# Performance Analysis of Two Hop Amplify-and-Forward Systems with Interference at the Relay

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***Abstract*—We analyze the performance of a two hop channel state information (CSI)-assisted amplify-and-forward system, with co-channel interference at the relay. The system's outage probability and the average bit error rate (BER) in the presence of Rayleigh faded multiple interferers are investigated. We derive an exact expression for the outage probability and an accurate bound for the system's average BER. Simulation results show the validity of the analysis and point out the effect of interference.**

***Index Terms*—Relay, co-channel interference, outage probability, average bit error rate, two hop networks.**

## I. INTRODUCTION

ALTHOUGH amplify-and-forward (AF) relaying has now become a well studied protocol in the literature, (see e.g. [1]–[3]), most of the existing work has only considered thermal noise-limited conditions with no interference. In many practical scenarios however, interference present in relay networks can cause severe performance degradation [4].

Few works that have studied the impact of interference on the AF and the decode-and-forward (DF) relaying performance have assumed interference either at the relay(s) or the destination(s) [4], [5]. In [4], the impact of interference on the performance of a channel state information (CSI)-assisted AF relay network has been investigated. Their asymptotic analysis has shown that interference limits the system's diversity gain. The authors of [5], have analyzed the performance gains of a half-duplex multi-user cooperative network where the relay-destination slot is reused. Thus that slot becomes an interfering scenario at each destination. Their results in terms of the average sum-throughput show that by tuning the number of interfering relays and the target signal-to-noise ratio (SNR) between relay and destination, significant gains can be obtained. In [6] and [7], the performance of DF opportunistic relaying in the presence of interference has been investigated. More specifically, closed-form expressions for the outage probability have been derived. Recently, in [8], the outage probability of a fixed gain AF relay system in Rayleigh fading with interference at the destination has been investigated.

The main purpose of this letter is to study the outage probability and the average bit error rate (BER) of the CSI-assisted AF protocol with interference at the relay. We consider multiple interferers and derive an exact expression for the system's outage probability. Using an approximation to the end-to-end (e2e) signal-to-interference and noise ratio (SINR), we also derive an accurate bound for the error probability. The impact of interferer power on the performance is illustrated through analytical and simulation results.

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## II. SYSTEM MODEL

We consider a communication system, where a source, $S$, communicates with a destination, $D$ through a relay, $R$. All nodes are equipped with a single antenna. It is assumed that $S$ does not have a direct link to $D$. The communication in the system is divided into two orthogonal time intervals. In the first time interval, $S$ sends its symbol $s_0$ to $R$. In the second time interval, $R$ communicates with $D$. We assume that $R$ operates in an interference limited environment, and that the interference at $D$ is negligible.

At $R$, the received signal in the presence of $N_{\mathrm{I}}$ number of interferers, each with an average power, $P_j$, and additive white Gaussian noise (AWGN) can be expressed as

$$y_r = \sqrt{P_s} h_{sr} s_0 + \sum_{j=1}^{N_{\mathrm{I}}} \sqrt{P_j} h_j s_j + n_r, \quad (1)$$

where $P_s$ is the transmit power, $h_{sr}$ is the complex channel between $S$ and $R$ with average fading power $\Omega_{sr}$, $\{h_j\}_{j=1}^{N_{\mathrm{I}}}$ are the channels from the interferers to $R$, the transmitted symbols, $s_0$ and interfering symbols $s_j$ are assumed to have zero mean and unit variance and $n_r$ is the AWGN at $R$ satisfying $E[|n_r|^2] = \sigma_r^2$. The expectation operator is $E[\cdot]$. It is assumed that $\{h_j\}_{j=1}^{N_{\mathrm{I}}}$ are independent not necessarily identically distributed random variables (RVs) with $E[|h_j|^2] = \Omega_j$. They are also independent of $h_{sr}$ and $h_{rd}$, where $h_{rd}$ is the complex channel between $R$ and $D$ with $E[|h_{rd}|^2] = \Omega_{rd}$. All links are assumed to be subject to Rayleigh fading.

