Energy Efficient Dynamic Resource Optimization in NOMA System

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Abstract

Non-orthogonal multiple access (NOMA) with successive interference cancellation (SIC) is a promising technique for next generation wireless communications. Using NOMA, more than one user can access the same frequency-time resource simultaneously and multi-user signals can be separated successfully using SIC. In this paper, resource allocation algorithms for subchannel assignment and power allocation for a downlink NOMA network are investigated. Different from the existing works, here, energy efficient dynamic power allocation in NOMA networks is investigated. This problem is explored using the Lyapunov optimization method by considering the constraints on minimum user quality of service (QoS), the maximum transmit power limit. Based on the framework of Lyapunov

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optimization, the problem of energy efficient optimization can be broken down into three subproblems. Two of which are linear and the rest can be solved by introducing Lagrangian function. The mathematical analysis and simulation results confirm that the proposed scheme can achieve a significant utility performance gain and the energy efficiency and delay tradeoff is derived as $[\mathrm{O}(1/V), \mathrm{O}(V)]$ with V as a control parameter under maintaining the queue stability.

Index Terms

NOMA, Lyapunov optimization, power allocation, subchannel assignment

I. Introduction

In the past decade, orthogonal frequency division multiple access (OFDMA) has been widely studied and has been adopted in 4th generation (4G) mobile communication systems [1], [2]. However, orthogonal channel access in OFDMA is becoming a limiting factor of spectrum efficiency since each subchannel can only be used by at most one user in each time slot. With the explosive growth of smart mobile devices and the increasing demands for higher spectral efficiency, non-orthogonal multiple access (NOMA) has been proposed to mitigate the consequent heavy loading at the base station (BS) [3]–[5]. NOMA is a promising technique to realize the massive connectivity in 5th generation (5G) mobile networks because NOMA can achieve significant improvement in spectral efficiency with a lower receiver complexity by allowing multiple users to share the same subchannel in the power domain [6]–[8].

The use of NOMA will result in inter-user interference since multiple users will share the same resources [9]. Successive interference cancellation (SIC) can be applied at the end-user receivers to mitigate this interference [10]. NOMA can achieve a capacity region that significantly outperforms orthogonal multiple access schemes by power domain multiplexing at the transmitter and SIC at the receivers [11]. The outage performance of NOMA was evaluated in [12], while in [13], the authors investigated the system sum-rate of multiuser NOMA single-carrier systems as well as proposing a suboptimal power allocation and presenting a precoder design. Moreover, the authors addressed fairness considerations and posed a max-min fairness problem for NOMA. In [15], an optimal power allocation strategy for the energy-efficiency maximization of NOMA in single-carrier systems was investigated. The authors in [16] investigated the subchannel assignment and power allocation using the difference of convex programming method. A suboptimal algorithm was proposed to solve an uplink scheduling problem with fixed transmission power for uplink NOMA in [17] and a greedy-based algorithm was proposed to improve the throughput in uplink NOMA in [18]. In [19], a new spectrum and energy efficient mmWave transmission scheme which integrates the concept of NOMA with beamspace MIMO was proposed to break the fundamental limit. In [20], a hierarchical power control solution to improve the spectrum and energy efficiency in NOMA-enabled vehicular small cell networks was proposed by performing the joint optimization of cell association and power control. In [21], a novel resource allocation design was investigated for NOMA enhanced heterogeneous networks. However, the study of energy efficient subchannel assignment and power allocation in multiple small cells with NOMA under the constraints of minimum QoS requirements and cross interference has not been well studied.

On contrary to the typically full buffer assumptions and snapshot-based models, delay is a key metric to measure the QoS. The congestion performance for delay-tolerant services and the stochastic and time-varying features of traffic arrivals should be considered in realistic wireless networks. By utilizing Lyapunov optimization, which is an useful method for handling queue-aware radio resource allocation problems, [22], [23] considered the impacts of stochastic traffic arrivals and time-varying channel conditions on the system performance to stabilize the queues of networks when optimizing performance metrics. In [22], the authors applied the framework of Lyapunov optimization to balance the average throughput and average delay in energy efficient OFDMA heterogeneous cloud networks. The authors in [23] adopted the Lyapunov optimization framework in a two-tier OFDMA heterogeneous network under the hybrid access mode to solve the dynamic optimization problem of resource allocation. In [24], an energy efficient resource allocation algorithm was proposed to provide fairness among different small cell base stations(SCBSs) based on the Nash bargaining solution. However, most of the existing works considered resource allocation using Lyapunov optimization only in OFDMA systems. And to the best of the authors' knowledge, energy efficient resource allocation for time-varying NOMA networks has not been well studied in the previous works.

In this paper, we investigate the subchannel and power allocation respectively in a multiple downlink NOMA network by considering energy efficiency, quality of service (QoS) requirements, power limits, and queue stability. The main contributions of this paper are summarized as follows:

- Development of a novel energy efficient NOMA network optimization framework: This framework jointly considers energy efficiency maximization, QoS requirements, queue stability and power limits in the optimization of NOMA. Moreover, the queue stability, utility and average queue length performance are studied through both analytical and simulation results.
- Design of a subchannel assignment algorithm based on matching theory: We model the subchannel-user matching problem as a two-sided matching process and propose a subchannel assignment algorithm based on matching method.
- Design of a power allocation algorithm with multiple constraints: We formulate a power allocation problem for NOMA as a mixed integer programming problem. A minimum QoS requirement is employed to provide reliable transmission for users. The energy efficiency optimization problem is decomposed into three subproblems with time-averaged variables and instantaneous variables. We solve the three subproblems and propose a power allocation algorithm using Lyapunov optimization, where the convergence of the proposed algorithm is also demonstrated via simulations.

II. System Model and Problem Formulation

A. Basic Notation

As shown in Fig. 1, we consider a time-varying downlink multiple cell NOMA network in which N SCBSs exist and each SCBS transmits the signals to each set of mobile users denoted by $\mathcal{U} = \{1, 2, ..., U\}$. The available bandwidth is divided by BS into a set of subchannels which is denoted by $\mathcal{N} = \{1, 2, ..., N\}$. The NOMA system is assumed to operate in a slotted time mode with unit time slots $t \in \{0, 1, 2....\}$, where the time slot t refers to the time interval $[t, t+1)$. We assume that the BSs have full knowledge of the channel state information and denote by $g_{k,u,n}(t)$ the channel gain between the kth SCBS and user u on subchannel n at time slot t. We set $a_{k,u,n}(t) = 1$ when the subchannel n of SCBS k is allocated to user u at time slot t; otherwise, $a_{k,u,n}(t) = 0$. $[x]^+$ means the larger one between x and zero and denote x^T as the transpose of x. $E\{\cdot\}$ denotes the expectation.

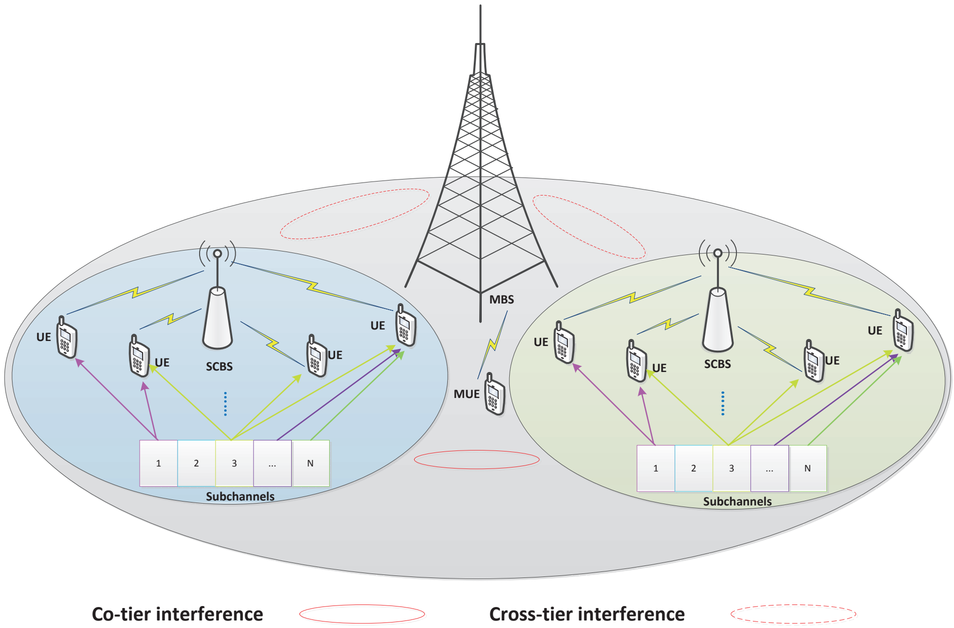

Fig. 1. The architecture of the NOMA network.

