# Amplify-and-Forward Relaying with Optimal and Suboptimal Transmit Antenna Selection 

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#### Abstract

Antenna selection schemes offer a good complexity versus performance tradeoff for amplify-and-forward (AF) relay network implementation. In this paper, we consider a dual-hop channel state information assisted AF relay network with a direct source-destination link and investigate the performance of two antenna selection schemes (one optimal and another suboptimal). We derive analytical expressions for the systems' rate outage probabilities and the average bit error rate (BER) that match perfectly with simulation based results in the medium to high signal-to-noise ratio (SNR) regimes. We also investigate the channel capacity of the two schemes by deriving tight upper bounds. In order to gain further insights, simple high SNR approximations for the outage probability and the average BER have also been developed. Our theoretical analysis shows that the performance gap between the optimal and suboptimal schemes largely diminishes in the high SNR regime by increasing the number of antennas at the source. Since the suboptimal scheme has a near optimal performance as well as low signaling overhead compared to the optimal scheme, it seems to be a promising solution for implementing future cellular networks expecting to support relay based communications.


Index Terms-Antenna selection, relay networks, amplify-andforward, average bit error rate, channel capacity.

## I. Introduction

RELAY based systems can offer significant performance benefits, including increased spatial diversity, enhanced throughput and extended coverage [1]. As a result, future mobile broadband communication networks such as 3GPP LTE-Advanced, IEEE 802.16j and IEEE 802.16 m are expected to support relay based communications [2]. Amplify-andForward (AF) relaying is one of the simplest techniques and uses knowledge of the source to relay channel for amplification purposes [3]. This kind of relay is widely identified as a variable gain or channel state information (CSI)-assisted relay [3], [4], [6].

[^0]The benefits of employing multiple antennas at radio terminals is now well understood. To date, a large number of papers have appeared on various aspects of multipleinput multiple-output (MIMO) AF relay systems (see for example [3], [7]-[9]). However, the improved performance gains of using MIMO processing in these systems brings a corresponding increase in hardware complexity since multiple radio frequency chains must be implemented.

Transmit antenna selection is a practical way of reducing the system complexity while achieving the full diversity order of the AF MIMO relay channel [9], [10]. In the existing point-topoint communication systems, a single transmit antenna which maximizes the signal-to-noise ratio (SNR) at the receiver output is selected [11]. However, in relay systems, selecting a single best antenna at each node (source, relay and the destination) for maximizing the end-to-end (e2e) SNR is not a trivial problem to solve[9], [10]. Finding the optimal antenna configuration that yields the maximum the e2e SNR requires regular SNR measurements on the source-destination, sourcerelay and relay-destination links. These SNR values must be reported to one terminal (e.g. the destination) to enable a decision to be made and then the decision must be conveyed to the source terminal. This antenna selection strategy clearly requires a significant amount of SNR estimation time slots and feedback bits.

Dual-hop networks with antenna selection techniques have been proposed in the prior literature (see for example, [9], [10], [12]-[15]). In [12], the performance of a dual-hop AF relay system with transmit antenna selection at the source and maximal ratio combining (MRC) at the destination has been investigated. In [13] and [14], a dual-hop AF MIMO relay network with the best antenna selected at the source and the destination have been considered. In [15], fast antenna selection algorithms to maximize the capacity of AF and DF MIMO relay systems have been proposed. All of the work in [12]-[15] has ignored the direct link from the source to the destination. Such an assumption reduces the complexity of the antenna selection problem in some relay network configurations since maximizing individual link SNRs separately can maximize the overall received SNR at the destination. To the best of authors' knowledge, only a few papers have considered antenna selection in relay networks those that have direct paths [9], [10]. In [9] and [10], optimal and suboptimal SNR-based transmit antenna selection rules at the source/relay nodes for the AF half-duplex MIMO relay channel have been derived.

In this paper, we consider the performance of transmit
antenna selection for an AF relay system with a sourcedestination direct link. The system considered employs multiple antennas at the source while both the relay and the destination are equipped with a single antenna. This scenario can be directly applied to current cellular networks where the use of multiple antennas in a base station is reasonable, but the use of multiple antennas at mobile terminals and or relays may be prohibitive due to the terminal size and power constraints. Among possible antenna selection schemes in the network, we select and analytically study the rate outage probability of the optimal scheme along with another suboptimal scheme. In this suboptimal scheme, antenna selection is performed based only on the source-to-destination link channel reducing the signaling overhead compared to the optimal scheme. It should be mentioned that in this network, a suboptimal selection strategy based only on the source-relay link performs much worse than selection based only on the source-destination link, since the former has a much lower diversity order [16]. Furthermore, this also shows that not all possible suboptimal antenna selection schemes provide near optimal performance. Both the optimal and the source-destination based selection schemes achieve the highest possible diversity order of the system. To gain insights into their performance, we also derive high SNR approximations for both the optimal and the near optimal schemes. In [10], the rate outage performance of different antenna selection schemes has also been studied using a simulation based methodology. In contrast, our approach is analytically based and has the advantage of being able to show how the outage probability is affected by system and network parameters. Additionally, using the cumulative density function (cdf) expressions derived for the total received SNR, we have developed expressions for other performance measures such as the system's average bit error rate and the channel capacity. Using exhaustive Monte Carlo simulations we have confirmed the correctness of our analysis.

The rest of the paper is organized as follows. In Section II, the system model is presented. To investigate the rate outage probability, in Section III cdf expressions and high SNR approximations are derived. In Section IV, numerical results supported by simulations are presented. Finally, we conclude in Section V.

## II. System Description

In the cooperative wireless network of Fig. 1, a single source, $S$, employs a relay, $R$, for information transmission to a destination, $D$. We assume that a direct path between $S$ and $D$ is also available. Thus at $D$, the signals received via the direct $(S-D)$ and the relayed $(S-R-D)$ paths are combined and spatial diversity gains can be obtained. We assume that $S$ is equipped with $N_{t}$ antennas while both $R$ and $D$ are equipped with a single antenna. This model is reasonable for a downlink mobile communication system, since the base station can be equipped with more than one antenna while the mobile stations may not be equipped with more than one antenna due to size constraints [17]-[19].

Transmission in this network is accomplished in two phases: In the first phase, $S$ transmits its zero mean and unit variance signal $x$ towards $R$ and $D$. In the next phase, $R$ forwards an amplified version of its received signal, $y_{R}$, to $D$. If $S$


Fig. 1. System model. The source selects a single antenna, $k$, to communicate with the destination.
transmits using the $i$ th antenna, $i=1, \ldots, N_{t}$, the received signals at $D$ from the direct and relayed links are given by

$$
\begin{align*}
& y_{D 1}=\sqrt{P_{1}} h_{S D, i} x+n_{D 1},  \tag{1}\\
& y_{D 2}=h_{R D} G y_{R}+n_{D 2}, \tag{2}
\end{align*}
$$

where $P_{1}$ is the source transmit power, $h_{S D, i}$ is the channel between the $i$ th source antenna and $D, h_{R D}$ is the channel between $R$ and $D$ and $G$ is the relay gain. All links are assumed to be subject to independent Rayleigh fading, therefore we model the channels between the $i$ th source antenna and $D, h_{S D, i}$, for $i=1, \ldots, N_{t}$ as mutually independent and identically distributed (i.i.d) complex Gaussian random variables (RVs) with zero mean and average power, $\Omega_{S D}$. Similarly the channels between the $i$ th source antenna and $R, h_{S R, i}$, for $i=1, \ldots, N_{t}$ are modeled as mutually i.i.d complex Gaussian RVs with zero mean and average power, $\Omega_{S R}$. Moreover, we model $h_{R D}$ as an independent complex Gaussian RV with zero mean and average power, $\Omega_{R D}$. The assumption that the coefficients $h_{S D, i}$ for $i=1, \ldots, N_{t}$ are identically distributed and $h_{S R, i}$ for $i=1, \ldots, N_{t}$ are identically distributed is reasonable since all antennas are located at the source. We denote the additive white Gaussian noise (AWGN) at $D, n_{D}$, as complex Gaussian RVs with zero mean and variance $\sigma_{D}^{2}$. The received signal at $R$ during the first time slot, $y_{R}$, is given by

$$
\begin{equation*}
y_{R}=\sqrt{P_{1}} h_{S R, i} x+n_{R} \tag{3}
\end{equation*}
$$

where the complex AWGN at the relay is denoted by $n_{R}$ with zero mean and variance, $\sigma_{R}^{2}$. In this paper we employ the so called CSI-assisted relay gain. Therefore, $G$ is given by [4], [5]

$$
\begin{equation*}
G=\sqrt{\frac{P_{2}}{P_{1}\left|h_{S R, i}\right|^{2}+\sigma_{R}^{2}}}, \tag{4}
\end{equation*}
$$

where $P_{2}$ is the relay transmit power.
The destination has two observations containing $x$ and assuming that it employs a minimum mean squared error (MMSE) receiver, the e2e instantaneous SNR at $D$ can be written as [9]

$$
\begin{equation*}
\gamma_{i}=\gamma_{S D, i}+\frac{\gamma_{S R, i} \gamma_{R D}}{\gamma_{S R, i}+\gamma_{R D}+1} \tag{5}
\end{equation*}
$$

where $\gamma_{S D, i}=\frac{P_{1}}{\sigma_{D}^{2}}\left|h_{S D, i}\right|^{2}, \gamma_{S R, i}=\frac{P_{1}}{\sigma_{R}^{2}}\left|h_{S R, i}\right|^{2}, \gamma_{R D}=$ $\frac{P_{2}}{\sigma_{D}^{2}}\left|h_{R D}\right|^{2}$. The e2e SNR at the destination is the summation of the direct link SNR, $\gamma_{\mathcal{D}}$, and the relayed link SNR, $\gamma_{\mathcal{R}}$.

