# Sensing Time Optimization and Power Control for Energy Efficient Cognitive Small Cell With Imperfect Hybrid Spectrum Sensing

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#### I. INTRODUCTION

Abstract-Cognitive radio enabled small cell network is an emerging technology to address the exponential increase of mobile traffic demand in next generation mobile communications. Recently, many technological issues, such as resource allocation and interference mitigation pertaining to cognitive small cell network have been studied, but most studies focus on maximizing spectral efficiency. Different from the existing works, we investigate the power control and sensing time optimization problem in a cognitive small cell network, where the cross-tier interference mitigation, imperfect hybrid spectrum sensing, and energy efficiency are considered. The optimization of energy efficient sensing time and power allocation is formulated as a nonconvex optimization problem. We solve the proposed problem in an asymptotically optimal manner. An iterative power control algorithm and a near optimal sensing time scheme are developed by considering imperfect hybrid spectrum sensing, cross-tier interference mitigation, minimum data rate requirement, and energy efficiency. Simulation results are presented to verify the effectiveness of the proposed algorithms for energy efficient resource allocation in the cognitive small cell network.

Index Terms-Cognitive small cell, OFDMA, power control, resource allocation, sensing time optimization.

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EMAND for mobile data traffic is increasing exponentially due to the wide usage of smart mobile devices and data-centric applications in mobile Internet. As a promising technology in the fifth-generation (5G) mobile communications, small cell can offload heavy traffics from primary macrocells by shortening the distance between basestation and users. Since small cell can effectively improve the coverage and spatial reuse of spectrum by deploying low-power access points, it is not surprising that small cell has attracted much research interests in both industry and academia. However, the benefits of small cell deployments come with a number of fundamental challenges, which include spectrum access, resource allocation and interference mitigation [1]–[7].

Cognitive radio is also an emerging technology to improve the efficiency of spectrum access in the 5G networks [10]. The cognitive capabilities can improve the spectrum efficiency, radio resource utilization, and interference mitigation by efficient spectrum sensing, interference sensing, and adaptive transmission. Therefore, a cognitive radio enabled small cell network can further improve the system performance by co-existing with a macrocell network [9]. There are three ways for cognitive small cell to access the spectrum potentially used by primary macrocell: 1) spectrum sharing, where cognitive small cell can share the spectrum with primary macrocell; 2) opportunistic spectrum access, where cognitive small cell can opportunistically access the spectrum that is detected to be idle; 3) hybrid spectrum sensing, where cognitive small cell senses the channel status and optimizes the power allocation based on the spectrum sensing result. In this paper, cognitive radio enabled small cell architecture is designed to opportunistically access the spectrum via cognitive small basestation (CSBS).

Orthogonal frequency division multiple access (OFDMA) working jointly with cognitive small cell can improve spectrum efficiency and energy efficiency via resource allocation and interference mitigation [11]. Wang et al. [12] investigated the resource allocation problems based on multistage stochastic programming for stringent quality of service (QoS) requirements of real-time streaming scalable videos in cognitive small cell networks. The issues on spectrum sensing and interference mitigation were studied in [13], where an interference coordination approach was adopted. Opportunistic cooperation between cognitive small cell users and primary macrocell users was proposed for cognitive small cell networks based on a generalized Lyapunov optimization technique [14].

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In [15], a spectrum-sharing scheme between primary macrocell and secondary small cell was investigated, and bounds on maximum intensity of simultaneously transmitting cognitive small cell that satisfies a given per-tier outage constraint in these schemes were theoretically derived using a stochastic geometry model. In [16], interferences due to different interfering sources were analyzed within cognitive-empowered small cell networks, and a stochastic dual control approach was introduced for dynamic sensing coordination and interference mitigation without involving global and centralized control efforts. Moreover, energy efficient resource allocation has also been investigated for cognitive radio and small cell. In [17], the energy efficiency aspect of spectrum sharing and power allocation was studied using a Stackelberg game in heterogeneous cognitive radio networks with femtocells. While in [18], Nash equilibrium of a power adaptation game was derived to reduce energy consumption. Moreover, interference temperature limits, originated from the cognitive radio, were used in [19] to mitigate cross-tier interferences between macrocell and small cell.

However, among those existing works, the problem of both sensing time optimization and power control in cognitive small cell has not been well investigated. Although some works [20] have been performed to optimize of sensing time and power allocation in cognitive small cell networks, these work mainly focused on throughput maximization rather than energy efficiency. Moreover, most of the existing works do not consider the hybrid spectrum sensing based cognitive small cell. To the best of the authors' knowledge, the problem of sensing time and power control in cognitive small cell considering cross-tier interference mitigation, energy efficiency, and imperfect hybrid spectrum sensing has not been investigated. A preliminary investigation on this research problem was published in [21], and this work extends [21] in the following ways: (1) the maximum tolerable interference level for primary macrocell is now considered; (2) we take into account the minimum transmit data rate constraint in order to guarantee the QoS for small cell; (3) the detailed optimization algorithm of sensing time is presented now; (4) simulation results under multiple angles are provided to verify the proposed methods. In this paper, we study optimization of sensing time and power control in OFDMA based cognitive small cell by considering energy efficiency, QoS requirement, cross-tier interference limitation and imperfect hybrid spectrum sensing. The main contributions of this paper can be summarized as follows.

- Design a novel energy efficient OFDMA cognitive small cell optimization framework: This is a new approach by considering energy efficiency maximization, cross-tier interference mitigation, imperfect hybrid spectrum sensing, and user QoS requirements in the design of OFDMA cognitive small cell optimization framework. We formulate a sensing time and power control problem in cognitive small cell as a non-convex optimization problem.
- Make use of imperfect hybrid spectrum sensing and crosstier interference temperature limit: The hybrid spectrum sensing, which combines spectrum sharing access and opportunistic spectrum access, is considered in the opti-

mization problem. The power control policy is adaptive to the spectrum detection result of the subchannel state. Moreover, cross-tier interference temperature limit is also taken into consideration in the design of the resource allocation optimization in order to mitigate the interference from cognitive small cell to primary macrocell.

 Develop an energy efficient power control algorithm with multiple constraints: Given a sensing time, the power control optimization problem in fractional form is transformed into subtractive form. We propose an energy efficient power control algorithm to solve the transformed optimization problem. A minimum QoS requirement is employed to provide reliable transmission for cognitive small cell. Energy efficiency is taken into account in the design of power control and sensing time optimization problem. The non-convex optimization problem is then solved in an alternating optimal manner.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

# A. System Model

We consider an OFDMA cognitive small cell network where a co-channel cognitive small cell is overlaid on a primary macrocell. We focus on resource allocation in the downlink of the cognitive small cell. The OFDMA system has a bandwidth of B, which is divided into N subchannels. The channel fading of each subcarrier is assumed the same within a subchannel, but may vary across different subchannels. The channel model for each subchannel includes path loss and frequency-nonselective Rayleigh fading. Note that we focus on resource allocation in the downlink of cognitive small cell. Before small cell accesses the spectrum licensed to primary macrocell, CSBS performs spectrum sensing to determine the occupation status of the subchannels. In each time frame, the cognitive small cell can sense N subchannels by energy detection based spectrum sensing. The CSBS adapts the transmit power based on the spectrum sensing result. The  $\mathcal{H}_n^o$  is the hypothesis that the *n*th subchannel is occupied by the primary macrocell. The  $\tilde{\mathcal{H}}_n^o$  represents the spectrum sensing result that the *n*th subchannel is occupied by primary macrocell. The  $\mathcal{H}_n^{v}$  is the hypothesis that the *n*th subchannel is not occupied by primary macrocell. The  $\mathcal{H}_n^v$  represents the spectrum sensing result that the *n*th subchannel is not occupied by primary macrocell. The probabilities of the false alarm and mis-detection on subchannel *n* are  $q_n^f$  and  $q_n^m$ , respectively. We assume that the user signal of primary macrocell is a complex-valued phase shift keying (PSK) signal, and the noise at CSBS is circularly symmetric complex Gaussian (CSCG) with mean zero and variance  $\sigma^2$ . According to [22], the probability of mis-detection  $q_n^m$  can be approximated by

$$q_n^m(\varepsilon_n,\tau) = 1 - Q\left(\left(\frac{\varepsilon_n}{\sigma^2} - \gamma_n - 1\right)\sqrt{\frac{\tau f}{2\gamma_n + 1}}\right)$$
(1)

where  $\varepsilon_n$  is a chosen threshold of energy detector on subchannel n;  $\tau$  is the spectrum sensing time;  $\gamma_n$  is the received signal-to-noise ratio (SNR) of the primary macrocell user measured at the CSBS on subchannel n; f is the sampling frequency;



Fig. 1. Frame structure of cognitive small cell networks.

the standard Gaussian Q-function is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp(-t^2/2) dt.$$
 (2)

The probability of false alarm  $q_n^f$  can be approximated by [22]

$$q_n^f(\varepsilon_n, \tau) = Q((\frac{\varepsilon_n}{\sigma^2} - 1)\sqrt{\tau f})$$
$$= Q(\sqrt{2\gamma_n + 1} \left( Q^{-1}(\hat{q}_n^d) \right) + \sqrt{\tau f}\gamma_n) \quad (3)$$

where  $\widehat{q}_n^d$  is the target probability of detection.

