Performance of Beamforming in Correlated MISO Systems with Estimation Error and Feedback Delay

Abdulla Firag, *Member, IEEE*, Peter J. Smith, *Senior Member, IEEE*, Himal A. Suraweera, *Member, IEEE*, and Arumugam Nallanathan, *Senior Member, IEEE*

***Abstract*—This paper analyzes the exact average symbol error rate (SER), outage probability and ergodic capacity performance of beamforming in spatially correlated multiple-input single output systems with channel estimation error and feedback delay. We derive the joint distribution function of two correlated quadratic forms and employ the result to obtain expressions for the cumulative distribution function, probability density function, moment generating function and moments of the signal-to-noise ratio. Using these expressions, we investigate the exact SER applicable for a large number of modulation schemes, outage probability and the ergodic capacity of the system. The results show that the average SER performance is sensitive to both feedback delay, channel estimation and spatial correlation at the transmitter. Furthermore, feedback delay causes an error floor and has the most degrading impact on performance. We also present Monte Carlo simulation results as verification of our analytical results.**

***Index Terms*—Beamforming, spatial correlation, channel estimation error, feedback delay, symbol error rate, ergodic capacity.**

I. Introduction

Transmit beamforming systems have recently received much attention due to capacity improvements and their ability to mitigate the severe effects of fading through diversity [1], [2]. In beamforming systems, the signal-to-noise ratio (SNR) maximization is achieved by providing channel state information (CSI) to the transmitter [3]. In frequency division duplex (FDD) systems, such knowledge is provided by the feedback of CSI from the receiver to the transmitter.

The performance of practical systems suffers from many forms of CSI imperfections [4]. The most common sources of imperfection are channel estimation errors and feedback delay. An information theoretic approach to multiple-input single output (MISO) transmit beamforming with imperfect feedback has been presented in [5]. In [6], the effects of delayed and limited feedback on the error performance of MISO systems have been investigated. Analyzing the combined effects of channel estimation errors and outdated feedback has been the subject of several recent publications [4], [7], [8]. The results found in [4]-[7] have been based on spatially independent and identically-distributed (i.i.d) channels.

Also, in a practical scenario, spatial constraints limit the size of an antenna array. In such a situation, an increase in the number of antenna elements introduces reductions in inter-element spacing and correlation among antenna elements arises [9]. The effect of antenna correlation on different aspects of performance for various multi-antenna systems has been addressed previously. The main performance measures evaluated have been channel capacity [10], symbol error rate (SER) and outage probability [11], [12]. In [13], an optimal beamforming structure considering noisy channel estimates has been proposed. Moreover, the error rate performance of this system over correlated Rayleigh fading channels has been analyzed.

Although there is a large body of existing results available for MISO systems, only a few studies have investigated beamforming performance considering the joint effects of channel estimation error and feedback delay under spatial correlation (See for e.g. [14] and [15]). In [14], a simple codebook design algorithm for the spatially correlated Rayleigh fading MISO channels with channel estimation error has been presented. In [15], the ergodic capacity of beamforming with spatial correlation, channel estimation error and feedback delay has been investigated. However, the authors of [15] have not considered other important performance measures such as the SER and the outage probability. In order to facilitate a comprehensive performance analysis, the major challenge is to evaluate the statistics of the SNR. In the literature, a statistical characterization of the SNR has not been forthcoming and therefore new results must be derived. In this paper we fill this gap and present new performance results for spatially correlated MISO systems with beamforming. In particular, first we derive expressions for the probability density function (pdf), the cumulative distribution function (cdf), the moment generating function (MGF) and the moments of the SNR. These expressions enable us to investigate the system's outage probability and the average SER applicable for a large number of modulation schemes. In addition, we also present a tight upper bound expression for the ergodic capacity. The analytical results are useful in several ways. The closed-form performance results accelerate system level simulations and may be used in adaptive systems and scheduling. They also lead to simplified results in Sec. IV-B and the methodology in Appendix A has applications to other communication systems. One example is the upper bound calculation of the SINR

Manuscript received May 16, 2010; revised February 13, 2011; accepted May 9, 2011. The associate editor coordinating the review of this paper and approving it for publication was M. Pun.

A. Firag and P. J. Smith are with the Department of Electrical and Computer Engineering, University of Canterbury, Christchurch, New Zealand (e-mail: {abdulla.firag, peter.smith}@canterbury.ac.nz).

H. A. Suraweera is with Engineering Product Development, Singapore University of Technology and Design, Singapore (e-mail: himalsuraweera@sutd.edu.sg).