Unfortunately (5) is not mathematically tractable. No closed-form cdf or probability density function (pdf) expressions for the e2e SNR exist. These are required to analytically study the system performance ${ }^{1}$. As a solution, previous literature has adopted a tight upper bound to (5) given by [4], [21] as

$$
\begin{equation*}
\gamma_{i} \leq \gamma_{S D, i}+\min \left\{\gamma_{S R, i}, \gamma_{R D}\right\} \tag{6}
\end{equation*}
$$

Our subsequent analysis relies on this upper bound and using this bound, the statistics of $\gamma_{i}$ can be evaluated conveniently. Moreover, in Section IV we provide extensive simulation results to complement the analysis.

We now describe the transmit antenna selection schemes in detail.

Scheme (a) Optimal Antenna Selection: The optimal transmit antenna selection scheme maximizes the instantaneous post-processing SNR at $D$ by selecting a single best antenna at the source. Mathematically, transmit antenna $k$ is selected according to

$$
\begin{equation*}
k=\arg \max _{i}\left\{\gamma_{i}\right\} \tag{7}
\end{equation*}
$$

and the e2e SNR under the optimal antenna selection scheme is given by

$$
\begin{equation*}
\gamma=\gamma_{k} \tag{8}
\end{equation*}
$$

Using (6), the e2e SNR given in (8) can be tightly upper bounded as

$$
\begin{equation*}
\gamma_{\mathrm{ub} 1}=\gamma_{S D, k}+\min \left\{\gamma_{S R, k}, \gamma_{R D}\right\} \tag{9}
\end{equation*}
$$

This scheme can achieve the maximum possible diversity order of this system which is equal to $N_{t}+1$.
Scheme (b) Antenna Selection Based on the $S-D$ Link: A simple solution to reduce the signaling overhead is to employ an alternative antenna selection scheme whose operation is based on maximizing $\gamma_{\mathcal{D}}$ [10]. In other words, we can select a single antenna at the source, based only on the $S-D$ channel information.

Consider the selection strategy based only on the $S-D$ link. Transmit antenna $k$ is selected according to

$$
\begin{equation*}
k=\arg \max _{i}\left\{\gamma_{S D, i}\right\} \tag{10}
\end{equation*}
$$

Therefore, the e2e SNR at $D$ is given by

$$
\begin{equation*}
\gamma=\gamma_{S D, k}+\frac{\gamma_{S R, k} \gamma_{R D}}{\gamma_{S R, k}+\gamma_{R D}+1} \tag{11}
\end{equation*}
$$

Now the e2e SNR given in (11) can be tightly upper bounded as

$$
\begin{equation*}
\gamma_{\mathrm{ub} 2}=\gamma_{S D, k}+\min \left\{\gamma_{S R, k}, \gamma_{R D}\right\} \tag{12}
\end{equation*}
$$

Note that the diversity order of this scheme is also $N_{t}+1$. This motivates us to analytically compare the performance of the system under Schemes (a) and (b).

[^1]Scheme (a): An antenna selection decision can be made at $D$. A decision made at $D$ needs $2 N_{t}$ time slots and $\left\lceil\log _{2}\left(N_{t}\right)\right\rceil$ bits. This results from each of the $N_{t}$ source antennas requiring two time slots to signal and $D$ requiring $\left\lceil\log _{2}\left(N_{t}\right)\right\rceil$ bits, to convey the selected antenna index to $S$.

Scheme (b): $D$ implements an antenna selection decision using $N_{t}$ time slots and $\left\lceil\log _{2}\left(N_{t}\right)\right\rceil$ bits. This results from each of the $N_{t}$ source antennas requiring a single time slot to signal and $D$ requiring $\left\lceil\log _{2}\left(N_{t}\right)\right\rceil$ bits, to convey the selected antenna index to $S$.

In both schemes, the required feedback from $D$ to $S$ can be efficiently implemented in practice. For example, in many wireless systems, such as 3GPP HSPA, LTE etc use high rate feedback of CSI for selecting data transmission rates and also for identifying handover opportunities. Also, IEEE 802.16 m and LTE Advanced standards address relay networks, hence signaling mechanisms established there could be exploited.

## III. Performance Analysis

In this section we investigate important performance measures when transmit antenna selection Schemes $(a)$ and (b) are implemented. This includes the outage probability, average BER and the channel capacity. We also present simple high SNR approximations.

## A. Outage Probability

The instantaneous mutual information of the considered system is given by

$$
\begin{equation*}
\mathcal{I}=\frac{1}{2} \log _{2}(1+\gamma) \tag{13}
\end{equation*}
$$

Note that the pre-log factor $\frac{1}{2}$ is due to the two time slots employed in the relay protocol. The outage probability can be defined as the probability that $\mathcal{I}$ falls below the required rate, $R_{0}$. Mathematically, the probability, $\operatorname{Pr}\left(\mathcal{I}<R_{0}\right)$, is given by

$$
\begin{equation*}
P_{o}=F_{\gamma}\left(2^{2 R_{0}}-1\right) \tag{14}
\end{equation*}
$$

where $F_{\gamma}(\cdot)$ is the cdf of the e2e SNR. In order to derive the outage probability, next we will develop cdf expressions for Schemes (a) and (b).

Scheme (a): Consider the cdf of the e2e SNR, $F_{\gamma}(z)$, given by

$$
\begin{equation*}
\operatorname{Pr}[\gamma<z]=\operatorname{Pr}\left[\gamma_{S D, k}+\frac{\gamma_{S R, k} \gamma_{R D}}{\gamma_{S R, k}+\gamma_{R D}+1}<z\right] \tag{15}
\end{equation*}
$$

Conditioned upon $\gamma_{R D}=v$, the cdf in (15) can be rewritten as

$$
\begin{equation*}
F_{\gamma}(z)=E_{v}\left[\operatorname{Pr}\left(\gamma_{S D, k}+\frac{\gamma_{S R, k} v}{\gamma_{S R, k}+v+1}<z\right)\right] \tag{16}
\end{equation*}
$$

where $E_{v}[\cdot]$ denotes the expectation operator with respect to the RV $v$. Using the upper bound in (9) gives the corresponding version of (16) as

$$
\begin{align*}
F_{\gamma_{\text {ub1 }}}(z) & =E_{v}\left[\operatorname{Pr}\left(\gamma_{S D, k}+\min \left(\gamma_{S R, k}, v\right)<z\right)\right]  \tag{17}\\
& =E_{v}\left[\operatorname{Pr}\left(\gamma_{S D, i}+\min \left(\gamma_{S R, i}, v\right)<z\right)^{N_{t}}\right]
\end{align*}
$$

To proceed further, we define $X=\gamma_{S D, i}, Y=\gamma_{S R, i}$ and $Z=X+\min (Y, v)$. Also the pdfs of $\gamma_{S D, i}, \gamma_{S R, i}$
and $\gamma_{R D}$ are given by $f_{\gamma_{S D, i}}(x)=\frac{1}{\bar{\gamma}_{S D}} e^{-\frac{x}{\gamma_{S D}}}, f_{\gamma_{S R, i}}(x)=$ $\frac{1}{\bar{\gamma}_{S R}} e^{-\frac{x}{\bar{\gamma}_{S R}}}$ and $f_{\gamma_{R D}}(x)=\frac{1}{\bar{\gamma}_{R D}} e^{-\frac{\bar{\gamma}_{S D}}{\bar{\gamma}_{R D}}}$, respectively where $\bar{\gamma}_{S D}=\frac{P_{1} \Omega_{S D}}{\sigma_{D}^{2}}, \bar{\gamma}_{S R}=\frac{P_{1} \Omega_{S R}}{\sigma_{R}^{2}}$ and $\bar{\gamma}_{R D}=\frac{P_{2} \Omega_{R D}}{\sigma_{D}^{2}}$.

In order to obtain an expression for $F_{\gamma_{\mathrm{ubl}}}(z)$, we will first derive an expression for the cdf of the RV, $Z$. The cdf of $Z$, $F_{Z}(z)$, can be written as

$$
\begin{align*}
F_{Z}(z) & =\operatorname{Pr}(X<z-\min (Y, v))  \tag{18}\\
& =\frac{1}{\bar{\gamma}_{S R}} \int_{0}^{\infty} \operatorname{Pr}(X<z-\min (y, v)) e^{-\frac{y}{\gamma_{S R}}} d y
\end{align*}
$$

To simplify the integral in (18) let us define the two domains, $\mathcal{D}_{1}=\{\min (y, v)>z\}$ and $\mathcal{D}_{2}=\{\min (y, v)<z\}$. In $\mathcal{D}_{1}$, $v>z$ and $y>z$. In $\mathcal{D}_{2}, v<z$ or $y<z$. Note that if $v>z$, $\mathcal{D}_{1}=\{y>z\}$ and $\mathcal{D}_{2}=\{y<z\}$. If $v<z, \mathcal{D}_{1}=\{\emptyset\}$ and $\mathcal{D}_{2}=\{y>0\}$.

