ is the power amplifier efficiency and $\sigma_{\text{feed}}$ is the loss incurred by the antenna feeder. Hence, the total power consumption, $P_{ns}$, for network $n$ BSs is defined as

$$P_{ns} = \sum_{m \in M_{ns}} \sum_{k \in K_{ns}} \rho_{nsm}^k P_{nsm}^k \xi (1 - \sigma_{\text{feed}}) + P_{RF} + P_{BB} (1 - \sigma_{\text{DC}}) (1 - \sigma_{\text{MS}}) (1 - \sigma_{\text{cool}})$$

where $\sigma_{\text{DC}}$, $\sigma_{\text{MS}}$, and $\sigma_{\text{cool}}$ are the losses incurred by DC-DC power supply, main supply and active cooling, respectively.

**C. Packet Dropping Probability and Average Packet Delay**

To analyze the dropping probability and the average packet delay, in this section, we introduce the queuing model for each MT $m$. The service time for each packet at MT $m$, denoted by $X_m$, follows a distribution given by Appendix A. Assume that the queuing buffer is modeled as an M/G/1 system [35], and that the packet arrivals at the MT $m$ follow Poisson process with an average rate of $\lambda_m$ (packets/s). $Q_m(t)$ denotes the number of the packets in the queue buffer for MT $m$ at the beginning of time slot $t$; $\pi_m(t)$ is the number of dropped packets at the beginning of time slot $t$ owing to exceeding the maximum packet delay; and $v_m(t)$ is the packet arrival rate during the time slot $t$. In the work, the video traffic is modeled by a Poisson process. Moreover, the queue length (in number of packets), $Q_m(t + 1)$, for MT $m$ at the beginning of time slot $(t + 1)$ is given as below with the length of each packet denoted by $P_L$ (in bits per packet) [36].

$$Q_m(t + 1) = Q_m(t) - \pi_m(t) - \left[ \frac{T_0 \sum_{n \in N} \sum_{s \in S_n} \sum_{k \in K_{ns}} \rho_{nsm}^k R_{nsm}^k}{P_L} \right] + v_m(t) T_0$$

where $[x]$ rounds the elements of $x$ to the nearest integer less than or equal to $x$. If we use the transmission rate rather than the secrecy rate in (4), the average packet dropping probability and
the average packet delay can not be guaranteed. This is due to the fact that the transmission rate is larger than the secrecy rate. If we adopt the transmission rate, the analysis results for the average packet dropping probability and the average packet delay are smaller than the practical results. In fact, the effective transmission rate, i.e., the secrecy rate, is smaller than the transmission rate with the Shannon capacity equation. Consequently, we adopt the secrecy rate in (4) to analyze the average packet dropping probability and the average packet delay.

The average packet dropping probability for MT $m$ is required not to exceed the maximum packet dropping probability, i.e.,

$$\bar{u}_m \leq u_m, \forall m \in \mathcal{M},$$

where $\bar{u}_m$ is the average packet dropping probability for MT $m$, and $u_m$ is the maximum permissive packet dropping probability for MT $m$.

The average packet delay for MT $m$ is also required to be no larger than the maximum packet delay, i.e.,

$$\bar{d}_m \leq d_m, \forall m \in \mathcal{M},$$

where $\bar{d}_m$ is the average packet delay for MT $m$, and $d_m$ is the maximum packet delay for MT $m$.

Due to the fact that the required average packet delay and average packet dropping probability at the link layer are not directly related to the subchannel and power allocations at the physical layer, we analyze their relationship in the following two subsections.

1) Packet Dropping Probability: At the beginning of each time slot, a packet will be dropped if the packet exceeds the maximum packet delay. The average packet dropping probability, $\bar{\pi}_m$, for MT $m$ is defined as (7) [35].

$$\bar{\pi}_m = \frac{E[\pi_m(t)]}{E[v_m(t)] T_0}.$$  (7)

where $E[\pi_m(t)]$ can be obtained from Eq. (4).

In order to satisfy the average packet dropping probability constraint in (5) and the equation (4), the required minimum secrecy transmission rate of MT $m$ satisfies

$$\sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{S}_n} \sum_{k \in \mathcal{K}_n} \rho^k_{nsm} P^k_{nsm} \geq \alpha'_m,$$  (8)

where $\alpha'_m = P_L (1 - u_m) E[v_m(t)]$. 
2) Average Packet Delay: The average waiting time for an M/G/1 system is defined as below [35].

\[
E\{W_m\} = \frac{E\{V_m(t)\} E\{X_m^2\}}{2\left(1 - E\{V_m(t)\} E\{X_m\}\right)}. \tag{9}
\]

With \(E\{X_m^2\} \geq E^2\{X_m\}\), the lower bound of the average waiting time, \(E\{W_m\}\), is given by:

\[
E\{W_m\} \geq \frac{E\{V_m(t)\} (E^2\{X_m\})}{2\left(1 - E\{V_m(t)\} E\{X_m\}\right)}. \tag{10}
\]