A. Nallanathan is with the Centre for Telecommunications Research, King's College London, London WC2R 2LS, United Kingdom (e-mail: arumugam.nallanathan@kcl.ac.uk).

Digital Object Identifier 10.1109/TWC.2011.060811.100829

in signal-to-leakage- and-noise ratio based systems [16]. Our results show that the performance of the system is sensitive to the three factors: feedback delay, channel estimation and special correlation at the transmitter. Having higher correlation at the transmitter and bigger channel estimation error reduces the performance of the system, but does not change the diversity order. However, the feedback delay causes an error floor and has the most degrading impact on performance. Monte Carlo simulation results are also presented to confirm the validity of our analysis.

The rest of the paper is organized as follows. In section II we introduce the system model. The SNR of the system is statistically characterized in Section III. In Section IV, new closed-form expressions for the outage probability, SER and the ergodic capacity are presented. These results are confirmed in Section V using Monto Carlo simulations. Finally, we conclude this paper in Section VI.

II. System Model

We consider a MISO system with N_t transmit antennas at the base station (BS) and one receive antenna at the mobile station (MS) with transmit beamforming as shown in Fig. 1. The received signal at the MS given by [4]

$$y = \boldsymbol{w}^\dagger \boldsymbol{h} s + n, \tag{1}$$

where s is the transmitted symbol with $E\{|s|^2\} = a^2$, $\boldsymbol{w}$ is the $N_t \times 1$ beamforming vector, n is the noise modeled by a zero mean circularly symmetric complex Gaussian (ZMCSCG) random variable with variance σ_n^2, and $\boldsymbol{h}$ is the $N_t \times 1$ channel gain vector. Here, $E\{\cdot\}$ denotes expectation. We assumed that there is insufficient scattering around the BS, resulting in spatial correlations at the BS. Therefore, assuming a flat fading Rayleigh channel, the channel gain vector, $\boldsymbol{h}$, for the MISO system can be modeled as

$$\boldsymbol{h} = \boldsymbol{R}^{1/2} \boldsymbol{h}_0, \tag{2}$$

where the $N_t \times 1$ vector, $\boldsymbol{h}_0$, has i.i.d ZMCSCG entries with unit variance and $\boldsymbol{R}$ is the $N_t \times N_t$ transmit antenna correlation matrix. We assume that $\boldsymbol{R}$ is a positive definite Hermitian matrix and can be eigen-decomposed as $\boldsymbol{R} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^\dagger$, where $\boldsymbol{\Lambda} = \mathrm{diag}[\lambda_1, \ldots, \lambda_{N_t}]$ is a diagonal matrix with λ_i as the ith eigenvalue of $\boldsymbol{R}$. However, the results presented in this paper can easily be extended to a positive semi-definite $\boldsymbol{R}$.

For transmit beamforming, the BS requires CSI. In this paper, we consider imperfect CSI due to both channel estimation and feedback delay. In [17], the channel estimation process is modeled as

$$\boldsymbol{h}[k] = \widehat{\boldsymbol{h}}[k] + \boldsymbol{e}_e[k], \tag{3}$$

where $\widehat{\boldsymbol{h}}$ is the channel estimate and $\boldsymbol{e}_e$ is the estimation error. Where convenient we omit the dependence on time and drop the $[k]$ argument for ease of notation. Assuming the minimum mean-square-error (MMSE) channel estimation method, the covariance of the channel estimation error is given by [17], [18] as

$$E\{\boldsymbol{e}_e \boldsymbol{e}_e^\dagger\} = (\boldsymbol{R}^{-1} + \eta_e \boldsymbol{I}_{N_t})^{-1}, \tag{4}$$

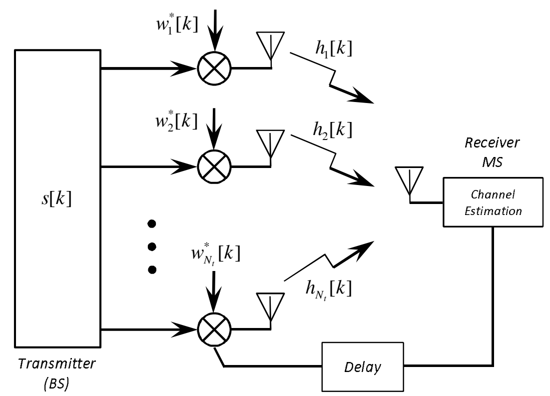

Fig. 1. System model of beamforming with channel estimation error and feedback delay.