In the second time interval, $R$ forwards $y_r$ to $D$ after multiplying it with the gain, $G$. Hence, the received signal at $D$ is given by

$$y_d = h_{rd} G \left( \sqrt{P_s} h_{sr} s_0 + \sum_{j=1}^{N_{\mathrm{I}}} \sqrt{P_j} h_j s_j + n_r \right) + n_d, \quad (2)$$

where $n_d$ is the AWGN at $D$ satisfying $E[|n_d|^2] = \sigma_d^2$. In the presence of interference, $G$ can be expressed as [4]

$$G = \sqrt{\frac{P_r}{P_s |h_{sr}|^2 + \sum_{j=1}^{N_{\mathrm{I}}} P_j |h_j|^2 + \sigma_r^2}}. \quad (3)$$

As a result, the SINR of the decision variable can be written as

$$\gamma_{eq} = \frac{P_s|h_{sr}|^2|h_{rd}|^2}{|h_{rd}|^2\left(\sum_{j=1}^{N_\mathrm{I}} P_j|h_j|^2 + \sigma_{sr}^2\right) + \frac{\sigma_{rd}^2}{G^2}}. \tag{4}$$

Now substituting (3) into (4) and since $R$ is interference limited (the effect of $n_r$ is negligible) it can be shown that the SINR is given by

$$\gamma_{eq1} = \frac{\gamma_1\gamma_2}{\gamma_{INF}(\gamma_2+1)+\gamma_1}, \tag{5}$$

where $\gamma_1 = P_s|h_{sr}|^2$, $\gamma_2 = \frac{P_r}{\sigma_d^2}|h_{rd}|^2$ and $\gamma_{INF} = \sum_{j=1}^{N_\mathrm{I}} P_j|h_j|^2$.

## III. Performance Analysis

### A. Outage Probability

The outage probability, $P_o$ is defined as the probability that $\gamma_{eq1}$ drops below an acceptable SNR threshold $\gamma_{\text{th}}$, or mathematically

$$P_o = \Pr(\gamma_{eq1} < \gamma_{\text{th}}) = F_{\gamma_{eq1}}(\gamma_{\text{th}}), \tag{6}$$

where $\Pr(\cdot)$ denotes the probability. To derive the outage probability of $\gamma_{eq1}$, conditioned on $\gamma_2$ and $\gamma_{INF}$, we first express the cumulative distribution function (cdf) of $\gamma_{eq1}$ as

$$F_{\gamma_{eq1}}(\gamma_{\text{th}}) = \int_0^\infty \Pr\left(\gamma_1 \le \frac{\gamma_{\text{th}}(y+1)z}{y-\gamma_{\text{th}}}\right) \times f_{\gamma_2}(y) f_{\gamma_{INF}}(z)\,dydz. \tag{7}$$

Applying algebraic manipulations to (7) yields

$$F_{\gamma_{eq1}}(\gamma_{\text{th}}) = 1 - \int_0^\infty \Pr\left(\gamma_1 \ge \frac{\gamma_{\text{th}}(w+\gamma_{\text{th}}+1)z}{w}\right) \times f_{\gamma_2}(w+\gamma_{\text{th}}) f_{\gamma_{INF}}(z)\,dwdz. \tag{8}$$

In order to evaluate (8), the cdf and the probability density function (pdf) of the RVs $\gamma_1, \gamma_2$ and $\gamma_{INF}$, respectively are needed. We note that the complementary cdf of $\gamma_1$ and the pdf of $\gamma_2$ can be expressed as, $C_{\gamma_1}(x) = e^{-\frac{x}{\bar{\gamma}_1}}$ and $f_{\gamma_2}(x) = \frac{1}{\bar{\gamma}_2}e^{-\frac{x}{\bar{\gamma}_2}}$, where $\bar{\gamma}_1 = P_s\Omega_{sr}$ and $\bar{\gamma}_2 = \frac{P_r\Omega_{rd}}{\sigma_d^2}$ respectively. Let's denote by $\eta_1, \eta_2, \ldots, \eta_r$ the distinct values of $\eta_j = P_j\Omega_j$ with multiplicities $\nu_1, \nu_2, \ldots, \nu_r$, respectively, such that $\sum_{k=1}^r \nu_k = N_\mathrm{I}$. It is possible to show that the pdf of $\gamma_{INF}$ is given by [10]