Combining \(\bar{d}_m = E\{W_m\} + E\{X_m\}\), we obtain the upper bound of the average service time as follows:

\[
E\{X_m\} \leq \frac{1 + d_m E\{V_m(t)\} - \sqrt{1 + (d_m E\{V_m(t)\})^2}}{E\{V_m(t)\}}. \tag{11}
\]

After some manipulations, the average delay requirement given by (6) with the equation (4) is transformed into a constraint in terms of the required secrecy transmission rate of MT \(m\) expressed as:

\[
\sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{S}_n} \sum_{k \in \mathcal{K}_{ns}} \rho_{nsm}^k P_{nsm}^k \geq \alpha_m'' \tag{12}
\]

where \(\alpha_m'' = \frac{P_L E\{V_m(t)\}}{1 + d_m E\{V_m(t)\} - \sqrt{1 + (d_m E\{V_m(t)\})^2}}\).

In a sum, combining (8) and (12), an equivalent minimum transmission rate constraint is given by:

\[
\sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{S}_n} \sum_{k \in \mathcal{K}_{ns}} \rho_{nsm}^k P_{nsm}^k \geq \alpha_m \tag{13}
\]

where \(\alpha_m = \max(\alpha_m', \alpha_m'')\).

D. Optimization Formulation

In this paper, we investigate the security-aware resource allocation problem with packet-QoS constraints for heterogeneous multi-homing networks. Based on the aforementioned, the security-aware resource scheduler jointly allocates subchannel and power for different MTs across different radio interfaces, subject to the total power constraint at each BS, the packet dropping probability constraint as well the average packet delay constraint. The subchannel allocation at network \(n\) BS \(s\) should fall in the feasible region, i.e.,

\[
\sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{S}_n} \sum_{m \in \mathcal{M}_n} \rho_{nsm}^k \leq 1, \text{ and } \rho_{nsm}^k \geq 0, \forall n \in \mathcal{N}, s \in \mathcal{S}_n, k \in \mathcal{K}_{ns} \tag{14}
\]
where $\rho_{nsm}^k$ is a continuous variable. The total power consumption, $P_{ns}$, at network $n$ BS $s$ satisfies the maximum power constraint, i.e.,

$$\sum_{m \in M_{ns}} \sum_{k \in K_{ns}} \rho_{nsm}^k P_{nsm}^k \frac{\xi(1-\sigma_{cool})}{(1-\sigma_{DC})(1-\sigma_{MS})(1-\sigma_{cool})} + P_{RF} + P_{BB} \leq P_{Ts}^n, \forall n \in \mathcal{N}, s \in \mathcal{S}_n \tag{15}$$

where $P_{Ts}^n$ is the total available power at network $n$ BS $s$.

Define the secrecy energy efficiency in bits/sec/Watt, denoted by $\eta$, as the ratio of the total achievable secrecy transmission rate over the total power consumption, i.e.,

$$\eta = \frac{\sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{S}_n} \sum_{m \in M_{ns}} \sum_{k \in K_{ns}} \rho_{nsm}^k P_{nsm}^k}{\sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{S}_n} P_{ns}} \tag{16}$$

Consequently, the security-aware energy-efficient subchannel and power allocation problem for the considered downlink multi-homing network is formulated as:

$$\text{OP1} \max_{\rho_{nsm}^k, P_{nsm}^k} \eta \tag{17}$$

s.t. (13), (14), (15), $P_{nsm}^k \geq 0$,

where problem (17) is shown to be a fractional programming.

### III. SECURITY-AWARE CROSS-LAYER RESOURCE ALLOCATION

In this section, we solve the security-aware energy-efficient resource allocation problem (17) with packet-QoS-aware guarantee for heterogeneous multi-homing networks. Firstly, we transform the fractional programming problem (17) into the tri-convex programming with the polynomial form. Then, we adopt the dual decomposition method and the binary search method to obtain the security-aware subchannel and power allocation. Finally, we propose a security-aware subchannel and power allocation algorithm.

#### A. Equivalent Transformation in Epigraph Form

Problem (17) has an equivalent transformation via its epigraph form [37], i.e.,

$$\text{OP2} \max_{\rho_{nsm}^k, P_{nsm}^k} y \tag{18}$$

s.t. $y \leq \eta$, (13), (14), (15)

$$P_{nsm}^k \geq 0, y \geq 0.$$
Such a transformation can guarantee the equal relation of (17) and (18). Collect $\rho_{nsm}^k$ and $P_{nsm}^k$ into two vectors $B$ and $P$. (18) can be analyzed geometrically as an optimization problem in the graph space $\{B, P, y\}$. Consequently, we maximize $y$ over the epigraph $\{B, P, y\}$, where $B$ is the subchannel allocation set, and $P$ is the power allocation set. Additionally, the first constraint in (18) can be transformed into

$$\sum_{n \in N} \sum_{s \in S_n} \sum_{m \in M_n} \sum_{k \in K_{nsm}} \rho_{nsm}^k P_{nsm}^k - y \sum_{n \in N} \sum_{s \in S_n} P_{ns} \geq 0.$$  (19)

where problem (18) is a tri-convex optimization problem\(^2\), which can be proven in appendix B. Since problem (18) is a tri-convex optimization problem, we solve the variables $y$, $P_{nsm}^k$, and $\rho_{nsm}^k$, separately. Firstly, the variables $P_{nsm}^k$, and $\rho_{nsm}^k$ are solved via the dual decomposition method with the fixed variable $y$. Then, $y$ is obtained via binary search method with the given variables $P_{nsm}^k$ and $\rho_{nsm}^k$.