where η_e is the average SNR at the receiver during the channel estimation phase and $\boldsymbol{I}_{N_t}$ is the $N_t \times N_t$ identity matrix. From the properties of MMSE estimation, $\widehat{\boldsymbol{h}}[k]$ and $\boldsymbol{e}_e[k]$ are uncorrelated and, therefore, using (3), the covariance matrix, $E\{\widehat{\boldsymbol{h}}\widehat{\boldsymbol{h}}^\dagger\} = \boldsymbol{R} - (\boldsymbol{R}^{-1} + \eta_e \boldsymbol{I}_{N_t})^{-1}$. Now, considering delay in the beamforming process we have an estimate $\widehat{\boldsymbol{h}}[k]$ fed back to the transmitter and used at time $k + D$. From [4] a simple Markov chain model of the channel state evolution is presented whereby the current channel is related to the past channel as

$$\boldsymbol{h}[k+D] = \rho_d \boldsymbol{h}[k] + \sqrt{1 - \rho_d^2}\, \boldsymbol{e}_d[k+D]. \tag{5}$$

In (5), ρ_d is the correlation between $\boldsymbol{h}[k]$ and $\boldsymbol{h}[k+D]$ and $\boldsymbol{e}_d$ is an error term due to temporal changes in the channel. As the error, $\boldsymbol{e}_d$, is only due to delay, then both $\boldsymbol{h}[k+D]$ and $\boldsymbol{h}[k]$ have the same covariance matrix, $E\{\boldsymbol{h}\boldsymbol{h}^\dagger\} = \boldsymbol{R}$. Also, from the Markov chain model, $\boldsymbol{e}_d[k+D]$ and $\boldsymbol{h}[k]$ are uncorrelated and, hence, $\boldsymbol{e}_d$ has the covariance $E\{\boldsymbol{e}_d \boldsymbol{e}_d^\dagger\} = \boldsymbol{R}$. Now, substituting (3) into (5), we have

$$\begin{aligned} \boldsymbol{h}[k+D] &= \rho_d \widehat{\boldsymbol{h}}[k] + \rho_d \boldsymbol{e}_e[k] + \sqrt{1-\rho_d^2}\, \boldsymbol{e}_d[k+D] \\ &\triangleq \rho_d \widehat{\boldsymbol{h}}[k] + \boldsymbol{e}[k+D]. \end{aligned} \tag{6}$$

As $\boldsymbol{e}_d$, $\widehat{\boldsymbol{h}}$ and $\boldsymbol{e}_e$ are uncorrelated, then from (6), the covariance of the total error vector, $\boldsymbol{e}$ can be found as

$$\begin{aligned} E\{\boldsymbol{e}\boldsymbol{e}^\dagger\} &= (1-\rho_d^2)(\boldsymbol{R} - (\boldsymbol{R}^{-1} + \eta_e \boldsymbol{I}_{N_t})^{-1}) + (\boldsymbol{R}^{-1} + \eta_e \boldsymbol{I}_{N_t})^{-1} \\ &= (1-\rho_d^2)\boldsymbol{R} + \rho_d^2 (\boldsymbol{R}^{-1} + \eta_e \boldsymbol{I}_{N_t})^{-1}. \end{aligned} \tag{7}$$

From [4], the optimal beamforming vector, $\boldsymbol{w}$, for transmit beamforming is $\boldsymbol{w} = \widehat{\boldsymbol{h}}/|\widehat{\boldsymbol{h}}|$. Hence, at time $k+D$, the received signal at the MS is given by

$$\begin{aligned} y[k+D] &= \frac{\widehat{\boldsymbol{h}}[k]^\dagger}{|\widehat{\boldsymbol{h}}[k]|} \boldsymbol{h}[k+D] s[k+D] + n[k+D] \\ &= \frac{\rho_d\, s[k+D]}{|\widehat{\boldsymbol{h}}[k]|^{-1}} + \frac{\widehat{\boldsymbol{h}}[k]^\dagger}{|\widehat{\boldsymbol{h}}[k]|} \boldsymbol{e}[k+D] s[k+D] + n[k+D]. \end{aligned} \tag{8}$$

Treating the error term in (8) as additional noise, the received signal (for simplicity all time indexes are ignored) can be rewritten as

$$y = \rho_d |\widehat{\boldsymbol{h}}| s + \widetilde{n}, \tag{9}$$

where $\widetilde{n} = \frac{\widehat{\boldsymbol{h}}^\dagger}{|\widehat{\boldsymbol{h}}|}\boldsymbol{e}s + n$.

III. Statistical Characterization of the SNR

In this section we derive a statistical characterization of the SNR which is required for performance evaluation of the system. For this we first derive the joint pdf of two correlated quadratic forms, and from that exact closed-form results for the cdf, pdf, MGF and moments of the SNR are obtained.