The frame structure of cognitive small cell network is shown in Fig. 1 [22]. As can be seen from Fig. 1, a spectrum sensing duration/time  $\tau$  is inserted in the beginning of each frame. The CSBS adapts its transmit power based on the spectrum sensing decision made in the beginning of each frame. If the subchannel *n* detected to be idle  $(\tilde{\mathcal{H}}_n^o)$ , cognitive small cell can transmit high power  $P_{s,n}^o$ ; if the subchannel *n* detected to be occupied  $(\tilde{\mathcal{H}}_n^o)$ , cognitive small cell can transmit low power  $P_{s,n}^o$  in order to mitigate the interference caused to primary macrocell. This approach is called hybrid spectrum sensing, and it is different from the opportunistic spectrum access and the spectrum sharing approach. Based on Shannon's capacity formula, when the spectrum sensing result is idle, the achievable capacity on subchannel *n* in small cell is given by

$$R_{v,n} = \log_2\left(1 + \frac{g_{ss,n} \cdot P_{s,n}^v}{\sigma^2}\right) \tag{4}$$

where  $g_{ss,n}$  is the channel gain of subchannel *n* between small cell user and CSBS. If the spectrum sensing result is active/occupied, the achievable capacity on subchannel *n* in small cell is given by

$$R_{o,n} = \log_2 \left( 1 + \frac{g_{ss,n} \cdot P_{s,n}^o}{g_{ms,n} \cdot P_{m,n}^o + \sigma^2} \right)$$
(5)

where  $g_{ms,n}$  is the channel gain of subchannel *n* between macrocell basestation (MBS) and CSBS;  $P_{m,n}^{o}$  is the transmit power of MBS on subchannel *n*.

In a cognitive heterogeneous network, which typically consists of a cognitive small cell and primary macrocell, imperfect spectrum sensing of CSBS can cause severe co-channel interference to the primary macrocell, and thus degrade the performance of the heterogeneous cognitive small cell networks. Since it is the CSBS that determines whether a subchannel is occupied by primary macrocell or not, four different cases are to be considered as follows.

• Case 1: subchannel *n* is vacant in primary macrocell, and the spectrum sensing decision made by CSBS is vacant;

- Case 2: subchannel *n* is vacant in primary macrocell, but the spectrum sensing decision made by CSBS is occupied;
- Case 3: subchannel *n* is occupied in primary macrocell, but the spectrum sensing decision made by CSBS is vacant;
- Case 4: subchannel *n* is occupied in primary macrocell, and the spectrum sensing decision made by CSBS is occupied.

For the first and fourth cases, the CSBS makes the correct decisions. On the other hand, the second case is false alarm, and the third case is mis-detection. Therefore, we can calculate the achievable capacities on subchannel n in small cell for the four different cases as

$$R_{1,n} = \log_2\left(1 + \frac{g_{ss,n} \cdot P_{s,n}^o}{\sigma^2}\right),\tag{6}$$

$$R_{2,n} = \log_2\left(1 + \frac{g_{ss,n} \cdot P_{s,n}^o}{\sigma^2}\right),\tag{7}$$

$$R_{3,n} = \log_2 \left( 1 + \frac{g_{ss,n} \cdot P_{s,n}}{g_{ms,n} \cdot P_{m,n} + \sigma^2} \right), \tag{8}$$

$$R_{4,n} = \log_2 \left( 1 + \frac{g_{ss,n} \cdot P_{s,n}}{g_{ms,n} \cdot P_{m,n} + \sigma^2} \right).$$
(9)

Our objective is to maximize energy efficiency of cognitive small cell networks by optimizing sensing time and power allocation. The energy efficiencies of those four cases are defined as follows

$$\eta_{1,n} = \frac{R_{1,n}}{P_{s,n}^v + P_c},\tag{10}$$

$$\eta_{2,n} = \frac{R_{2,n}}{P_{s,n}^o + P_c},\tag{11}$$

$$\eta_{3,n} = \frac{R_{3,n}}{P_{s,n}^v + P_c},\tag{12}$$

$$\eta_{4,n} = \frac{R_{4,n}}{P_{s,n}^o + P_c}$$
(13)

where  $P_c$  is the constant circuit power consumption which includes lowpass filter, mixer for modulation, frequency synthesizer, and digital-to-analog converter [24], and  $P_c$  is assumed to be independent of the transmitted power [25].

The average energy efficiency of subchannel n in our hybrid spectrum sensing scheme is

$$\eta_n = P(\mathcal{H}_n^v)(1 - q_n^f(\varepsilon_n, \tau))\eta_{1,n} + P(\mathcal{H}_n^v)q_n^f(\varepsilon_n, \tau)\eta_{2,n} + P(\mathcal{H}_n^o)q_n^m(\varepsilon_n, \tau)\eta_{3,n} + P(\mathcal{H}_n^o)(1 - q_n^m(\varepsilon_n, \tau))\eta_{4,n}$$
(14)

where  $P(\mathcal{H}_n^v)$  and  $P(\mathcal{H}_n^o)$  are the probabilities of vacant status and occupied status of subchannel *n*, respectively.

Let us first investigate the constraints in the proposed optimization framework. Since the resource allocation is performed in CSBS, the transmit power of CSBS on subchannel n is constrained by

$$\sum_{n=1}^{N} \begin{bmatrix} P(\mathcal{H}_{n}^{v})(1-q_{n}^{f}(\varepsilon_{n},\tau))P_{s,n}^{o} \\ + P(\mathcal{H}_{n}^{v})q_{n}^{f}(\varepsilon_{n},\tau)P_{s,n}^{o} \\ + P(\mathcal{H}_{n}^{o})q_{n}^{m}(\varepsilon_{n},\tau)P_{s,n}^{o} \\ + P(\mathcal{H}_{n}^{o})(1-q_{n}^{m}(\varepsilon_{n},\tau))P_{s,n}^{o} \end{bmatrix} \frac{T-\tau}{T} \leq P_{max} \quad (15)$$

where  $P_{max}$  is the maximum average transmit power of CSBS.

Since primary macrocells play a fundamental role in providing cellular coverage, macrocell users' QoS should not be affected by cognitive small cell's deployment. Therefore, to implement cross-tier interference protection, we impose an average interference power limit to constrain the crosstier interference suffered by macrocell. Let  $I_n^{th}$  denote the maximum tolerable interference level on subchannel *n* for the macrocell user. We have

$$P(\mathcal{H}_{n}^{o}) \cdot g_{sm,n} \left[ \begin{array}{c} q_{n}^{m}(\varepsilon_{n},\tau) P_{s,n}^{o} \\ +(1-q_{n}^{m}(\varepsilon_{n},\tau)) P_{s,n}^{o} \end{array} \right] \frac{T-\tau}{T} \leq I_{n}^{th}, \quad \forall n$$

$$\tag{16}$$

where  $g_{sm,n}$  is the channel power gain from small cell to macrocell user on subchannel *n*.

In order to guarantee the QoS for small cell, we introduce a minimum transmit data rate constraint

$$\left\{ \begin{array}{l} P(\mathcal{H}_{n}^{v})(1-q_{n}^{J}(\varepsilon_{n},\tau))R_{1,n}+P(\mathcal{H}_{n}^{v})q_{n}^{J}(\varepsilon_{n},\tau)R_{2,n} \\ +P(\mathcal{H}_{n}^{o})q_{n}^{m}(\varepsilon_{n},\tau)R_{3,n}+P(\mathcal{H}_{n}^{o})(1-q_{n}^{m}(\varepsilon_{n},\tau))R_{4,n} \end{array} \right\}$$
  
$$\geq R_{\min}$$
(17)

where  $R_{min}$  is the minimum transmit data rate requirement of each subchannel.

For a target detection probability of  $\hat{q}_n^d$  on subchannel *n*, substituting  $\hat{q}_n^d$  into (1), we get

$$\left(\frac{\varepsilon_n}{\sigma^2} - \gamma_n - 1\right) \sqrt{\frac{\tau f}{2\gamma_n + 1}} = Q^{-1}\left(\widehat{q}_n^d\right). \tag{18}$$

Therefore, for a given sensing time  $\hat{\tau}$ , the detection threshold  $\varepsilon_n$  can be determined as

$$\varepsilon_n = \left(\sqrt{\frac{2\gamma_n + 1}{\hat{\tau}f}}Q^{-1}\left(\widehat{q}_n^d\right) + \gamma_n + 1\right)\sigma^2, \quad \forall n.$$
(19)

#### **B.** Problem Formulation

In this paper, our aim is to maximize the cognitive small cell's energy efficiency while protecting QoS of the primary macrocell users. We assume that the cross-tier interference power limit is sent by a primary MBS periodically. This process requires little overhead in the primary macrocell. In this case, the sensing time optimization and power control in primary macrocell are not part of our optimization. Thus, the corresponding sensing time optimization and power allocation problem for downlink CSBS can be mathematically formulated as

$$\max_{\{\tau, \mathbf{P}_{s}^{\upsilon}, \mathbf{P}_{s}^{o}\}} \sum_{n=1}^{N} \frac{T-\tau}{T} \eta_{n} \left(\tau, P_{s,n}^{\upsilon}, P_{s,n}^{o}\right)$$
(20)

where  $\mathbf{P}_{s}^{o} = [p_{s,n}^{o}]_{1 \times N}$  and  $\mathbf{P}_{s}^{o} = [p_{s,n}^{o}]_{1 \times N}$  are the power allocation vectors of the *N* subchannels in cognitive small cell. Constraint *C*1 limits the maximum transmit power of each CSBS to  $P_{max}$ ; *C*2 sets the tolerable interference power level on each subchannel of the macrocell user on subchannel *n*; *C*3 represents minimum QoS requirement of each subchannel;

C4 represents the non-negative power constraint of the transmit power on each subchannel; C5 expresses the constraint of sensing time in each frame.

