Energy-Efficient Chance-Constrained Resource Allocation for Multicast Cognitive OFDM Network

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Abstract—In this paper, an energy-efficient resource allocation problem is modeled as a chance-constrained programming for multicast cognitive orthogonal frequency division multiplexing (OFDM) network. The resource allocation is subject to constraints in service quality requirements, total power, and probabilistic interference constraint. The statistic channel state information (CSI) between cognitive-based station (CBS) and primary user (PU) is adopted to compute the interference power at the receiver of PU, and we develop an energy-efficient chance-constrained subcarrier and power allocation algorithm. Support vector machine (SVM) is employed to compute the probabilistic interference constraint. Then, the chance-constrained resource allocation problem is transformed into a deterministic resource allocation problem, and Zoutendijk's method of feasible direction is utilized to solve it. Simulation results demonstrate that the proposed algorithm not only achieves a tradeoff between energy efficiency and satisfaction index, but also guarantees the probabilistic interference constraint very well.

Index Terms—Multicast cognitive OFDM network, energy efficiency, SVM, Zoutendijk's method of feasible direction.

I. INTRODUCTION

COGNITIVE radio (CR) network can solve the spectral resource scarcity problem, where CR user is permitted to access the PU's spectrum by controlling the interference power [1]–[5]. Since video traffic is becoming more and more popular in recent years, the future wireless communication system requires high transmission rate. In addition, OFDM and multicast technologies can further enhance the spectral efficiency [6], [7]. Hence, multicast cognitive OFDM network improve quality of experience (QoE) for CR users greatly.

In multicast cognitive OFDM network, there are some challenges to design the resource allocation algorithms [8]–[11]. For example, the same data is transmitted from the CBS to multiple CR users at the same subcarriers in a multicast group, and it leads to the mismatching data rates for different CR users in the same multicast group, due to their asymmetric channel

gains. With maximizing the expected sum rate, a risk-return model is used to design a distributed joint subcarrier and power allocation algorithm [8]. The multicast model in [8] is usually based on the full buffer traffic and it does not consider the nature of limited traffic. Taking this into account, a distributed resource allocation algorithm, based on the Lagrangian dual decomposition, is proposed [9]. In [10], [11], multiple description coding is combined with multicast cognitive OFDM network, and two heuristic distributed resource allocation algorithms to maximize the weighted sum rate are proposed.

One limitation with the existing radio resource allocation algorithms in [8]–[11] is that they only maximize the spectral efficiency. However, the energy efficiency is very important due to steadily rising energy consumption in the communication network and environmental concerns. On the other hand, the perfect CSI is assumed in [8]–[11], and the interference power at the receiver of PU can be calculated precisely. However, the cooperation between cognitive network (CN) and primary network (PN) is not perfect, which is assumed in unicast cognitive OFDM network [12]. This leads it is difficult to estimate the CSI between CR users and primary base station precisely, and only the statistic CSI between CR users and primary base station can be used. Additionally, PU does not belong to the management of CBS, and channel estimate between CBS and PU increases the control overhead and the management complexity. Hence, we adopt the statistic CSI between CBS and PU to perform the resource allocation algorithm like [12]. Chance-constrained programming is developed by Charnes, and it offers a powerful mean of modeling stochastic wireless network by specifying a confidence level [13]. Since the statistic CSI between CBS and PU is adopted, the subcarrier and power allocation based on the statistic CSI for multicast cognitive OFDM network is casted into a chance-constrained programming problem. Different from the determined resource allocation problem, the probabilistic constraint needs to be calculated.

In this paper, we propose an energy-efficient chance-constrained resource allocation algorithm for multicast OFDM network. Specially, we summarize the contributions of this paper as follows:

(i) An energy-efficient subcarrier and power allocation problem is formulated as a chance-constrained programming for multicast cognitive OFDM network, with minimum required QoS and the probabilistic interference constraints.

(ii) Using the SVM method, the energy-efficient chance-constrained resource allocation problem is transformed into a deterministic optimization problem, which can be solved by Zoutendijk's method of feasible direction.
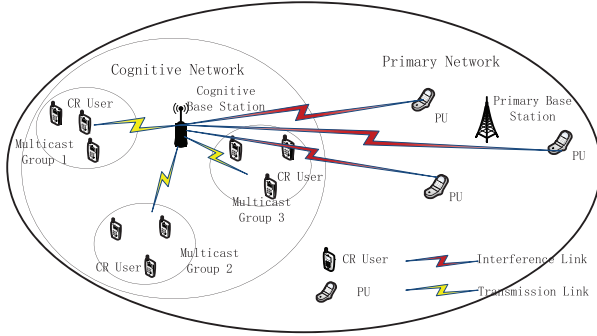
Fig. 1. Multicast cognitive OFDM network.

(iii) The performance of the proposed algorithm is evaluated in comparison with upper bounder $\epsilon \to 0$, lower bounder $\epsilon \to 1$, max-min algorithm and unicast case. Simulation results demonstrate the proposed algorithm not only improves the energy efficiency, the total throughput and QoS satisfaction index, but also satisfy the probabilistic interference constraint.

The rest of the paper is organized as follows. The system model and energy-efficient resource allocation problem are presented in section II and III, respectively. The chance-constrained resource allocation algorithm is in section IV. Section V gives the computational complexity and signaling overhead. Simulation results and conclusions are given in section VI and VII, respectively.