First consider the case of $v<z ; F_{Z}(z)$ can be written as

$$
\begin{equation*}
F_{Z}(z)=\frac{1}{\bar{\gamma}_{S R}} \int_{0}^{\infty}\left(1-e^{-\frac{(z-\min (y, v))}{\bar{\gamma}_{S D}}}\right) e^{-\frac{y}{\bar{\gamma}_{S R}}} d y \tag{19}
\end{equation*}
$$

Equation (19) can be re-expressed as

$$
\begin{align*}
F_{Z}(z) & =1-\frac{1}{\bar{\gamma}_{S R}} \int_{0}^{v} e^{-\frac{z-y}{\bar{\gamma}_{S D}}} e^{-\frac{y}{\gamma_{S R}}} d y  \tag{20}\\
& -\frac{1}{\bar{\gamma}_{S R}} \int_{v}^{\infty} e^{-\frac{z-v}{\gamma_{S D}}} e^{-\frac{y}{\gamma_{S R}}} d y
\end{align*}
$$

Simplifying (20) yields

$$
\begin{align*}
F_{Z}(z) & =1-\bar{\gamma}_{S D} e^{-\frac{z}{\gamma_{S D}}}\left(\frac{1-e^{-\left(\frac{\bar{\gamma}_{S D}-\bar{\gamma}_{S R}}{\gamma_{S D} \bar{\gamma}_{S R}}\right) v}}{\bar{\gamma}_{S D}-\bar{\gamma}_{S R}}\right)  \tag{21}\\
& -e^{-\frac{z-v}{\bar{\gamma}_{S D}}-\frac{v}{\bar{\gamma} S R}} \\
& \stackrel{\text { def }}{=} \mathcal{F}(v, z)
\end{align*}
$$

Next consider the case of $v>z$; Since $\operatorname{Pr}(X<0)=0$, we have $y<z$ and $\min (y, v)=y$. Now using (18), $F_{Z}(z)$ is given by

$$
\begin{align*}
F_{Z}(z) & =\frac{1}{\bar{\gamma}_{S R}} \int_{0}^{z}\left(1-e^{-\frac{(z-\min (y, v))}{\bar{\gamma}_{S D}}}\right) e^{-\frac{y}{\bar{\gamma}_{S R}}} d y  \tag{22}\\
& =\frac{1}{\bar{\gamma}_{S R}} \int_{0}^{z} e^{-\frac{y}{\gamma_{S R}}} d y-\frac{1}{\bar{\gamma}_{S R}} \int_{0}^{z} e^{\frac{-z+y}{\bar{\gamma}_{S D}}} e^{-\frac{y}{\gamma_{S R}}} d y
\end{align*}
$$

Equation (22) can be re-expressed as

$$
\begin{equation*}
F_{Z}(z)=1-e^{-\frac{z}{\bar{\gamma}_{S R}}}-\frac{e^{-\frac{z}{\bar{\gamma}_{S D}}}}{\bar{\gamma}_{S R}} \int_{0}^{z} e^{-\left(\frac{\overline{\bar{\gamma}} S D-\bar{\gamma}_{S R}}{\bar{\gamma}_{S D} \overline{\bar{\gamma}}_{S R}}\right) y} d y \tag{23}
\end{equation*}
$$

Simplifying the integral in (23) we get

$$
\begin{align*}
F_{Z}(z) & =1-e^{-\frac{z}{\bar{\gamma}_{S R}}}-\frac{\bar{\gamma}_{S D}\left(e^{-\frac{z}{\gamma_{S D}}}-e^{-\frac{z}{\bar{\gamma}_{S R}}}\right)}{\bar{\gamma}_{S D}-\bar{\gamma}_{S R}}  \tag{24}\\
& \stackrel{\text { def }}{=} \mathcal{G}(z)
\end{align*}
$$

Since $F_{\gamma_{\mathrm{ub1}}}(z)=E_{v}\left[\operatorname{Pr}(Z<z)^{N_{t}}\right]$, the cdf of the e2e SNR can be written as

$$
\begin{align*}
F_{\gamma_{\mathrm{ubl}}}(z) & =\frac{1}{\bar{\gamma}_{R D}} \int_{0}^{z}(\mathcal{F}(v, z))^{N_{t}} e^{-\frac{v}{\bar{\gamma}_{R D}} d v}  \tag{25}\\
& +\frac{1}{\bar{\gamma}_{R D}} \int_{z}^{\infty}(\mathcal{G}(z))^{N_{t}} e^{-\frac{v}{\gamma_{R D}} d v}
\end{align*}
$$

Equation (25) can be simplified as

$$
\begin{align*}
F_{\gamma_{\mathrm{ubl}}}(z) & =\frac{1}{\bar{\gamma}_{R D}} \int_{0}^{z}\left(p+q e^{-\omega v}\right)^{N_{t}} e^{-\frac{v}{\gamma_{R D}}} d v  \tag{26}\\
& +(\mathcal{G}(z))^{N_{t}} e^{-\frac{z}{\gamma_{R D}}}
\end{align*}
$$

where

$$
\begin{aligned}
p & =1-\frac{\bar{\gamma}_{S D}}{\bar{\gamma}_{S D}-\bar{\gamma}_{S R}} e^{-\frac{z}{\bar{\gamma}_{S D}}} \\
q & =\frac{\bar{\gamma}_{S R}}{\bar{\gamma}_{S D}-\bar{\gamma}_{S R}} e^{-\frac{z}{\bar{\gamma}_{S D}}}
\end{aligned}
$$

and

$$
\begin{equation*}
\omega=\frac{\bar{\gamma}_{S D}-\bar{\gamma}_{S R}}{\bar{\gamma}_{S D} \bar{\gamma}_{S R}} \tag{29}
\end{equation*}
$$

We can rewrite (26) using the binomial theorem as

$$
\begin{align*}
F_{\gamma_{\mathrm{ubl}}}(z) & =(\mathcal{G}(z))^{N_{t}} e^{-\frac{z}{\bar{\gamma}_{R D}}}  \tag{30}\\
& +\frac{1}{\bar{\gamma}_{R D}} \sum_{j=0}^{N_{t}}\binom{N_{t}}{j} p^{N_{t}-j} q^{j} \int_{0}^{z} e^{-\left(\frac{1}{\gamma_{R D}}+j \omega\right) v} d v
\end{align*}
$$

After simplifying the integral, the cdf of the e2e SNR in Scheme (a) can be expressed as

$$
\begin{align*}
F_{\gamma}(z) & \geq F_{\gamma_{\mathrm{ubl}}}(z)=(\mathcal{G}(z))^{N_{t}} e^{-\frac{z}{\gamma_{R D}}}  \tag{31}\\
& +\frac{1}{\bar{\gamma}_{R D}} \sum_{j=0}^{N_{t}}\binom{N_{t}}{j} p^{N_{t}-j} q^{j}\left(\frac{1-e^{-\left(\frac{1}{\gamma_{R D}}+j \omega\right) z}}{1 / \bar{\gamma}_{R D}+j \omega}\right)
\end{align*}
$$

Finally, the outage probability of Scheme (a) can be obtained from (31) by substituting $z=2^{2 R_{0}}-1$.

Scheme ( $b$ ): We now derive an expression for $F_{\gamma_{\text {ub } 2}}(z)$ which is a tight lower bound to outage probability of the system when Scheme ( $b$ ) is implemented. Let $\tilde{X}=\gamma_{S D, k}$ and $\tilde{Y}=$ $\min \left\{\gamma_{S R, k}, \gamma_{R D}\right\}$. First we will obtain an expression for the e2e SNR pdf. Since $\tilde{X}$ and $\tilde{Y}$ are independent, the pdf of the e2e SNR given by their sum, can be obtained using the convolution integral as

$$
\begin{equation*}
f_{\gamma_{\mathrm{ub} 2}}(z)=\int_{0}^{z} f_{\tilde{X}}(y) f_{\tilde{Y}}(z-y) d y \tag{32}
\end{equation*}
$$

where $f_{\tilde{X}}(\cdot)$ and $f_{\tilde{Y}}(\cdot)$ denote the pdfs of $\tilde{X}$ and $\tilde{Y}$ respectively. The pdf of $\tilde{X}$ can be found using the cdf expression of [22, Eq. (9.344)] and is given by

$$
\begin{equation*}
f_{\tilde{X}}(y)=\frac{N_{t}}{\bar{\gamma}_{S D}} \sum_{n=0}^{N_{t}-1}(-1)^{n}\binom{N_{t}-1}{n} e^{-\frac{n+1}{\bar{\gamma}_{S D}} y} \tag{33}
\end{equation*}
$$

In Scheme (b) selection of $k$ leads to a random selection of the $S-R$ channel and therefore, $\gamma_{S R, k}$ is an exponential RV. Noting that the minimum of two exponential RVs is also another exponential RV, $\tilde{Y}$ is distributed as

$$
\begin{equation*}
f_{\tilde{Y}}(y)=\frac{\bar{\gamma}_{S R}+\bar{\gamma}_{R D}}{\bar{\gamma}_{S R} \bar{\gamma}_{R D}} e^{-\left(\frac{\bar{\gamma}_{S R}+\bar{\gamma}_{R D}}{\bar{\gamma}_{S R}}\right) y} \tag{34}
\end{equation*}
$$

We rewrite (32) after substituting (33) and (34) as

$$
\begin{align*}
f_{\gamma_{\mathrm{ub} 2}}(z) & =\frac{N_{t}}{\bar{\gamma}_{S D}}\left(\frac{\bar{\gamma}_{S R}+\bar{\gamma}_{R D}}{\bar{\gamma}_{S R} \bar{\gamma}_{R D}}\right) e^{-\left(\frac{\bar{\gamma}_{S R}+\bar{\gamma}_{R D}}{\bar{\gamma}_{S R} \bar{\gamma}_{R D}}\right) z}  \tag{35}\\
& \times \sum_{n=0}^{N_{t}-1}(-1)^{n}\binom{N_{t}-1}{n} \int_{0}^{z} e^{-\left(\frac{n+1}{\gamma_{S D}}-\frac{\bar{\gamma}_{S R}+\bar{\gamma}_{R D}}{\gamma_{S R} \bar{\gamma}_{R D}}\right) y} d y
\end{align*}
$$