Note that the optimization problem in (20) under the constraints of (21), as shown at the top of the next page, is non-convex with respect to  $\{\tau, \mathbf{P}_s^v, \mathbf{P}_s^o\}$ . Therefore, we first investigate the problem of energy efficient power control given the sensing time  $\hat{\tau}$ .

## III. ENERGY EFFICIENT RESOURCE OPTIMIZATION IN ONE COGNITIVE SMALL CELL

#### A. Transformation of the Optimization Problem

Given the sensing time  $\hat{\tau}$ , the power control problem in (20) under the constraints of (21) can be classified as a non-linear fractional programming problem. Since the joint optimization problem of  $\mathbf{P}_s^v$  and  $\mathbf{P}_s^o$  in (20) can be decoupled into two separate subproblems, namely one for  $\mathbf{P}_s^v$  and the other for  $\mathbf{P}_s^o$ . We first deal with the subproblem related to  $\mathbf{P}_s^v$ . Since subchannels in (20) are independent, we define a nonnegative variable  $\eta_{13,n}^*$  for the sum of average energy efficiencies on subchannel *n* in Case 1 and Case 3 as

$$\eta_{13,n}^{*} = \frac{\left\{ \begin{array}{l} P(\mathcal{H}_{n}^{v})(1 - q_{n}^{f}(\varepsilon_{n}, \hat{\tau}))R_{1,n}(\hat{\tau}, \tilde{P}_{s,n}^{v}) \\ + P(\mathcal{H}_{n}^{o})q_{n}^{m}(\varepsilon_{n}, \hat{\tau})R_{3,n}(\hat{\tau}, \tilde{P}_{s,n}^{v}) \end{array} \right\}}{\tilde{P}_{s,n}^{v} + P_{c}}$$
(22)

where  $P_{s,n}^{v}$  is the optimal solution to the problem of (20) under the constraints of (21). We introduce Theorem 1 as follows:

*Theorem 1:*  $\eta_{13,n}^*$  is achieved if and only if (23), as shown at the top of the next page.

where the  $P_{s,n}^{v}$  in (23) is one of the feasible solutions to (20) under the constraints of (21).

*proof:* 1) Suppose that  $\eta^*_{13,n}$  is the optimal solution of (22), as shown at the top of the next page, the inequality can be obtained as (24), shown at the top of the next page.

Hence, we have (25), as shown at the top of the next page. Therefore, we can conclude that

$$\max_{\substack{P_{s,n}^{v} \\ P_{s,n}^{v}}} \left\{ \begin{array}{l} P(\mathcal{H}_{n}^{v})(1-q_{n}^{f}(\varepsilon_{n},\hat{\tau}))R_{1,n}(\hat{\tau},P_{s,n}^{v}) \\ +P(\mathcal{H}_{n}^{o})q_{n}^{m}(\varepsilon_{n},\hat{\tau})R_{3,n}(\hat{\tau},P_{s,n}^{v}) \\ -\eta_{13,n}^{*}(P_{s,n}^{v}+P_{c}) \end{array} \right\} = 0$$

That is, eq. (23) is achieved.

2) Suppose that  $P_{s,n}^{v}$  is a solution to the problem of (23). The definition of (23) implies that (26), as shown at the top of the next page.

Therefore, we can obtain

$$\frac{\left\{\begin{array}{l}P(\mathcal{H}_{n}^{v})(1-q_{n}^{f}(\varepsilon_{n},\hat{\tau}))R_{1,n}(\hat{\tau},\tilde{P}_{s,n}^{v})\\+P(\mathcal{H}_{n}^{o})q_{n}^{m}(\varepsilon_{n},\hat{\tau})R_{3,n}(\hat{\tau},\tilde{P}_{s,n}^{v})\end{array}\right\}}{\tilde{P}_{s,n}^{v}+P_{c}}=\eta_{13,n}^{*}$$
(27)

and

$$\frac{\left\{\begin{array}{l}P(\mathcal{H}_{n}^{v})(1-q_{n}^{f}(\varepsilon_{n},\hat{\tau}))R_{1,n}(\hat{\tau},P_{s,n}^{v})\\+P(\mathcal{H}_{n}^{o})q_{n}^{m}(\varepsilon_{n},\hat{\tau})R_{3,n}(\hat{\tau},P_{s,n}^{v})\end{array}\right\}}{P_{s,n}^{v}+P_{c}}\leq\eta_{13,n}^{*}.$$
 (28)

s.t. 
$$C1: \sum_{n=1}^{N} \left[ \begin{array}{c} P(\mathcal{H}_{n}^{v})(1-q_{n}^{f}(\varepsilon_{n},\tau))P_{s,n}^{v} + P(\mathcal{H}_{n}^{v})q_{n}^{f}(\varepsilon_{n},\tau)P_{s,n}^{o} \\ + P(\mathcal{H}_{n}^{o})q_{n}^{m}(\varepsilon_{n},\tau)P_{s,n}^{v} + P(\mathcal{H}_{n}^{o})(1-q_{n}^{m}(\varepsilon_{n},\tau))P_{s,n}^{o} \end{array} \right] \frac{T-\tau}{T} \leq P_{max}$$

$$C2: P(\mathcal{H}_{n}^{o}) \cdot g_{sm,n} \left[ q_{n}^{m}(\varepsilon_{n},\tau)P_{s,n}^{v} + (1-q_{n}^{m}(\varepsilon_{n},\tau))P_{s,n}^{o} \right] \frac{T-\tau}{T} \leq I_{n}^{th}, \quad \forall n$$

$$C3: P(\mathcal{H}_{n}^{v})(1-q_{n}^{f}(\varepsilon_{n},\tau))R_{1,n} + P(\mathcal{H}_{n}^{v})q_{n}^{f}(\varepsilon_{n},\tau)R_{2,n} \\ + P(\mathcal{H}_{n}^{o})q_{n}^{m}(\varepsilon_{n},\tau)R_{3,n} + P(\mathcal{H}_{n}^{o})(1-q_{n}^{m}(\varepsilon_{n},\tau))R_{4,n} \geq R_{\min}, \quad \forall n$$

$$C4: P_{s,n}^{v} \geq 0, P_{s,n}^{o} \geq 0, \quad \forall n$$

$$C5: 0 \leq \tau \leq T$$

$$(21)$$

$$\max_{P_{s,n}^{\upsilon}} \left\{ P(\mathcal{H}_{n}^{\upsilon})(1-q_{n}^{f}(\varepsilon_{n},\hat{\tau}))R_{1,n}(\hat{\tau},P_{s,n}^{\upsilon}) + P(\mathcal{H}_{n}^{o})q_{n}^{m}(\varepsilon_{n},\hat{\tau})R_{3,n}(\hat{\tau},P_{s,n}^{\upsilon}) - \eta_{13,n}^{*}(P_{s,n}^{\upsilon}+P_{c}) \right\} \\ = P(\mathcal{H}_{n}^{\upsilon})(1-q_{n}^{f}(\varepsilon_{n},\hat{\tau}))R_{1,n}(\hat{\tau},P_{s,n}^{\upsilon}) + P(\mathcal{H}_{n}^{o})q_{n}^{m}(\varepsilon_{n},\hat{\tau})R_{3,n}(\hat{\tau},P_{s,n}^{\upsilon}) - \eta_{13,n}^{*}(P_{s,n}^{\upsilon}+P_{c}) = 0$$
(23)

$$\eta_{13,n}^{*} = \frac{P(\mathcal{H}_{n}^{v})(1-q_{n}^{f}(\varepsilon_{n},\hat{\tau}))R_{1,n}(\hat{\tau},\tilde{P}_{s,n}^{v}) + P(\mathcal{H}_{n}^{o})q_{n}^{m}(\varepsilon_{n},\hat{\tau})R_{3,n}(\hat{\tau},\tilde{P}_{s,n}^{v})}{\tilde{P}_{s,n}^{v} + P_{c}} \\ \geq \frac{P(\mathcal{H}_{n}^{v})(1-q_{n}^{f}(\varepsilon_{n},\hat{\tau}))R_{1,n}(\hat{\tau},P_{s,n}^{v}) + P(\mathcal{H}_{n}^{o})q_{n}^{m}(\varepsilon_{n},\hat{\tau})R_{3,n}(\hat{\tau},P_{s,n}^{v})}{P_{s,n}^{v} + P_{c}}$$
(24)

$$\begin{cases} P(\mathcal{H}_{n}^{v})(1-q_{n}^{f}(\varepsilon_{n},\hat{\tau}))R_{1,n}(\hat{\tau},\tilde{P}_{s,n}^{v}) + P(\mathcal{H}_{n}^{o})q_{n}^{m}(\varepsilon_{n},\hat{\tau})R_{3,n}(\hat{\tau},\tilde{P}_{s,n}^{v}) - \eta_{13,n}^{*}\left(\tilde{P}_{s,n}^{v} + P_{c}\right) = 0\\ P(\mathcal{H}_{n}^{v})(1-q_{n}^{f}(\varepsilon_{n},\hat{\tau}))R_{1,n}(\hat{\tau},P_{s,n}^{v}) + P(\mathcal{H}_{n}^{o})q_{n}^{m}(\varepsilon_{n},\hat{\tau})R_{3,n}(\hat{\tau},P_{s,n}^{v}) - \eta_{13,n}^{*}\left(P_{s,n}^{v} + P_{c}\right) \leq 0. \end{cases}$$
(25)

$$P(\mathcal{H}_{n}^{\upsilon})(1-q_{n}^{f}(\varepsilon_{n},\hat{\tau}))R_{1,n}(\hat{\tau},P_{s,n}^{\upsilon})+P(\mathcal{H}_{n}^{o})q_{n}^{m}(\varepsilon_{n},\hat{\tau})R_{3,n}(\hat{\tau},P_{s,n}^{\upsilon})-\eta_{13,n}^{*}\left(P_{s,n}^{\upsilon}+P_{c}\right)$$