II. SYSTEM MODEL

Consider a primary base station to communicate with N primary users at M OFDM subcarriers, and a CBS is allowed to transmit G downlink traffic flows. The interference temperature model is adopted to guarantee PU communication as long as the total interference power at the receiver of PU is below the interference threshold [14]. Each CR user receives one traffic flow at most, and belongs to one multicast group. Let $\mathbf{K}_g$ and $|\mathbf{K}_g|$ $(g = 1, 2, \ldots, G)$ denote the CR user set of multicast group g and its cardinality, respectively. Especially, if $|\mathbf{K}_g| = 1$, it is simplified to an unicast network. All CR users belong to the set $\mathbf{K}_{CR} = \cup_{g=1}^{G} \mathbf{K}_g$ and $|\mathbf{K}_{CR}|$ is the total number of CR users in multicast cognitive OFDM network. Let W denote the total available bandwidth, and $W_m = W/M$ denote the bandwidth of each subcarrier. Multicast cognitive OFDM network is depicted in Fig. 1. The resource allocation mechanism adopts the perfect CSI between CBS and CR users and the statistic CSI between CBS and primary users. In addition, we adopt a slow-fading channel model in CN, and the channel conditions remain unchanged during the resource allocation period. With the perfect CSI in CN, it is possible to determine the maximum transmission rate, at which an individual CR user can reliably receive data. The signal to interference noise rate (SINR), $\alpha_{k,m}^g$, for the kth CR user over the mth subcarrier in the gth multicast group is defined by

$$\alpha_{k,m}^g = h_{k,m}^g \Big/ \left[\Gamma_{k,m}^g \left(I_m + \eta \right) \right] \quad (1)$$

$$h_{k,m}^g = D_{k,g}^{-\alpha} \left| f_{k,m}^g \right|^2 \quad (2)$$

where the channel power gain, $h_{k,m}^g$, between CBS and the kth CR user over the mth subcarrier in the gth multicast group is defined by (2), $D_{k,g}$ is the distance from the kth CR user in the gth multicast group to the CBS, α is the path loss exponent, and the small-scale fading, $f_{k,m}^g$, of the kth CR user over the mth subcarrier in the gth multicast group follows the complex Gaussian distribution. Moreover, I_m is the interference power at the receiver of CBS over the mth subcarrier, η is the background noise power, and the capacity gap, $\Gamma_{k,m}^g$, of the kth CR user over the mth subcarrier in the gth multicast group is defined by

$$\Gamma_{k,m}^g = -\frac{\ln\left(5\overline{\text{BER}}_{k,m}^g\right)}{1.5} \quad (3)$$

where $\text{BER}_{k,m}^g$ is the target bit error rate of the kth CR user over the mth subcarrier in the gth multicast group [15], [16].

Compared with the unicast cognitive OFDM network, a subcarrier is allowed to serve many CR users in multicast group for multicast cognitive OFDM network, and the same data can be transmitted from the CBS to multiple CR users in a multicast group at the same subcarriers. This leads to the mismatching data rates attainable by individual CR users of the multicast group. In order to guarantee the multicast service, the smallest rate of all the CR users in a multicast group is enforced to adopt by CBS, and the minimum SINR $\beta_{g,m}^{\min}$ over the mth subcarrier in the gth multicast group is defined by

$$\beta_{g,m}^{\min} = \min_{k \in \mathbf{K}_g} \alpha_{k,m}^g. \quad (4)$$

Consequently, the minimum transmission rate, $b_{g,m}^{\min}$, over the mth subcarrier in the gth multicast group is defined by

$$b_{g,m}^{\min} = \frac{W}{M} \log_2\left(1 + p_{g,m}\beta_{g,m}^{\min}\right) \quad (5)$$

where $p_{g,m}$ is the transmission power of CBS over the mth subcarrier in the gth multicast group [17].

Since all CR users receive the same data rate in a multicast group, the total rate of the gth multicast group over the mth subcarrier is defined by

$$R_{g,m} = \sum_{k \in \mathbf{K}_g} b_{g,m}^{\min} = |\mathbf{K}_g| \, b_{g,m}^{\min}. \quad (6)$$

Considers the downlink cognitive multicast OFDM network, and CBS access the spectrum in the underlay mode. That means cognitive network share the same spectrum with PN and CBS controls the transmission power to guarantee PU's communication. This interference power control model is adopted in many literature, e.g., [12]. Although the other interference power control model for cognitive OFDM network is investigated to utilize the spectrum hole, and consider the cross channel interference, e.g., [14]. But the energy-efficient chance-constrained resource allocation problem for cognitive multicast OFDM network is complicated, and we adopt the simple interference power control model in [12].

III. PROBLEM FORMULATION

Consider the downlink energy-efficient resource allocation problem based on chance-constrained programming for

multicast cognitive OFDM network. The resource allocation is operated in a centralized manner. To prevent the unacceptable performance degradation of PUs, the interference temperature model based on the underlay mode is adopted, and the interference power at the receiver of PU is carefully controlled under a given threshold.

Let $\rho_{g,m}$ denote the subcarrier allocation indicator for the gth multicast group over the mth subcarrier. For example, $\rho_{g,m} = 1$ represents the mth subcarrier is allocated to the gth multicast group, and each subcarrier can be only allocated to one multicast group at most, i.e.,

$$\sum_{g=1}^{G} \rho_{g,m} \leq 1, \rho_{g,m} \geq 0, \forall m, g. \quad (7)$$

Let P_{fix} denote the a fixed power capturing the power consumption at the power supply, P_{total} denote the total power for multicast cognitive OFDM network, and ς denote the power amplifier efficiency [18]. In order to guarantee the feasible of power allocation, we add the constraint

$$P_{fix} + \frac{\sum_{g=1}^{G} \sum_{m=1}^{M} \rho_{g,m} p_{g,m}}{\varsigma} \leq P_{total}, p_{g,m} \geq 0, \forall m, g. \quad (8)$$

The total achieved data rate, $R_g = \sum_{m=1}^{M} \rho_{g,m} R_{g,m}$, by the gth multicast group should satisfy the minimum rate requirement, $R_g^{\min}$, i.e.,

$$R_g \geq R_g^{\min}, \forall g. \quad (9)$$

The instantaneous channel coefficient, h_m^n, between CBS and the nth PU over the mth subcarrier follows the complex Gaussian distribution, and the channel power gain, $g_m^n = |h_m^n|^2$, follows the exponential distribution, i.e.,

$$f_{g_m^n}(\eta) = \frac{1}{\sigma_n} \exp\left(-\frac{\eta}{\sigma_n}\right) \quad (10)$$

where $\sigma_n = (d_n/d_0)^{-\alpha} s_n$ is the long-term average channel gain between CBS and the nth PU, d_n is the distance between CBS and the nth PU, d_0 is the reference distance, α is the amplitude path-loss exponent and the shadow fading, s_n, between CBS and the nth PU follows the log-normal distribution.