A. cdf of the SNR

In this section, we calculate the cdf of the SNR. From (9), the overall SNR of the transmit beamforming system can be given as

$$SNR = \frac{\rho_d^2 |\widehat{\boldsymbol{h}}|^2 a^2}{E\{|\widetilde{n}|^2\}}, \tag{10}$$

where

$$E\{|\widetilde{n}|^2\} = \sigma_n^2 + a^2 \frac{\widehat{\boldsymbol{h}}^\dagger E\{\boldsymbol{e}\boldsymbol{e}^\dagger\}\widehat{\boldsymbol{h}}}{|\widehat{\boldsymbol{h}}|^2}. \tag{11}$$

As $\widetilde{n}$ is a function of s, the SNR presented here is valid for constant amplitude modulations such as BPSK, 4-QAM. For other modulations, SNR and SER results may be computed conditional on the signal amplitude and then averaged. Hence, the analysis presented here is general and can be extended to multi-level constellations. The channel estimate, $\widehat{\boldsymbol{h}}$, can be expressed as $\widehat{\boldsymbol{h}} = \boldsymbol{T}^{1/2}\boldsymbol{u}$, where $\boldsymbol{T} = \boldsymbol{R} - (\boldsymbol{R}^{-1} + \eta_e \boldsymbol{I}_{N_t})^{-1}$ and $\boldsymbol{u}$ has i.i.d. ZMCSCG entries with unit variance. Using the eigenvalue decomposition of $\boldsymbol{R}$, $\boldsymbol{R} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^\dagger$, $E\{\boldsymbol{e}\boldsymbol{e}^\dagger\}$ and $\boldsymbol{T}$ can be expressed as

$$E\{\boldsymbol{e}\boldsymbol{e}^\dagger\} = \boldsymbol{U}\boldsymbol{D}_1\boldsymbol{U}^\dagger \quad \text{and} \quad \boldsymbol{T} = \boldsymbol{U}\boldsymbol{D}_2\boldsymbol{U}^\dagger, \tag{12}$$

where $\boldsymbol{D}_1 = (1-\rho_d^2)\boldsymbol{\Lambda} + \rho_d^2(\boldsymbol{\Lambda}^{-1} + \eta_e \boldsymbol{I}_{N_t})^{-1}$ and $\boldsymbol{D}_2 = \boldsymbol{\Lambda} - (\boldsymbol{\Lambda}^{-1} + \eta_e \boldsymbol{I}_{N_t})^{-1}$ are diagonal matrices. Now, the overall SNR can be given as

$$SNR = \frac{\rho_d^2 a^2 \widetilde{\boldsymbol{u}}^\dagger \boldsymbol{D}_2 \widetilde{\boldsymbol{u}}}{\sigma_n^2 + a^2 \frac{\widetilde{\boldsymbol{u}}^\dagger \boldsymbol{D}_1 \boldsymbol{D}_2 \widetilde{\boldsymbol{u}}}{\widetilde{\boldsymbol{u}}^\dagger \boldsymbol{D}_2 \widetilde{\boldsymbol{u}}}}, \tag{13}$$

where $\widetilde{\boldsymbol{u}} = \boldsymbol{U}^\dagger \boldsymbol{u}$ has i.i.d. ZMCSCG entries with unit variance and we have used the results $\widehat{\boldsymbol{h}} = \boldsymbol{T}^{1/2}\boldsymbol{u}$ and $\boldsymbol{T}^{1/2} = \boldsymbol{U}\boldsymbol{D}_2^{1/2}\boldsymbol{U}^\dagger$. Then, the cdf of the SNR can be calculated as

$$F_{SNR}(r) = \mathrm{P}[SNR < r] = \mathrm{P}\left[\frac{\rho_d^2 \widetilde{\boldsymbol{u}}^\dagger \boldsymbol{D}_2 \widetilde{\boldsymbol{u}}}{\frac{\sigma_n^2}{a^2} + \frac{\widetilde{\boldsymbol{u}}^\dagger \boldsymbol{D}_1 \boldsymbol{D}_2 \widetilde{\boldsymbol{u}}}{\widetilde{\boldsymbol{u}}^\dagger \boldsymbol{D}_2 \widetilde{\boldsymbol{u}}}} < r\right]. \tag{14}$$