### B. Security-aware Resource Allocation with Packet QoS Constraint

In this section, we solve the security-aware resource allocation problem with packet QoS guarantee for heterogeneous multi-homing networks. Firstly, we obtain the solutions $P_{nsm}^k$ and $\rho_{nsm}^k$ via the dual decomposition method. With the fixed variable $y$, the Lagrangian function for problem (18) is

$$L \left( u, v_{ns}^k, w_{ns}, l_m, P_{nsm}^k \right) = y + \sum_{n \in N} \sum_{s \in S_n} v_{ns}^k f_1 + \sum_{n \in N} \sum_{s \in S_n} w_{ns} f_2 + \sum_{m \in M} l_m f_3 + u f_4$$  (20)

where $v_{ns}^k$, $w_{ns}$, $l_m$, and $u$ are Lagrangian multipliers associated with for (14), (15), (13) and (19), respectively.

The dual function, $h \left( u, v_{ns}^k, w_{ns}, l_m \right)$, is

$$h \left( u, v_{ns}^k, w_{ns}, l_m \right) = \max_{\rho_{nsm}^k, P_{nsm}^k} \left\{ L \left( u, v_{ns}^k, w_{ns}, l_m, \rho_{nsm}, P_{nsm} \right) \right\} \text{ s.t. } \rho_{nsm}^k \geq 0, P_{nsm}^k \geq 0.$$  (21)

Additionally, the dual problem is

$$\text{OP3} \min_{u, v_{ns}^k, w_{ns}, l_m} h \left( u, v_{ns}^k, w_{ns}, l_m \right) \text{ s.t. } u \geq 0, v_{ns}^k \geq 0, w_{ns} \geq 0, l_m \geq 0.$$  (22)

\(^2\)In this work, the tri-convex optimization problem is similar with bi-convex, i.e., the optimization problem is convex with any one variable when fixed the other two variables.
For network $n$ BSs, the Lagrangian function (20) can be simplified to
\[
L_{ns} = y + u \left( \sum_{m \in M_{ns}} \sum_{k \in K_{ns}} \rho_{nsm}^k P_{nsm}^k - y P_{ns} \right) - \sum_{m \in M_{ns}} \sum_{k \in K_{ns}} \rho_{nsm}^k v_{nsm}^k - P_{ns} w_{ns} + l_m \sum_{m \in M_{ns}} \sum_{k \in K_{ns}} \rho_{nsm}^k P_{nsm}^k.
\] (23)

Consequently, each BS could solve its own utility maximization problem, i.e.,
\[
\text{OP4: \max_{\rho_{nsm}^k, P_{nsm}^k} L_{ns}}
\] (24)
\[
\text{s.t. } \rho_{nsm}^k \geq 0, P_{nsm}^k \geq 0.
\]

The optimum power allocation, $P_{nsm}^k$, for fixed values $\rho_{nsm}^k$, $y$, $u$, $v_{nsm}$, $w_{ns}$ and $l_m$ can be calculated with (25) by applying KKT condition on the optimization problem (24).
\[
\frac{\partial L_{ns}}{\partial P_{nsm}^k} = 0.
\] (25)

From (25), we can obtain
\[
P_{nsm}^k = \left[ \sqrt{\frac{b^2 - 4ac}{2a}} - \frac{b}{2a} \right]^+
\] (26)

and
\[
\begin{aligned}
a &= h_{nsm}^k g_{nsk} \\
b &= (h_{nsm}^k + g_{nsk}) B_{ns} n_0 \\
c &= (B_{ns} n_0)^2 - \frac{(h_{nsm}^k - g_{nsk})(B_{ns})^2 n_0}{A \ln 2} \\
A &= \frac{(w_{ns} + uy)}{(u + l_m)} f_\sigma \\
f_\sigma &= \frac{1}{\xi(1-\sigma_{feed})(1-\sigma_{DC})(1-\sigma_{MS})(1-\sigma_{cool})}
\end{aligned}
\] (27)

where $[x]^+$ is a projection of $x$ on the positive orthant.