B. System Model

In NOMA, one user can receive signals from the BS through multiple subchannels and one subchannel can be allocated to multiple users at the same time slot t. Since the user j on subchannel n causes interference to the other users on the same subchannel, each user j adopts SIC after receiving the superposed signals to demodulate the target message. As shown in [25], without constraints on the specific power split, one user's data can be successfully decoded by another user whose channel gain is better via superposition coding with SIC. Since user u with higher channel gain can only decode the signals of user i with worse channel gain, the interference signals caused by user j whose channel gain is better than user u cannot be decoded and will be treated as noise. Thus, after SIC, the interference for user u caused by other users of same SCBS k on the same subchannel n is given by

$$I_{k,u,n}(t) = \sum\nolimits_{i \in \left\{ S_{k,n} | g_{k,i,n} > g_{k,u,n} \right\}} a_{k,i,n}(t) p_{k,i,n}(t) g_{k,u,n}(t) \tag{1}$$

where $S_{k,n}$ is the set of users of SCBS k on subchannel n. Modeling this residual interference as additional AWGN, we can use the Shannon's capacity formula to write the capacity of user

$u \in \mathcal{U} = \{1, 2, ..., U\}$ of SCBS k on the nth subchannel at time slot t as

$$R_{k,u,n}(t) = \frac{B}{N} a_{k,u,n}(t) \log_2 \left(1 + \frac{p_{k,u,n}(t) g_{k,u,n}(t)}{\sigma_{k,n}^2 + I_{k,u,n}(t) + \tilde{I}_{k,u,n}(t)} \right), \forall k \in \mathcal{K}, u \in \mathcal{U}, n \in \mathcal{N}. \quad (2)$$

where B and ${\sigma_n}^2$ are the bandwidth of the system and the noise variance respectively. $\tilde{I}_{k,u,n}(t)$ is the total of the co-tier interference caused by other SCBSs to SCBS k and cross-tier interference caused by macro BS to SCBS k which is given by:

$$\tilde{I}_{k,u,n}(t) = \sum_{l \neq k}^{K} \sum_{u=1}^{U} a_{l,u,n} p_{l,u,n} g_{l,k,u,n} + \sum_{u=1}^{U} p_{u,n}^{M} g_{k,u,n}^{M}, \forall k \in \mathcal{K}, u \in \mathcal{U}, n \in \mathcal{N}. \quad (3)$$

where $g_{l,k,u,n}$ as the channel gain on subchannel n of user u in SCBS l to SCBS k. $p_{u,n}^{M}$ and $g_{k,u,n}^{M}$ are the power allocation and channel gain on subchannel n of user u in macro BS to SCBS k respectively. The capacity of user u of SCBS k at time slot t can be written as

$$R_{k,u}(t) = \sum_{n=1}^{N} R_{k,u,n}(t), \forall k \in \mathcal{K}, u \in \mathcal{U}. \quad (4)$$

In order to specify the QoS of users, we let $\hat{R}_{k,u}$ be the QoS requirement in terms of the minimum capacity of user u which is thus given as

$$C1 : R_{k,u}(t) \geq \hat{R}_{k,u}, \forall k \in \mathcal{K}, u \in \mathcal{U}. \quad (5)$$

When the queue stability of the SCBS is guaranteed, we can only focus on the performance optimization with queue stability. This is because the SCBS can be assumed to be fixed over a longer duration than the scheduling slot of subchannels. In this NOMA network, the separate buffering queue $Q_{k,u}(t)$ is maintained for each user U of SCBS k. The random traffic arrivals for $Q_{k,u}(t)$ are denoted by $A_{k,u}(t)$ whose peak arrival is $A_{k,u}^{\max}$ at time slot t. $A_{k,u}(t)$ is independent and identically distributed (i.i.d.) over slots with the constant expectation. The exogenous arrival rates may be outside of the network capacity region in practice for the reasons that the statistic of $A_{k,u}(t)$ is usually unknown to SCBSs and the achievable capacity region is usually difficult to estimate. Then, a transport flow control mechanism is needed to keep the traffic queues be stabilized. Denote $r_{k,u}(t)$ as the admitted data rate out of the potentially substantial traffic arrivals for user u of SCBS k which satisfy $0 \leq r_{k,u}(t) \leq A_{k,u}(t)$ obviously. Therefore, we denote the traffic buffering queues for user u as

$$Q_{k,u}(t+1) = [Q_{k,u}(t) - R_{k,u}(t)]^{+} + r_{k,u}(t), \forall k \in \mathcal{K}, u \in \mathcal{U}. \quad (6)$$

The time averaged throughput in terms of the data arrivals for users in SCBS k is defined as $\bar{r}_{k,u} = \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T} r_{k,u}(t)$.

Denote by $p_{k,u}(t)$ and $p_{tot}(t)$ the instantaneous power of user u of SCBS k at time slot t and the total power consumption of all the SCBSs at time slot t respectively, which can be written as

$$p_{k,u}(t) = \sum_{n=1}^{N} p_{k,u,n}(t) + p_k^C \tag{7}$$

and

$$p_{tot}(t) = \sum_{k=1}^{K} \sum_{u=1}^{U} p_{k,u}(t) \tag{8}$$

where p_k^C accounts for the circuit power of SCBS k. The average and instantaneous power constraints of user u of SCBS k are denoted by $P_{k,u}$ and $\hat{P}_{k,u}$, which can be written as

$$\bar{p}_{k,u} = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t} p_{k,u}(\tau) \leq P_{k,u}, \forall k \in K, u \in U \tag{9}$$

$$p_{k,u}(t) \leq \hat{P}_{k,u}, \forall k \in \mathcal{K}, u \in \mathcal{U}. \tag{10}$$

We define the function $g_R(\cdot)$ as the revenue obtained by the throughput which is a non-decreasing concave utility function. Let η_{EE} denote the energy efficiency (EE) which is defined as the ratio of the profit brought by the long-term utility of average throughput to the corresponding long-term total power consumption. It can be written as

$$\eta_{EE} = \frac{\sum_{k=1}^{K} \sum_{u=1}^{U} g_R\left(\bar{r}_{k,u}\right)}{\bar{p}_{tot}}. \tag{11}$$

where $\bar{p}_{tot} = \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T} p_{tot}(t)$.

C. Optimization Problem Formulation

In this subsection, when considering all constraints, the utility function is expressed as

$$\begin{aligned}
&\max_{\mathcal{P}} \eta_{EE} = \frac{\sum_{k=1}^{K} \sum_{u=1}^{U} g_R\left(\bar{r}_{k,u}\right)}{\bar{p}_{tot}} \\
&s.t. C1 \\
&C2 : \bar{p}_{k,u} = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t} p_{k,u}(\tau) \leq P_{k,u}, \forall k \in K, u \in U \\
&C3 : p_{k,u}(t) \leq \hat{P}_{k,u}, \forall k \in K, u \in U \\
&C4 : \widetilde{R_{k,u}} = \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T} R_{k,u}(t) \geq \bar{r}_{k,u}, \forall k \in \mathcal{K}, u \in \mathcal{U}.
\end{aligned} \tag{12}$$

where $\mathcal{P}$ denotes the power allocation policy and the constraint $C1$ ensures the QoS of users; $C2$ is limit of the maximum average transmit power of user u of SCBS k; $C3$ is the maximum instantaneous transmit power of user u of SCBS k; and $C4$ ensures the stability of user u in SCBS k. We define the optimal EE η_{EE}^{opt} as

$$\eta_{EE}^{opt} = \frac{\sum_{k=1}^{K}\sum_{u=1}^{U} g_R\left(\bar{r}_{k,u}(\mathcal{P}^*)\right)}{\bar{p}_{tot}(\mathcal{P}^*)} = \max_{\mathcal{P}} \frac{\sum_{k=1}^{K}\sum_{u=1}^{U} g_R\left(\bar{r}_{k,u}(\mathcal{P})\right)}{\bar{p}_{tot}(\mathcal{P})} \tag{13}$$

where $\mathcal{P}^*$ denotes the optimal power allocation policy that yields η_{EE}^{opt}. We can classify the utility function in (12) as a nonlinear fractional program. We introduce Theorem 1 as follows.

Theorem 1: The optimal EE η_{EE}^{opt} can be reached if and only if

$$\begin{aligned}
&\max_{\mathcal{P}} \sum_{k=1}^{K}\sum_{u=1}^{U} g_R\left(\bar{r}_{k,u}(\mathcal{P})\right) - \eta_{EE}^{opt}\bar{p}_{tot}(\mathcal{P}) \\
&= \sum_{k=1}^{K}\sum_{u=1}^{U} g_R\left(\bar{r}_{k,u}(\mathcal{P}^*)\right) - \eta_{EE}^{opt}\bar{p}_{tot}(\mathcal{P}^*) = 0 \\
&for \sum_{k=1}^{K}\sum_{u=1}^{U} g_R\left(\bar{r}_{k,u}(\mathcal{P})\right) \geq 0, \bar{p}_{tot}(\mathcal{P}) \geq 0.
\end{aligned} \tag{14}$$

Proof: Please refer to Appendix A.

According to Theorem 1, an equivalent objective function in subtractive form is existed for the (non-convex) optimization problem (12) which can be classified as nonlinear fractional program. Then, the formulation (12) can be rewritten in the more tractable form

$$\begin{aligned}
&\max_{\mathcal{P}} \sum_{k=1}^{K}\sum_{u=1}^{U} g_R\left(\bar{r}_{k,u}(\mathcal{P})\right) - \eta_{EE}\bar{p}_{tot}(\mathcal{P}) \\
&s.t. C1, C2, C3, C4.
\end{aligned} \tag{15}$$

III. Energy Efficient Optimization Using Lyapunov Optimization

In this section, the subchannel assignment is investigated in the NOMA network and the optimization problem in (15) is solved based on Lyapunov optimization.