Let $I_{\max}^n$ denote the interference threshold for the nth PU, ε denote the desired lower-bound on the probability that the interference threshold is not exceeded. Since the interference power constraint is modeled as the probabilistic interference constraint, i.e.,

$$\sum_{n=1}^{N} \Pr\left\{ \sum_{g=1}^{G} \sum_{m=1}^{M} \frac{\rho_{g,m} p_{g,m} g_m^n}{\varsigma} < I_{\max}^n \right\} \geq \varepsilon \quad (11)$$

where $\Pr\{\bullet\}$ is the possibility.

Define the energy efficiency of CBS for multicast cognitive OFDM network as a ratio of the achieved data rate to the power consumption, and the optimization objective maximizes

the energy efficiency. Hence, the chance-constrained resource allocation problem is formulated as (12).

$$\max_{\rho_{g,m}, p_{g,m}} \frac{\sum_{g=1}^{G} \sum_{m=1}^{M} \rho_{g,m} R_{g,m}}{P_{fix} + \frac{\sum_{g=1}^{G} \sum_{m=1}^{M} \rho_{g,m} p_{g,m}}{\varsigma}}$$

$$s.t.: C1. R_g \geq R_g^{\min}, \forall g$$

$$C2. \sum_{n=1}^{N} \Pr\left\{ \sum_{g=1}^{G} \sum_{m=1}^{M} \frac{\rho_{g,m} p_{g,m} g_m^n}{\varsigma} < I_{\max}^n \right\} \geq \varepsilon$$

$$C3. P_{fix} + \frac{\sum_{g=1}^{G} \sum_{m=1}^{M} \rho_{g,m} p_{g,m}}{\varsigma} \leq P_{total}$$

$$C4. \sum_{g=1}^{G} \rho_{g,m} \leq 1, \forall m$$

$$C5. \rho_{g,m} \geq 0, p_{g,m} \geq 0, \forall m, g. \quad (12)$$

IV. CHANCE-CONSTRAINED ENERGY-EFFICIENT RESOURCE ALLOCATION ALGORITHM

In this section, we firstly adopt SVM to calculate the probabilistic interference constraint. Then, the chance-constrained resource allocation problem is transformed into the deterministic resource allocation problem. Finally, Zoutendijk's method of feasible direction are adopted to solve the deterministic resource allocation problem.

A. Calculate the Probabilistic Interference Constraint by SVM

In order to compute the probabilistic interference constraint (11), we define a probabilistic function, $U_{pro}(\mathbf{x}_i)$, as

$$U_{pro}(\mathbf{x}_i) = \sum_{n=1}^{N} \Pr\left\{ \sum_{g=1}^{G} \sum_{m=1}^{M} \frac{\rho_{g,m} p_{g,m} g_m^n}{\varsigma} < I_{\max}^n \right\} \quad (13)$$

where $\mathbf{x}_i = [\rho_i, \mathbf{p}_i]$ is the input sample matrix of the function $U_{pro}(\mathbf{x}_i)$, $\rho_i = [\rho_{g,m}]$ is the ith subcarrier allocation matrix, and $\mathbf{p}_i = [p_{g,m}]$ is the ith power allocation matrix.

Since $U_{pro}(\mathbf{x}_i)$ is difficult to compute, we adopt least squares support vector machine (LS-SVM) to estimate $U_{pro}(\mathbf{x}_i)$. LS-SVM works with a least square cost function, and the solution follows from a linear Karush-Kuhn-Tucker system [19]. Let $\Pr_i$ denote the ith output sample of $U_{pro}(\mathbf{x}_i)$, which is computed by the stochastic simulation method [20], [21], N_T denote the number of training samples, and $\{\mathbf{x}_i, \Pr_i\}_{i=1}^{N_T}$ denote the training data set.

In order to minimize the bias between the output sample $\Pr_i$ and the estimated value, we define a function, $U_{pro}^{est}(\mathbf{x}_i)$, by (14). In addition, the empirical risk $C_{emp}(\omega, B)$, which depicts the bias between the output sample $\Pr_i$ and the estimated value, $U_{pro}^{est}(\mathbf{x}_i)$, is defined by (15).

$$U_{pro}^{est}(\mathbf{x}_i) = \omega\phi(\mathbf{x}_i) + B \tag{14}$$

$$C_{emp}(\omega, B) = \frac{1}{N_T}\sum_{i=1}^{N_T}\left|Pr_i - U_{pro}^{est}(\mathbf{x}_i)\right| \tag{15}$$

where ω is the weight vector, B is the bias term, and $\phi(\bullet)$ is a mapping function, which is defined by the kernel function, $K(\mathbf{x}_i, \mathbf{x}_j)$, in (16). Here, we adopt the radial basis function (17) as $K(\mathbf{x}_i, \mathbf{x}_j)$ [22].

$$K(\mathbf{x}_i, \mathbf{x}_j) = \phi(\mathbf{x}_i)^T\phi(\mathbf{x}_j) \tag{16}$$

$$K(\mathbf{x}_i, \mathbf{x}_j) = \exp\left(-\frac{|\mathbf{x}_i - \mathbf{x}_j|^2}{2\sigma^2}\right) \tag{17}$$

where σ is the scale parameter.