$$\leq P(\mathcal{H}_{n}^{\upsilon})(1-q_{n}^{f}(\varepsilon_{n},\hat{\tau}))R_{1,n}(\hat{\tau},\tilde{P}_{s,n}^{\upsilon})+P(\mathcal{H}_{n}^{o})q_{n}^{m}(\varepsilon_{n},\hat{\tau})R_{3,n}(\hat{\tau},\tilde{P}_{s,n}^{\upsilon})-\eta_{13,n}^{*}\left(\tilde{P}_{s,n}^{\upsilon}+P_{c}\right)=0$$
or
$$\begin{cases} P(\mathcal{H}_{n}^{\upsilon})(1-q_{n}^{f}(\varepsilon_{n},\hat{\tau}))R_{1,n}(\hat{\tau},P_{s,n}^{\upsilon})+P(\mathcal{H}_{n}^{o})q_{n}^{m}(\varepsilon_{n},\hat{\tau})R_{3,n}(\hat{\tau},P_{s,n}^{\upsilon})-\eta_{13,n}^{*}\left(P_{s,n}^{\upsilon}+P_{c}\right)\leq0\\ P(\mathcal{H}_{n}^{\upsilon})(1-q_{n}^{f}(\varepsilon_{n},\hat{\tau}))R_{1,n}(\hat{\tau},\tilde{P}_{s,n}^{\upsilon})+P(\mathcal{H}_{n}^{o})q_{n}^{m}(\varepsilon_{n},\hat{\tau})R_{3,n}(\hat{\tau},\tilde{P}_{s,n}^{\upsilon})-\eta_{13,n}^{*}\left(\tilde{P}_{s,n}^{\upsilon}+P_{c}\right)\leq0 \end{cases}$$
(26)

According to *Theorem 1*, the optimization problem of (23) under the constraints of (21) has the same solution of the optimization problem of (22) under the constraints of (21). Similarly, the objective function with respect to  $\mathbf{P}_s^o$  in fractional form can also be transformed to a subtractive form by introducing a non-negative variable  $\eta_{24,n}^*$ .

#### B. Iterative Energy Efficiency Maximization Algorithm

To solve the transformed optimization problem in the subtractive form under the constraints of (20), we propose Algorithm 1.

As shown in Algorithm 1, in each iteration of the outer loop, the lth inner loop power control problem is given as (29)

$$\max_{\{\mathbf{P}_{s}^{o},\mathbf{P}_{s}^{o}\}} \sum_{n=1}^{N} \frac{T-\hat{\tau}}{T} \begin{cases} P(\mathcal{H}_{n}^{o})(1-q_{n}^{J}(\varepsilon_{n},\hat{\tau}))R_{1,n}(\hat{\tau},P_{s,n}^{o}) \\ +P(\mathcal{H}_{n}^{o})q_{n}^{m}(\varepsilon_{n},\hat{\tau})R_{3,n}(\hat{\tau},P_{s,n}^{o}) \\ +P(\mathcal{H}_{n}^{o})q_{n}^{m}(\varepsilon_{n},\hat{\tau})R_{2,n}(\hat{\tau},P_{s,n}^{o}) \\ +P(\mathcal{H}_{n}^{o})(1-q_{n}^{m}(\varepsilon_{n},\hat{\tau}))R_{4,n}(\hat{\tau},P_{s,n}^{o}) \\ -\eta_{13,n}(l)(P_{s,n}^{o}+P_{c}) \\ -\eta_{24,n}(l)(P_{s,n}^{o}+P_{c}) \end{cases} \end{cases}$$
(29)

s.t. C1 - C4. (30)

Since the optimization problem of (29) under the constraints

of (30) is convex with respect to  $\mathbf{P}_s^{o}$  and  $\mathbf{P}_s^{o}$ . The Lagrangian function is given by (31), as shown at the top of the next page.

where  $\lambda$ ,  $\mu_n$  and  $\nu_n$  are the Lagrangian multipliers (also called dual variables) vectors for the constraints C1, C2 and C3 in (21), respectively. Thus, the Lagrangian dual function is defined as

$$g(\lambda, \boldsymbol{\mu}, \boldsymbol{\nu}) = \max_{\mathbf{P}_{s}^{\nu}, \mathbf{P}_{s}^{\rho}} L(\mathbf{P}_{s}^{\nu}, \mathbf{P}_{s}^{\rho}, \lambda, \boldsymbol{\mu}, \boldsymbol{\nu}).$$
(32)

The dual problem can be expressed as

$$\min_{\lambda,\boldsymbol{\mu},\boldsymbol{\nu}\geq 0} g(\lambda,\boldsymbol{\mu},\boldsymbol{\nu}).$$
(33)

Using Lagrangian function and the Karush-Kuhn-Tucker (KKT) conditions, we can obtain the near optimal solution of  $\tilde{P}_{s,n}^{v}$  on subchannel *n* as

$$\tilde{P}_{s,n}^{\nu} = \left[\frac{A_{\nu,n} + \sqrt{B_{\nu,n}}}{2}\right]^+ \tag{34}$$

where  $[x]^+ = \max\{x, 0\}$ , and (35), (36), as shown at the top of the next page. Similar to  $\widetilde{P}_{s,n}^{v}$ , we can obtain the near optimal

$$L(\mathbf{P}_{s}^{v}, \mathbf{P}_{s}^{o}, \lambda, \boldsymbol{\mu}, \boldsymbol{\nu}) = \left\{ \sum_{n=1}^{N} \frac{T - \hat{\tau}}{T} \left[ (P(\mathcal{H}_{n}^{v})(1 - q_{n}^{f}(\varepsilon_{n}, \hat{\tau}))R_{1,n}(\hat{\tau}, P_{s,n}^{v}) + P(\mathcal{H}_{n}^{o})q_{n}^{m}(\varepsilon_{n}, \hat{\tau})R_{3,n}(\hat{\tau}, P_{s,n}^{v}) - \eta_{13,n}(l)P_{s,n}^{v}}{+ P(\mathcal{H}_{n}^{v})q_{n}^{f}(\varepsilon_{n}, \hat{\tau})R_{2,n}(\hat{\tau}, P_{s,n}^{o}) + P(\mathcal{H}_{n}^{o})(1 - q_{n}^{m}(\varepsilon_{n}, \hat{\tau}))R_{4,n}(\hat{\tau}, P_{s,n}^{o}) - \eta_{24,n}(l)P_{s,n}^{o}} \right] \right\} \\ - \lambda \left\{ \sum_{n=1}^{N} \left[ P(\mathcal{H}_{n}^{v})(1 - q_{n}^{f}(\varepsilon_{n}, \hat{\tau}))P_{s,n}^{v} + P(\mathcal{H}_{n}^{v})q_{n}^{f}(\varepsilon_{n}, \hat{\tau})P_{s,n}^{o} + \right] \frac{T - \hat{\tau}}{T} - P_{\max} \right\} \\ - \lambda \left\{ \sum_{n=1}^{N} \left[ P(\mathcal{H}_{n}^{v})q_{n}^{m}(\varepsilon_{n}, \hat{\tau})P_{s,n}^{v} + P(\mathcal{H}_{n}^{v})q_{n}^{f}(\varepsilon_{n}, \hat{\tau}))P_{s,n}^{o} \right] \frac{T - \hat{\tau}}{T} - P_{\max} \right\} \\ - \sum_{n=1}^{N} \mu_{n} \left\{ \left[ \sum_{n=1}^{N} P(\mathcal{H}_{n}^{o})g_{sm,n}(q_{n}^{m}(\varepsilon_{n}, \hat{\tau})P_{s,n}^{v} + (1 - q_{n}^{m}(\varepsilon_{n}, \hat{\tau}))P_{s,n}^{o}) \right] \frac{T - \hat{\tau}}{T} - I_{n}^{th} \right\} \\ + \sum_{n=1}^{N} \nu_{n} \left\{ \left[ P(\mathcal{H}_{n}^{v})(1 - q_{n}^{f}(\varepsilon_{n}, \tau))R_{1,n} + P(\mathcal{H}_{n}^{v})q_{n}^{f}(\varepsilon_{n}, \tau)R_{2,n} \\ + P(\mathcal{H}_{n}^{o})q_{n}^{m}(\varepsilon_{n}, \tau)R_{3,n} + P(\mathcal{H}_{n}^{o})(1 - q_{n}^{m}(\varepsilon_{n}, \tau))R_{4,n} \right] - R_{\min} \right\}$$
(31)

$$A_{v,n} = \frac{(1+v_n)(P(\mathcal{H}_n^v)(1-q_n^{f}(\varepsilon_n,\hat{\tau})) + P(\mathcal{H}_n^o)q_n^m(\varepsilon_n,\hat{\tau}))}{\ln 2(\eta_{13,n}(l) + \lambda(P(\mathcal{H}_n^v)(1-q_n^{f}(\varepsilon_n,\hat{\tau})) + P(\mathcal{H}_n^o)q_n^m(\varepsilon_n,\hat{\tau})) + \mu_n g_{sm,n} P(\mathcal{H}_n^o)q_n^m(\varepsilon_n,\hat{\tau}))} - \frac{2\sigma^2 + g_{ms,n} P_{m,n}^o}{g_{ss,n}},$$

$$B_{v,n} = A_{v,n}^2 - \frac{4}{g_{ss,n}} \cdot \left\{ \frac{\sigma^4 + \sigma^2 g_{ms,n} P_{m,n}^o}{g_{ss,n}} - \frac{4}{g_{ss,n}} \cdot \left\{ \frac{\sigma^4 + \sigma^2 g_{ms,n} P_{m,n}^o}{g_{ss,n}} - \frac{(1+v_n)[P(\mathcal{H}_n^v)(1-q_n^f(\varepsilon_n,\hat{\tau})(\sigma^2 + g_{ms,n} P_{m,n}^o) + P(\mathcal{H}_n^o)q_n^m(\varepsilon_n,\hat{\tau})\sigma^2]}{\ln 2(\eta_{13,n}(l) + \lambda(P(\mathcal{H}_n^v)(1-q_n^f(\varepsilon_n,\hat{\tau}) + P(\mathcal{H}_n^o)q_n^m(\varepsilon_n,\hat{\tau})) + \mu_n g_{sm,n} P(\mathcal{H}_n^o)q_n^m(\varepsilon_n,\hat{\tau}))} \right\}$$

$$(36)$$

solution of  $\widetilde{P}^{o}_{s,n}$  on subchannel *n* as

$$\tilde{P}^{o}_{s,n} = \left[\frac{A_{o,n} + \sqrt{B_{o,n}}}{2}\right]^+ \tag{37}$$

where (38) and (39), as shown at the top of the next page.