A. Subchannel Matching

We assume that all the users of BS can transmit on the subchannel n arbitrarily at time slot t in a NOMA system. Considering the complexity of decoding and the fairness of users, each subchannel can only be allocated to at most D_n users and each user can only occupy at most D_u subchannels at the same time. We assume that $N * D_n \geq U * D_u$. The dynamic matching between the users and the subchannels of SCBS k is considered as a many-many matching process between the set of U users and the set of N subchannels. User u is matched with subchannel n at time slot t if $a_{k,u,n}(t) = 1$. Based on the channel state information, we assume user u prefers channel n_1 over n_2 if and only if $g_{k,u,n_1} > g_{k,u,n_2}$. Then, the preference lists of the users of SCBS k can be denoted by

$$\Pr ef\_(K,U) = [\Pr ef\_U(1), ..., \Pr ef\_U(u), ... \Pr ef\_U(U)]^T \tag{16}$$

where $\Pr ef\_(K,U)(u)$ is the preference list of user u of SCBS k which is in the descending order of channel gains of subchannels. To reduce the complexity, we propose a suboptimal matching algorithm for subchannel allocation as Algorithm 1. For each user u of SCBS k, the matching request is send to its most preferred subchannel according to its preference list. Then the preferred subchannel decides whether accept the user or not according to the users's channel gain. For each subchannel n, there is at most D_n number of accepted users who has higher channel gain than the rejected users. The rejected user on subchannel n will remove the subchannel n from its preference list and its most preferred subchannel is changed. The matching request is send again until all the users of each SCBS matched D_u subchannels and the matching is achieved.

B. The Queues of Lyapunov Optimization

Because $g_R(\bar{r}_{k,u})$ is related to the time averaged throughput, we define auxiliary variables $\gamma_{k,u}$ for the traffic arrival of user u in SCBS k which satisfies $\bar{\gamma}_{k,u} \le \bar{r}_{k,u}$ and $0 \le \gamma_{k,u} \le A_{k,u}^{\max}$. Therefore, the optimization problem in (15) can be rewritten as

$$\begin{aligned} & \max_{\mathcal{P}} \sum_{k \in K} \sum_{u \in U} \overline{g_R(\gamma_{k,u}) - \eta_{EE} p_{tot}} \\ & s.t. \mathrm{C1, C2, C3, C4} \\ & C5 : \bar{\gamma}_{k,u} \le \bar{r}_{k,u}, 0 \le \gamma_{k,u} \le A_{k,u}^{\max} \end{aligned} \tag{17}$$

where $\bar{\gamma}_{k,u} = \lim\limits_{T \to \infty} \frac{1}{T} \sum\limits_{t=0}^{T} \gamma_{k,u}(t)$ and $\overline{g_R(\gamma_{k,u}) - \eta_{EE} p_{tot}} = \lim\limits_{T \to \infty} \frac{1}{T} \sum\limits_{t=0}^{T} (g_R(\gamma_{k,u}(t)) - \eta_{EE} p_{tot})$. The equivalent of problems (15) and (17) can be proved by appendix B. To satisfy the averaged throughput constraint in $C5$, we denote by $H_{k,u}(t)$ the virtual queue for user u in SCBS k at time slot t. We get

$$H_{k,u}(t+1) = [H_{k,u}(t) - r_{k,u}(t)]^+ + \gamma_{k,u}(t), \forall k \in \mathcal{K}, u \in \mathcal{U}. \tag{18}$$

Algorithm 1 Suboptimal Matching Algorithm for Subchannel Allocation

Initialize the matched list $S_{k,n}$ and $S_{k,u}$ to denote the number of users matched with subchannel $n\,(\forall n \in \{1,2,...,N\})$ and the number of subchannels matched with user $u\,(\forall n \in \{1,2,...,U\})$ in SCBS k, respectively;
Initialize preference lists $\Pr ef\_U(k,u)$ for all the users of SCBS k according to channel state information;
Initialize the set of not fully matched users $S_{U\_F}(k,u)$ to denote users of SCBS k who have not been matched with D_u subchannels;
while $S_{U\_F}(k,u) \neq \phi$ **do**
  for $u = 1$ to U **do**
    if $S_{k,u} < D_u$ **then**
      User u of SCBS k sends a matching request to its most preferred subchannel $\widehat{n}$ according to $\Pr ef\_U(k,u)$;
      if $S_{k,\widehat{n}} < D_{k,\widehat{n}}$ **then**
        Set $a_{k,u,\widehat{n}} = 1$, $S_{k,u} = S_{k,u} + 1$ and $S_{k,\widehat{n}} = S_{k,\widehat{n}} + 1$;
      else if $S_{k,\widehat{n}} = D_{k,\widehat{n}}$ **then**
        Find the minimum channel gain of users $g_{k,\widehat{u},\widehat{n}}$ on channel $\widehat{n}$ in SCBS k and compare it with $g_{k,u,\widehat{n}}$;
        if $g_{k,\widehat{u},\widehat{n}} < g_{k,u,\widehat{n}}$ **then**
          Set $a_{k,u,\widehat{n}} = 1$, $a_{k,\widehat{u},\widehat{n}} = 0$, $S_{k,u} = S_{k,u} + 1$, and $S_{k,\widehat{u}} = S_{k,\widehat{u}} - 1$;
        else
          Remove subchannel $\widehat{n}$ from the $\Pr ef\_U(k,u)$ and find the next $\widehat{n}$ of user u in SCBS k according to $\Pr ef\_U(k,u)$.
        end if
      end if
    end if
  end for
end while

Similarly, in order to satisfy the constraint $C2$, virtual power queues for the user u in SCBS k are defined. We denote $Z_{k,u}(t)$ as the queue arrival with the transmit power $p_{k,u}(t)$, and get

$$Z_{k,u}(t+1) = [Z_{k,u}(t) - P_{k,u}]^{+} + p_{k,u}(t), \forall k \in \mathcal{K}, u \in \mathcal{U}. \tag{19}$$

C. The Formulation of Lyapunov Optimization

Denote $\Phi(t) = [Q(t), H(t), Z(t)]$ as the matrix of all the queues. We define the Lyapunov function as a scalar metric of queue congestion

$$L\left(\Phi\left(t\right)\right) = \frac{1}{2}\left\{\sum_{k=1}^{K}\sum_{u=1}^{U}\left(Q_{k,u}(t)^2 + H_{k,u}(t)^2 + Z_{k,u}(t)^2\right)\right\}. \tag{20}$$

We introduce a Lyapunov drift in this subsection for pushing the Lyapunov function to a lower congestion state and keep both the actual and virtual queues stable

$$\Delta(\Phi(t)) = E\left\{L(\Phi(t+1)) - L(\Phi(t))\right\}. \tag{21}$$

According to Lyapunov optimization, by subtracting $VE\left\{\sum\limits_{u\in\mathcal{U}} g_R(\gamma_{k,u}) - \eta_{EE} p_{tot}\right\}$ from both sides of (21), we get

$$\begin{array}{l}
\Delta\left(\Phi\left(t\right)\right) - VE\left\{\sum\limits_{k=1}^{K}\sum\limits_{u=1}^{U} g_R\left(\gamma_{k,u}\right) - \eta_{EE} p_{tot}\right\}\\
\le C + \sum\limits_{k=1}^{K}\sum\limits_{u=1}^{U} E\left\{H_{k,u}(t)\gamma_{k,u} - V g_R\left(\gamma_{k,u}\right)\right\}\\
+VE\left(\eta_{EE}\left\{\sum\limits_{k=1}^{K}\left\{\sum\limits_{u=1}^{U} p_{k,u}(t) + p_k^C\right\}\right\}\right)\\
-\sum\limits_{k=1}^{K}\sum\limits_{u=1}^{U}\left\{H_{k,u}(t) - Q_{k,u}(t)\right\}E\left\{r_{k,u}(t)\right\} - \sum\limits_{k=1}^{K}\sum\limits_{u=1}^{U} Q_{k,u}(t)E\left\{R_{k,u}(t)\right\}\\
-\sum\limits_{k=1}^{K}\sum\limits_{u=1}^{U} Z_{k,u}(t)E\left\{P_{k,u}(t) - p_{k,u}(t)\right\}
\end{array} \tag{22}$$

where V is an arbitrarily positive control parameter which represents the emphasis on utility maximization compared to queue stability and C is a finite constant that satisfies

$$\begin{array}{l}
C \ge \frac{1}{2}E\left\{\sum\limits_{k=1}^{K}\sum\limits_{u=1}^{U}\left(r_{k,u}(t)^2 + R_{k,u}(t)^2\right)\right\}\\
+\frac{1}{2}E\left\{\sum\limits_{k=1}^{K}\sum\limits_{u=1}^{U}\left[\left(r_{k,u}(t) - \gamma_{k,u}(t)\right)^2 + \left(P_{k,u}(t) - p_{k,u}(t)\right)^2\right]\right\}.
\end{array} \tag{23}$$

1) The solution of virtual variables: The optimal choice of γ_u to minimize (22) can be made by solving

$$\begin{aligned} &\max V g_R(\gamma_{k,u}) - H_{k,u}(t)\gamma_{k,u} \\ &s.t. 0 \le \gamma_{k,u} \le A_{k,u}^{\max} \end{aligned} \tag{24}$$

and the solution is

$$\gamma_{k,u}(t) = \min\left\{\frac{V}{H_{k,u}(t)}, A_{k,u}^{\max}\right\}. \tag{25}$$

2) The solution of actual traffic arrival: In order to minimize (22), the optimal actual traffic arrival can be achieved by maximizing the expression as follows:

$$\begin{aligned} &\max \left\{H_{k,u}(t) - Q_{k,u}(t)\right\} E\left\{r_{k,u}(t)\right\} \\ &s.t. 0 \le r_{k,u}(t) \le A_{k,u}(t). \end{aligned} \tag{26}$$

