Estimation accuracy of multi-cell massive multiple-input multiple-output systems in correlated Rician fading channel

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The performance analysis of a multi-user multi-cell massive multiple-input multiple-output system that applies a conventional channel estimation technique, namely the linear minimum mean square error over correlated Rician fading is investigated. Based on the analysis, it is found that increasing the line-of-sight component can enhance the estimation accuracy.

Introduction: The major targets for 5th generation (5G) are to achieve 1000 times the system capacity, 10 times the spectral efficiency, energy efficiency and data rate, and 25 times the average cell throughput [1]. Massive multiple-input multiple-output (MIMO) is a promising technology that allows the attainment of 5G targets. The major limiting factor in massive MIMO is the availability of accurate, instantaneous channel state information (CSI) at the base station. The CSI is typically acquired by transmitting predefined pilot signals and estimating the channel coefficients from the received signals. The instantaneous channel matrix is acquired from the received pilot signal by applying an appropriate channel estimation algorithm. However, the necessary pilot reuse in cellular networks creates spatially correlated inter-cell interference, known as pilot contamination, which reduces the channel estimation performance and spectral efficiency [1, 2]. Thus far, most of the studies of massive MIMO systems assume the channel condition to be an independent and identically distributed Rayleigh fading. To evaluate massive MIMO in more realistic scenarios, we need models that capture important massive MIMO channel characteristics, such as a Rician fading channel that is used to model the direct line-of-sight paths in mobile radio channels and indoor wireless system [3]. As millimetre-wave (mm-wave) enjoys near line-of-sight (LOS) propagation that would mitigate the effect of pilot contamination, it is being considered as a candidate for new radio bands for 5G mobile communication systems [3, 4]. Furthermore, the effect of correlation should be considered, as the majority of the previous studies assumed that the channels are independent, but in more realistic environments the antennas are not sufficiently separated and the propagation environment does not provide a sufficient amount of rich scattering [5]. Zhang *et al.* [3, 6, 7] analysed the uplink (UL) rate of multi-cell massive MIMO in Rician fading, while Yue *et al.* [8] studied the downlink (DL) rate for multi-cell massive MIMO in Rician fading.

Motivated by the above reasons, this Letter provides the performance analysis of a multi-cell multi-user massive MIMO system using the linear minimum mean square error (LMMSE) channel estimation in correlated Rician fading channels. The work is shown through theoretical analysis and computer simulations.

System model: Following the system model in [9], we consider a multi-cell massive MIMO with L cells. Each cell consists of M antennas at the base station, N single antennas users, and the system operates in the time-division duplex mode to exploit the channel reciprocity. Assuming a block fading structure, each block begins with a UL pilot, followed by UL data transmission. The system then toggles to the DL and begins with the UL data transmission; the coherence period ends the DL data transmission. The UL channel is used for pilot-based channel estimation, and the received signal at the base station is expressed as

$$\boldsymbol{y} = x_{\mathrm{p}}\boldsymbol{h} + \boldsymbol{n}, \tag{1}$$

where x_{p} is a pilot signal that is used for channel estimation, and the term $\boldsymbol{n}$ is an ergodic process that consists of independent receiver noise $\boldsymbol{n}_{\mathrm{noise}}$ of zero mean and ζ_{BS}^2 variance, as well as potential interference $\boldsymbol{n}_{\mathrm{interf}}$ from other simultaneous transmissions. We assume that $\boldsymbol{n}_{\mathrm{interf}}$ has zero mean and that S is the covariance matrix during pilot transmission, $\boldsymbol{h}$ is the block of fading of the fast fading matrix between the base station and the user equipment and the average power is $P^{\mathrm{UE}} = E[|x_{\mathrm{p}}|^2]$ [9]. The fast fading matrix $\boldsymbol{h}$ is modelled as a correlated Rician fading channel, and can be written as [7]

$$\boldsymbol{h}_{\mathrm{Rician}} = [K(K+1)^{-1}]^{1/2} + h[(K+1)^{-1}]^{1/2} \tag{2}$$

where K is the Rice factor, which represents the ratio of the power of the deterministic component to the power of the fading component [10].

Mean squared error (MSE) approximation: The MSE of the LMMSE estimator will be derived as in [9, 11]. The appropriate relationship of the LMMSE estimator for the Rician fading channel that minimises the estimation error of the channel matrix $\boldsymbol{h}$ can be given as [12–14]

$$\hat{\boldsymbol{h}} = M + (\boldsymbol{y} - Mx_{\mathrm{p}})A, \tag{3}$$

where A values are the complex weights chosen to minimise the MSE between the true value of the channel and the estimated channel, and M is the mean value of the channel, and can be given as in [15] as

$$M = \sqrt[2]{K/K+1}, \tag{4}$$

considering

$$Z = E[\boldsymbol{y}\boldsymbol{y}^{\mathrm{H}}] = P^{\mathrm{UE}}R_{\mathrm{Rayleigh}} + S + \zeta_{\mathrm{BS}}^2 I, \tag{5}$$

where the channel covariance matrix R for Rayleigh and Rician fading can be written as

$$R_{\mathrm{Rayleigh}} = E[\boldsymbol{h}_{\mathrm{Rayleigh}}\boldsymbol{h}_{\mathrm{Rayleigh}}^{\mathrm{H}}] \tag{6}$$

and

$$R_{\mathrm{Rician}} = E[\boldsymbol{h}_{\mathrm{Rician}}\boldsymbol{h}_{\mathrm{Rician}}^{\mathrm{H}}]. \tag{7}$$