With the fixed values $P_{nsm}^k$, $y$, $u$, $v_{nsm}$, $w_{ns}$, and $l_m$, the Lagrangian function $L_{ns}$ for network $n$ BSs is linear with respect to the variable $\rho_{nsm}^k$, and $\rho_{nsm}^k$ falls in the interval $[0, 1]$. Hence, when $\partial L_{ns} / \partial \rho_{nsm}^k > 0$, the maximum value can be obtained via $\rho_{nsm}^k = 1$. On the other hand, when $\partial L_{ns} / \partial \rho_{nsm}^k < 0$, the maximum value can be obtained via $\rho_{nsm}^k = 0$.

\[
\frac{\partial L_{ns}}{\partial P_{nsm}^k} = H_{nsm}^k - v_{nsm}^k
\] (28)

and
\[
H_{nsm}^k = R_{nsm}^k (l_m + y) - (w_{ns} + uy) P_{nsm}^k f_\sigma.
\] (29)

Subchannel $k$ at network $n$ BSs is assigned to MT $m$ with the largest $H_{nsm}^k$, that is,
\[
\rho_{nsm}^k = \max_m H_{nsm}^k.
\] (30)
The optimum values $u$, $w_{ns}$ and $l_m$ can be calculated by solving the dual problem (22). For a differentiable dual function, a gradient descent method can be applied to calculate the values for $u$, $w_{ns}$ and $l_m$, i.e.,

$$
\begin{align*}
    w_{ns}(i+1) &= [w_{ns}(i) - \varepsilon_1(i) f_2]^+ \\
    l_m(i+1) &= [l_m(i) - \varepsilon_2(i) f_3]^+ \\
    u(i+1) &= [u(i) - \varepsilon_3(i) f_4]^+
\end{align*}
$$

(31)

where $i$ is the iteration index. Since the subchannel allocation is given in (30), it is not necessary to update the Lagrangian variable $v^k_{ns}$. $\varepsilon_1(i)$, $\varepsilon_2(i)$, and $\varepsilon_3(i)$ are the step sizes of iteration $i$, and the step sizes should satisfy the condition,

$$
\sum_{i=1}^{\infty} \varepsilon_j(i) = \infty, \lim_{i \to \infty} \varepsilon_j(i) = 0, \forall j \in \{1, 2, 3\}.
$$

(32)

where $\varepsilon_j(i)$ can be set with $\frac{1}{i}$.

The gradient of (31) satisfies the Lipchitz continuity condition [38], and the resource allocation $P^k_{nsm}$ and $\rho^k_{nsm}$ in (26) and (30) converges to the optimum solution [38]. The security-aware cross-layer resource allocation can be obtained by algorithm 1 and algorithm 2. Algorithm 1 is the outer loop, and algorithm 2 is the inner loop. In algorithm 1, the variable $y$ is solved based on the binary search method with the given variables $P^k_{nsm}$ and $\rho^k_{nsm}$. If $y \leq \eta$, set $Y_{\text{min}}(j) = y(j)$; else, $Y_{\text{max}}(j) = y(j)$. In algorithm 2, the power allocation and subchannel allocation $P^k_{nsm}$ and $\rho^k_{nsm}$ are obtained via the dual decomposition method with the fixed variable $y$. Additionally, the power allocation and subchannel allocation in algorithm 2 is a recursive algorithm and converges to the optimum solutions by updating the Lagrangian multipliers via (31) with the fixed $y$ [38]. $\varepsilon_y$ is an arbitrarily small positive number, and $I_{\text{max}}$ is the maximum iteration number. $l_m(i)$, $u(i)$ and $w_{ns}(i)$ are the Lagrangian multipliers at the $i$th iteration. $Y_{\text{min}}(j)$, $Y_{\text{max}}(j)$, and $y(j)$ are the variable values at the $i$th iteration, and $y(j-1)$ is the variable value at the $(i-1)$th iteration.

**IV. Heuristic Algorithm**

Although algorithm 1 and algorithm 2 can obtain the security-aware subchannel and power allocation, it has enormous computational complexity. This motivates us to develop a heuristic algorithm for heterogeneous multi-homing networks to reduce the computational complexity. In
Algorithm 1: Security-Aware Cross-Layer Resource Allocation Algorithm

Require: \( u_m, d_m, P_L \) and \( T_0 \).

Ensure: \( y, \rho_{nsm}^k \) and \( P_{nsm}^k \).

1: Initialize \( \varepsilon_y, \alpha_m, Y_{\text{min}}(j) = 0, Y_{\text{max}}(j), y(j) = Y_{\text{max}}(j)/2, \) and \( j = 1 \).
2: repeat
3: Use Algorithm 2 to obtain \( \rho_{nsm}^k \) and \( P_{nsm}^k \).
4: Set \( j = j + 1 \), and update \( Y_{\text{min}}(j), Y_{\text{max}}(j), \) and \( y(j) = (Y_{\text{min}}(j) + Y_{\text{max}}(j))/2. \)
5: if \( |y(j) - y(j - 1)| \leq \varepsilon_y \), then
6: \( y = y(j) \). Output \( y, \rho_{nsm}^k \) and \( P_{nsm}^k \).
7: else
8: Go to step 3.
9: end if
10: until

this section, we design a heuristic algorithm firstly. Then, the computational complexities for the
and heuristic algorithms are analyzed.