We get the optimal solution as

$$r_{k,u}(t) = \begin{cases} A_{k,u}(t), if H_{k,u}(t) - Q_{k,u}(t) \succ 0 \\ 0, else. \end{cases} \tag{27}$$

3) Power allocations: In order to minimize (22), we first obtain the optimal solution of the virtual variables and the actual traffic arrival, then we can minimize the remaining part of (22) and denote it as

$$\begin{aligned} &\min\left\{\begin{array}{l} VE\left(\eta_{EE}\left\{\sum\limits_{k=1}^{K}\left\{\sum\limits_{u=1}^{U} p_{k,u}(t) + p_k^C\right\}\right\}\right) + \sum\limits_{k=1}^{K}\sum\limits_{u=1}^{U} Z_{k,u}(t)p_{k,u}(t) \\ -\sum\limits_{k=1}^{K}\sum\limits_{u=1}^{U} Q_{k,u}(t)E\left\{R_{k,u}(t)\right\} \end{array}\right\} \\ &= VE\left(\eta_{EE}\left\{\sum_{k=1}^{K}\left\{\sum_{u=1}^{U}\sum_{n=1}^{N} a_{k,u,n}(t)p_{k,u,n}(t)\right\} + p_k^C\right\}\right) \\ &+ \sum_{k=1}^{K}\sum_{u=1}^{U} Z_{k,u}(t)\left\{\sum_{n=1}^{N} a_{k,u,n}(t)p_{k,u,n}(t)\right\} \\ &- \sum_{k=1}^{K}\sum_{u=1}^{U} Q_{k,u}(t)E\left\{\sum_{n=1}^{N} R_{k,u,n}(t)\right\}. \end{aligned} \tag{28}$$

Let $\omega_{k,u,n}(t) = a_{k,u,n}(t)p_{k,u,n}(t), \forall k \in \mathcal{K}, u \in \mathcal{U}, n \in \mathcal{N}$; then we can get

$$R_{k,u,n}(t) = \frac{B}{N}a_{k,u,n}(t)\log_2\left(1 + \frac{\omega_{k,u,n}(t)g_{k,u,n}(t)}{a_{k,u,n}(t)\left(\sigma_{k,n}^2 + I_{k,u,n}(t) + \tilde{I}_{k,u,n}(t)\right)}\right). \tag{29}$$

Then, we can rewrite (28) as follows

$$\begin{aligned}
&\min VE\left(\eta_{EE}\left\{\sum_{k=1}^{K}\sum_{u=1}^{U}p_{k,u}(t)\right\}\right)-\sum_{k=1}^{K}\left\{\sum_{u=1}^{U}Q_{k,u}(t)E\left\{R_{k,u}(t)\right\}+\sum_{u=1}^{U}Z_{k,u}(t)p_{k,u}(t)\right\}\\
&=VE\left(\eta_{EE}\left\{\sum_{k=1}^{K}\sum_{u=1}^{U}\sum_{n=1}^{N}\omega_{k,u,n}(t)\right\}\right)+\sum_{k=1}^{K}\sum_{u=1}^{U}Z_{k,u}(t)\left\{\sum_{n=1}^{N}\omega_{k,u,n}(t)\right\}\\
&-\sum_{k=1}^{K}\sum_{u=1}^{U}Q_{k,u}(t)E\left\{\frac{B}{N}\sum_{n=1}^{N}a_{k,u,n}(t)\log_2\left(1+\frac{\omega_{k,u,n}(t)g_{k,u,n}(t)}{a_{k,u,n}(t)\left(\sigma_{k,n}^2+I_{k,u,n}(t)+\tilde{I}_{k,u,n}(t)\right)}\right)\right\}
\end{aligned} \tag{30}$$

Since (30) is convex, to satisfy the series of constraints, the Lagrange function of the problem (30) can be expressed by

$$\begin{aligned}
&F(\lambda,\beta)=\min L(\lambda,\beta)\\
&=VE\left(\eta_{EE}\left\{\sum_{k=1}^{K}\sum_{u=1}^{U}\sum_{n=1}^{N}\omega_{k,u,n}(t)\right\}\right)+\sum_{k=1}^{K}\sum_{u=1}^{U}Z_{k,u}(t)\left\{\sum_{n=1}^{N}\omega_{k,u,n}(t)\right\}-\\
&\sum_{k=1}^{K}\sum_{u=1}^{U}Q_{k,u}(t)E\left\{\sum_{n=1}^{N}\frac{B}{N}a_{k,u,n}(t)\log_2\left(1+\frac{\omega_{k,u,n}(t)g_{k,u,n}(t)}{a_{k,u,n}(t)\left(\sigma_{k,n}^2+I_{k,u,n}(t)+\tilde{I}_{k,u,n}(t)\right)}\right)\right\}\\
&+\sum_{k=1}^{K}\sum_{u=1}^{U}\lambda_{k,u}(t)\left\{\sum_{n=1}^{N}\omega_{k,u,n}(t)-\hat{P}_{k,u}\right\}\\
&+\sum_{k=1}^{K}\sum_{u=1}^{U}\beta_{k,u}(t)\left\{\hat{R}_{k,u}-\sum_{n=1}^{N}\frac{B}{N}a_{k,u,n}(t)\log_2\left(1+\frac{\omega_{k,u,n}(t)g_{k,u,n}(t)}{a_{k,u,n}(t)\left(\sigma_{k,n}^2+I_{k,u,n}(t)+\tilde{I}_{k,u,n}(t)\right)}\right)\right\},
\end{aligned} \tag{31}$$

where λ , β are the Lagrange multiplier vectors for the constraints in (17). Taking the first order derivation of $F(\lambda,\beta)$ with respect to $\omega_{k,u,n}(t)$, we can get the optimal power allocation as follows

$$p_{k,u,n}(t)=\frac{\omega_{k,u,n}(t)}{a_{k,u,n}(t)}=\frac{\frac{B}{N}\left(Q_{k,u}(t)+\beta_{k,u}(t)\right)}{\ln 2(V\eta_{EE}+Z_{k,u}(t)+\lambda_{k,u}(t))}-\frac{\sigma_{k,n}^2+I_{k,u,n}(t)+\tilde{I}_{k,u,n}(t)}{g_{k,u,n}(t)}. \tag{32}$$

Based on the subgradient method [27], the master dual problem in (32) can be solved by

$$\begin{aligned}
&\lambda_{k,u}^{l+1}=[\lambda_{k,u}^{l}-\varepsilon_1^l(\hat{P}_{k,u}-\sum_{n=1}^{N}\omega_{k,u,n}(t))]^{+},\forall k\in K,u\in U;\\
&\beta_{k,u}^{l+1}=[\beta_{k,u}^{l}-\varepsilon_2^l(\sum_{n=1}^{N}R_{k,u,n}-R_{k,u})]^{+},\forall k\in K,u\in U.
\end{aligned} \tag{33}$$

After the time interval T, we get the average queue length $\overline{Q}$, total average capacity R^{ave}, average power consumption P^{ave} and the average energy efficient EE η_{EE}^{ave} as

$$\bar{Q}=\frac{1}{T}\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{u=1}^{U}Q_{k,u}(t) \tag{34}$$

$$R^{ave}=\frac{1}{T}\sum_{t=1}^{T}U_{tot}(t) \tag{35}$$

Algorithm 2 Lyapunov Optimization based Resource Allocation Algorithm

Initialize the $a_{k,u,n}(t)$ using suboptimal Algorithm 1;
Initialize $p_{k,u,n}(t)$ using equal power allocation;
Initialize the value of $A_{k,u}^{\max}$ and $Q_{k,u}(t)$;
For each time slot, calculate the auxiliary variables $\gamma_{k,u}(t)$ for each time slot and admitted traffic $r_{k,u}(t)$ by solving (25), (27) respectively;
repeat
    Obtain the optimal allocation of power in the current time slot according to (32);
    Update Lagrangian multipliers of λ, β by solving (33);
    Solve (12) using the Dinkelbach method in [1] to get the optimal value of η_{EE} in an iteration way;
until Convergence or certain stopping criteria is met
Calculate the traffic queue $Q_{k,u}(t)$ and the virtual queues of $H_{k,u}(t)$ and $Z_{k,u}(t)$ of next time slot by solving (6), (18), (19) respectively.

$$\bar{P} = \frac{1}{T}\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{u=1}^{U} p_{k,u}(t) \tag{36}$$

$$\eta_{EE}^{ave} = \frac{1}{T}\sum_{t=1}^{T}\eta_{EE}. \tag{37}$$

The above proposed approach based on Lyapunov optimization for solving the EE optimization problem in (19) can be summarized in Algorithm 2.

D. Complexity Analysis

The complexity of the proposed algorithms is analyzed in this subsection. The worst-case complexity of proposed suboptimal Algorithm 1 is $\mathrm{O}(KUN{+}1/2K\lfloor U/D_n\rfloor^2)$ in which the finding of preference lists is $\mathrm{O}(KUN)$. The optimal algorithm of subchannel matching can only be obtained by searching over all possible combinations of users whose computational complexity can be approximated as $\mathrm{O}(K\frac{[U!]^{Du}}{{D_n}^N})$. It can be found from the analysis above, proposed suboptimal subchannel matching Algorithm 1 has a much lower polynomial complexity than the exhaustive search. In Algorithm 2, the calculation of (32) for each user in each SCBS on each subchannel entails KUN operations. Then the complexity of Algorithm 2 is $\mathrm{O}(TLKUN)$ where L is the number of iterations each time slot. As the complexity of Algorithm 2 increases with the number of users and the convergence of EE is influenced by the control parameter V, the Algorithm 2 may be practical for a middle-scale realtime network and the value of V should be well chosen. Furthermore, cellular users are always distributed in a cluster-way. Therefore, the cellular users in a cell can be divided into several clusters, and the proposed algorithm can be used in each cluster. Subchannels are not shared between different clusters. Since the number of users in each cluster is small, the complexity of the proposed algorithms will be further reduced.