Simplifying the integral in (35), the pdf can be expressed in closed-form as

$$
\begin{align*}
f_{\gamma_{\mathrm{ub} 2}}(z) & =\frac{N_{t}}{\bar{\gamma}_{S D}}\left(\frac{\bar{\gamma}_{S R}+\bar{\gamma}_{R D}}{\bar{\gamma}_{S R} \bar{\gamma}_{R D}}\right)  \tag{36}\\
& \times \sum_{n=0}^{N_{t}-1} \frac{(-1)^{n}\binom{N_{t}-1}{n}\left(e^{-\left(\frac{\bar{\gamma}_{S R}+\bar{\gamma}_{R D}}{\bar{\gamma}_{S R} \bar{\gamma}_{R D}}\right) z}-e^{-\frac{n+1}{\bar{\gamma}_{S D}} z}\right)}{\frac{n+1}{\bar{\gamma}_{S D}}-\frac{\bar{\gamma}_{S R}+\bar{\gamma}_{R D}}{\bar{\gamma}_{S R} \bar{\gamma}_{R D}}} .
\end{align*}
$$

Now it is easy to obtain the cdf, $F_{\gamma_{\mathrm{ub} 2}}(z)=\int_{0}^{z} f_{\gamma_{\mathrm{ub} 2}}(z) d z$. The cdf of $\gamma_{\mathrm{ub} 2}$ is given by

$$
\begin{align*}
F_{\gamma}(z) & \geq F_{\gamma_{\mathrm{ub} 2}}(z)  \tag{37}\\
& =\frac{N_{t}}{\bar{\gamma}_{S D}} \sum_{n=0}^{N_{t}-1} \frac{(-1)^{n}\binom{N_{t}-1}{n}}{\frac{n+1}{\bar{\gamma}_{S D}}-\frac{\bar{\gamma}_{S R}+\bar{\gamma}_{R D}}{\bar{\gamma}_{S R} \bar{\gamma}_{R D}}}\left(1-e^{-\left(\frac{\bar{\gamma}_{S R}+\bar{\gamma}_{R D}}{\gamma_{S R} \gamma_{R D}}\right) z}\right. \\
& \left.-\frac{\bar{\gamma}_{S D}}{n+1}\left(\frac{1}{\bar{\gamma}_{S R}}+\frac{1}{\bar{\gamma}_{R D}}\right)\left(1-e^{-\left(\frac{n+1}{\bar{\gamma}_{S D}}\right) z}\right)\right) .
\end{align*}
$$

The outage probability of Scheme (b) can be obtained from (37) by substituting $z=2^{2 R_{0}}-1$.

## B. High SNR Outage Approximations

Although the outage expressions of Schemes (a) and (b) given by (31) and (37) are valid for arbitrary SNR, it is difficult to obtain further insights. For example, it is interesting to know how system and network parameters such as $N_{t}$ and SNR imbalances among $\bar{\gamma}_{S D}, \bar{\gamma}_{R S}$ and $\bar{\gamma}_{R D}$ influence the system's outage performance. Such information can not be directly obtained from (31) or (37). Along these lines, we have developed simple high SNR approximations, whose derivations can be found in the Appendix.

Scheme (a): At high SNR, the system's outage probability is given by ${ }^{2}$

$$
\begin{align*}
P_{o / a}^{\infty} & =\frac{1}{\mu_{1} \mu_{2}\left(N_{t}+1\right)}\left(\frac{2^{2 R_{0}}-1}{\bar{\gamma}_{S D}}\right)^{N_{t}+1}  \tag{38}\\
& +o\left(\left(\frac{2^{2 R_{0}}-1}{\bar{\gamma}_{S D}}\right)^{N_{t}+2}\right)
\end{align*}
$$

where $\mu_{1}=\frac{\bar{\gamma}_{S R}}{\bar{\gamma}_{S D}}$ and $\mu_{2}=\frac{\bar{\gamma}_{R D}}{\bar{\gamma}_{S R}}$.
Scheme (b): At high SNR, the system's outage probability is given by

$$
\begin{align*}
P_{o / b}^{\infty} & =\frac{1+\mu_{2}}{\mu_{1} \mu_{2}\left(N_{t}+1\right)}\left(\frac{2^{2 R_{0}}-1}{\bar{\gamma}_{S D}}\right)^{N_{t}+1}  \tag{39}\\
& +o\left(\left(\frac{2^{2 R_{0}}-1}{\bar{\gamma}_{S D}}\right)^{N_{t}+2}\right) .
\end{align*}
$$

Clearly, (38) and (39) show that both Schemes (a) and (b) achieve the same diversity order of $N_{t}+1$. The reason for this is that the relay-destination link in the relayed path acts as the bottleneck. In other words, in Scheme (a), although the best source-relay link is selected, due the single antenna relaydestination link, the relayed path does not provide a diversity gain (i.e., diversity is one). In Scheme (b), we totally ignore the relayed path meaning that the diversity is again one. However, in both schemes, the source-destination direct path offers the

[^2]full spatial degrees of freedom and is equal to the total number of transmit antennas (since the destination employs a single antenna). As a result combining the relayed and direct paths, both schemes provide the same diversity order.

When the outage probability is plotted on a log scale against $\bar{\gamma}_{S D}$ in dB, the high SNR penalty, $\delta$, of Scheme (b) compared to Scheme (a) can be expressed as $^{3}$

$$
\begin{equation*}
\delta=\frac{10}{N_{t}+1} \log \left(1+\mu_{2}\right) \quad \mathrm{dB} \tag{40}
\end{equation*}
$$

To see this, for Schemes (a) and (b) we write

$$
\begin{align*}
\log \left(P_{o / a}^{\infty}\right) & =\log \left(\frac{1}{\mu_{1} \mu_{2}\left(N_{t}+1\right)}\right)  \tag{41}\\
& -\left(N_{t}+1\right) \log \left(\frac{\bar{\gamma}_{S D}}{2^{2 R_{0}}-1}\right)
\end{align*}
$$

and

$$
\begin{align*}
\log \left(P_{o / b}^{\infty}\right) & =\log \left(\frac{1+\mu_{2}}{\mu_{1} \mu_{2}\left(N_{t}+1\right)}\right)  \tag{42}\\
& -\left(N_{t}+1\right) \log \left(\frac{\bar{\gamma}_{S D}}{2^{2 R_{0}}-1}\right)
\end{align*}
$$

respectively. Now equating (41) and (42), simplifying and noting that what we require is an expression for $10\left(\log \left(\left.\bar{\gamma}_{S D}\right|_{\text {Scheme (a) }}-\log \left(\left.\bar{\gamma}_{S D}\right|_{\text {Scheme (b) }}\right)\right.\right.$ yields the desired result in (40).

The high SNR performances of both schemes are governed by $N_{t}, \mu_{1}$ and $\mu_{2}$. Clearly, in both schemes, we see that increasing these parameters has a positive effect on the outage probability. On the other hand, (40) shows that $\delta$ depends on $N_{t}$ and $\mu_{2}$. We see that the performance loss for large $N_{t}$ and small $\mu_{2}$ is marginal.

## C. Average BER

In this subsection we analyze the average BER of Schemes (a) and (b). For many modulation formats used in wireless applications, the average BER can be expressed as [23]

$$
\begin{align*}
P_{e} & =E[a Q(\sqrt{b \gamma})]  \tag{43}\\
& =\frac{a}{\sqrt{2 \pi}} \int_{0}^{\infty} F_{\gamma}\left(\frac{t^{2}}{b}\right) e^{-\frac{t^{2}}{2}} d t
\end{align*}
$$

where $Q(x)=\frac{1}{\sqrt{2 \pi}} \int_{x}^{\infty} e^{-\frac{y^{2}}{2}} d y$ is the Gaussian $Q$-function and $a, b>0$ are determined by specific constellations. Some example modulation schemes for which (43) apply include binary phase shift keying (BPSK) modulation, $a=1$, and $b=$ 2, and Quadrature phase shift keying (QPSK), $a=1$ and $b=$ 1. Our results also provide the approximate BER for M-ary phase-shift keying (M-PSK), $a=1$, and $b=2 \sin ^{2}(\pi / M)$ ). Moreover, the following BER derivations can also be trivially extended to square/rectangular M-QAM, for which $P_{e}$ can be written as a weighted sum of $a Q(\sqrt{b \gamma})$ terms [24].