The security-aware heuristic algorithm is given in algorithm 3, which contains two stages. At
the first stage, the total power at each BS is allocated equally among different MTs and among
different radio interfaces at each MT, and MT \( m^* \) with the minimum secrecy transmission rate
is selected. Additionally, the \( k^* \)th subchannel in network \( n^* \) BS \( s^* \) with the largest secrecy
transmission rate is selected for MT \( m^* \). The subchannel allocation procedure is repeated until
all MTs satisfy the minimum secrecy transmission rate requirement. At the second stage, the
\( k^* \)th subchannel in network \( n^* \) BS \( s^* \) with the maximum energy efficiency margin is selected
for MT \( m^* \), and the subchannel allocation procedure is repeated until all the subchannels are
allocated or the network energy efficiency can not be decreased. \( R_{nsm}^k \) is the temporary secrecy
transmission rate for network \( n \) BS \( s \) MT \( m \) over the \( k \)th subchannel, \( \eta_{nsm}^k \) is the temporary
network energy efficiency when the subchannel is allocated to network \( n \) BS \( s \) MT \( m \) over the
\( k \)th subchannel, and \( \eta_{nsm}^k \) is the network energy efficiency when the subchannel is allocated to
network \( n \) BS \( s \) MT \( m \) over the \( k \)th subchannel. \( K_{\text{remain}} \) is the remaining vacant subchannel
set. \( R_m \) is the secure transmission rate for MT \( m \).
Algorithm 2 Subchannel and Power Allocation Algorithm with Fixed $y$

Require: $P_{ns}^T$, $B_{ns}$, $\alpha_m$, and $y(j)$.

Ensure: $\rho_{nsm}^k$ and $P_{nsm}^k$.

1: Initialize $l_m(i)$, $u(i)$, $w_{ns}(i)$, $\rho_{nsm}^k$, $P_{nsm}^k$, $I_{\text{max}}$, and $i = 1$.

2: repeat
3: Calculate $\rho_{nsm}^k$, $P_{nsm}^k$, and $R_{nsm}^k$.
4: if $\alpha_m \leq \sum_{n \in N} \sum_{s \in S_n} \sum_{k \in k_{ns}} R_{nsm}^k$ and $i = I_{\text{max}}$ then
5: Go to step 10.
6: else
7: Set $i = i + 1$, and update $w_{ns}(i)$, $u(i)$, and $l_m(i)$. Go to step 3.
8: end if
9: until
10: Output $\rho_{nsm}^k$ and $P_{nsm}^k$.

V. Performance Evaluation

This section presents the simulation results for algorithm, i.e., algorithm 1 combined with algorithm 2. We consider a geographical region that is covered by a macrocell BS and two femtocell BSs. The macrocell BS has a coverage area of 500 m, while each femtocell BS has a coverage area of 21 m. There exists an eavesdropper in heterogeneous multi-homing networks, and the distance from the eavesdropper to the macrocell BS is 900 m. Additionally, the distances from the eavesdropper to the two femtocell BSs are both 922 m, and the distances from the macrocell BS to the two femtocell BSs are both 200 m. The fast fading follows the Rayleigh fading. Due to the overlapped coverage among macrocell BS and two femtocell BSs, three service areas can be distinguished. In the first and second areas, MTs can get service from both macrocell BS and one femtocell BS. In the third service area, MTs can get service only from macrocell BS. There are both 16 subchannels at the physical layer for macrocell and femtocell.

The maximum packet dropping probability is $u_m = 0.01$, and the packet length is $P_L = 1000$ bit. The one-side noise spectral density is $n_0 = -174$ dBm/Hz, and the time slot is $T_0 = 40$ ms. The power consumption parameters are adopted in [34]. In order to compare with the and heuristic algorithms, two benchmark algorithms, i.e., optimal power allocation algorithm and optimal
Algorithm 3 Heuristic Security-Aware Algorithm.

Require: $P_{ns}^T$, $B_{ns}$, $\alpha_m$, $\mathcal{K}_{remain}$, $P_L$, and $T_0$.

Ensure: $\rho_{nsm}^k$ and $P_{nsm}^k$.

1: Initialize $\rho_{nsm}^k$, $R_{nsm}^k$, $P_{nsm}^k$, $\eta_{nsm}^k$, $\eta_t^k$, and $R_{nsm}^k$.

2: repeat

3: Find $m^* = \min_{m \in \mathcal{M}} R_m$ and $(n^*, s^*, k^*) = \max_{(n,s,k) \in \mathcal{N}, \mathcal{S}_n, \mathcal{K}_{ns}} R_{nsm}^k$.

4: if $\alpha_m^* \leq R_m^*$ then

5: Update $\rho_{n^*s^*m^*}^k = 1$, $\mathcal{K}_{remain} = \mathcal{K}_{remain} - (n^*, s^*, k^*)$, and go to step 9.

6: end if

7: until

8: repeat

9: Find $(n^*, s^*, k^*, m^*) = \max_{(n,s,k,m) \in \mathcal{N}, \mathcal{S}_n, \mathcal{K}_{ns}, \mathcal{M}} (\eta_{nsm}^k - \eta_t^k)$.