IV. Performance Analysis

In this section, we analyze the performance bounds of the proposed Algorithm 2 based on the Lyapunov optimization.

A. Stability of Queues

In this work, all the actual queues $Q_{k,u}(t)$ and virtual queues of $H_{k,u}(t)$ and $Z_{k,u}(t)$ are mean rate stable. We assume the expectation of $P_{tot}(t)$ and $U_{tot}(t)$ are bound by

$$P_{\min} \leq E\left\{P_{tot}(t)\right\} \leq P_{\max} \tag{38}$$

$$R_{\min} \leq E\left\{U_{tot}(t)\right\} \leq R_{\max} \tag{39}$$

where $P_{\min}$, $P_{\max}$, $R_{\min}$ and $R_{\max}$ are finite constants. From the boundedness assumptions, there is positive constant C satisfying

$$\Delta(\Phi(t)) \leq C. \tag{40}$$

It can be written as

$$E\left\{L(\Phi(t+1))\right\} - E\left\{L(\Phi(t))\right\} \leq C. \tag{41}$$

Considering telescoping sums over $t \in \{0, 1, ..., T-1\}$ in above inequality, we can get

$$E\left\{L(\Phi(T))\right\} - E\left\{L(\Phi(0))\right\} \leq TC. \tag{42}$$

Using (20), we can rewrite (42) as

$$E\left\{Z_{k,u}(T)^2\right\} \leq 2TC + 2E\left\{L(\Phi(0))\right\}. \tag{43}$$

Given $D(|Z_{k,u}(T)|) = E(|Z_{k,u}(T)|^2) - [E(|Z_{k,u}(T)|)]^2 \geq 0$, we get $E(|Z_{k,u}(T)|^2) \geq [E(|Z_{k,u}(T)|)]^2$. Thus we get

$$E(|Z_{k,u}(T)|) \leq \sqrt{2TC + 2E\left\{L(\Phi(0))\right\}}. \tag{44}$$

Taking the limit $T \to \infty$ and dividing T at the same time, we have

$$\lim_{T\to\infty} \frac{E(|Z_{k,u}(T)|)}{T} = 0. \tag{45}$$

Therefore, the queues of $Z_{k,u}(T)$ are mean rate stable. Similarly, we can also prove the queues $Q_{k,u}(T)$ and $H_{k,u}(T)$ are mean rate stable.

B. The Utility and Average Queue Length Performance

The EE performance and the average queue length performance obtained by Algorithm 2 is given in Theorem 2.

Theorem 2: Utilizing the proposed optimization solution, η_{EE} is bounded by

$$\eta_{EE} \geq \eta_{EE}^{opt} - \frac{B}{VP_{\min}}. \tag{46}$$

The performance of the average network queue length is bounded by

$$\bar{Q} \leq \frac{B + V(R_{\max} - \eta_{EE}^{opt} P_{\min})}{\varepsilon} \tag{47}$$

where η_{EE}^{opt} is the theoretical maximum achievable utility of all achievable solutions.

Proof: Please refer to Appendix C.

With the consideration of $\eta_{EE} \leq \eta_{EE}^{opt}$, which naturally holds from (13), the bound of EE is $\eta_{EE}^{opt} - \frac{B}{VP_{\min}} \leq \eta_{EE} \leq \eta_{EE}^{opt}$. Therefore, η_{EE} can arbitrarily approach η_{EE}^{opt} by setting large enough V to make $\frac{B}{VP_{\min}}$ arbitrarily small. The bound of average queue backlog $\bar{Q}$ increases linearly in V according to (47). Theorem 2 shows that there exists an $[\mathrm{O}(1/V), \mathrm{O}(V)]$ utility-backlog tradeoff that leads to Little's Theorem.

V. Simulation Results and Discussion

In this section, simulation results are given to evaluate the performance of the proposed algorithms. In the considered NOMA network, there are K SCBSs which consists U users distributed randomly in a circular with radius of $30\ m$ and the SCBSS are distributed randomly in the circular of MBS whose radius is $500\ m$. The noise power spectral density is -174

dBm/Hz. We consider a normalized bandwidth which is $B = 1\ Hz$. For the simulation, the maximum average transmit power and the maximum instantaneous transmit power of user U in SCBS k is set as $2.5/U\ Watts$ and $2.505/U\ Watts$ respectively. We assume the circuit power of user u is $1\ Watt$. The time interval T is set as 5000 slots which means traffic arrival rates is $15\ bps/Hz$. We assume that each user can access only one subchannel in the OFDMA scheme.

In Fig. 2, the average queue length versus time t is evaluated under different values of control parameter V. The user QoS requirement is set as $\hat{R}_u = 2\ bps/Hz$ and the maximum number of matched subchannels of each user is $D_u = 2$ with the maximum number of matched users of each subchannel is $D_n = 2$. It is shown that, the value of the average queue length increases with time t and gradually fluctuates around a certain fixed value. And for the same value of t, a smaller value of V leads to a smaller value of average queue length which satisfy with the Theorem 2. This conclusion can also be obtained in Fig. 9.

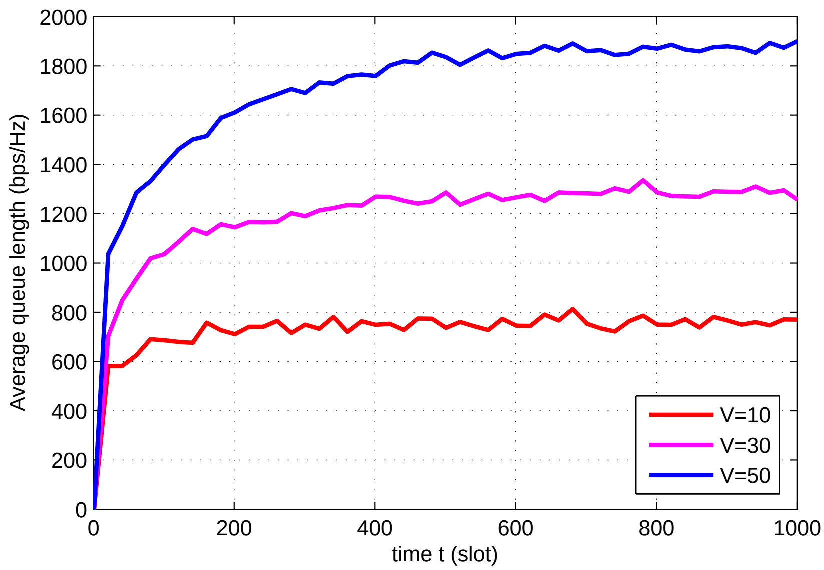

Fig. 2. Average queue length versus simulation time length.

Fig. 3 shows the EE performance versus time t with the same constraints of Fig. 2 except the QoS constraint is $\hat{R}_u = 1\ bps/Hz$ instead. It can be observed that the EE performance will gradually fluctuates around a certain fixed value with the increase of t which illustrates the convergence of EE with time t. And the larger value of U lead to smaller convergence of EE.

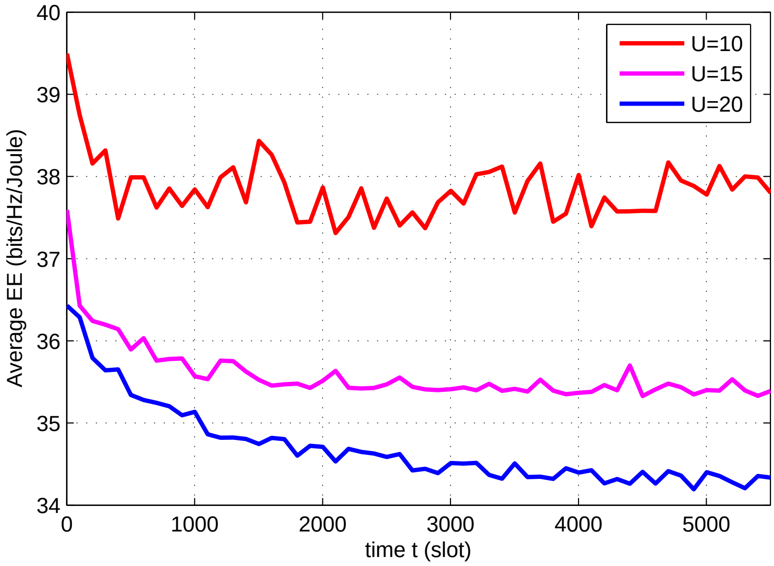

Fig. 3. EE performance versus simulation time length.

This is because the need of subchannels will increase with the increase of user number which lead to the decrease of bandwidth of each subchannel.

As shown in Fig. 4, the EE performance is evaluated with the simulation time slot t. In each time slot t, the algorithms in Fig. 4 have the equal total power consumption. That is, the total power consumption is time varied for each algorithm. With the increase of time t, the EE will gradually fluctuates around a certain fixed value. From Fig. 4, we observe that the performance of $V = 100$ which is the proposed resource allocation algorithms with the control parameter $V = 100$, including subchannel assignment and power allocation, is much better than the algorithm of NOMA-EQ in [16]. This is because the algorithm of NOMA-EQ is limited of equal power allocation of each subchannel in NOMA system. For different subchannel allocation schemes, the EE performance using exhaustive search which denoted as NOMA-Opt is better than using the suboptimal algorithm we proposed.