[^3]Scheme (a): By substituting (31) into (43) system's average BER can be tightly lower bounded as

$$
\begin{align*}
& P_{e} \geq \tilde{P}_{e / a}  \tag{44}\\
& =\frac{a}{\sqrt{2 \pi}} \int_{0}^{\infty}\left(1-\frac{e^{-\frac{t^{2}}{b \bar{\gamma}_{S D}}}}{1-\frac{\bar{\gamma} \bar{\gamma}_{S R}}{\bar{\gamma}_{S D}}}-\frac{e^{-\frac{t^{2}}{b \bar{\gamma}_{S R}}}}{1-\frac{\bar{\gamma}_{S D}}{\bar{\gamma}_{S R}}}\right)^{N_{t}} \\
& \times e^{-\left(\frac{1}{b \bar{\gamma}_{R D}}+\frac{1}{2}\right) t^{2}} d t \\
& +\frac{a}{\sqrt{2 \pi}} \sum_{j=0}^{N_{t}}\binom{N_{t}}{j} \int_{0}^{\infty}\left(1-\frac{e^{-\frac{t^{2}}{b \bar{\gamma}_{S D}}}}{\omega \bar{\gamma}_{S R}}\right)^{N_{t}-j} \\
& \times \frac{e^{-\left(\frac{j}{b \bar{\gamma} \bar{\gamma}_{S D}}+\frac{1}{2}\right) t^{2}}}{\left(\omega \bar{\gamma}_{S D}\right)^{j}}\left(\frac{1-e^{-\left(\frac{1}{\gamma_{R D}}+j \omega\right) \frac{t^{2}}{b}}}{1+j \omega \bar{\gamma}_{R D}}\right) d t .
\end{align*}
$$

Next, we apply the binomial expansion to simplify (44) and obtain

$$
\begin{align*}
\tilde{P}_{e / a} & =\frac{a}{\sqrt{2 \pi}} \sum_{j=0}^{N_{t}} \frac{\binom{N_{t}}{j}}{\left(\omega \bar{\gamma}_{S D}\right)^{j}} \sum_{k=0}^{N_{t}-j}\binom{N_{t}-j}{k} \frac{(-1)^{k}}{\left(\omega \bar{\gamma}_{S R}\right)^{k}}  \tag{45}\\
& \times \int_{0}^{\infty} e^{-\left(\frac{j}{\gamma_{S R}}+\frac{1}{\gamma_{R D}}+\frac{k}{\gamma_{S D}}+\frac{b}{2}\right) \frac{t^{2}}{b}} d t \\
& +\frac{a}{\sqrt{2 \pi}} \sum_{j=0}^{N_{t}} \frac{\binom{N_{t}}{j}}{\left(\omega \bar{\gamma}_{S D}\right)^{j}\left(1+j \omega \bar{\gamma}_{R D}\right)} \\
& \times \sum_{k=0}^{N_{t}-j}\binom{N_{t}-j}{k} \frac{(-1)^{k}}{\left(\omega \bar{\gamma}_{S R}\right)^{k}} \\
& \times \int_{0}^{\infty}\left(1-e^{-\left(\frac{1}{\bar{\gamma}_{R D}}+j \omega\right) \frac{t^{2}}{b}}\right) e^{-\left(\frac{j+k}{\bar{\gamma}_{S D}}+\frac{b}{2}\right) \frac{t^{2}}{b}} d t .
\end{align*}
$$

Finally, using the following integral result from [25]

$$
\begin{equation*}
\mathcal{J}_{1}=\int_{0}^{\infty} e^{-q^{2} x^{2}} d x=\frac{\sqrt{\pi}}{2 q}, \quad[q>0] \tag{46}
\end{equation*}
$$

$\tilde{P}_{e / a}$ can be evaluated in closed-form as

$$
\begin{align*}
& \tilde{P}_{e / a}=\frac{a}{2} \sum_{j=0}^{N_{t}} \frac{\binom{N_{t}}{j}}{\left(\omega \bar{\gamma}_{S D}\right)^{j}}  \tag{47}\\
& \times \sum_{k=0}^{N_{t}-j} \frac{(-1)^{k}\binom{N_{t}-j}{k}}{\left(\omega \bar{\gamma}_{S R}\right)^{k} \sqrt{1+\frac{2 j}{b \bar{\gamma}_{S R}}+\frac{2}{b \bar{\gamma}_{R D}}+\frac{2 k}{b \bar{\gamma}_{S D}}}} \\
& +\frac{a}{2} \sum_{j=0}^{N_{t}} \frac{\binom{N_{t}}{j}}{\left(\omega \bar{\gamma}_{S D}\right)^{j}\left(1+j \omega \bar{\gamma}_{R D}\right)} \sum_{k=0}^{N_{t}-j}\binom{N_{t}-j}{k} \frac{(-1)^{k}}{\left(\omega \bar{\gamma}_{S R}\right)^{k}} \\
& \times\left(\frac{1}{\sqrt{1+\frac{2(j+k)}{b \bar{\gamma}_{S D}}}}-\frac{1}{\sqrt{1+\frac{2}{b \bar{\gamma}_{R D}}+\frac{2 j \omega}{b}+\frac{2(j+k)}{b \bar{\gamma}_{S D}}}}\right)
\end{align*}
$$

Scheme (b): Substituting (37) into (43), the average BER of Scheme (b) can be expressed as

$$
\begin{align*}
& P_{e} \geq \tilde{P}_{e / b}=\frac{a N_{t}}{2 \bar{\gamma}_{S D}} \sum_{n=0}^{N_{t}-1} \frac{(-1)^{n}\binom{N_{t}-1}{n}}{\frac{n+1}{\bar{\gamma}_{S D}}-\frac{\bar{\gamma}_{S R}+\bar{\gamma}_{R D}}{\bar{\gamma}_{S R} \bar{\gamma}_{R D}}}  \tag{48}\\
& \quad-\frac{a N_{t}}{\sqrt{2 \pi} \bar{\gamma}_{S D}} \sum_{n=0}^{N_{t}-1} \frac{(-1)^{n}\binom{N_{t}-1}{n}}{\frac{n+1}{\bar{\gamma}_{S D}}-\frac{\bar{\gamma}_{S R}+\bar{\gamma}_{R D}}{\bar{\gamma}_{S R} \bar{\gamma}_{R D}}} \tag{55}
\end{align*}
$$

Scheme (a): Using the pdf of $\gamma_{\mathrm{ub} 1}, f_{\gamma_{\mathrm{ubl}}}(x)$, the exact $C$ can be tightly upper bounded by solving the following integral

$$
C_{a}=\frac{1}{2 \ln 2} \int_{0}^{\infty} \ln (1+x) f_{\gamma_{\mathrm{ub} 1}}(x) d x
$$

$$
\begin{align*}
f_{\gamma_{\text {bl }}}(z) & =\sum_{j=0}^{N_{t}} \frac{\binom{N_{t}}{j}}{\left(\omega \bar{\gamma}_{S D}\right)^{j}} \sum_{k=0}^{N_{t}-j} \frac{(-1)^{k-1}\binom{N_{t}-j}{k}\left(\frac{j}{\bar{\gamma}_{S R}}+\frac{1}{\bar{\gamma}_{R D}}+\frac{k}{\bar{\gamma} S D}\right)}{\left(\omega \bar{\gamma}_{S R}\right)^{k}} e^{-\left(\frac{j}{\overline{\gamma_{S R}}}+\frac{1}{\bar{\gamma}_{R D}}+\frac{k}{\gamma_{S D}}\right) z}  \tag{56}\\
& +\sum_{j=0}^{N_{t}} \frac{\binom{N_{t}}{j}}{\left(\omega \bar{\gamma}_{S D}\right)^{j}\left(1+j \omega \bar{\gamma}_{R D}\right)} \sum_{k=0}^{N_{t}-j} \frac{(-1)^{k-1}\left(N_{t}-j\right.}{k} \begin{array}{l}
\left(\omega \bar{\gamma}_{S R}\right)^{k}
\end{array} \\
& \times\left(\frac{j+k}{\bar{\gamma}_{S D}} e^{-\left(\frac{j+k}{\gamma_{\gamma D}}\right) z}-\left(\frac{1}{\bar{\gamma}_{R D}}+j \omega+\frac{j+k}{\bar{\gamma}_{S D}}\right) e^{-\left(\frac{1}{\gamma_{R D}}+j \omega+\frac{j+k}{\bar{\gamma}_{S D}}\right) z}\right) .
\end{align*}
$$

$$
\begin{align*}
C \leq C_{a} & =\frac{1}{2 \ln (2)} \sum_{j=0}^{N_{t}} \frac{\binom{N_{t}}{j}}{\left(\omega \bar{\gamma}_{S D}\right)^{j}} \sum_{k=0}^{N_{t}-j} \frac{(-1)^{k-1}\binom{N_{t}-j}{k}\left(\frac{j}{\bar{\gamma}_{S R}}+\frac{1}{\bar{\gamma}_{R D}}+\frac{k}{\bar{\gamma}_{S D}}\right)}{\left(\omega \bar{\gamma}_{S R}\right)^{k}}  \tag{57}\\
& \times \int_{0}^{\infty} \ln (1+x) e^{-\left(\frac{j}{\bar{\gamma}_{S R}}+\frac{1}{\bar{\gamma}_{R D}}+\frac{k}{\bar{\gamma}_{S D}}\right) x} d x+\frac{1}{2 \ln (2)} \sum_{j=0}^{N_{t}} \frac{\binom{N_{t}}{j}}{\left(\omega \bar{\gamma}_{S D}\right)^{j}\left(1+j \omega \bar{\gamma}_{R D}\right)} \\
& \times \sum_{k=0}^{N_{t}-j} \frac{(-1)^{k-1}\binom{N_{t}-j}{k}}{\left(\omega \bar{\gamma}_{S R}\right)^{k}}\left(\frac{j+k}{\bar{\gamma}_{S D}} \int_{0}^{\infty} \ln (1+x) e^{-\left(\frac{j+k}{\bar{\gamma}_{S D}}\right) x} d x\right. \\
& \left.-\left(\frac{1}{\bar{\gamma}_{R D}}+j \omega+\frac{j+k}{\bar{\gamma}_{S D}}\right) \int_{0}^{\infty} \ln (1+x) e^{-\left(\frac{1}{\bar{\gamma}_{R D}}+j \omega+\frac{j+k}{\gamma_{S D}}\right) x} d x\right) .
\end{align*}
$$