Either the ellipsoid or the subgradient method can be adopted in updating the dual variables [27]. Here, we choose the subgradient method to update the dual variables, and the update formulas are (40), (41), (42), as shown at the top of the next page, where  $\vartheta_1^l$ ,  $\vartheta_2^l$  and  $\vartheta_3^l$  denote the step size of iteration l ( $l \in \{1, 2, ..., L_{max}\}$ ) for  $\lambda$ ,  $\mu$ ,  $\nu$  respectively, and  $L_{max}$  is the maximum number of iterations. Meanwhile, the step size must meet the following conditions

$$\sum_{l=1}^{\infty} \vartheta_i^l = \infty, \lim_{l \to \infty} \vartheta_i^l = 0, \ \forall i \in \{1, 2, 3\}.$$
(43)

Algorithm 1 is proposed to optimize the power  $P_{s,n}^{o}$  and  $P_{s,n}^{o}$  of (20) given the sensing time  $\hat{\tau}$ . In Algorithm 1, the process of power control is decomposed to inner loop problem and outer loop problem. In each iteration, the  $\eta_{13,n}^{*}(l)$  and  $\eta_{24,n}^{*}(l)$  can be found through outer loop, the inner loop control problem is solved by outer loop results  $\eta_{13,n}^{*}(l)$  and  $\eta_{24,n}^{*}(l)$ , the Lagrangian method and eqs. (34), (37).

The near optimal sensing time scheme can be found in Algorithm 2 based on a one-dimensional exhaustive search. Algorithm 2 is proposed to optimize the sensing time in (20) when we have obtained the optimal power through Algorithm 1. Therefore, running Algorithm 1 with  $\hat{\tau}(l)$  to obtain the optimal power  $\tilde{P}_{s,n}^{v}$  and  $\tilde{P}_{s,n}^{o}$  have to be firstly done in Algorithm 2. Then the optimal sensing time is found based on a one-dimensional exhaustive search.

# C. Complexity Analysis

The computational complexity of the proposed algorithms is analyzed in this subsection. Suppose the subgradient method used in Algorithm 1 needs  $\Delta_1$  iterations to converge, the updates of  $\lambda$  need O(1) operations,  $\mu$  and  $\nu$  need O(N)operations each. The method used in Algorithm 1 to calculate  $\eta_{13,n}^*$  and  $\eta_{24,n}^*$  on each subchannel in a small cell need  $\Delta_2$ iterations to converge. The total complexity of Algorithm 1 is thus  $O(N^2 \Delta_1 \Delta_2)$ .  $\Delta_1$  and  $\Delta_2$  can be small enough if the initial values of  $\lambda$ ,  $\mu$  and  $\nu$  are well chosen, together with suitable values of iteration step sizes. In Algorithm 2, finding the optimal sensing time for each subchannel requires O(L)operations. Therefore, the total computational complexity of Algorithm 2 is O(NL) for the network with N subchannels.

## IV. ENERGY EFFICIENT RESOURCE OPTIMIZATION IN MULTIPLE COGNITIVE SMALL CELLS

# A. Proposed Algorithms' Application in Multiple Cognitive Small Cells

In this subsection, we investigate the energy efficient resource optimization in multiple cognitive small cells. The aforementioned method is applied to optimize the energy efficiency in multiple cognitive small cells network, where the

$$A_{o,n} = \frac{(1 + \nu_n)(P(\mathcal{H}_n^{o})q_n^{f}(\varepsilon_n, \hat{\tau}) + P(\mathcal{H}_n^{o})(1 - q_n^{m}(\varepsilon_n, \hat{\tau})))}{\ln 2(\eta_{24,n}(l) + \lambda(P(\mathcal{H}_n^{o})q_n^{f}(\varepsilon_n, \hat{\tau}) + P(\mathcal{H}_n^{o})(1 - q_n^{m}(\varepsilon_n, \hat{\tau}))) + \mu_n g_{sm,n} P(\mathcal{H}_n^{o})(1 - q_n^{m}(\varepsilon_n, \hat{\tau})))} - \frac{2\sigma^2 + g_{ms,n} P_{m,n}^{o}}{g_{ss,n}},$$

$$B_{o,n} = A_{o,n}^2 - \frac{4}{g_{ss,n}} \cdot \left\{ \frac{\sigma^4 + \sigma^2 g_{ms,n} P_{m,n}^{o}}{g_{ss,n}} - \frac{4}{g_{ss,n}} \cdot \left\{ \frac{\sigma^4 + \sigma^2 g_{ms,n} P_{m,n}^{o}}{g_{ss,n}} - \frac{(1 + \nu_n)[P(q(\ell), \sigma_n^{f}(\varepsilon_n, \hat{\tau})(\sigma^2 + \sigma_n - P_{m,n}^{0})] + P(q(\ell), (1 - \sigma_n^{m}(\varepsilon_n, \hat{\tau}))(\sigma^2 + \sigma_n - P_{m,n}^{0})] + P(q(\ell), (1 - \sigma_n^{m}(\varepsilon_n, \hat{\tau}))(\sigma^2 + \sigma_n - P_{m,n}^{0})] + P(q(\ell), (1 - \sigma_n^{m}(\varepsilon_n, \hat{\tau}))(\sigma^2 + \sigma_n - P_{m,n}^{0}) + P(q(\ell), (1 - \sigma_n^{m}(\varepsilon_n, \hat{\tau}))(\sigma^2 + \sigma_n - P_{m,n}^{0})] + P(q(\ell), (1 - \sigma_n^{m}(\varepsilon_n, \hat{\tau}))(\sigma^2 + \sigma_n - P_{m,n}^{0}) + P(q(\ell), (1 - \sigma_n^{m}(\varepsilon_n, \hat{\tau}))(\sigma^2 + \sigma_n - P_{m,n}^{0}) + P(\sigma_n^{0})(\sigma^2 + \sigma_n - P_{m,n}$$

$$\frac{(1+\eta_n)(1+\eta_$$

$$\lambda^{l+1} = \lambda^l - \vartheta_1^l \left( P_{max} - \sum_{n=1}^N \left[ \begin{array}{c} P(\mathcal{H}_n^v)(1 - q_n^f(\varepsilon_n, \tau)) P_{s,n}^v + P(\mathcal{H}_n^v) q_n^f(\varepsilon_n, \tau) P_{s,n}^o + \\ P(\mathcal{H}_n^o) q_n^m(\varepsilon_n, \tau) P_{s,n}^v + P(\mathcal{H}_n^o)(1 - q_n^m(\varepsilon_n, \tau)) P_{s,n}^o + \\ \end{array} \right] \frac{T - \tau}{T} \right)$$
(40)

$$\mu_n^{l+1} = \mu_n^l - \vartheta_2^l \left( I_n^{th} - P(\mathcal{H}_n^o) \cdot g_{sm,n} \left[ q_n^m(\varepsilon_n, \tau) P_{s,n}^v + (1 - q_n^m(\varepsilon_n, \tau)) P_{s,n}^o \right] \frac{T - \tau}{T} \right), \quad \forall n$$

$$\tag{41}$$

$$\nu_n^{l+1} = \nu_n^l - \vartheta_3^l \left( \left\{ \begin{array}{l} P(\mathcal{H}_n^v)(1 - q_n^f(\varepsilon_n, \tau))R_{1,n} + P(\mathcal{H}_n^o)q_n^f(\varepsilon_n, \tau)R_{2,n} + \\ P(\mathcal{H}_n^o)q_n^m(\varepsilon_n, \tau)R_{3,n} + P(\mathcal{H}_n^o)(1 - q_n^m(\varepsilon_n, \tau)R_{4,n} \end{array} \right\} \frac{T - \tau}{T} - R_{\min} \right), \quad \forall n$$

$$\tag{42}$$

$$\max_{\{\tau, \mathbf{P}_{ms}^{v}, \mathbf{P}_{ms}^{0}\}} \sum_{k=1}^{K} \sum_{n=1}^{N} \frac{T - \tau_{k}}{T} \eta_{k,n} \left(\tau_{k}, P_{s,k,n}^{0}, P_{s,k,n}^{0}\right) \tag{44}$$
s.t.  $C1: \sum_{n=1}^{N} \left[ P(\mathcal{H}_{k,n}^{v})(1 - q_{k,n}^{f}(\varepsilon_{k,n}, \tau_{k}))P_{s,k,n}^{v} + P(\mathcal{H}_{k,n}^{o})q_{k,n}^{f}(\varepsilon_{k,n}, \tau_{k})P_{s,k,n}^{o}} \right] \frac{T - \tau_{k}}{T} \leq P_{max}$ 