To minimize the empirical risk $C_{emp}(\omega, B)$, we formulate the optimization problem (18) as

$$\min_{\omega, \mathbf{B}, \xi_i} J(\omega, B, \xi_i) = \frac{1}{2}\omega^T\omega + \frac{1}{2}\beta\left\{\sum_{i=1}^{N_T}\xi_i^2\right\}$$

$$s.t.: \text{C1}.Pr_i = \omega\phi(\mathbf{x}_i) + B + \xi_i, i = 1, \ldots, N_T \tag{18}$$

where ξ_i is the slack variable of the ith constraint of C1 in (18) and β is a positive real constant.

In $J(\omega, B, \xi_i)$, the first item is defined according to the least square criterion, and the second item is defined to minimize the error between the estimated value and the accurate value. In addition, the constraint C1 in (18) establishes the relationship between the estimated value and accurate value via the slack variable ξ_i.

Obviously, (18) is a convex problem, and we adopt the Lagrange multiplier method to solve it. Hence, the Lagrange function $f_J\left(\omega, B, \xi_i, \gamma_i^\xi\right)$ for (18) is defined by (19).

$$f_J\left(\omega, B, \xi_i, \gamma_i^\xi\right) = J(\omega, B, \xi_i) + \gamma_i^\xi\left(Pr_i - \omega\phi(\mathbf{x}_i) - B - \xi_i\right). \tag{19}$$

According to the Karush-Kuhn-Tucker condition, we can yield (20) by differentiating $f_J\left(\omega, B, \xi_i, \gamma_i^\xi\right)$ with respect to ω, B, ξ_i and γ_i^ξ, respectively.

$$\begin{cases} \frac{\partial f_J\left(\omega, B, \xi_i, \gamma_i^\xi\right)}{\partial\omega} = 0 \\ \frac{\partial f_J\left(\omega, B, \xi_i, \gamma_i^\xi\right)}{\partial B} = 0 \\ \frac{\partial f_J\left(\omega, B, \xi_i, \gamma_i^\xi\right)}{\partial\xi_i} = 0 \\ \frac{\partial f_J\left(\omega, B, \xi_i, \gamma_i^\xi\right)}{\partial\gamma_i^\xi} = 0. \end{cases} \tag{20}$$

By solving (20), we can obtain

$$\begin{cases} \omega = \sum_{i=1}^{N_T}\gamma_i^\xi\phi(\mathbf{x}_i) \\ \gamma^\xi = \mathbf{R}^{-1}\left(\mathbf{Pr} - B\overrightarrow{1}\right) \\ B = \left(\overrightarrow{1}^T\mathbf{R}^{-1}\mathbf{Pr}\right)\Big/\left(\overrightarrow{1}^T\mathbf{R}^{-1}\overrightarrow{1}\right) \\ \mathbf{R} = \Omega + \mathbf{I}/\beta \end{cases} \tag{21}$$

where $\mathbf{I}$ denotes the unit matrix, $\mathbf{R}^{-1}$ denotes the inverse matrix of $\mathbf{R}$, γ_i^ξ is the ith Lagrange multiplier, $\overrightarrow{1} = [1, \ldots, 1]^T$, $\mathbf{Pr} = \left[Pr_1, \ldots, Pr_{N_T}\right]^T$, $\gamma^\xi = \left[\gamma_1^\xi, \ldots, \gamma_{N_T}^\xi\right]^T$, and $\Omega = \left[K(\mathbf{x}_i, \mathbf{x}_j)\right]\Big|_{1\leq i\leq N_T}^{1\leq j\leq N_T}$.

Hence, (14) can be rewritten by

$$U_{pro}^{est}(\mathbf{x}_i) = \sum_{j=1}^{N_T}\gamma_j^\xi K(\mathbf{x}_i, \mathbf{x}_j) + B. \tag{22}$$

B. Chance-Constrained Resource Allocation Algorithm

In order to convert the chance-constrained resource allocation problem into the deterministic resource allocation problem, the constraint (11) can be rewritten by

$$\sum_{j=1}^{N_T}\gamma_j^\xi K(\mathbf{x}_i, \mathbf{x}_j) + B \geq \varepsilon \tag{23}$$

According to (12) and (23), the deterministic resource allocation problem can be expressed by

$$\min_{\rho_{g,m}, p_{g,m}} -\frac{\sum_{g=1}^{G}\sum_{m=1}^{M}\rho_{g,m}R_{g,m}}{P_{fix} + \frac{\sum_{g=1}^{G}\sum_{m=1}^{M}\rho_{g,m}p_{g,m}}{\varsigma}}$$

$$s.t.: \text{C1}.R_g - R_g^{\min} \geq 0, \forall g$$

$$\text{C2}.\sum_{j=1}^{N_T}\gamma_j^\xi K(\mathbf{x}_i, \mathbf{x}_j) + B - \varepsilon \geq 0, \forall n$$

$$\text{C3}.P_{total} - \left(P_{fix} + \frac{\sum_{g=1}^{G}\sum_{m=1}^{M}\rho_{g,m}p_{g,m}}{\varsigma}\right) \geq 0$$

$$\text{C4}.1 - \sum_{g=1}^{G}\rho_{g,m} \geq 0, \forall m$$

$$\text{C5}.\rho_{g,m} \geq 0, \forall m, g$$

$$\text{C6}.p_{g,m} \geq 0, \forall m, g. \tag{24}$$

Since the optimization problem (24) is an NP-hard problem, we can solve it by the Zoutendijk's method as long as we can find a feasible descent direction in each iteration. The advantage of Zoutendijk's method is the dimension of the problem can be reduced due to the variable elimination, and it can also utilize the special structure of the problem [23], [24].