10: if $\mathcal{K}_{remain} \neq \emptyset$ then

11: Update $\rho_{n^*s^*m^*}^k = 1$, $\mathcal{K}_{remain} = \mathcal{K}_{remain} - (n^*, s^*, k^*)$, and go to step 9.

12: else

13: Go to step 16.

14: end if

15: until

16: Output $\rho_{nsm}^k$ and $P_{nsm}^k$.

subchannel allocation algorithm, are proposed. In the optimal power allocation algorithm, each BS allocates the transmission power to minimize the consumed energy per bit based on the dual decomposition method and the binary search method with the fixed subchannel allocation. In the optimal subchannel allocation algorithm, each BS allocates the subchannels to minimize the consumed energy per bit according to the dual decomposition method and the binary search method with the fixed power allocation. Since the formulated tri-convex programming problem is not a rigorous convex optimization problem, the strictly optimal solution is hardly obtained. Consequently, we use the exhausted search to find the asymptotically optimal solution in a small-scale scenario. Please refer to Fig. 2-Fig. 6 in Section V of the revised paper.

We evaluate the impact of total bandwidth at macrocell BS on energy efficiency and hetero-
geneous networks throughput in Fig. 2 and Fig. 3, respectively. The heterogeneous networks throughput is defined as \( \sum_{n \in N} \sum_{s \in S_n} \sum_{m \in \mathcal{M}_n} \sum_{k \in \mathcal{K}_m} \rho_{nsm}^k R_{nsm} \). The number of MTs in each macrocell and each femtocell are both \( M = 5 \). The total bandwidth for each femtocell BS is 2 MHz, and the total available power for each macrocell BS and each femtocell BS are 3 W and 1W, respectively. Moreover, the arrival rate of video packet is \( 1.5 \times 10^3 \) packets/s, and the maximum packet delay is \( d_m = 30 \) ms. From Fig. 2, we can see that the energy efficiency for the algorithm is largest and that for the heuristic algorithm is smallest. The energy efficiency for four algorithms increase with the total bandwidth at each macrocell BS, which can be explained that more bandwidth resource can improve the energy efficiency. Since the energy efficiency for the optimal power allocation algorithm outperforms that of the optimal subchannel allocation algorithm, the power allocation in the algorithm contributes the most impact to improve the energy efficiency. In Fig. 3, it can be seen that the heterogeneous networks throughput for the optimal subchannel allocation algorithm outperforms that for the algorithm. This is due to the fact that the algorithm improves the energy efficiency at the cost of heterogeneous networks throughput. Additionally, we can see that the energy efficiency and the heterogeneous networks throughput for the proposed algorithm are very close to that of exhausted search algorithm in Fig. 2 and Fig. 3.

We evaluate the impact of packet arrival rate on energy efficiency and heterogeneous networks throughput in Fig. 4 and Fig. 5, respectively. The number of MTs in each macrocell and each femtocell are both \( M = 5 \). The total bandwidth resources for each femtocell BS and each macrocell BS is 2 MHz and 10 MHz, respectively. The maximum packet delay is \( d_m = 30 \) ms. The total available power for each macrocell BS and each femtocell BS are 3 W and 1W, respectively. From Fig. 4 and Fig. 5, we can see that the energy efficiency for the algorithm, the optimal power allocation algorithm, and the optimal subchannel allocation algorithm decrease with the packet arrival rate, while the heterogeneous networks throughput for the algorithm, the optimal power allocation algorithm, and the optimal subchannel allocation algorithm decrease with the packet arrival rate. This is because increasing the packet arrival rate for each MT means increasing the required minimum transmission rate to maintain the target packet dropping rate and packet delay. Additionally, increasing the required minimum transmission rate needs more resources to guarantee the packet-level QoS. Consequently, the energy efficiency and throughput for the algorithm, the optimal power allocation algorithm, and the optimal subchannel allocation algorithm decrease. Since the heterogeneous networks throughput for the heuristic algorithm
Figure 2. Energy efficiency vs. total bandwidth at macrocell BS.

grows with the packet arrival rate, the energy efficiency for heuristic algorithm increases. Additionally, we can see that the energy efficiency and the heterogeneous networks throughput for the proposed algorithm 1 are very close to that of exhausted search algorithm in Fig. 4 and Fig. 5. On the other hand, the exhausted search algorithm sacrifices the throughput to improve the energy efficiency slightly.

We evaluate the impact of maximum average packet delay on energy efficiency and heterogeneous networks throughput in Fig. 6 and Fig. 7, respectively. The number of MTs in each macrocell and each femtocell are both $M = 5$. The total bandwidth resources for each femtocell BS and each macrocell BS is 2 MHz and 10 MHz, respectively. The total available power for each macrocell BS and each femtocell BS are 3 W and 1W, respectively. Moreover, the arrival rate of video packet is $1.5 \times 10^3$ packets/s. In Fig. 6 and Fig. 7, we can see that the energy efficiency for the algorithm, the optimal power allocation algorithm, and the optimal subchannel allocation algorithm increase with the maximum average packet delay, and the heterogeneous networks throughput for the algorithm, the optimal power allocation algorithm, and the optimal subchannel
allocation algorithm grow with the maximum average packet delay. This can be explained that reducing the requirement of packet-level QoS can make more resources to improve the energy efficiency. Although the heterogeneous networks throughput for the optimal subchannel allocation algorithm is largest, it is obtained at the cost of sacrificing the energy efficiency. Since the heterogeneous networks throughput for the heuristic algorithm reduces with the maximum average packet delay, the energy efficiency for heuristic algorithm decreases with the maximum average packet delay.