In order to derive the capacity we first need an expression for the pdf of $\gamma_{\mathrm{ub} 1}$. This can be easily obtained using the cdf of $\gamma_{\mathrm{ub} 1}$ given in (31). Therefore, by differentiating (31) with respect to $z$ we obtain the pdf of $\gamma_{\mathrm{ub} 1}$ as (56). Now substituting (56) into (55) we write (57). Using the closed-form expression for the integral ${ }^{4}$

$$
\begin{equation*}
\mathcal{J}_{2}=\int_{0}^{\infty} \ln (1+\beta x) e^{-s x} d x=\frac{e^{\frac{s}{\beta}}}{s} E_{1}\left(\frac{s}{\beta}\right) \tag{58}
\end{equation*}
$$

given in [25], where $E_{1}(x)=\int_{x}^{\infty} \frac{e^{-t}}{t} d t$ is the exponential integral function of order one, the mean capacity can be tightly upper bounded as (59) where

$$
\Im= \begin{cases}0, & j=k=0 \\ 1, & \text { else }\end{cases}
$$

Scheme (b): We substitute (36) into (55) and express the upper bound for the mean channel capacity as (60). The integrals in (60) can be evaluated in closed-form using (58) to yield

$$
\begin{align*}
C_{b} & =\frac{N_{t}}{2 \ln (2) \bar{\gamma}_{S D}} e^{\frac{\bar{\gamma}_{S R}+\bar{\gamma}_{R D}}{\bar{\gamma}_{S R} \bar{\gamma}_{R D}}} E_{1}\left(\frac{\bar{\gamma}_{S R}+\bar{\gamma}_{R D}}{\bar{\gamma}_{S R} \bar{\gamma}_{R D}}\right)  \tag{61}\\
& \times \sum_{n=0}^{N_{t}-1} \frac{(-1)^{n}\binom{N_{t}-1}{n}}{\frac{n+1}{\bar{\gamma}_{S D}}-\frac{\bar{\gamma}_{S R}+\bar{\gamma}_{R D}}{\bar{\gamma}_{S R} \bar{\gamma}_{R D}}}-\frac{N_{t}}{2 \ln (2)}\left(\frac{\bar{\gamma}_{S R}+\bar{\gamma}_{R D}}{\bar{\gamma}_{S R} \bar{\gamma}_{R D}}\right) \\
& \times \sum_{n=0}^{N_{t}-1} \frac{(-1)^{n}\binom{N_{t}-1}{n} e^{\frac{n+1}{\gamma_{S D}}}}{\left(\frac{n+1}{\bar{\gamma}_{S D}}-\frac{\bar{\gamma}_{S R}+\bar{\gamma}_{R D}}{\bar{\gamma}_{S R}}\right)(n+1)} E_{1}\left(\frac{n+1}{\bar{\gamma}_{S D}}\right) .
\end{align*}
$$

## IV. Numerical Results and Comparisons

This section presents the numerical and simulation results for the system's rate outage probability, average BER and the mean channel capacity.

[^4]

Fig. 2. Scheme (a): Outage probability versus $\bar{\gamma}_{S D}$ for different $N_{t} . \mu_{1}=$ 1.25 and $\mu_{2}=0.5$.

Figs. 2 and 3 show the outage probability for Scheme (a) and Scheme ( $b$ ) respectively. The target rate, $R_{0}=1 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}$. The analytical results are plotted using Eqs. (31) and (37) respectively, while the high SNR approximations correspond to Eqs. (38) and (39). In order to verify our analytical results, we have also plotted simulation based results. We see that the analytical results obtained by employing the tight bound match perfectly with the simulations in the medium to high SNR regimes. Furthermore, it is observed that the high-SNR approximation curves converge to the analytical ones in the medium to high SNR regimes. As expected, increasing the number of antennas has a positive effect on reducing the outage probability. However, the reduction of outage probability

$$
\begin{aligned}
C_{a} & =\frac{1}{2 \ln (2)} \sum_{j=0}^{N_{t}} \frac{\binom{N_{t}}{j}}{\left(\omega \bar{\gamma}_{S D}\right)^{j}} \sum_{k=0}^{N_{t}-j} \frac{(-1)^{k-1}\binom{N_{t}-j}{k}}{\left(\omega \bar{\gamma}_{S R}\right)^{k}} e^{\left(\frac{j}{\bar{\gamma}_{S R}}+\frac{1}{\bar{\gamma}_{R D}}+\frac{k}{\bar{\gamma}_{S D}}\right)} E_{1}\left(\frac{j}{\bar{\gamma}_{S R}}+\frac{1}{\bar{\gamma}_{R D}}+\frac{k}{\bar{\gamma}_{S D}}\right) \\
& +\frac{1}{2 \ln (2)} \sum_{j=0}^{N_{t}} \frac{\binom{N_{t}}{j}}{\left(\omega \bar{\gamma}_{S D}\right)^{j}\left(1+j \omega \bar{\gamma}_{R D}\right)} \sum_{k=0}^{N_{t}-j} \frac{(-1)^{k-1}\binom{N_{t}-j}{k}}{\left(\omega \bar{\gamma}_{S R}\right)^{k}} \\
& \times\left(\Im e^{\left(\frac{j+k}{\bar{\gamma} S D}\right)} E_{1}\left(\frac{j+k}{\bar{\gamma}_{S D}}\right)-e^{\left(\frac{1}{\bar{\gamma}_{R D}}+j \omega+\frac{j+k}{\bar{\gamma}_{S D}}\right)} E_{1}\left(\frac{1}{\bar{\gamma}_{R D}}+j \omega+\frac{j+k}{\bar{\gamma}_{S D}}\right)\right)
\end{aligned}
$$

$$
\begin{align*}
C_{b} & =\frac{N_{t}}{2 \ln (2) \bar{\gamma}_{S D}}\left(\frac{\bar{\gamma}_{S R}+\bar{\gamma}_{R D}}{\bar{\gamma}_{S R} \bar{\gamma}_{R D}}\right) \sum_{n=0}^{N_{t}-1} \frac{(-1)^{n}\binom{N_{t}-1}{n}}{\frac{n+1}{\bar{\gamma}_{S D}}-\frac{\bar{\gamma}_{S R}+\bar{\gamma}_{R D}}{\bar{\gamma}_{S R} \bar{\gamma}_{R D}}}  \tag{60}\\
& \times\left(\int_{0}^{\infty} \ln (1+x) e^{-\left(\frac{\bar{\gamma}_{S R}+\bar{\gamma}_{R D}}{\bar{\gamma}_{S R} \bar{\gamma}_{R D}}\right) x} d x-\int_{0}^{\infty} \ln (1+x) e^{-\frac{n+1}{\bar{\gamma}_{S D}} x} d x\right)
\end{align*}
$$



Fig. 3. Scheme (b): Outage probability versus $\bar{\gamma}_{S D}$ for different $N_{t} . \mu_{1}=$ 0.5 and $\mu_{2}=1.25$.
decreases for higher $N_{t}$. Since in practice, adding an extra antenna element attracts extra system costs, this number of antennas/performance tradeoff is useful in selecting a desirable value for $N_{t}$.

In Fig. 4 we compare the theoretical rate outage performance of Scheme (a) and Scheme ( $b$ ) by varying the number of antennas, $N_{t}$, with $\mu_{1}=\mu_{2}=1.5$. The plots are based on Eqs. (31) and (37) respectively. To avoid clutter, we have not plotted the high SNR and simulation based results. In all cases, as expected, Scheme (a) has a better outage performance in all SNR regimes. However, the performance gap between Schemes (a) and (b) is marginal and decreases when $N_{t}$ is increased as predicted by our analysis. Thus, in this network, if a small performance loss is tolerable, Scheme (b) which has a low signaling overhead can be employed to realize the cooperative diversity gains. In Fig. 4, $\delta$ values obtained using (40) an outage probability of $\approx 10^{-6}$ are also shown. These values perfectly matched simulated system outages (not shown here). When $N_{t}$ is increased from 2 to 5 , we see that the relative performance difference between the schemes drops


Fig. 4. Outage probability versus $\bar{\gamma}_{S D}$ of Schemes (a) and (b) for different $N_{t}, \mu_{1}=\mu_{2}=1.5$.
from 1.3 dB to 0.66 dB .
Figure 5 shows the average BER of Scheme (a) with 4QAM modulation and $N_{t}=2$. Four different values of $\mu_{1}$ and $\mu_{s}$ have been considered to illustrate the impact of SNR imbalance on the system's error probability. In addition to the simulated BER, analytical and high SNR results are also shown in Fig. 5. We observe that the analytical results from (47) match the simulations very well except at low SNRs given by $\bar{\gamma}_{S D}<5 \mathrm{~dB}$. High SNR results calculated from (50) are also useful to predict system's error performance accurately in the regimes of $\bar{\gamma}_{S D}>15 \mathrm{~dB}$ and in some cases, as low as $\bar{\gamma}_{S D}>10 \mathrm{~dB}$. High values of $\mu_{1}$ and $\mu_{2}$ improve the BER as expected. However, relative performance gains become marginal as $\mu_{1}$ and $\mu_{2}$ are increased. This can be seen from plots for $\left[\mu_{1}=\mu_{2}=1.5\right.$ and 5] and $\left[\mu_{1}=\mu_{2}=5\right.$ and 10].