In Zoutendijk's method of feasible direction, a feasible starting solution is selected, and an iterative solution is obtained by

$$\mathbf{x}_{i+1} = \mathbf{x}_i + \lambda_i\mathbf{d}_i \tag{25}$$

where $\mathbf{d}_i$ is the moving direction, λ_i is the moving distance and $\mathbf{x}_{i+1}$ is the final solution obtained at the end of the ith iteration.

In order to guarantee the feasible direction, $\mathbf{d}_i$ needs to satisfy

$$\nabla f (\mathbf{x}_i)^T \mathbf{d}_i < 0 \qquad (26)$$

$$\nabla g_k (\overline{\mathbf{x}})^T \mathbf{d}_i > 0, k \in I (\mathbf{x}_i) \qquad (27)$$

where $I (\mathbf{x}_i) = \{k \, | g_k (\mathbf{x}_i) = 0 \, , k = 1, \ldots, 6\}$ is the index set, $f (\mathbf{x}_i)$ and $g_k (\mathbf{x}_i)$ are defined by

$$f (\mathbf{x}_i) = -\sum_{g=1}^{G} \sum_{m=1}^{M} \left\{ \frac{\sum_{g=1}^{G} \sum_{m=1}^{M} \rho_{g,m} R_{g,m}}{P_{fix} + \frac{\sum_{g=1}^{G} \sum_{m=1}^{M} \rho_{g,m} p_{g,m}}{\varsigma}} \right\}. \qquad (28)$$

$$g_k (\mathbf{x}_i) = \begin{cases} R_g - R_g^{\min} & , k = 1 \\ \sum_{j=1}^{N_T} \gamma_j^{\xi} K (\mathbf{x}_i, \mathbf{x}_j) + B - \varepsilon & , k = 2 \\ P_{total} - \left(P_{fix} + \frac{\sum_{g=1}^{G} \sum_{m=1}^{M} \rho_{g,m} p_{g,m}}{\varsigma} \right) & , k = 3 \\ 1 - \sum_{g=1}^{G} \rho_{g,m} & , k = 4 \\ \rho_{g,m} & , k = 5 \\ p_{g,m} & , k = 6. \end{cases} \qquad (29)$$

The Zoutendijk's method of feasible direction is described in Algorithm 1.

V. COMPUTATIONAL COMPLEXITY AND SIGNALING OVERHEAD

A. Computational Complexity

In the proposed algorithm, the computational complexity is $O (3GM)$ in step 1. In step 2, the computational complexity is $O (2GM)$. In step 3, the computational complexity is $O (A_I GM)$, where A_I is the number of iterations to solve (31). In step 4, the computational complexity is $O (GM)$. In step 5, the computational complexity is $O (C_I GM)$, where C_I is the number of iterations to solve (32). In step 6, the computational complexity is $O (2GM)$. Hence, the total computational complexity is $O ((8 + A_I + C_I) GM)$.

In order to compare with proposed algorithm, we adopt the resource allocation algorithm based on max-min criterion in [25]. In max-min algorithm, the total computational complexity is $O \left((J_I + K_I) M \sum_{g=1}^{G} |\mathbf{K}_g| \right)$. J_I and K_I are the number of iterations for the bandwidth allocation and power allocation, respectively.

B. Signaling Overhead

In the proposed algorithm, the CSI between CBS and CR users need to be fed back from CR users each time slot. Additionally, the subcarrier allocation solution $\rho_{g,m}$ and rate allocation solution $R_{g,m}$ are allocated to CR

Algorithm 1. Zoutendijk's method of feasible direction.

Require: The total power P_{total}, the minimum rate requirement $R_g^{\min}$, the desired lower-bound on the probability ε, the interference threshold $I_{\max}^n$ and other parameters.

Ensure: For each resource allocation, return the subcarrier allocation result $\rho_{g,m}$ and the power allocation result $p_{g,m}$.

1: Initialize a feasible point $\mathbf{x}_1$ and $i = 1$; Set ε_1, ε_2 and ε_3 with arbitrary small positive numbers. Calculate $f (\mathbf{x}_1)$ and $g_k (\mathbf{x}_1), k = 1, 2, \ldots, 6$.

2: If $g_k (\mathbf{x}_i) > 0, \forall k$, set the search direction $\mathbf{d}_i$ with (30) and go to 5; otherwise, go to 3.

$$\mathbf{d}_i = -\nabla f (\mathbf{x}_i). \qquad (30)$$

3: Find the feasible direction $\mathbf{d}_i$ by solving the optimization problem (31).

$$\begin{aligned} \min_{\mathbf{d}_i} &: -z \\ s.t. &: \text{C1}. \nabla g_k (\mathbf{x}_i)^T \mathbf{d}_i \geq -z, j \in I (\mathbf{x}_i) \\ &\text{C2}. \nabla f (\mathbf{x}_i)^T \mathbf{d}_i \leq z \\ &\text{C3}. -1 \leq d_n \leq 1. \end{aligned} \qquad (31)$$

where d_n is the nth component of $\mathbf{d}_i$.

4: If $z^* \leq \varepsilon_1$, terminate the iteration by taking $\mathbf{x}_{opt} = \mathbf{x}_i$. If $z^* > \varepsilon_1$, go to 5.