We evaluate the impact of the number of MTs at macrocell on energy efficiency and heterogeneous networks throughput in Fig. 8 and Fig. 9, respectively. The number of MTs in each femtocell is $M = 5$. The total bandwidth resources for each femtocell BS and each macrocell BS is 2 MHz and 5 MHz, respectively. The total available power for each macrocell BS and each femtocell BS are 3 W and 1W, respectively. Moreover, the arrival rate of video packet is $1.5 \times 10^3$ packets/s, and the maximum packet delay is $d_m = 30$ ms. From Fig. 8 and Fig. 9, we can see that the heterogeneous networks throughput for the four algorithms increase with
the number of MTs at macrocell. On the other hand, the energy efficiency for these algorithms increase with the number of MTs at macrocell. This is due to the fact that the multiuser diversity helps the users with the better channel state information to obtain the resources. Consequently, increasing the number of MTs at macrocell improves the energy efficiency and the spectrum efficiency for heterogeneous multi-homing networks.

We evaluate the impact of the number of MTs at macrocell on computational complexity in Table I. In Fig. 10, we depict the convergence speed for the proposed security-awareness cross-layer resource allocation algorithm. The number of MTs in each femtocell is $M = 5$. The total bandwidth resources for each femtocell BS and each macrocell BS is 2 MHz and 5 MHz, respectively. The total available power for each macrocell BS and each femtocell BS are 3 W and 1W, respectively. Moreover, the arrival rate of video packet is $1.5 \times 10^3$ packets/s, and the maximum packet delay is $d_m = 30$ ms. In table I, we can see that the heuristic algorithm reduces the computational complexity at the cost of sacrificing the performance. On the other hand, the proposed algorithm 1 can fast converge to the stable solution. In Fig. 2-Fig. 10, it can
be concluded that the proposed security-aware subchannel and power allocation algorithm not only improves the energy efficiency for heterogenous networks efficiently, but also obtains the better throughput.

VI. Conclusions

In this paper, we studied the security-aware energy-efficient cross-layer resource allocation problem for heterogeneous multi-homing networks. Each BS adjusted the subchannel assignment
and power allocation to CSI at the physical layer and queueing state information at the link layer. The objective was to maximize the secrecy energy efficiency subject to the average packet delay, the average packet dropping probability, and the total available power constraints. The above optimization problem was modelled as a fractional programming, and an equivalent epigraph form was exploited to transform the factional programming into a tri-convex programming problem. The security-aware resource allocation algorithm was designed leveraging Lagrangian dual decomposition method, and a security-aware heuristic resource allocation algorithm was also proposed to reduce the computation complexity. Simulation results demonstrated that the proposed security-aware subchannel and power allocation scheme can improve the secrecy energy efficiency significantly in comparison with other benchmark schemes.

Figure 6. Energy efficiency vs. maximum average packet delay.
APPENDIX A

The service time, $X_m$, for MT $m$ is determined by the maximum service time among different radio interfaces, i.e.,

$$X_m = \max_{n \in \mathcal{N}, s \in \mathcal{S}_n} \{X_{nsm}\}$$

where $X_{nsm}$ is the service time for network $n$ BS $s$ MT $m$.

Since the service time, $X_{nsm}$, for different radio interfaces are independent, the CDF of service time, $X_m$, is defined by (34).

$$\Pr (X_m \leq T_m) = \Pr \left( \max_{n \in \mathcal{N}, s \in \mathcal{S}_n} \{X_{nsm}\} \leq T_m \right)$$

$$= \prod_{n \in \mathcal{N}, s \in \mathcal{S}_n} \Pr \{X_{nsm} \leq T_m\}$$

(34)

In (34), $\Pr \{X_{nsm} \leq T_m\}$ is the probability of the service time, $X_{nsm}$, and is defined by (35).

$$\Pr (X_{nsm} \leq T_m) = \Pr (X_{nsm} \leq T_m | R_{nsm} \geq r_{nsm}) \Pr (R_{nsm} \geq r_{nsm})$$

$$+ \Pr (X_{nsm} \leq T_m | R_{nsm} < r_{nsm}) \Pr (R_{nsm} < r_{nsm})$$

$$= \Pr (X_{nsm} \leq T_m | R_{nsm} \geq r_{nsm}) \Pr (R_{nsm} \geq r_{nsm})$$

(35)
where $r_{nsm}$ is the target transmission rate for network $n$ BS $s$ MT $m$.

Since $\Pr(X_{nsm} \leq T_m | R_{nsm} \geq r_{nsm})$ and $\Pr(R_{nsm} \geq r_{nsm})$ follow the general distributions, $\Pr(X_{nsm} \leq T_m)$ follows the general distribution.