$$f_{\gamma_{INF}}(z) = \sum_{k=1}^{r}\sum_{\ell=1}^{\nu_k} \frac{\phi_{k\ell}}{(j-1)!\eta_k^\ell} z^{\ell-1} e^{-\frac{z}{\eta_k}}, \tag{9}$$

where the coefficients $\phi_{k\ell}$ are given by

$$\phi_{k\ell} = \frac{(-1)^{\nu_k-\ell}}{\eta^{\nu_k-\ell}} \sum_{\tau(k,\ell)} \prod_{\substack{m=1\\ m\neq k}} \binom{\nu_m+q_m-1}{q_m} \frac{\eta_m^{q_m}}{\left(1-\frac{\eta_m}{\eta_k}\right)^{\nu_m+q_m}}, \tag{10}$$

with $\tau(k,\ell)$ being defined as the set of multi-indices $q = (q_1, q_2, \ldots, q_r)$ such that $\tau(k,\ell) := \{q = (q_1, q_2, \ldots, q_r) \in \mathbb{N}^r : q_k = 0, \sum_{m=1}^r q_m = \nu_k - \ell\}$.

Now substituting the cdf/pdfs of $\gamma_1, \gamma_2$ and $\gamma_{INF}$ into (8) we get

$$F_{\gamma_{eq1}}(\gamma_{\text{th}}) = 1 - \frac{e^{-\frac{\gamma_{\text{th}}}{\bar{\gamma}_2}}}{\bar{\gamma}_2}\sum_{k=1}^{r}\sum_{\ell=1}^{\nu_k}\frac{\phi_{k\ell}}{(\ell-1)!\eta_k^\ell} \times \int_0^\infty z^{\ell-1}e^{-\left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_1}+\frac{1}{\eta_k}\right)z}dz \int_0^\infty e^{-\frac{(\gamma_{\text{th}}^2+\gamma_{\text{th}})z}{\bar{\gamma}_1 w}-\frac{w}{\bar{\gamma}_2}}dw. \tag{11}$$

Equation (11) can be evaluated with the help of the identities [9, Eq. (2.3.16.1)] and [9, Eq. (2.16.8.4)]. The outage probability in closed-form is given by

$$F_{\gamma_{eq1}}(\gamma_{\text{th}}) = 1 - e^{-\frac{\gamma_{\text{th}}}{\bar{\gamma}_2}} \sum_{k=1}^{r} \frac{1+\gamma_{\text{th}}}{\left(1+\frac{\bar{\gamma}_1}{\eta_k\gamma_{\text{th}}}\right)\bar{\gamma}_2} \times \sum_{\ell=1}^{\nu_k} \frac{\phi_{k\ell}\ell!}{\left(1+\frac{\eta_k\gamma_{\text{th}}}{\bar{\gamma}_1}\right)^\ell} U\left(\ell+1, 2; \frac{1+\gamma_{\text{th}}}{\left(1+\frac{\bar{\gamma}_1}{\eta_k\gamma_{\text{th}}}\right)\bar{\gamma}_2}\right), \tag{12}$$

where $U(a,b;z)$ is the confluent hypergeometric function of the second kind defined as $U(a,b;z) = \frac{1}{\Gamma(a)}\int_0^\infty e^{-zt}t^{a-1}(1+t)^{b-a-1}dt$.

In several special cases (12) can be further simplified. For example, when all $N_\mathrm{I}$ interferers are identical ($\eta_j = \eta$ for all $j$), $\gamma_{INF}$, becomes a central $\chi^2$ RV with $2N_\mathrm{I}$ degrees of freedom and (12) simplifies to

$$F_{\gamma_{eq1}}(\gamma_{\text{th}}) = 1 - \frac{(1+\gamma_{\text{th}})e^{-\frac{\gamma_{\text{th}}}{\bar{\gamma}_2}}N_\mathrm{I}!}{\left(1+\frac{\bar{\gamma}_1}{\eta\gamma_{\text{th}}}\right)\left(1+\frac{\eta\gamma_{\text{th}}}{\bar{\gamma}_1}\right)^{N_\mathrm{I}}\bar{\gamma}_2} \times U\left(N_\mathrm{I}+1, 2; \frac{1+\gamma_{\text{th}}}{\left(1+\frac{\bar{\gamma}_1}{\eta\gamma_{\text{th}}}\right)\bar{\gamma}_2}\right). \tag{13}$$

### B. Average Bit Error Rate

We now derive expressions for the average BER of the system under consideration. Our results apply for all modulation formats that have a BER expression of the form:

$$P_b = a\,E[Q(\sqrt{b\gamma_{eq1}})], \tag{14}$$

where $a, b > 0$ and $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{y^2}{2}}dy$ is the Gaussian $Q$-function. The average BER of modulation schemes that are in the form of (14) include binary/quadrature phase shift keying (BPSK/QPSK), frequency shift keying (FSK) and $M$-ary quadrature amplitude modulation ($M$-QAM).