5: Find a step length λ_i along the direction $\mathbf{d}_i$ and obtain a new point $\mathbf{x}_{i+1}$ according to (25). λ_i is obtained by solving the one dimensional search problem, i.e.,

$$\begin{aligned} \min_{\lambda_i} &: f (\mathbf{x}_i + \lambda_i \mathbf{x}_i) \\ s.t. &: \text{C1}.0 \leq \lambda_i \leq \overline{\lambda}. \end{aligned} \qquad (32)$$

where $\overline{\lambda}$ is defined by

$$\overline{\lambda} = \sup \{\lambda \, | g_k (\mathbf{x}_i + \lambda \mathbf{d}_i) \geq 0, k = 1, \ldots, 6\}. \qquad (33)$$

6: Calculate . If $f (\mathbf{x}_i)$, $f (\mathbf{x}_{i+1})$, $\mathbf{x}_i$ and $\mathbf{x}_{i+1}$ satisfy (34) and (35), terminate the iteration by $\mathbf{x}_{opt} = \mathbf{x}_{i+1}$; Otherwise, set $i \leftarrow i + 1$ and repeat 2-6.
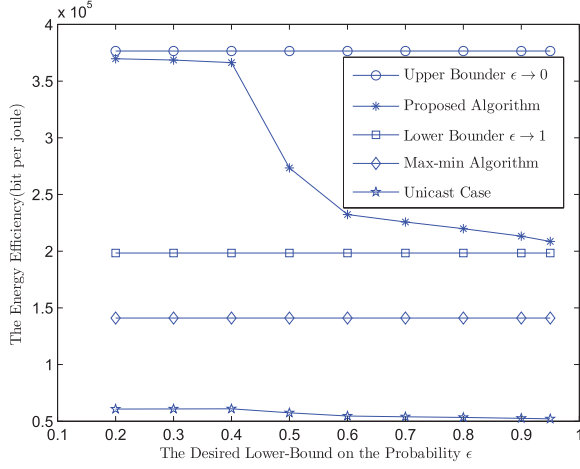
$$\left| \frac{f (\mathbf{x}_i) - f (\mathbf{x}_{i+1})}{f (\mathbf{x}_i)} \right| \leq \varepsilon_2 \qquad (34)$$

$$\|\mathbf{x}_i - \mathbf{x}_{i+1}\| \leq \varepsilon_3 \qquad (35)$$

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users from CBS. Hence, the total signaling overhead is

$$O \left(M \sum_{g=1}^{G} |\mathbf{K}_g| + 2MG \right).$$

VI. SIMULATION RESULTS AND DISCUSSIONS

This section presents the simulation results for the proposed algorithm in multicast cognitive OFDM network. Consider a geographical region, which is covered by a PN and a CN. The CBS has a coverage area with the radius 200 m, and CR

Fig. 2. The energy efficiency vs. the desired lower-bound on the probability ε.

Fig. 3. The spectral efficiency vs. the desired lower-bound on the probability ε.

users are randomly located in the cell. In addition, there are 20 CR users in multicast cognitive OFDM network. In PN, there are two PUs and one primary base station. Moreover, the distances between CBS and two PUs are [110, 150] m. In CN and PN, the modulation technology at the physical layer adopts the OFDM technology and the number of subcarriers is 128. The path loss exponent is $\alpha = 3$. The other simulation parameters are $W = 2$ MHz, $G = 5$, $P_{total} = 45$ watts, $P_{fix} = 15$ watts, $\varsigma = 0.1$, $\overline{BER}_{k,m}^g = 1 \times 10^{-4}$, $I_k = N\left(0, 1 \times 10^{-9}\right)$ watts and $\eta = N\left(0, 1 \times 10^{-12}\right)$ watts. In order to compare with the proposed algorithm, we adopt upper bounder $\varepsilon \to 0$, lower bounder $\varepsilon \to 1$, max-min algorithm, and unicast case. In the upper bounder $\varepsilon \to 0$, the exhaust search method is utilized to maximize the energy efficiency and guarantee the actual probability of not exceeding I_{max}^n towards 0 to obtain the upper bounder. In the lower bounder $\varepsilon \to 1$, the exhaust search method is adopted to maximize the energy efficiency and guarantee the actual probability of not exceeding I_{max}^n towards 1 to obtain the lower bounder. In the max-min algorithm, the resource allocation algorithm is based on max-min criterion [26], [27], the subcarrier is allocated to user with the minimum throughput, and the total power is defined by P_{total}/G to guarantee PU communication. In the unicast case, it is the special case with $|\mathbf{K}_g| = 1$ for multicast cognitive OFDM network, and the Zoutendijk's method of feasible direction is adopted to solve the chance-constrained resource allocation problem.