**APPENDIX B**

Proof: We prove (18) is a tri-convex programming problem with the variables $\rho_{nsm}^k$, $P_{nsm}^k$ and $y$. Define the objective function and the constraints in (18) as the functions $g_1$, $f_1$, $f_2$, $f_3$, and
Figure 9. Heterogeneous networks throughput vs. number of MTs at macrocell.

\[
\begin{align*}
    f_4, \text{ i.e.,} & \\
    g_1 &= y \\
    f_1 &= 1 - \sum_{n \in N} \sum_{s \in S_n} \sum_{m \in M_{ns}} \rho_{nsm}^k \\
    f_2 &= P_{ns}^T - P_{ns} \\
    f_3 &= \sum_{n \in N} \sum_{s \in S_n} \sum_{k \in K_n} \rho_{nsm}^k \alpha_m \\
    f_4 &= \sum_{n \in N} \sum_{s \in S_n} \sum_{m \in M_{ns}} \sum_{k \in K_{nms}} \rho_{nsm}^k \beta_{nsm} - y \sum_{n \in N} \sum_{s \in S_n} P_{ns}
\end{align*}
\]

The second derivatives of \(g_1\), \(f_1\), \(f_2\), \(f_3\) and \(f_4\) with respective to \(\rho_{nsm}^k\) are

\[
\begin{align*}
    \frac{\partial^2 g_1}{\partial (\rho_{nsm}^k)^2} &= 0 \\
    \frac{\partial^2 f_1}{\partial (\rho_{nsm}^k)^2} &= 0 \\
    \frac{\partial^2 f_2}{\partial (\rho_{nsm}^k)^2} &= 0 \\
    \frac{\partial^2 f_3}{\partial (\rho_{nsm}^k)^2} &= 0 \\
    \frac{\partial^2 f_4}{\partial (\rho_{nsm}^k)^2} &= 0
\end{align*}
\]
Given the variables $P_{nk_{nsm}}$ and $y$, the objective function is concave on $\rho_{nsm}^k$, and the constraints are concave according to (37). Hence, (18) is a convex programming problem with the variable $\rho_{nsm}^k$.

The second derivatives of $g_1$, $f_1$, $f_2$, $f_3$ and $f_4$ with respective to $P_{nk_{nsm}}$ are

$$
\begin{align*}
\frac{\partial^2 g_1}{\partial (P_{nk_{nsm}})} &= 0 \\
\frac{\partial^2 f_1}{\partial (P_{nk_{nsm}})} &= 0 \\
\frac{\partial^2 f_2}{\partial (P_{nk_{nsm}})} &= 0 \\
\frac{\partial^2 f_3}{\partial (P_{nk_{nsm}})} &= \frac{\rho_{nsm}^k B_{nk_{nsm}} g_{nk_{nk}}^2}{(B_{nk_{nsm}0} + P_{nk_{nsm}} g_{nk_{nk}})^2 \ln 2} - \frac{\rho_{nsm}^k B_{nk_{nsm}} h_{nk_{nk}}^2}{(B_{nk_{nsm}0} + P_{nk_{nsm}} h_{nk_{nk}})^2 \ln 2} \leq 0 \\
\frac{\partial^2 f_4}{\partial (P_{nk_{nsm}})} &= \frac{\rho_{nsm}^k B_{nk_{nsm}} g_{nk_{nk}}^2}{(B_{nk_{nsm}0} + P_{nk_{nsm}} g_{nk_{nk}})^2 \ln 2} - \frac{\rho_{nsm}^k B_{nk_{nsm}} h_{nk_{nk}}^2}{(B_{nk_{nsm}0} + P_{nk_{nsm}} h_{nk_{nk}})^2 \ln 2} \leq 0
\end{align*}
$$

(38)

Given the variables $\rho_{nsm}^k$ and $y$, the objective function is concave on $P_{nk_{nsm}}$, and the constraints are concave according to (38). Hence, (18) is a convex programming problem with the variable $P_{nk_{nsm}}$. 

Figure 10. Energy efficiency vs. iterative number.
The second derivatives of $g_1, f_1, f_2, f_3$ and $f_4$ with respective to $y$ are

\[
\begin{align*}
\frac{\partial^2 g_1}{\partial y^2} &= 0 \\
\frac{\partial^2 f_1}{\partial y^2} &= 0 \\
\frac{\partial^2 f_2}{\partial y^2} &= 0 \\
\frac{\partial^2 f_3}{\partial y^2} &= 0 \\
\frac{\partial^2 f_4}{\partial y^2} &= 0
\end{align*}
\]

(39)

Given the variables $\rho^k_{nsm}$ and $P^k_{nsm}$, the objective function is concave on $y$, and the constraints are concave according to (39). Hence, (18) is a convex programming problem with the variable $y$. Consequently, problem (18) is a tri-convex programming problem with variables $y$, $\rho^k_{nsm}$ and $P^k_{nsm}$.

REFERENCES