We consider

$$\gamma_{eq2} = \min\left(\frac{\gamma_1}{\gamma_{INF}}, \gamma_2\right), \tag{15}$$

given in [4] instead of $\gamma_{eq1}$ for mathematical tractability. Thus, employing $\gamma_{eq2}$ we express the average BER as

$$P_b \approx a\int_0^\infty Q(\sqrt{b\gamma})f_{\gamma_{eq2}}(\gamma)d\gamma \approx \frac{a}{\sqrt{2\pi}}\int_0^\infty F_{\gamma_{eq2}}\left(\frac{\gamma^2}{b}\right)e^{-\frac{\gamma^2}{2}}d\gamma. \tag{16}$$

Before evaluating the average BER using (16), we first derive an expression for $F_{\gamma_{eq2}}(z)$. For this, first consider, $F_{\frac{\gamma_1}{\gamma_{INF}}}(\gamma_{\text{th}})$. It is easy to show that

$$F_{\gamma_{eq2}}(\gamma_{\text{th}}) = 1 - C_{\frac{\gamma_1}{\gamma_{INF}}}(\gamma_{\text{th}})C_{\gamma_2}(\gamma_{\text{th}}). \tag{17}$$

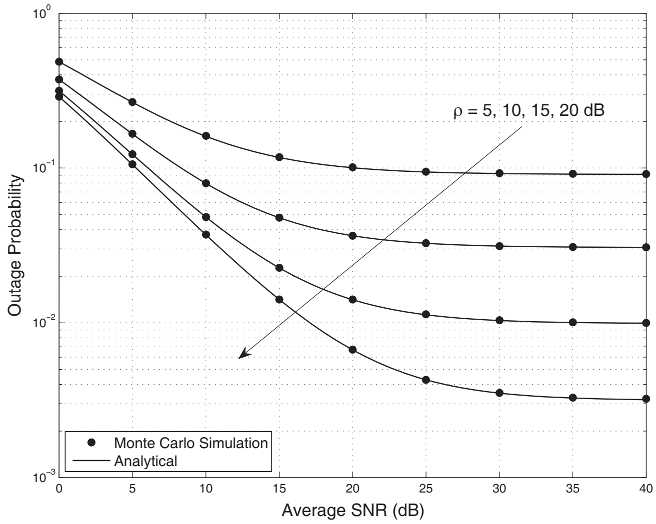


Fig. 1. Outage probability for different $\rho$ with $\gamma_{\text{th}} = -5$ dB.

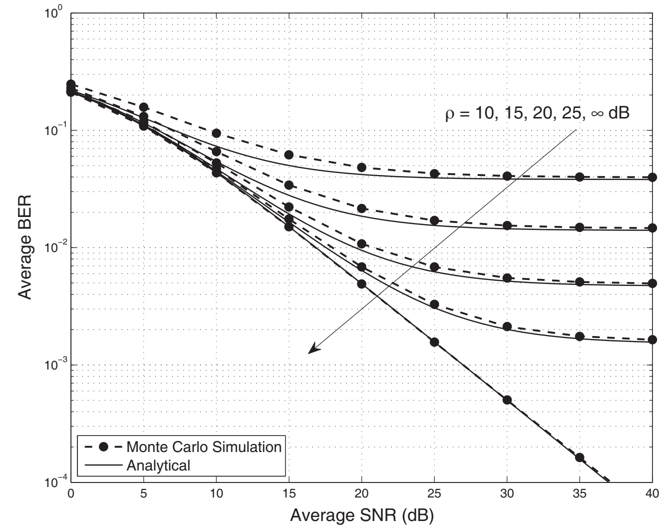


Fig. 2. Average BER using 4-QAM modulation for different $\rho$.