$$C2: P(H_{k,n}^{o}) \cdot g_{k,n}^{sm} \left[ q_{k,n}^{m}(\varepsilon_{k,n}, \tau_{k}) P_{s,k,n}^{v} + (1 - q_{k,n}^{m}(\varepsilon_{k,n}, \tau_{k}))P_{s,k,n}^{o}} \right] \frac{T - \tau_{k}}{T} \leq I_{k,n}^{th}, \forall k, n$$

$$C3: P(\mathcal{H}_{k,n}^{v})(1 - q_{k,n}^{f}(\varepsilon_{k,n}, \tau_{k}))R_{k,n}^{1} + P(\mathcal{H}_{k,n}^{o})q_{k,n}^{f}(\varepsilon_{k,n}, \tau_{k}))R_{k,n}^{2} = R_{min}$$

$$C4: \sum_{j=1, j \neq k}^{K} \frac{T - \tau_{k}}{T} g_{k,j,n} \left\{ P(\mathcal{H}_{j,n}^{o}) \left[ q_{j,n}^{m}(\varepsilon_{j,n}, \tau_{j}) P_{s,j,n}^{v} + (1 - q_{n,j}^{m}(\varepsilon_{j,n}, \tau_{j}))P_{s,j,n}^{o} \right] + \right\} \leq \gamma_{k,n}^{th}, \forall n, k$$

$$C5: P_{s,k,n}^{o} \geq 0, P_{s,k,n}^{o} \geq 0, \forall n, k$$

$$C6: 0 \leq \tau_{k} \leq T, \forall k \tag{45}$$

$$\eta_{k,n}^{13^*} = \frac{P(\mathcal{H}_{k,n}^{\upsilon})(1 - q_{k,n}^{J}(\varepsilon_{k,n}, \hat{\tau}_{k}))R_{k,n}^{1}(\hat{\tau}_{k}, P_{s,k,n}^{\upsilon}) + P(\mathcal{H}_{k,n}^{o})q_{k,n}^{m}(\varepsilon_{k,n}, \hat{\tau}_{k})R_{k,n}^{3}(\hat{\tau}_{k}, P_{s,k,n}^{\upsilon})}{\tilde{P}_{s,k,n}^{\upsilon} + P_{c}}$$
(46)

interference between small cells will be considered. In multiple cognitive small cells, to maximize the total energy efficiency with the consideration of co-tier interference mitigation, the problem in (20) under the constraints of (21) can be formulated as (44), as shown at the top of this page, where  $\tau = [\tau]_{1 \times K}$  is the sensing time vector of *K* cognitive small cells; *N* is the number of subchannels in each small cell;  $\mathbf{P}_{\mathbf{ms}}^{\mathbf{v}} = \begin{bmatrix} P_{s,k,n}^{v} \end{bmatrix}_{K \times N}$  and  $\mathbf{P}_{\mathbf{ms}}^{\mathbf{o}} = \begin{bmatrix} P_{s,k,n}^{o} \end{bmatrix}_{K \times N}$ are the power allocation vectors of the *N* subchannels in *K* cognitive small cells. Constraint *C*1 limits the maximum transmit power of each CSBS to  $P_{max}$ ; C2 sets the tolerable interference power level for each small cell on each subchannel of the macrocell user on subchannel *n*; C3 represents minimum QoS requirement of each subchannel; C4 represents the tolerable interference power level from other small cells, where  $\gamma_{k,n}^{th}$  denotes the co-tier interference limits on *n*th subchannel in *k*th small cell; C5 represents the non-negative power constraint of the transmit power on each subchannel; C6 expresses the constraint of sensing time in each frame. Algorithm 1 Proposed Energy-Efficient Power Control Algorithm

- 1: Initialize the maximum number of iterations  $L_{\text{max}}$  and convergence tolerance  $\varepsilon_n$ ;
- 2: Set  $\eta_{13,n}(1) = 0$ ,  $\eta_{24,n}(1) = 0$ , l = 0;
- 3: Initialize power allocation with an equal power distribution and begin the outer loop;
- 4: for n = 1 to N do
- 5: repeat
- 6: a) The inner loop power control problem is solved with outer loop results  $\eta_{13,n}^*(l)$ ,  $\eta_{24,n}^*(l)$ , the Lagrangian method and eqs. (34), (37);
- 7: b) Then, we can obtain the power control solution  $\underline{P}_{s,n}^{v}(l)$  and  $\underline{P}_{s,n}^{o}(l)$ ;
- 8: **if**  $(P(\mathcal{H}_n^v)(1 q_n^f(\varepsilon_n, \hat{\tau}))R_{1,n}(\hat{\tau}, \underline{P}_{s,n}^v(l)) + P(\mathcal{H}_n^o)q_n^m(\varepsilon_n, \hat{\tau})R_{3,n}(\hat{\tau}, \underline{P}_{s,n}^v(l)) \eta_{13,n}(l)(\underline{P}_{s,n}^v(l) + P_c)) < \varepsilon_\eta$  then
- 9: Convergence= true;  $\widetilde{P}_{s,n}^{v} = \underline{P}_{s,n}^{v}(l)$  $\eta_{13,n}^{*} = f$

10:  

$$\frac{P(H_n^v)(1-q_n^J(\varepsilon_n,\hat{\tau}))R_{1,n}(\hat{\tau},P_{s,n}^v)}{+P(H_n^o)q_n^m(\varepsilon_n,\hat{\tau})R_{3,n}(\hat{\tau},\tilde{P}_{s,n}^v)} \frac{\tilde{P}_{s,n}^v+P_c}{\tilde{P}_{s,n}^v+P_c}$$

12: 
$$\begin{array}{l} \eta_{13,n}(l+1) = \\ P(H_n^v)(1 - q_n^f(\varepsilon_n, \hat{\tau}))R_{1,n}(\hat{\tau}, \underline{P}_{s,n}^v(l)) \\ + P(H_n^o)q_n^m(\varepsilon_n, \hat{\tau})R_{3,n}(\hat{\tau}, \underline{P}_{s,n}^o(l)) \\ \end{array}$$

- 13: Convergence= **false**, l = l + 1;
- 14: **end if**
- 15: **if**  $(P(\mathcal{H}_n^o)q_n^f(\varepsilon_n, \hat{\tau})R_{2,n}(\hat{\tau}, \underline{P}_{s,n}^o(l)) + P(\mathcal{H}_n^o)(1 q_n^m(\varepsilon_n, \hat{\tau}))R_{4,n}(\hat{\tau}, \underline{P}_{s,n}^o(l)) \eta_{24,n}(l)(\underline{P}_{s,n}^o(l) + P_c)) < \varepsilon_\eta$  **then**

16: Convergence= **true**; 
$$P_{s,n}^{o} = \underline{P}_{s,n}^{o}(l)$$
  
 $P(H_{n}^{v})q_{n}^{f}(\varepsilon_{n}, \hat{\tau})R_{2,n}(\hat{\tau}, \tilde{P}_{s,n}^{o})$   
 $+ \underline{P}(H_{n}^{v})(1 - \underline{\sigma}_{n}^{m}(\varepsilon_{n}, \hat{\tau}))R_{n}(\hat{\tau}, \tilde{P}_{s,n}^{o})$ 

17: 
$$\eta_{24,n}^* = \frac{+P(H_n^o)(1-q_n^m(\varepsilon_n,\tau))R_{4,n}(\tau,P_{s,n}^o)}{\tilde{P}_{s_n}^o + P_c}$$

18: else  $n_{24}$  (l+1) -

$$\begin{array}{rcl}
& & & & & & \\
19: & & & & P(H_n^o)q_n^{f}(\varepsilon_n, \hat{\tau})R_{2,n}(\hat{\tau}, \underline{P}_{s,n}^o(l)) \\
& & & & & \\
& & & & & \\
20: & & & & \\
20: & & & & \\
\end{array} \xrightarrow{P_{s,n}^o(l) + P_c} P_{s,n}^o(l) + P_c \\
\end{array}$$

21: end if 22: until Convergence= true or  $l = L_{max}$ 23: end for

The problem in (44) under the constraints of (45) can be solved using the method proposed in Section III. First of all, similar to the problem in (20), the problem of power control in (44) under the constraints of (45), as shown at the top of the previous page, is decoupled into two separate subproblems respect to  $\mathbf{P}_{\mathbf{ms}}^{\mathbf{v}}$  and  $\mathbf{P}_{\mathbf{ms}}^{\mathbf{o}}$  respectively when the sensing time  $\hat{\tau}_k$ is given. We first deal with the subproblem respect to  $\mathbf{P}_{\mathbf{ms}}^{\mathbf{v}}$ . The variable  $\eta_{k,n}^{13^*}$  is defined as where  $\eta_{k,n}^{13^*}$  represents the sum of average energy efficiencies on the *n*th subchannel of the *k*th small cell in Case 1 and Case 3.  $\tilde{P}_{s,k,n}^{v}$  is the optimal solution to the problem of (44) under the constraints of (45).