Figure 6 compares the average BER of Schemes (a) and (b) for $N_{t}=2$ and $N_{t}=3$ respectively. As expected, in both schemes increasing the number of antennas, $N_{t}$, has a positive effect on the error performance since the achievable diversity order is increased from three to four. For both values


Fig. 5. Scheme (a): Average BER versus $\bar{\gamma}_{S D}$ with 4-QAM modulation, $N_{t}=2$ antennas.


Fig. 6. Average BER versus $\bar{\gamma}_{S D}$ of Schemes (a) and (b) with 4-QAM modulation. $\mu_{1}=\mu_{2}=1.5$.
of $N_{t}$, Scheme (a) compared to Scheme (b) only shows a slight performance advantage.

Fig 7 shows the mean channel capacity of Scheme (a). As expected, when $\bar{\gamma}_{S D}$ and $N_{t}$ are increased, the capacity improves. For example, when $\bar{\gamma}_{S D}$ is increased from 0 dB to 40 dB , in the case of $N_{t}=4$ the mean capacity is increased from $1 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}$ to $7.4 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}$. However, capacity improvement when $N_{t}$ is increased from two to four is marginal at all SNR. Also by comparing plots for $N_{t}=[2,3]$ and $N_{t}=[3,4]$ we see that the relative capacity gains reduces. Simulation results shown in Fig. 7 again confirm the tightness of the derived capacity bounds.

Finally, we make some remarks on optimal and suboptimal antenna selection performance when all terminals have multiple antennas, i.e., $R$ and $D$ are also equipped with $N_{r}$ and $N_{d}$ antennas respectively. In [9] it has been shown that the diversity order of such a system equals, $N_{t} N_{d}+N_{r} \min \left(N_{t}, N_{d}\right)$. First assume that $N_{d}=1$; This situation may be the case in some infrastructure based networks with fixed relays where


Fig. 7. Channel capacity versus $\bar{\gamma}_{S D}$ of Scheme (a). $\mu_{1}=\mu_{2}=1.5$.
the relay has the functionality of a mini base station. The optimal scheme achieves a diversity order of $N_{t}+N_{r}$. Under this configuration, the suboptimal antenna scheme can only achieve a same diversity order of $N_{t}=N_{r}$, if $R$ deploys hybrid signal processing techniques such as MRC/antenna selection [12] or MRC/transmit beamforming. Suboptimal antenna selection is only capable of achieving a diversity order of $N_{t}+1$. This is because selecting a single antenna at $S$ only based on the $S-D$ link will yield a random selection of the $S-D$ link and thus would not yield any diversity gains in the relayed link. The diversity order of the system when $N_{d}>1$ with suboptimal antenna selection and antenna selection/diversity combining is $N_{t} N_{d}+1$ and $N_{t} N_{d}+N_{r}$. Therefore, the performance of these schemes will be very much inferior to the optimal antenna selection scheme, especially for a "square" $S-D$ link, i.e, $\operatorname{large} \min \left(N_{t}, N_{d}\right)$.

## V. CONCLUSIONS

Antenna selection is a simple way of improving the performance of AF relaying while maintaining a low hardware complexity. In this paper, we analytically investigated and compared the performance of two transmit antenna selection schemes in a dual-hop AF relay network. Using a tight upper bound, we derived expressions for the SNR cdf. These expressions were next utilized to study important performance measures such as the outage probability, average BER and the mean channel capacity of the two schemes. In order to gain further insights, we also developed simple high SNR outage approximations. From the developed expressions, it is easy to see that when a large number of antennas at the source are deployed or $\mu_{2}$ is small, the performance gap between the suboptimal and optimal antenna selection schemes is marginal. Since the suboptimal scheme also has a low feedback overhead, it seems to be a promising solution for implementing future cellular networks expecting to support relay based communications.

## ApPENDIX <br> High SNR Analysis

Scheme (a): To obtain a first order expansion of $F_{\gamma}(z)$ we note that at high SNR

$$
\begin{equation*}
F_{\gamma}(z)=F_{\gamma_{\mathrm{ubl}}}(z) \tag{62}
\end{equation*}
$$

Therefore, consider the cdf $F_{\gamma_{\mathrm{ubl}}}(z)$ given in (31). To proceed, we substitute $\mu_{1}=\frac{\bar{\gamma} S R}{\bar{\gamma}_{S D}}$ and $\mu_{2}=\frac{\bar{\gamma} R D}{\bar{\gamma}_{S R}}$ in (31) which gives

$$
\begin{align*}
F_{\gamma_{\mathrm{ub1}}}(z) & =\left(1-e^{-\frac{z}{\mu_{1} \overline{\gamma_{S D}}}}-\frac{\left(e^{-\frac{z}{\gamma_{S D}}}-e^{-\frac{z}{\mu_{1} \bar{\gamma}_{S D}}}\right)}{1-\mu_{1}}\right)^{N_{t}}  \tag{63}\\
& \times e^{-\frac{z}{\mu_{1} \mu_{2} \bar{\gamma}_{S D}}} \\
& +\frac{1}{\mu_{1} \mu_{2} \overline{\gamma_{S D}}} \sum_{j=0}^{N_{t}}\binom{N_{t}}{j}\left(1-\frac{e^{-\frac{z}{\gamma_{S D}}}}{1-\mu_{1}}\right)^{N_{t}-j} \\
& \times\left(\frac{\mu_{1}}{1-\mu_{1}} e^{-\frac{z}{\gamma_{S D}}}\right)^{j}\left(\frac{1-e^{-\left(\frac{1}{\mu_{1} \mu_{2} \bar{\gamma}_{S D}}+j \omega\right) z}}{\frac{1}{\mu_{1} \mu_{2} \bar{\gamma}_{S D}}+j \omega}\right) .
\end{align*}
$$

Employing the Maclaurin series for the exponential function we obtain (64) where (65). We simplify (65) to obtain (66). Now as $A(z)$ contains higher powers of $z$ than $\left(N_{t}+1\right)$, we see that $B(z)$ will accurately describe the behavior of $F_{\gamma_{\mathrm{ubl}}}(z)$ around $z=0$.

Consider $B(z)$ given by

$$
\begin{aligned}
B(z) & =\frac{1}{\mu_{1} \mu_{2} \bar{\gamma}_{S D}} \sum_{j=0}^{N_{t}}\binom{N_{t}}{j}\left(1-\frac{\sum_{n=0}^{\infty} \frac{\left(-\frac{z}{\gamma}\right)^{n}}{n!}}{1-\mu_{1}}\right)^{N_{t}-j} \\
& \times\left(\frac{\mu_{1}}{1-\mu_{1}} \sum_{n=0}^{\infty} \frac{\left(-\frac{z}{\bar{\gamma}_{S D}}\right)^{n}}{n!}\right)^{j}\left(\frac{1-e^{-\left(\frac{1}{\mu_{1} \mu_{2} \bar{\gamma}_{S D}}+j \omega\right) z}}{\frac{1}{\mu_{1} \mu_{2} \bar{\gamma}_{S D}}+j \omega}\right) .
\end{aligned}
$$

It can be shown that

$$
\begin{equation*}
B(z)=\frac{z}{\mu_{1} \mu_{2} \bar{\gamma}_{S D}} \sum_{j=0}^{N_{t}}\binom{N_{t}}{j}\left(1-\frac{\sum_{n=0}^{\infty} \frac{\left(-\frac{z}{\bar{\gamma} S D}\right)^{n}}{n!}}{1-\mu_{1}}\right)^{N_{t}-j} \tag{68}
\end{equation*}
$$

$$
\times\left(\frac{\mu_{1}}{1-\mu_{1}} \sum_{n=0}^{\infty} \frac{\left(-\frac{z}{\bar{\gamma} S_{S D}}\right)^{n}}{n!}\right)^{j}
$$

Since the summation in (68) is the binomial expansion of $(p+q)^{N_{t}}$, we get

$$
\begin{equation*}
B(z)=\frac{z}{\mu_{1} \mu_{2} \bar{\gamma}_{S D}}\left(1-e^{-\frac{z}{\gamma_{S D}}}\right)^{N_{t}} \tag{69}
\end{equation*}
$$

We apply the binomial theorem to yield

$$
\begin{equation*}
B(z)=\frac{z}{\mu_{1} \mu_{2} \bar{\gamma}_{S D}} \sum_{l=0}^{N_{t}}\binom{N_{t}}{l}(-1)^{l} \sum_{n=0}^{\infty} \frac{\left(\frac{l z}{\bar{\gamma} S D}\right)^{n}}{n!} \tag{70}
\end{equation*}
$$

Once again, after several manipulations (70) can be written as

$$
\begin{equation*}
B(z)=\frac{1}{\mu_{1} \mu_{2}\left(N_{t}+1\right)}\left(\frac{z}{\bar{\gamma}_{S D}}\right)^{N_{t}+1}+o\left(\left(\frac{z}{\bar{\gamma}_{S D}}\right)^{N_{t}+2}\right) \tag{71}
\end{equation*}
$$

We see that $B(z)$ accurately describes the behavior of $F_{\gamma_{\mathrm{ubl}}}(z)$ around $z=0$ and therefore $F_{\gamma}(z)$ around $z=0$. Now substituting $z=2^{2 R_{0}-1}$ into (38) yields the desired result.