Figs. 2–3 depict the energy efficiency and the spectral efficiency vs. the desired lower-bound on the probability ε for different algorithms, respectively. The simulation conditions are $I_{max}^n = 1 \times 10^{-5}$ watts and $R_g^{min} = 1$ Mbps. As can be seen in Fig. 2, the proposed algorithm and upper bounder $\varepsilon \to 0$ can achieve the better energy efficiency than the other three algorithms. Moreover, the unicast case has the smallest energy efficiency. This is because it does not efficiently utilize the radio resource compared with the proposed algorithm. From Fig. 3, we can see that the spectral efficiency of proposed algorithm is between the upper bounder $\varepsilon \to 0$ and the lower bounder $\varepsilon \to 1$, and the spectral efficiency of proposed algorithm decreases along with the growth of ε. It can be explained that increasing the desired lower-bound on the probability ε
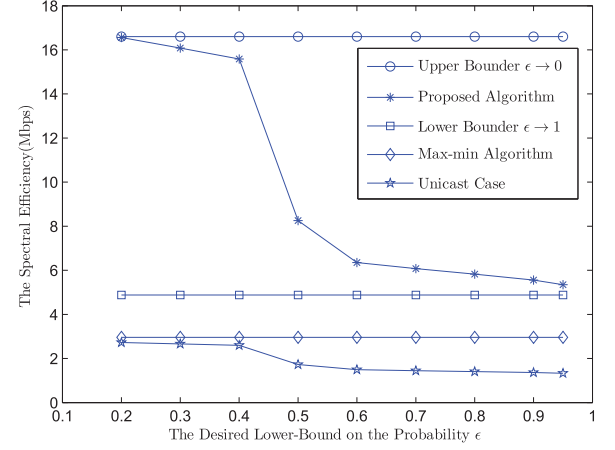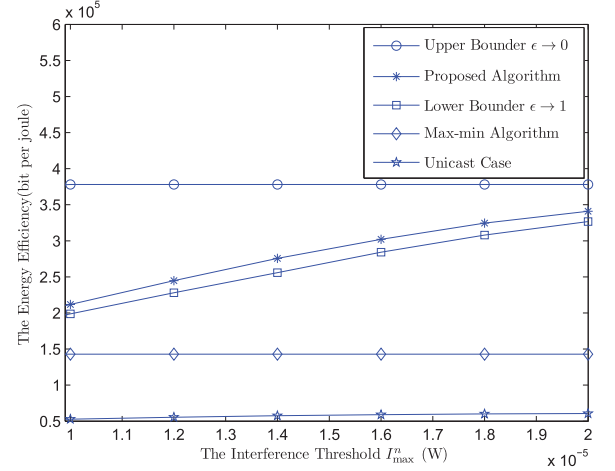
Fig. 4. The energy efficiency vs. the interference threshold I_{max}^n.
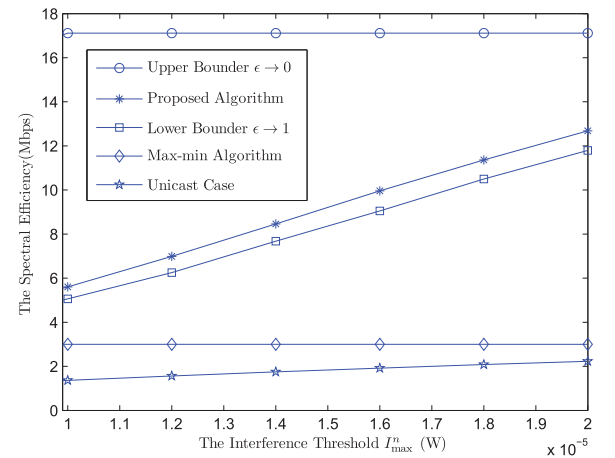
Fig. 5. The spectral efficiency vs. the interference threshold I_{max}^n.

strengthens the interference power constraint and reduces the available radio resource.

Figs. 4–5 depict the energy efficiency and spectral efficiency vs. the interference threshold I_{max}^n for the different algorithms, respectively. The simulation conditions are $\varepsilon = 0.9$ and $R_g^{min} =$
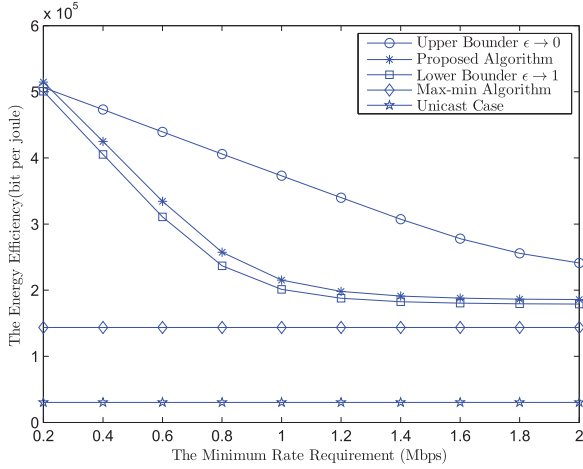
Fig. 6. The actual probability of not exceeding $I_{\max}^n$ vs. the interference threshold $I_{\max}^n$.
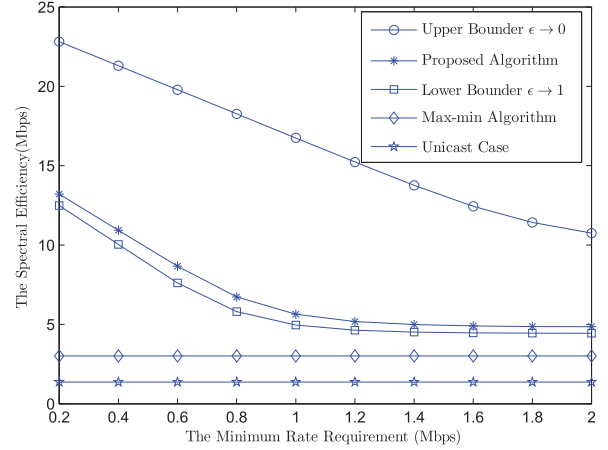

Fig. 7. The energy efficiency vs. the minimum rate requirement.

Fig. 8. The actual probability of not exceeding $I_{\max}^n$ vs. the minimum rate requirement.

1 Mbps. It can be seen in Fig. 4 and Fig. 5 that the energy efficiencies and spectral efficiencies of the five algorithms except for max-min algorithm increase when the interference threshold $I_{\max}^n$ grows, which can be explained that increasing the interference threshold $I_{\max}^n$ can provide more available transmission power in CBS. However, the max-min algorithm does not allocate the power and subcarrier according to the interference power constraint. Consequently, the energy efficiency and spectral efficiency for max-min algorithm remain unchanged when increasing the interference threshold $I_{\max}^n$. In addition, there is a special phenomenon in the upper bounder $\varepsilon \to 0$. When $I_{\max}^n \geq 1 \times 10^{-5}$ watts, the energy efficiency and spectral efficiency of upper bounder $\varepsilon \to 0$ remain unchanged. This is because CBS in the upper bounder $\varepsilon \to 0$ utilizes all transmission power and increasing the interference threshold $I_{\max}^n$ can not provide more available transmission power.