# Algorithm 2 Near Optimal Energy-Efficient Sensing Time Scheme

- 1: Initialize the maximum number of iterations  $L_{\text{max}}$  and convergence tolerance  $\varepsilon_{\tau}$
- 2: Set l = 0; Initialize  $\hat{\tau}(l)$ ;

# 3: repeat

4: Run Algorithm 1 with  $\hat{\tau}(l)$  to obtain the optimal power  $\tilde{P}_{s,n}^{v}$  and  $\tilde{P}_{s,n}^{o}$ ;

5: 
$$P_{s,n}^{o}(l) = P_{s,n}^{o}, P_{s,n}^{o}(l) = P_{s,n}^{o};$$
  
6:  $\widehat{\tau}(l) = \max_{\tau} \sum_{n=1}^{N} \frac{T-\tau}{T} \eta_n \left(\tau, P_{s,n}^{o}(l), P_{s,n}^{o}(l)\right);$   
7: **if**  $|\widehat{\tau}(l) - \widehat{\tau}(l-1)| \le \varepsilon_{\tau}$  **then**  
8: Convergence= **true**,  $\widetilde{\tau} = \widehat{\tau}(l);$ 

9: **else** 

Convergence= false, l = l + 1;

11: end if

10:

12: **until** Convergence= **true** or  $l = L_{max}$ 

Therefore, the optimization problem of (44) is transformed as optimization problem of (46), as shown at the top of the previous page, under the constraints of (45). Subsequently, Algorithm 1 is used to solve the transformed problem, and we can obtain the near optimal solution

$$\tilde{P}_{s,k,n}^{v} = \left[\frac{A_{v,k,n} + \sqrt{B_{v,k,n}}}{2}\right]^{+} \tag{47}$$

where (48) and (49), as shown at the top of the next page.

Similar to  $P_{s,k,n}^{v}$ , we can obtain the near optimal solution

$$\tilde{P}^{o}_{s,k,n} = \left[\frac{A_{o,k,n} + \sqrt{B_{o,k,n}}}{2}\right]^+ \tag{50}$$

where (51) and (52), as shown at the top of the next page. Finally, the near optimal sensing time for each small cell can be found in Algorithm 2 based on a one-dimensional exhaustive search.

#### B. Complexity Analysis

In this subsection, the computational complexity of the proposed algorithms in multiple small cells network is analyzed. Similar to the single small cell case, suppose the subgradient method used in Algorithm 1 needs  $\Delta_1$  iterations to converge, the updates of  $\lambda$  need O(N) operations,  $\mu$  and  $\nu$  need O(KN) operations each. The method used in Algorithm 1 to calculate  $\eta_{13,n}^*$  and  $\eta_{24,n}^*$  on each subchannel in a small cell need  $\Delta_2$  iterations to converge. The total complexity of Algorithm 1 is thus  $O(N^2 K^2 \Delta_1 \Delta_2)$ .  $\Delta_1$  and  $\Delta_2$  can be small enough if the values of iteration step sizes and initial values of  $\lambda$ ,  $\mu$  and  $\nu$  are well chosen. In Algorithm 2, finding the optimal sensing time for each subchannel requires O(L) operations. Therefore, the total computational complexity of Algorithm 2 is O(KNL).

#### V. SIMULATION RESULTS AND DISCUSSION

In this section, simulation results are presented to evaluate the performance of the proposed algorithms. The sampling frequency f is 6 MHz, T = 0.1 sec, N = 50, and  $\sigma^2 = 1 \times 10^{-4}$ . The channel gains are modeled as block

$$A_{v,k,n} = \frac{(1+v_{k,n})(P(\mathcal{H}_{k,n}^{v})(1-q_{k,n}^{f}(\varepsilon_{k,n},\hat{\tau}_{k})) + P(\mathcal{H}_{k,n}^{o})q_{k,n}^{m}(\varepsilon_{k,n},\hat{\tau}_{k}))}{\left\{ \frac{\ln 2(\eta_{k,n}^{13}(l) + \lambda_{k,n}(P(\mathcal{H}_{k,n}^{v})(1-q_{k,n}^{f}(\varepsilon_{k,n},\hat{\tau}_{k})) + P(\mathcal{H}_{k,n}^{o})q_{k,n}^{m}(\varepsilon_{k,n},\hat{\tau}_{k}))}{+\mu_{k,n}g_{k,n}^{sm}P(\mathcal{H}_{k,n}^{o})q_{k,n}^{m}(\varepsilon_{k,n},\hat{\tau}_{k})} - \frac{2\sigma^{2} + g_{k,n}^{ms}P_{m,n}^{o}}{g_{k,n}^{ss}} \right\} - \frac{2\sigma^{2} + g_{k,n}^{ms}P_{m,n}^{o}}{g_{k,n}^{ss}}$$
(48)

$$B_{v,k,n} = A_{v,k,n}^{2} - \frac{4}{g_{k,n}^{ss}} - \frac{4}{g_{k,n}^{ss}} - \frac{(1 + v_{k,n})[P(\mathcal{H}_{k,n}^{o})(1 - q_{k,n}^{f}(\varepsilon_{k,n}, \hat{\tau}_{k}))(\sigma^{2} + g_{k,n}^{ms}P_{m,k,n}^{o}) + P(\mathcal{H}_{k,n}^{o})q_{k,n}^{m}(\varepsilon_{k,n}, \hat{\tau}_{k})\sigma^{2}]}{g_{k,n}^{ss}} - \frac{(1 + v_{k,n})[P(\mathcal{H}_{k,n}^{o})(1 - q_{k,n}^{f}(\varepsilon_{k,n}, \hat{\tau}_{k}))(\sigma^{2} + g_{k,n}^{ms}P_{m,k,n}^{o}) + P(\mathcal{H}_{k,n}^{o})q_{k,n}^{m}(\varepsilon_{k,n}, \hat{\tau}_{k})\sigma^{2}]}{\left\{ \begin{array}{l} \ln 2(\eta_{k,n}^{13}(l) + \lambda_{k}(P(\mathcal{H}_{k,n}^{o})(1 - q_{k,n}^{f}(\varepsilon_{k,n}, \hat{\tau}_{k})) + P(\mathcal{H}_{k,n}^{o})q_{k,n}^{m}(\varepsilon_{k,n}, \hat{\tau}_{k}) \\ + \mu_{k,n}g_{k,n}^{ms}P(\mathcal{H}_{k,n}^{o})q_{k,n}^{m}(\varepsilon_{k,n}, \hat{\tau}_{k}) \\ - \zeta_{k,n}\sum_{j=1, j \neq k}^{K} g_{j,k,n} \Big[ P(\mathcal{H}_{j,n}^{o})(1 - q_{j,n}^{f}(\varepsilon_{j,n}, \hat{\tau}_{j})) + P(\mathcal{H}_{j,n}^{o})q_{j,n}^{m}(\varepsilon_{j,n}, \hat{\tau}_{j}) \Big] \right\}$$

$$(49)$$

$$A_{o,k,n} = \frac{(1+\nu_{k,n})P(\mathcal{H}_{k,n}^{o})q_{k,n}^{f}(\varepsilon_{k,n},\hat{\tau}_{k}) + P(\mathcal{H}_{k,n}^{o})(1-q_{k,n}^{m}(\varepsilon_{k,n},\hat{\tau}_{k}))}{\left[\frac{\ln 2(l_{k,n}^{24}(1)+\lambda_{k,n}P(\mathcal{H}_{k,n}^{o})q_{k,n}^{f}(\varepsilon_{k,n},\hat{\tau}_{k}) + P(\mathcal{H}_{k,n}^{o})(1-q_{k,n}^{m}(\varepsilon_{k,n},\hat{\tau}_{k}))}{-\xi_{k,n}\xi_{k,n}^{k}P(\mathcal{H}_{k,n}^{o})(1-q_{k,n}^{m}(\varepsilon_{k,n},\hat{\tau}_{k}))} - \frac{\xi_{k,n}^{k}g_{k,n}^{s}P(\mathcal{H}_{k,n}^{o})q_{j,n}^{f}(\varepsilon_{j,n},\hat{\tau}_{j}) + P(\mathcal{H}_{j,n}^{o})(1-q_{j,n}^{m}(\varepsilon_{j,n},\hat{\tau}_{j}))}{-\xi_{k,n}g_{k,n}^{s}}\right] - \frac{2\sigma^{2} + g_{k,n}^{s}P_{m,n}^{o}}{g_{k,n}^{ss}}$$

$$\left\{ \frac{\sigma^{4} + \sigma^{2}g_{k,n}^{ss}P_{m,k,n}^{o}}{-\frac{4}{g_{k,n}^{ss}}} - \frac{(1+\nu_{k,n})[P(\mathcal{H}_{k,n}^{o})q_{k,n}^{f}(\varepsilon_{k,n},\hat{\tau}_{k})(\sigma^{2} + g_{k,n}^{ms}P_{m,k,n}^{o}) + P(\mathcal{H}_{k,n}^{o})(1-q_{k,n}^{m}(\varepsilon_{k,n},\hat{\tau}_{k}))\sigma^{2}]}{\left[\frac{\ln 2(\eta_{k,n}^{13}(1)+\lambda_{k}(P(\mathcal{H}_{k,n}^{o})q_{k,n}^{f}(\varepsilon_{k,n},\hat{\tau}_{k}) + P(\mathcal{H}_{k,n}^{o})(1-q_{k,n}^{m}(\varepsilon_{k,n},\hat{\tau}_{k}))\sigma^{2}]}{\left[\frac{\ln 2(\eta_{k,n}^{13}(1)+\lambda_{k}(P(\mathcal{H}_{k,n}^{o})q_{k,n}^{f}(\varepsilon_{k,n},\hat{\tau}_{k}) + P(\mathcal{H}_{k,n}^{o})(1-q_{k,n}^{m}(\varepsilon_{k,n},\hat{\tau}_{k}))\sigma^{2}]}{\left[\frac{\ln 2(\eta_{k,n}^{13}(1)+\lambda_{k}(P(\mathcal{H}_{k,n}^{o})q_{k,n}^{f}(\varepsilon_{k,n},\hat{\tau}_{k})) + P(\mathcal{H}_{k,n}^{o})(1-q_{k,n}^{m}(\varepsilon_{k,n},\hat{\tau}_{k}))\sigma^{2}]}{\left[\frac{\ln 2(\eta_{k,n}^{13}(1)+\lambda_{k}(P(\mathcal{H}_{k,n}^{o})q_{k,n}^{f}(\varepsilon_{k,n},\hat{\tau}_{k})) + P(\mathcal{H}_{k,n}^{o})(1-q_{k,n}^{m}(\varepsilon_{k,n},\hat{\tau}_{k}))\sigma^{2}]}\right]}\right\}$$