By following (2) and (5), the channel covariance matrix R_{Rician} can be written as

$$R_{\mathrm{Rician}} = E\left[\frac{R_{\mathrm{Rayleigh}} + 2M\sqrt[2]{K} + K}{K+1}\right], \tag{8}$$

and the covariance matrix of interference during pilot transmission S can be expressed as

$$S = E[\boldsymbol{n}_{\mathrm{interf}}\boldsymbol{n}_{\mathrm{interf}}^{\mathrm{H}}], \tag{9}$$

the LMMSE estimate of $\boldsymbol{h}$ can be expressed as

$$\begin{aligned} \mathrm{MSE} &= \mathrm{argmin}_{(\mathrm{H})}\left\|\boldsymbol{h} - \hat{\boldsymbol{h}}\right\|_{\mathrm{F}}^2 \\ &= \mathrm{argmin}_{(\mathrm{w})}\|\boldsymbol{h} - M - (\boldsymbol{y} - Mx_{\mathrm{p}})\|_2^2 \\ &= \mathrm{Trace}[E[\boldsymbol{h} - M - (\boldsymbol{y} - Mx_{\mathrm{p}})A]^{\mathrm{H}}E[\boldsymbol{h} - M - (\boldsymbol{y} - Mx_{\mathrm{p}})A]]. \end{aligned} \tag{10}$$

The LMMSE estimator can be achieved by differentiating (9) with respect to A and equating to zero [9, 11, 12], and the result can be given as

$$A = x_{\mathrm{p}}^* R_{\mathrm{Rician}}(\sqrt[2]{P^{\mathrm{UE}}} + S + \zeta_{\mathrm{BS}}^2 I)^{-1}. \tag{11}$$

Finally, the estimated channel can be expressed as

$$\hat{\boldsymbol{h}} = M + (\boldsymbol{y} - Mx_{\mathrm{p}})x_{\mathrm{p}}^* R_{\mathrm{Rician}}(\sqrt[2]{P^{\mathrm{UE}}} + S + \zeta_{\mathrm{BS}}^2 I)^{-1}. \tag{12}$$

Therefore, the MSE can be written as

$$\mathrm{MSE} = 1 - \frac{\mathrm{trace}(x_{\mathrm{p}}A(\sqrt[2]{K+1}R_{\mathrm{Rician}} + 2K + K\sqrt[2]{K+1})}{\mathrm{trace}(\sqrt[2]{K+1}R_{\mathrm{Rician}} + 2K + K\sqrt[2]{K+1})}. \tag{13}$$

Numerical results: In this section, the MSE results over correlated Rician fading channels are provided. The relative MSE is to be normalised with the channel covariance matrix, and can be written as

$$\mathrm{MSE} = \left\|\frac{\boldsymbol{h} - \hat{\boldsymbol{h}}\|_{\mathrm{F}}^2}{\mathrm{trace}(R_{\mathrm{Rician}}).}\right. \tag{14}$$

The channel covariance matrix R_{Rayleigh} is generated by the exponential correlation model, as in [11, 16]. The system model considers the effect of large scale fading in terms of path losses and the shadowing effect of the LTE-system model, as in the 3GPP propagation model in [11, 17]. We assume the number of users is 10, the number of base station antennas is 100 and the correlation factor is 0.7. In Fig. 1, the relative MSE against the signal-to-noise ratio (SNR) in dB in the presence of the pilot contamination for K-Rice factor = 0, 1, 2 and 3 dB is plotted, and it can be clearly observed that the estimation accuracy of the LMMSE is enhanced as the LOS component increases via the increasing K-Rice fading factor.

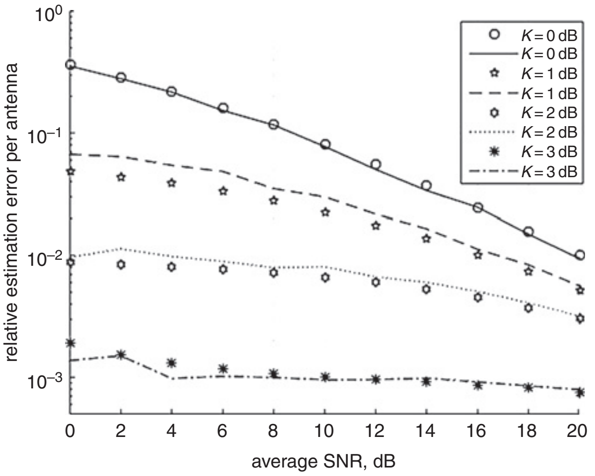

Fig. 1 *MSE of LMMSE channel estimator against average SNR for various Rician K-factors*

Conclusion: In this Letter, the performance analysis of a massive MIMO in a multi-cell scenario over correlated Rician fading has been investigated. It can be found that as the value of the K-Rice factor increases, the estimation accuracy in terms of the relative MSE of the LMMSE is enhanced. The results may assist the system designer to determine how the system and the channel parameter affect the reliability of the relative MSE of massive MIMO systems.

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