In Fig. 5, the performance of average EE is evaluated versus the parameter V with different values of D_n which is the number of users matched with each subchannel and $D_n = 1$ represents it's in a OFDMA scheme. The user QoS requirement is set as $\hat{R}_u = 2\ bps/Hz$ and the maximum number of matched subchannels of each user is $D_u = 1$. It is shown that, with the increase in the parameter V, the value of average EE increases and converges to a certain value both in NOMA at the same value of D_u and in OFDMA. It is seen that the average EE in NOMA is better than the average EE in OFDMA. And for the same value of V, a larger value of D_n leads to a larger value of average EE. This is because our proposed algorithm provides more freedom in the bandwidth allocation of assigned subchannels. For the same set of users under same value of D_u, the more larger of the valuse of D_n is, the bandwidth of each subchannel is more larger.

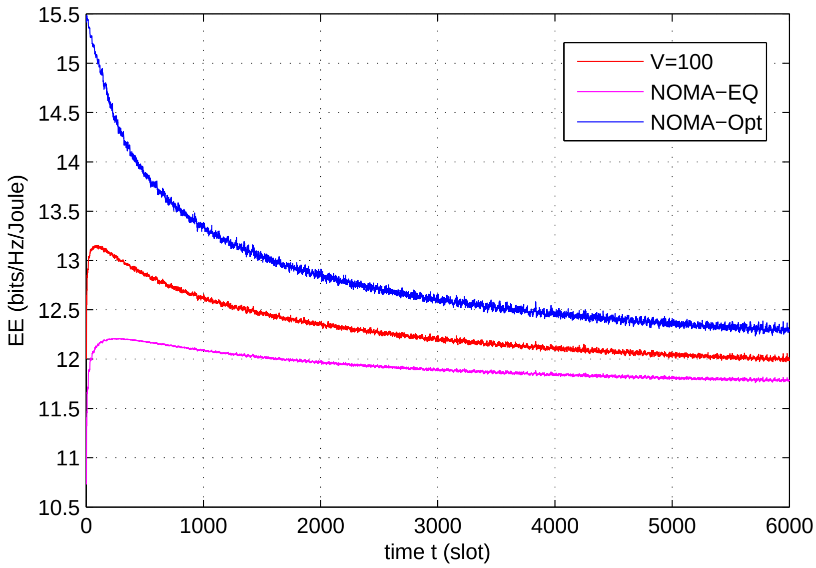

Fig. 4. EE performance versus simulation time length with different algorithms.

Fig. 6 shows the performance of average EE versus the parameter V with different values of D_u where the user QoS requirement is set as $\hat{R}_u = 2\ bps/Hz$ and the maximum number of matched users of each subchannel is $D_n = 2$. It is seen that, for arbitrary value of D_u, the value of average EE Arbitrary increases and converges to a certain value with the increase in the parameter V. For the same value of parameter V, a larger value of D_u leads to a larger value of average EE due to the various selection of subchannels.

Fig. 7 illustrates the convergence of the total average capacity versus the parameter V with different values of D_u. Different from the trend of average EE, for the same value of D_u, the value of the total average transmit capacity decreases and converges to a value with the increase of V. For the same value of V, a larger value of D_u results in a larger value of total average

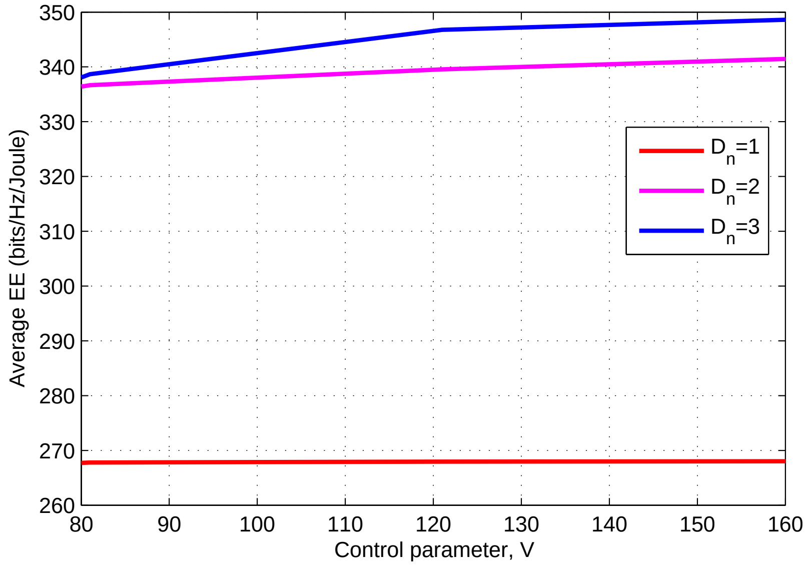

Fig. 5. Average EE performance versus the parameter V with different values of D_n.

transmit capacity. This is because every user has more selections in subchannel to guarantee the QoS requirement.

Fig. 8 shows average power consumption versus the parameter V with different values of D_u. The set of constraints is as Fig. 5 and Fig. 6. We see that the average power consumption decreases steadily with increasing V. For the same value of V, different from the trend of the total average capacity, a larger value of D_u results in a lower value of average power consumption. The average power consumption of $D_u = 3$ is 3% smaller than that of $D_u = 1$ when $V = 120$.

Fig. 9 depicts the average queue length versus the parameter V with different values of D_u. The user QoS requirement is set as $\hat{R}_u = 2\ bps/Hz$ and the maximum number of matched users of each subchannel is $D_n = 2$ in the NOMA scheme. With different values for D_u, the average queue length grows steadily with increasing V. For the same value of V, a larger value of D_u leads to a lower value of average queue length. It illustrates the users goes steadily faster with a larger value of D_u and this result also satisfy with the Theorem 2.

In Fig. 10, the average EE versus V with different QoS requirements is illustrated. We set the maximum number of allocated subchannels of each user is $D_u = 2$ and the maximum number of matched users of each subchannel is $D_n = 2$. As shown in Fig. 10, for the same value of

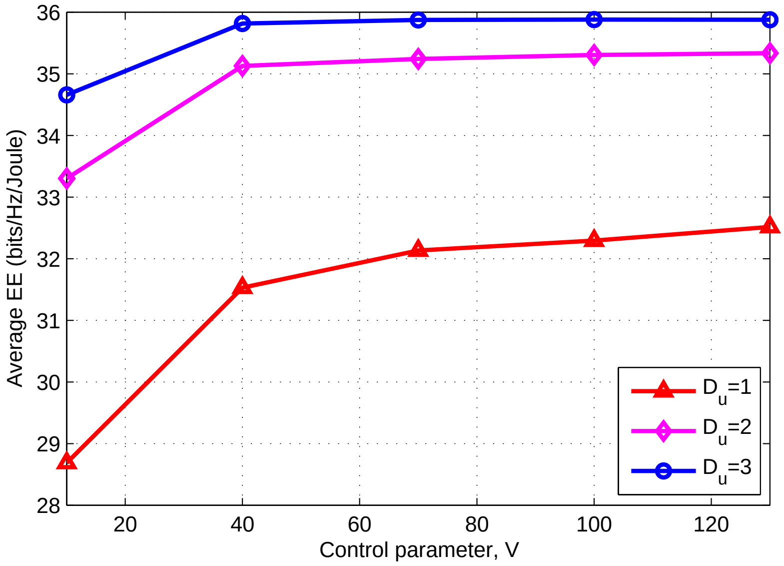

Fig. 6. Average EE performance versus the parameter V with different values of D_u.

V, a larger value of the QoS requirement results in a smaller converged value of average EE. A smaller value of QoS requirement leads to a larger value of EE due to the fact that a smaller value of QoS requirement enlarges the feasible region of the optimizing variable.

Fig. 11 shows the average power consumption versus V with different QoS requirements. The maximum number of allocated subchannels of each user is $D_u = 2$ and the maximum number of matched users of each subchannel is $D_n = 2$. As the parameter V increases, average power consumption continues to decrease. From Fig. 11, we can observe that for the same value of V, a larger value of the QoS requirement results in a larger value of average power consumption.

In Fig. 12, the total average transmit capacity is evaluated versus V with different QoS requirements. The mean traffic arrival rate $\alpha = 15\ bps/Hz$ and has the same constraints of Fig. 11. It can be observed that, the trend of this curve is similar to the average power consumption curves in Fig. 11. For the same parameter V, a larger value of the QoS requirement results in a larger value of the total average transmit capacity.

VI. Conclusion

We have investigated dynamic resource allocation in downlink NOMA networks. We have proposed a suboptimal subchannel assignment algorithm based on the two-side matching method.

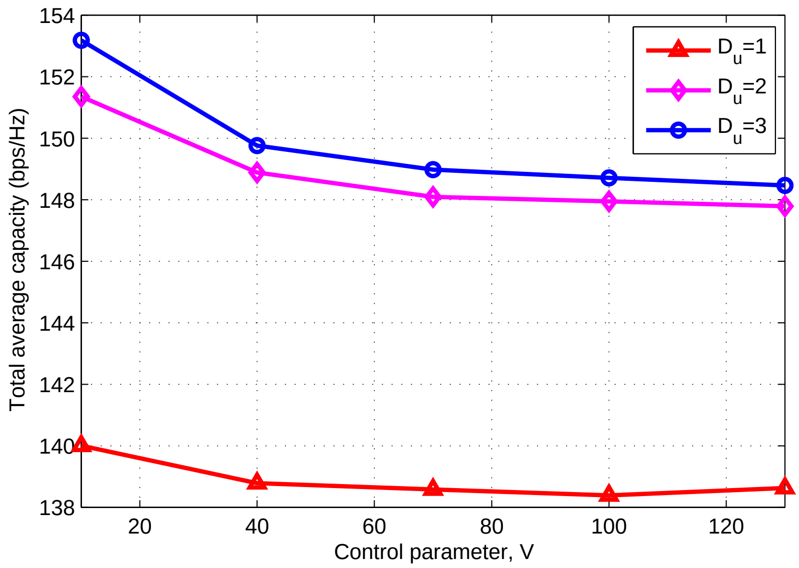

Fig. 7. The total average transmit capacity versus the parameter V with different values of D_u.