where $C_{\frac{\gamma_1}{\gamma_{INF}}}(z)$ and $C_{\gamma_2}(z)$ are complementary cdfs of the RVs, $\frac{\gamma_1}{\gamma_{INF}}$ and $\gamma_2$, respectively. $F_{\frac{\gamma_1}{\gamma_{INF}}}(\gamma_{\text{th}})$ can be expressed as

$$F_{\frac{\gamma_1}{\gamma_{INF}}}(\gamma_{\text{th}}) = 1 - \sum_{k=1}^{r}\sum_{\ell=1}^{\nu_k} \frac{\phi_{k\ell}\int_0^\infty z^{\ell-1} e^{-\left(\frac{\gamma_{\text{th}}}{\bar{\gamma}_1}+\frac{1}{\eta_k}\right)z} dz}{(\ell-1)!\eta_k^\ell} \tag{18}$$

$$= 1 - \sum_{k=1}^{r}\sum_{\ell=1}^{\nu_k} \phi_{k\ell}\left(1+\frac{\eta_k\gamma_{\text{th}}}{\bar{\gamma}_1}\right)^{-\ell}.$$

Now using (17), $F_{\gamma_{eq2}}(\gamma_{\text{th}})$, can be written as

$$F_{\gamma_{eq2}}(\gamma_{\text{th}}) = 1 - \sum_{k=1}^{r}\sum_{\ell=1}^{\nu_k} \phi_{k\ell}\left(1+\frac{\eta_k\gamma_{\text{th}}}{\bar{\gamma}_1}\right)^{-\ell} e^{-\frac{\gamma_{\text{th}}}{\bar{\gamma}_2}}. \tag{19}$$

Substituting (19) into (16), $P_b$ can be expressed as

$$P_b \approx \frac{a}{2}\left(1 - \sqrt{\frac{2}{\pi}} \sum_{k=1}^{r}\sum_{\ell=1}^{\nu_k} \phi_{k\ell} \int_0^\infty \frac{e^{-\left(\frac{1}{b\bar{\gamma}_2}+\frac{1}{2}\right)z^2}}{\left(1+\frac{\eta_k}{b\bar{\gamma}_1}z^2\right)^\ell} dz\right). \tag{20}$$

The integral in (20) can be evaluated in closed-form by first making a simple variable transformation and then using the identity [9, Eq. (2.3.6.9)]. Finally, $P_b$ is given by

$$P_b \approx \frac{a}{2}\left(1 - \sqrt{\frac{b\bar{\gamma}_1}{2}} \sum_{k=1}^{r}\sum_{\ell=1}^{\nu_k} \phi_{k\ell} \frac{U\left(\frac{1}{2}, \frac{3}{2}-\ell; \omega_k\right)}{\sqrt{\eta_k}}\right), \tag{21}$$

where $\omega_k = (2\bar{\gamma}_1 + b\bar{\gamma}_1\bar{\gamma}_2)/2\eta_k\bar{\gamma}_2$.

## IV. Numerical and Simulation Results

We verify the theoretical analysis in Section III through comparison with simulations and examine the effect of interference. In Figs. 1-2, we assume a single interferer ($N_{\text{I}} = 1$) and define $\rho = E[\gamma_1]/E[\gamma_{INF}]$. Fig. 1 shows the system's outage probability versus the average SNR ($\bar{\gamma}_2$) for different strengths of interference. The analytical results are from (12). In all cases, the outage probability decreases as the average SNR increases however, a floor at high SNR is clearly visible due to the impact of the interference. Fig. 2 shows the error probability of the system using 4-QAM modulation. The approximate analytical results from (21) closely follow the Monte Carlo simulation curves in the regimes of $\bar{\gamma}_2 > 15$ dB. The case of no interference ($\rho = \infty$ dB) is also plotted to demonstrate that interference can inflict significant performance losses, especially at high SNR.

## V. Conclusions

In this letter, we investigated the performance of a two hop CSI-assisted AF system with co-channel interference at the relay. We considered a general multiple interferer scenario and derived outage probability and average BER expressions over Rayleigh fading channels. A comparison with simulation results showed that the presented outage probability and average BER expressions accurately predict the system's performance.

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