Scheme (b): At high SNR, the outage probability of Scheme (b) can be obtained by deriving a first order expression for $F_{\gamma}(z)$. To this end, we begin by finding an exact expression for $F_{\gamma_{\mathcal{R}}}(z)$. Conditioned on the RV, $\gamma_{R D}, F_{\gamma_{\mathcal{R}}}(z)$ is given by

$$
\begin{align*}
F_{\gamma_{\mathcal{R}}}(z) & =\int_{0}^{\infty} \operatorname{Pr}\left(\frac{\gamma_{S R, k} w}{\gamma_{S R, k}+w+1}<z\right) f_{\gamma_{R D}}(w) d w  \tag{72}\\
& =\int_{0}^{z} \operatorname{Pr}\left(\gamma_{S R, k}>\frac{z(w+1)}{w-z}\right) f_{\gamma_{R D}}(w) d w \\
& +\int_{z}^{\infty} \operatorname{Pr}\left(\gamma_{S R, k} \leq \frac{z(w+1)}{w-z}\right) f_{\gamma_{R D}}(w) d w
\end{align*}
$$

After some manipulations it can be shown that

$$
\begin{equation*}
F_{\gamma_{\mathcal{R}}}(z)=1-\int_{0}^{\infty} P_{\gamma_{S R, k}}\left(z+\frac{z^{2}+z}{w}\right) f_{\gamma_{R D}}(z+w) d w \tag{73}
\end{equation*}
$$

where $P_{\gamma_{S R, k}}(x)=1-F_{\gamma_{S R, k}}(x)$ denotes the complementary cdf of $\gamma_{S R, k}$.

Simplifying (73) we get

$$
\begin{align*}
& F_{\gamma_{\mathcal{R}}}(z)=1-\frac{e^{-\left(\frac{1}{\bar{\gamma} S R}+\frac{1}{\gamma_{R D}}\right) z}}{\bar{\gamma}_{R D}} \int_{0}^{\infty} e^{-\frac{z^{2}+z}{\gamma_{R D} w}-\frac{w}{\bar{\gamma}_{R D}} d w}  \tag{74}\\
& =1-2 \sqrt{\frac{z^{2}+z}{\bar{\gamma}_{S R} \bar{\gamma}_{R D}}} e^{-\left(\frac{1}{\gamma_{S R}}+\frac{1}{\gamma_{R D}}\right) z} K_{1}\left(2 \sqrt{\frac{z^{2}+z}{\bar{\gamma}_{S R} \bar{\gamma}_{R D}}}\right)
\end{align*}
$$

where we have used [25, Eq. (3.471.9)] and $K_{1}(x)$ is the first order modified Bessel function of the second kind [30, Sec. (9.6)]. When $\nu>0$ is fixed and $x \rightarrow 0$, using the following approximation,

$$
\begin{equation*}
K_{\nu}(x) \simeq \frac{2^{\nu-1} \Gamma(\nu)}{x^{\nu}} \tag{75}
\end{equation*}
$$

it can be shown that at high SNR

$$
\begin{align*}
F_{\gamma_{\mathcal{R}}}(z) & =1-e^{-\left(\frac{1}{\gamma_{S R}}+\frac{1}{\gamma_{R D}}\right) z}  \tag{76}\\
& =\left(1+\frac{1}{\mu_{2}}\right)\left(\frac{z}{\bar{\gamma}_{S R}}\right)+o\left(\left(\frac{z}{\bar{\gamma}_{S R}}\right)^{2}\right) .
\end{align*}
$$

Similarly consider $F_{\gamma_{\mathcal{D}}}(z)$ given by

$$
\begin{align*}
F_{\gamma_{\mathcal{D}}}(z) & =1-N_{t} \sum_{l=0}^{N_{t}-1} \frac{(-1)^{l}}{1+l}\binom{N_{t}-1}{l} e^{-\frac{(1+l) z}{\gamma_{S D}}}  \tag{77}\\
& =1-N_{t} \sum_{l=0}^{N_{t}-1} \frac{(-1)^{l}}{1+l}\binom{N_{t}-1}{l} \sum_{n=0}^{\infty}(-1)^{n} \frac{\left(\frac{(1+l) z}{\bar{\gamma}_{S D}}\right)^{n}}{n!} \\
& =\left(\frac{z}{\bar{\gamma}_{S D}}\right)^{N_{t}}+o\left(\left(\frac{z}{\bar{\gamma}_{S D}}\right)^{N_{t}+1}\right) .
\end{align*}
$$

Finally, employing [31, Lemma 1] it can be shown that the high SNR cdf of $\gamma=\gamma_{\mathcal{R}}+\gamma_{\mathcal{D}}$ in terms of $\bar{\gamma}_{S D}$ can be written as

$$
\begin{equation*}
F_{\gamma}(z)=\frac{1+\frac{1}{\mu_{2}}}{\mu_{1}\left(N_{t}+1\right)}\left(\frac{z}{\bar{\gamma}_{S D}}\right)^{N_{t}+1}+o\left(\left(\frac{z}{\bar{\gamma}_{S D}}\right)^{N_{t}+2}\right) \tag{78}
\end{equation*}
$$

$$
\begin{align*}
F_{\gamma_{\mathrm{ubl}}}(z) & =\left(1-\sum_{n=0}^{\infty} \frac{\left(-\frac{z}{\mu_{1} \overline{\gamma_{S D}}}\right)^{n}}{n!}-\left(\frac{1}{1-\mu_{1}}\right)\right.  \tag{64}\\
& \left.\times\left(\sum_{n=0}^{\infty} \frac{\left(-\frac{z}{\bar{\gamma} S D}\right)^{n}}{n!}-\sum_{n=0}^{\infty} \frac{\left(-\frac{z}{\mu_{1} \bar{\gamma} S D}\right)^{n}}{n!}\right)\right)^{N_{t}} \sum_{n=0}^{\infty} \frac{\left(\frac{-z}{\mu_{1} \mu_{2} \overline{\gamma_{S D}}}\right)^{n}}{n!} \\
& +\frac{1}{\mu_{1} \mu_{2} \bar{\gamma}_{S D}} \sum_{j=0}^{N_{t}}\binom{N_{t}}{j}\left(1-\frac{\sum_{n=0}^{\infty} \frac{\left(-\frac{z}{\gamma_{S D}}\right)^{n}}{n!}}{1-\mu_{1}}\right)^{N_{t}-j} \\
& \times\left(\frac{\mu_{1}}{1-\mu_{1}} \sum_{n=0}^{\infty} \frac{\left(-\frac{z}{\bar{\gamma}_{S D}}\right)^{n}}{n!}\right)^{j}\left(\frac{1-e^{-\left(\frac{1}{\mu_{1} \mu_{2} \bar{\gamma}_{S D}}+j \omega\right) z}}{\frac{1}{\mu_{1} \mu_{2} \overline{\gamma_{S D}}}+j \omega}\right) \\
& =A(z)+B(z) .
\end{align*}
$$

$$
\begin{align*}
A(z) & =\left(1-\sum_{n=0}^{\infty} \frac{\left(-\frac{z}{\mu_{1} \overline{\gamma_{S D}}}\right)^{n}}{n!}-\left(\frac{1}{1-\mu_{1}}\right)\right.  \tag{65}\\
& \left.\times\left(\sum_{n=0}^{\infty} \frac{\left(-\frac{z}{\bar{\gamma} S D}\right)^{n}}{n!}-\sum_{n=0}^{\infty} \frac{\left(-\frac{z}{\mu_{1} \overline{\gamma_{S D}}}\right)^{n}}{n!}\right)\right)^{N_{t}} \sum_{n=0}^{\infty} \frac{\left(\frac{-z}{\mu_{1} \mu_{2} \bar{\gamma} S D}\right)^{n}}{n!} \\
& =\left(1+\frac{\mu_{1}}{1-\mu_{1}} \sum_{n=0}^{\infty} \frac{\left(-\frac{z}{\mu_{1} \overline{\gamma_{S D}}}\right)^{n}}{n!}-\frac{1}{1-\mu_{1}} \sum_{n=0}^{\infty} \frac{\left(-\frac{z}{\bar{\gamma} S D}\right)^{n}}{n!}\right)^{N_{t}} \sum_{n=0}^{\infty} \frac{\left(\frac{-z}{\mu_{1} \mu_{2} \bar{\gamma}_{S D}}\right)^{n}}{n!} .
\end{align*}
$$

$$
\begin{equation*}
A(z)=\left(\frac{\mu_{1}}{1-\mu_{1}} \sum_{n=2}^{\infty} \frac{\left(-\frac{z}{\mu_{1} \bar{\gamma} S D}\right)^{n}}{n!}-\frac{1}{1-\mu_{1}} \sum_{n=2}^{\infty} \frac{\left(-\frac{z}{\bar{\gamma} s D}\right)^{n}}{n!}\right)^{N_{t}} \sum_{n=0}^{\infty} \frac{\left(\frac{-z}{\mu_{1} \mu_{2} \bar{\gamma} \bar{\gamma}}\right)^{n}}{n!} . \tag{66}
\end{equation*}
$$

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[^1]:    ${ }^{1}$ Exact closed-form cdf/pdf expressions for $\gamma_{\mathcal{D}}$ and $\gamma_{\mathcal{R}}$ can be found. However, no closed-form expressions for the cdf/pdf of the total e2e SNR, $\gamma=\gamma_{\mathcal{D}}+\gamma_{\mathcal{R}}$, are known to exist [4], [20].

[^2]:    ${ }^{2} \mathrm{~A}$ function of $x, f(x)$, is represented as $o(x)$ if $\lim _{x \rightarrow 0} \frac{f(x)}{x}=0$.

[^3]:    ${ }^{3}$ Throughout the paper, the logarithm of $x$ to the base 10 is written as $\log (x)$.

[^4]:    ${ }^{4}$ Note that $E_{1}(x)=-\operatorname{Ei}(-x)$.