We have formulated the power allocation as a mixed integer programming problem by considering a minimum QoS requirement, and maximum power constraint. Based on the framework of Lyapunov optimization, the problem of energy efficient optimization was broken down into three subproblems, two of which are linear and the rest of which be solved via Lagrangian optimization. The mathematical analysis and simulation results have demonstrated the effectiveness of the proposed algorithms.

Appendix A

The proof of Theorem 1 is similar to the method in [26]. For the sake of notational simplicity, we define Θ as the set of of feasible solutions of the optimization problem in (12) and $\eta_{EE} = \frac{\sum\limits_{k=1}^{K}\sum\limits_{u=1}^{U} g_R\left(\bar{r}_{k,u}\right)}{\bar{p}_{tot}} = \frac{G(\mathcal{P})}{T(\mathcal{P})}$, respectively. Without loss of generality, we define η_{EE}^{opt} and $\{\mathcal{P}^*\} \in \Theta$ as the optimal energy efficiency and the optimal power allocation policy of the original objective function in (12), respectively. We can get the optimal energy efficiency as

$$\eta_{EE}^{opt} = \frac{G(\mathcal{P}^*)}{T(\mathcal{P}^*)} \geq \frac{G(\mathcal{P})}{T(\mathcal{P})}, \forall \{\mathcal{P}\} \in \Theta, \Rightarrow \begin{cases} G(\mathcal{P}) - \eta_{EE}^{opt} T(\mathcal{P}) \leq 0 \\ G(\mathcal{P}^*) - \eta_{EE}^{opt} T(\mathcal{P}^*) = 0 \end{cases}. \tag{48}$$

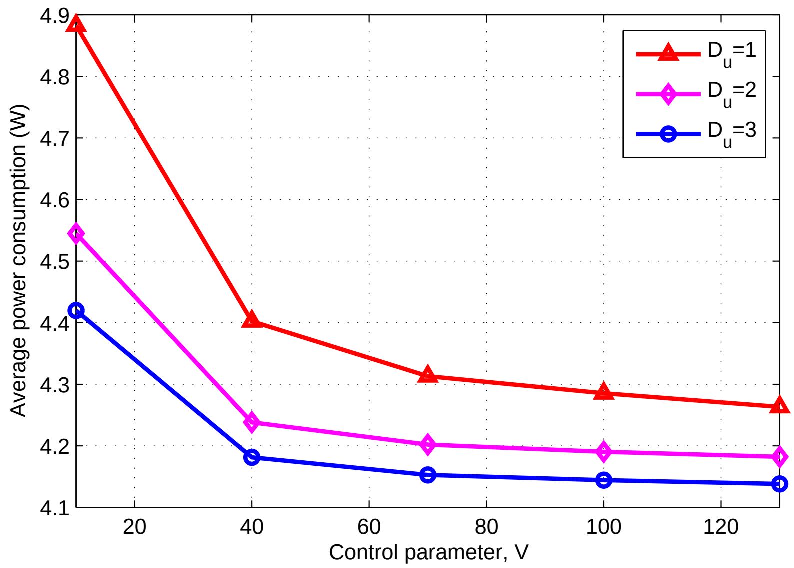

Fig. 8. The average power consumption versus the parameter V with different values of D_u.

Therefore, we can conclude that $\max_{p} G(\mathcal{P}) - \eta_{EE}^{opt} T(\mathcal{P}) = 0$ is achievable by power allocation policy $\{\mathcal{P}^*\}$ which completes the forward implication.

Then, the converse implication of Theorem 1 is proved as below. Suppose $\mathcal{P}_\circ^*$ is the optimal power allocation policy of the equivalent objective function such that

$$G(\mathcal{P}_\circ^*) - \eta_{EE}^{opt} T(\mathcal{P}_\circ^*) = 0 \tag{49}$$

Then, for any feasible power allocation policy $\{\mathcal{P}\} \in \Theta$, we can obtain the following inequality

$$G(\mathcal{P}) - \eta_{EE}^{opt} T(\mathcal{P}) \leq G(\mathcal{P}_\circ^*) - \eta_{EE}^{opt} T(\mathcal{P}_\circ^*) = 0. \tag{50}$$

The above inequality implies

$$\frac{G(\mathcal{P})}{T(\mathcal{P})} \leq \eta_{EE}^{opt}, \forall \{\mathcal{P}\} \in \Theta \tag{51}$$

and

$$\frac{G(\mathcal{P}_\circ^*)}{T(\mathcal{P}_\circ^*)} = \eta_{EE}^{opt} \tag{52}$$

It implies the optimal power allocation policy $\{\mathcal{P}_\circ^*\}$ for the equivalent objective function is also the optimal resource allocation policy for the original objective function.

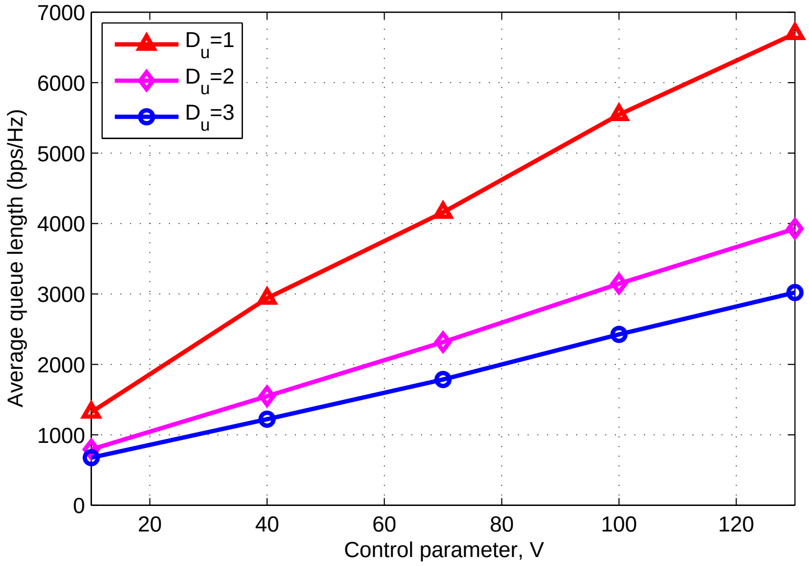

Fig. 9. The average queue length versus the parameter V with different values of D_u.

APPENDIX B

We denote Y_1 and Y_2 be the optimal utility of problems (15) and (17), respectively. The optimal solutions that achieve Y_1 and Y_2 are denoted by X_1 and X_2, respectively. Since the utility functions of (15) and (17) are non-decreasing concave function which can be expressed as $U(\cdot)$, by Jensen's inequality, we have

$$U\left(\overline{X_2}\right) \geq \overline{U}(X_2) = Y_2 \tag{53}$$

Due to the the solution X_2 satisfies the constraint $C5$, we can get

$$U\left(\overline{X_1}\right) \geq U\left(\overline{X_2}\right) \tag{54}$$

Moreover, since X_2 also satisfies the constraints of the problem (15), then we have

$$Y_1 \geq U\left(\overline{X_1}\right) \geq Y_2 \tag{55}$$

For X_1 is an optimal solution to the problem (15), it also satisfies the constraints $\mathrm{C1}-\mathrm{C4}$. By choosing $X_2 = \overline{X_1}$ at each slot, then we get

$$Y_2 \geq \overline{U}(X_2) = U\left(\overline{X_1}\right) = Y_1 \tag{56}$$

Therefore, $Y_1 = Y_2$ is proved and we can further conclude the equivalence of the problems (15) and (17).

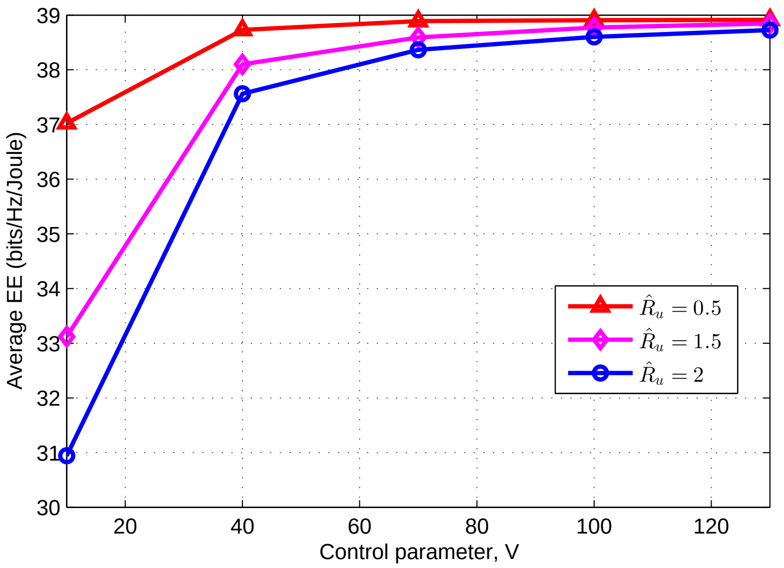

Fig. 10. Average EE performance versus the parameter V with different QoS requirements.