faded and exponentially distributed with mean of 0.1. The transmit power on each subchannel of primary macrocell is set at 25 mW. We assume that the QoS requirement of minimum data rate requirement is set as 0.3 bps/Hz. The target detection probability  $\hat{q}_n^d$  is set as 90% if not specified. In Figure 2, the convergence of Algorithm 1 is

evaluated with the  $P_{max} = 15$  dBm, the cross-tier interference limit  $I_n^{th} = -10$  dBm. As can be seen from Fig. 2, the average energy efficiency of small cell on each subchannel converges after 9 iterations. This result, together with the previous analysis, indicates that the proposed Algorithm 1 is practical in cognitive small cell.



Fig. 2. Convergence in terms of average energy efficiency of small cell on each subchannel versus the number of iterations.



Fig. 3. Average energy efficiency versus sensing time with different  $P_{max}$  values.

Figure 3 displays the average energy efficiency of each subchannel in cognitive small cell network when the sensing time increases from 0.0005 sec to 0.015 sec with  $P_{max} = 5, 10, 13, 15$  dBm, the cross-tier interference limit  $I_n^{th} = -10$  dBm. The relation between sensing time and the average energy efficiency of each subchannel is exhibited. As shown in Fig. 3, the average energy efficiency of each subchannel in cognitive small cell first increases and then drops when the sensing time is increased from 0.0005 sec to 0.015 sec. It is estimated that the near optimal sensing time is between 0.002 sec and 0.004 sec. Larger value of  $P_{max}$  results in higher average energy efficiency because a larger value of  $P_{max}$  enlarges the feasible region of the variables in the original optimization problem in (20)-(21).

Figure 4 shows the trend of average energy efficiency of each subchannel in cognitive small cell when  $P_{max}$  increases from 5 dBm to 25 dBm. The target detection probabilities  $\hat{q}_n^d = 0.8, 0.9$  and cross-tier interference limit  $I_n^{th} = -10$  dBm in Fig. 4. As shown in Fig. 4, the average energy efficiency of each subchannel of cognitive small cell increases when  $P_{max}$  is increased from 5 dBm to 25 dBm. Because a larger value of  $P_{max}$  results in a larger optimal power in (20)-(21). We can see that a larger target detection probability results in better performance of the optimal average energy efficiency



Fig. 4. Average energy efficiency versus  $P_{max}$  with different target detection probabilities.



Fig. 5. Average energy efficiency versus the cross-tier interference limit with different  $P_{max}$  values.

from Fig. 4. The reason is that a larger target detection probability makes it more accurate in detection of spectrum sensing.

Figure 5 shows the relation between cross-tier interference limit and the average energy efficiency of each subchannel with different target detection probability. As shown in Fig. 5, the average energy efficiency of each subchannel in cognitive small cell increases when  $I_n^{th}$  is changed from -15 dBm to 5 dBm. Similar to Fig. 3, this is because that a larger value of  $I_n^{th}$  can enlarges the feasible region of optimizing variable of power in (20)-(21).

Figure 6 shows the performance comparison of average spectral efficiency with different scheme. The proposed scheme is the combination of the proposed Algorithm 1 and the proposed near optimal sensing time scheme. Fixed sensing time scheme is the combination of the proposed Algorithm 1 and a random selected sensing time scheme. Fixed power scheme is the combination of equal power allocation and the proposed optimal sensing time scheme. As shown in Fig. 6, the average spectral efficiency of each subchannel in the cognitive small cell with  $P_{max}$  increases from 5 mW to 25 mW. However, the proposed scheme outperforms the fixed sensing time scheme and the fixed power scheme obviously.



Fig. 6. Performance comparison of different schemes in terms of average spectral efficiency of small cell on each subchannel.



Fig. 7. Performance comparison of different schemes in terms of average energy efficiency of small cell on each subchannel.

Figure 7 provides the energy efficiency performance comparison between proposed scheme and the other methods. In Fig. 7, the average energy efficiency of each subchannel in the cognitive small cell is shown when  $P_{max}$  increases from 8 mW to 18 mW, where the target detection probability  $\widehat{q}_n^d = 0.9$  and cross-tier interference limit  $I_n^{th} = -10$  dBm. The proposed scheme is the combination of the proposed Algorithm 1 and the proposed near optimal sensing time scheme. Fixed sensing time scheme is the combination of the proposed Algorithm 1 and a random selected sensing time scheme. Fixed power scheme is the combination of equal power allocation and the proposed optimal sensing time scheme. As shown in Fig. 7, the proposed scheme can achieve 15% higher energy efficiency than the fixed sensing time scheme. Fixed power scheme has the lowest curve, because of equal power allocation.

Figure 8 shows the relation between cross-tier interference limit and the optimal sensing time. As shown in Fig. 8, the near optimal sensing time decreases with an increase of  $I_n^{th}$ . Because when using KKT conditions related to C2, larger  $I_n^{th}$ 



Fig. 8. Near optimal sensing time versus the cross-tier interference limit with different  $P_{max}$  values.



Fig. 9. Average energy efficiency versus sensing time with different cross-tier interference limits.

results in larger optimized sensing time. Moreover, a larger value of  $P_{max}$  results in smaller optimized sensing time.

Figure 9 shows the relation between the sensing time and average energy efficiency of each subchannel in cognitive small cell network with different cross-tier interference limit. As shown in Fig. 9, similar to Fig.3, the average energy efficiency of each subchannel in cognitive small cell first increases and then drops as the sensing time is increased from 0.0005 sec to 0.015 sec. It is because that the near optimal sensing time is between 0.002 sec and 0.004 sec. Larger  $I_n^{th}$  value results in higher average energy efficiency since a larger of value of  $I_n^{th}$  leads to a larger optimization variable region in (20)-(21).

Figure 10 displays the trend of average energy efficiency of each subchannel in cognitive small cell when  $P_{max}$  increases from 5 dBm to 25 dBm with cross-tier interference limit  $I_n^{th} = -20, -5$  dBm and target detection probability  $\hat{q}_n^d = 0.9$ . Similar to Fig. 4, Fig. 10 shows that the average energy efficiency of each subchannel in cognitive small cell increases when  $P_{max}$  is increased from 5 dBm to 25 dBm. Besides, we conclude that larger cross-tier interference limit can result in improved performance in average energy efficiency.

Figure 11 shows the convergence performance of Algorithm 1 in the network consists of multiple cognitive small cells under the different circuit power  $P_c$ . As shown in Fig. 11, the total average energy efficiency on each



Fig. 10. Average energy efficiency versus  $P_{max}$  with different cross-tier interference limits.



Fig. 11. Convergence in terms of sum of the average energy efficiency of small cells versus the number of iterations.



Fig. 12. Sum of the average energy efficiency versus the number of small cells with different co-tier interference limits.

subchannel of all small cells converges after 12 iterations. The practical applicability of Algorithm 1 in the multiple cognitive small cells is demonstrated through this figure.

Figure 12 shows that the total average energy efficiency on each subchannel of all small cells versus the number of small cells in network with the co-tier interference limits



Fig. 13. Sum of the average energy efficiency versus the number of small cells with different cross-tier interference limits.

 $\gamma^{th} = -10, -20$  dBm, and  $P_{max} = 15$  dBm. As shown in Fig. 12, the total average energy efficiency on each subchannel of all small cells increase gradually with the increase of the number of small cells. However, the ratio of increase is diminishing, and it is caused by the co-tier interference among small cells. We can also see that a larger co-tier interference limits results in better performance of the optimal total average energy efficiency. It implies that our proposed method not only can optimize the energy efficiency but also can mitigate the co-tier interference in multiple cognitive small cells.

Figure 13 shows the total average energy efficiency on each subchannel of all small cells versus the number of small cells in network with the cross-tier interference limits  $I_n^{th} = -10, -13$  dBm, and  $P_{max} = 15$  dBm. We observe that the total average energy efficiency on each subchannel increases and then drops when the number of small cells is increased from 5 to 30. The slope of lines is diminishing. As shown in Fig. 13, the performance of the larger crosstier interference limits outperforms that of the smaller crosstier interference limit in terms of the total average energy efficiency on each subchannel. Therefore, we can say that our proposed scheme can mitigate the cross-tier interference when optimizing the energy efficiency.

#### VI. CONCLUSION

We investigated the power allocation and sensing time optimization problem in cognitive small cell where the cross-tier interference mitigation, imperfect spectrum sensing, and energy efficiency were considered. The energy efficient sensing time optimization and power allocation were modeled as a non-convex optimization problem. We transformed the fractional form non-convex optimization problem into an equivalent optimization problem in subtractive form. An iterative resource allocation algorithm was developed. Simulation results showed that the proposed algorithms not only converge within limited number of iterations, but also achieve improved performance than the existing schemes. In the future, we will extend our work into the multiple macrocells scenario [28].

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