Fig. 6 depicts the actual probability of not exceeding $I_{\max}^n$ vs. the interference threshold $I_{\max}^n$ for the different algorithms. In order to scale the precise of calculating the probabilistic interference, we introduce Benchmark, which is the probabilistic lower bounder of not exceeding the interference. The simulation conditions are the same as Figs. 4–5. It can be seen in Fig. 6 that the proposed algorithm, max-min algorithm, lower bounder $\varepsilon \to 1$ and unicast case can satisfy the probabilistic interference constraint. Although the upper bounder $\varepsilon \to 0$ has the better energy efficiency and spectral efficiency, the actual probability of not exceeding $I_{\max}^n$ for the upper bounder $\varepsilon \to 0$ is worse. Conversely, the max-min algorithm can satisfy the chance-constrained condition, but its energy efficiency and spectral efficiency are worse. In Fig. 6, it can also be seen that the actual probabilities of not exceeding $I_{\max}^n$ for the proposed algorithm and the unicast case are very close to Benchmark.

Fig. 7 depicts the energy efficiency vs. the minimum rate requirement for different algorithms. The simulation conditions are $\varepsilon = 0.9$, $I_{\max}^n = 1 \times 10^{-5}$ watts and $R_g^{\min} = 1$ Mbps. In Fig. 7, we can see that the multicast technology can significantly enhance the energy efficiency for cognitive OFDM network compared with the unicast case. In addition, the energy efficiency for the proposed algorithm, the upper bounder $\varepsilon \to$
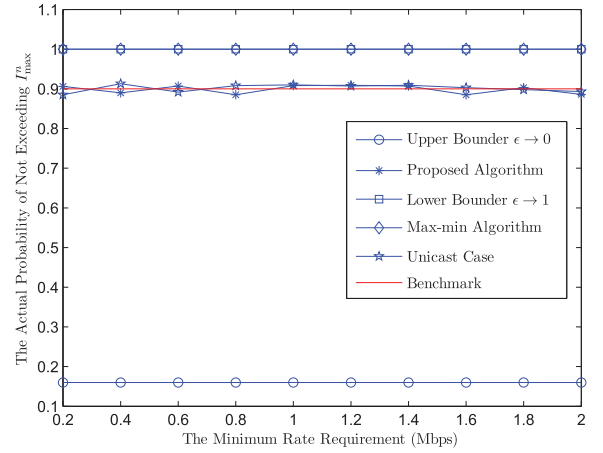
0, and lower bounder $\varepsilon \to 1$ decrease when the minimum rate requirement increases. This is due to the fact that the radio resource is not allocated to the multicast group with the highest energy efficiency, when the QoS for each group can not be satisfied. This leads to the loss of the resource utilizing efficiency.

Fig. 8 depicts the actual probability of not exceeding $I_{\max}^n$ and the satisfaction index vs. the minimum rate requirement for the different algorithms, respectively. The simulation conditions are the same as Fig. 7. The satisfaction index captures the resource allocation algorithm to satisfy the QoS requirements of multicast group. Specifically, the satisfaction index is defined as

$$SI = \mathrm{E}\left\{ 1_{R_g \geq R_g^{\min}} + 1_{R_g < R_g^{\min}} \frac{R_g}{R_g^{\min}} \right\} \qquad (36)$$

where $1_a = 1$ if a is satisfied, and 0 otherwise [28]. Fig. 8 shows that the proposed algorithm, lower bounder $\varepsilon \to 1$, and unicast case can satisfy the chance-constrained condition. Additionally, the actual probability of not exceeding $I_{\max}^n$ for the proposed algorithm are very close to the target lower bounder ε. This can make the proposed algorithm to improve the energy efficiency and spectral efficiency. Although the unicast case can
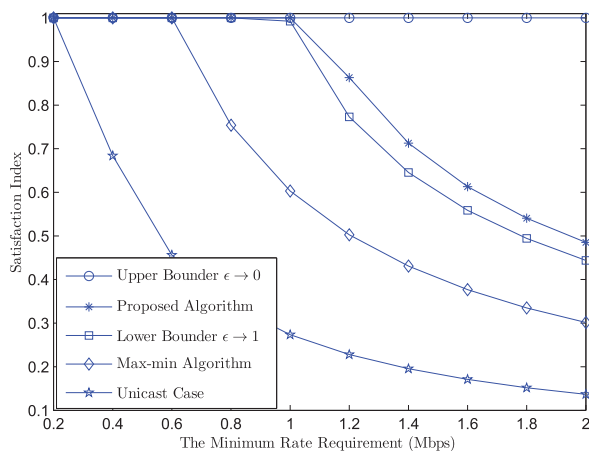
Fig. 9. The satisfaction index vs. the minimum rate requirement.

also satisfy the chance-constrained condition, the energy efficiency and spectral efficiency for the unicast case is lower than that of the proposed algorithm. The actual probability of not exceeding I_{max}^n for upper bounder $\varepsilon \to 0$ is not equal to 0, which can be explained that the total power in CBS is not enough large to make the actual probability of not exceeding I_{max}^n equal to 0. Fig. 9 shows that the satisfaction indexes of the lower bounder $\varepsilon \to 1$, proposed algorithm, max-min algorithm, and unicast case decrease along with the increase of the minimum rate requirement. This is because that the resource is not enough to satisfy the QoS requirement of multicast group. Compared with the max-min algorithm, the proposed algorithm increases the computational complexity, and the extra computational overhead is used to compute the chance-constrained condition. From Figs. 7–9, we can conclude the proposed algorithm achieves a tradeoff between the energy efficiency and probabilistic interference constraint.

VII. CONCLUSIONS

In this paper, we study the chance-constrained energy-efficient resource allocation problem for multicast cognitive OFDM network. The objective function maximizes the energy efficiency, and the constraint conditions include the probabilistic interference constraint and the total available power. In order to solve the above subcarrier and power allocation problem, we first define the uncertain function according to the probabilistic interference constraint. Then, the SVM is adopted to calculate it. Finally, the Zoutendijk's method of feasible direction is utilized. Simulation results demonstrate that the proposed algorithm not only improves the energy efficiency, spectral efficiency and satisfaction index, but also satisfies the chance-constrained condition.

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