Appendix C
Proof of Theorem 2

To prove the bounds on the EE and the average queue length, Lemma 1 is introduced below.

Lemma 1: For arbitrary arrival rates, a randomized stationary control policy Π exists and it chooses feasible control decision independent of current traffic queues and virtual queues. We get the following steady state values:

$$E[r_u^{\Pi}(t)] = r_u^* \tag{57}$$

$$E[R_u^{\Pi}(t)] \geq E[r_u^{\Pi}(t)] + \varepsilon = r_u^* + \varepsilon \tag{58}$$

$$E[\hat{P}_u - p_u^{\Pi}(t)] \leq \delta \tag{59}$$

$$E[U_{tot}^{\Pi}(t)] \geq E[p_{tot}^{\Pi}(t)](\eta_{EE}^{opt} - \delta). \tag{60}$$

Proof: The proof of Lemma 1 is similar to one found in [28].

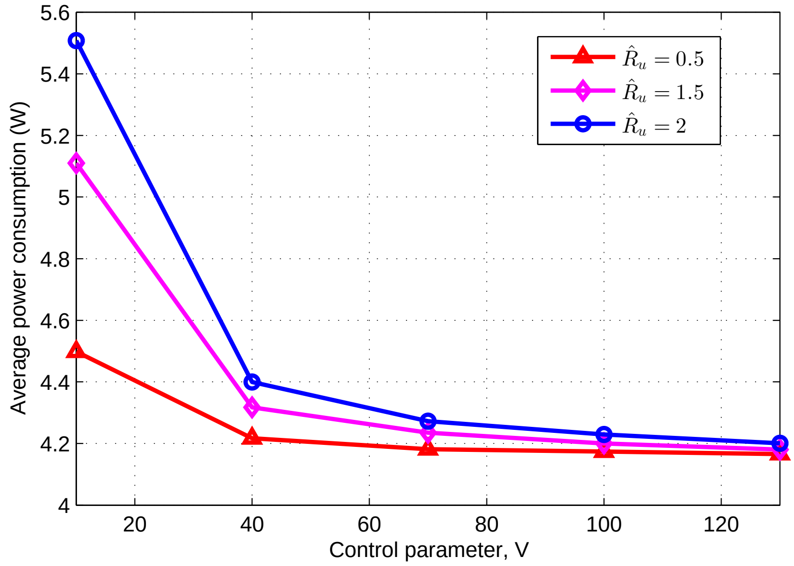

Fig. 11. The average power consumption versus the parameter V with different QoS requirements.

Substituting (57)-(60) into (22) and taking a limit $\delta \to 0$, we can get

$$\begin{aligned} &\Delta(\Phi(t)) - VE\left\{\sum_{u\in\mathcal{U}} g_R(\gamma_u) - \eta_{EE}p_{tot}\right\} \\ &\le B - V\eta_{EE}^{opt}E(p_{tot}^{\Pi}(t)) + V\eta_{EE}E(p_{tot}^{\Pi}(t)) \\ &-\varepsilon\sum_{u\in\mathcal{U}} Q_u(t). \end{aligned} \tag{61}$$

For $Q_u(t) \ge 0$, the inequality (61) can be further simplified to

$$\begin{aligned} &\Delta(\Phi(t)) - VE\left\{\sum_{u\in\mathcal{U}} g_R(\gamma_u) - \eta_{EE}p_{tot}\right\} \\ &\le B - V\eta_{EE}^{opt}E(p_{tot}^{\Pi}(t)) \\ &+V\eta_{EE}E(p_{tot}^{\Pi}(t)). \end{aligned} \tag{62}$$

Using telescoping sums over $t \in \{0, 1..., T-1\}$ and taking iterated expectation, we have

$$\begin{aligned} &\left\{ \begin{array}{l} E\left\{L(\Phi(T))\right\} - E\left\{L(\Phi(0))\right\} \\ -VE\left\{U_{tot} - \eta_{EE}(t)p_{tot}\right\} \end{array} \right\} \\ &\le T[B - V\eta_{EE}^{opt}E(p_{tot}^{\Pi}(t))\} \\ &+VE(p_{tot}^{\Pi}(t))\sum_{t=0}^{T-1} E\left\{\eta_{EE}\right\}. \end{aligned} \tag{63}$$

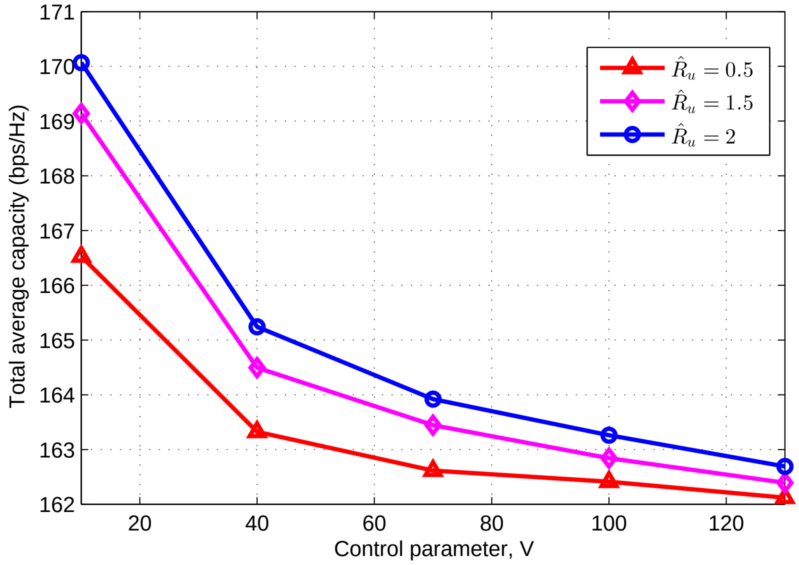

Fig. 12. The total average transmit capacity versus the parameter V with different QoS requirements.

Dividing (63) by VT, we get

$$\begin{aligned} & \tfrac{1}{T}\sum_{t=0}^{T-1} E\left\{\eta_{EE}(t)p_{tot}\right\} - \tfrac{1}{T}\sum_{t=0}^{T-1} E\left\{U_{tot}\right\} \\ & \leq \tfrac{B}{V} - \eta_{EE}^{opt}E(p_{tot}^{\Pi}(t)) \\ & + E(p_{tot}^{\Pi}(t))\tfrac{1}{T}\sum_{t=0}^{T-1} E\left\{\eta_{EE}\right\} + \tfrac{E\{L(\Phi(0))\}}{VT}. \end{aligned} \tag{64}$$

Taking the limit $T \to \infty$, one has

$$\begin{aligned} & \lim_{T\to\infty}[\tfrac{1}{T}\sum_{t=0}^{T-1} E\left\{\eta_{EE}(t)p_{tot}\right\} - \tfrac{1}{T}\sum_{t=0}^{T-1} E\left\{U_{tot}\right\}] \\ & = \bar{U}_{tot} - \bar{U}_{tot} = 0. \end{aligned} \tag{65}$$

We obtain

$$\frac{B}{V} - \eta_{EE}^{opt}E(p_{tot}^{\Pi}(t)) + \eta_{EE}E(p_{tot}^{\Pi}(t)) \geq 0. \tag{66}$$

Rearranging (66), we get

$$\eta_{EE} \geq \eta_{EE}^{opt} - \frac{B}{VE(p_{tot}^{\Pi}(t))} \geq \eta_{EE}^{opt} - \frac{B}{VP_{\min}}. \tag{67}$$

Similarly, taking iterated expectation and applying telescoping sums over $t \in \{0, 1, ..., T-1\}$

to (61), we get

$$\left\{ \begin{array}{l} E\left\{ L(\Phi(T)) \right\} - E\left\{ L(\Phi(0)) \right\} \\ -VE\left\{ U_{tot} - \eta_{EE}(t)p_{tot}(t) \right\} \end{array} \right\}$$
$$\begin{array}{l} \leq T[B - V\eta_{EE}^{opt}E(p_{tot}^{\Pi}(t))] \\ -\varepsilon \sum\limits_{t=0}^{T-1} [\sum\limits_{k\in K} \sum\limits_{u\in\mathcal{U}} E\left\{ Q_u(t) \right\}] \\ +VE(p_{tot}^{\Pi}(t)) \sum\limits_{t=0}^{T-1} E\left\{ \eta_{EE} \right\}. \end{array} \tag{68}$$

Dividing (68) by εT and taking a limit as $T \to \infty$, we obtain

$$\begin{array}{l} \bar{Q} = \lim\limits_{T\to\infty} \frac{1}{T} \sum\limits_{t=0}^{T-1} [\sum\limits_{u\in\mathcal{U}} E\left\{ Q_u(t) \right\}] \\ \leq \left\{ \begin{array}{l} \frac{B - V\eta_{EE}^{opt}E(p_{tot}^{\Pi}(t))}{\varepsilon} \\ +\frac{V}{\varepsilon} \lim\limits_{T\to\infty} \frac{1}{T} \sum\limits_{t=0}^{T-1} E\left\{ \eta_{EE}(t)p_{tot}(t) \right\} \end{array} \right\} \\ \leq \frac{B + V(R_{\max} - \eta_{EE}^{opt}P_{\min})}{\varepsilon}. \end{array} \tag{69}$$

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