Defining $X \triangleq \widetilde{\boldsymbol{u}}^\dagger \boldsymbol{D}_2 \widetilde{\boldsymbol{u}}$, $Y \triangleq \widetilde{\boldsymbol{u}}^\dagger \boldsymbol{D}_1 \boldsymbol{D}_2 \widetilde{\boldsymbol{u}}$, and $b = \rho_d^2 a^2$, the cdf can be expressed as

$$F_{SNR}(r) = \mathrm{P}\left[bX^2 - r\sigma_n^2 X < ra^2 Y\right]. \tag{15}$$

The random variables (RVs), X and Y can also be written as

$$X = \sum_{i=1}^{N_t} \psi_i |\widetilde{\boldsymbol{u}}_i|^2 \quad \text{and} \quad Y = \sum_{i=1}^{N_t} \beta_i |\widetilde{\boldsymbol{u}}_i|^2, \tag{16}$$

where $\psi_i = \lambda_i - (\lambda_i^{-1} + \eta_e)^{-1}$ and $\beta_i = (\lambda_i - (\lambda_i^{-1} + \eta_e)^{-1})((1-\rho_d^2)\lambda_i + \rho_d^2(\lambda_i^{-1} + \eta_e)^{-1})$. The RV, $|\widetilde{\boldsymbol{u}}_i|^2$ has an exponential distribution with a mean value of one, ie. $|\widetilde{\boldsymbol{u}}_i|^2 \sim \mathrm{Exp}(1)$ for $i = 1, 2, \ldots, N_t$. Now, the cdf in (15) can be rewritten as

$$F_{SNR}(r) = \mathrm{P}\left[\left(\sum_{i=1}^{N_t} \psi_i |\widetilde{\boldsymbol{u}}_i|^2\right)^2 < \sum_{i=1}^{N_t} r\phi_i |\widetilde{\boldsymbol{u}}_i|^2\right], \tag{17}$$

where $\phi_i = \frac{\sigma_n^2 \psi_i + a^2 \beta_i}{b}$. To evaluate the probability in (17) requires the joint pdf of X and $Z = \sum_{i=1}^{N_t} r\phi_i |\widetilde{\boldsymbol{u}}_i|^2$, denoted $f(x,z)$, where X and Z are two quadratic forms. The joint pdf, $f(x,z)$, can be given as follows.

Theorem 1. The joint pdf of $X = \sum_{i=1}^{N_t} \psi_i |\widetilde{\boldsymbol{u}}_i|^2$ and $Z = \sum_{i=1}^{N_t} r\phi_i |\widetilde{\boldsymbol{u}}_i|^2$ is given by

$$\begin{aligned} f(x,z) &= \sum_{i=1}^{N_t} \sum_{l=1, l\neq i}^{N_t} A_i \psi_i^{N_t-2} \frac{e^{\frac{-x}{\psi_i}}}{r} C_{l,i} e^{\frac{-1}{rB_{l,i}}\left(\frac{-xr\phi_i}{\psi_i} + z\right)} \\ &\quad \times \begin{cases} u\left(\frac{-xr\phi_i}{\psi_i} + z\right) u(x) & B_{l,i} > 0 \\ -u\left(\frac{xr\phi_i}{\psi_i} - z\right) u(x) & B_{l,i} < 0 \end{cases}, \end{aligned} \tag{18}$$

where $B_{l,i} = \phi_{l,i}/\psi_{i,l}$, $A_i = \prod_{l=1, l\neq i}^{N_t} 1/\psi_{i,l}$ and

$$C_{l,i} = B_{l,i}^{N_t-3} \prod_{m=1, m\neq l, m\neq i}^{N_t} (B_{l,i} - B_{m,i})^{-1}. \tag{19}$$

In (18), $\psi_{i,l} = \psi_i - \psi_l$ and $\phi_{l,i} = \psi_i \phi_l - \psi_l \phi_i$.

Proof: See Appendix A. ∎

To simplify the results presented here, we demonstrate that $B_{l,i}$ is always negative by showing that it can be given as

$$B_{l,i} = -\frac{\psi_i \psi_l \left(1 + (1-\rho_d^2)\eta_e(\lambda_i + \lambda_l + \eta_e \lambda_i \lambda_l)\right)}{\rho_d^2 \left(\eta_e(\lambda_i + \lambda_l) + \eta_e^2 \lambda_i \lambda_l\right)}. \tag{20}$$

A proof is presented in Appendix B. Hence, from (20) and considering the fact that $\rho_d^2 \leqslant 1$, we can conclude that $B_{l,i}$ is always negative for the scenario considered in this paper. Hence, hereafter we only present the results for the case when $B_{l,i}$ is negative. Now, using (18), the cdf of the SNR can be given as follows.

Theorem 2. The cdf of the SNR in (13) is given by

$$\begin{aligned} F_{SNR}(r) &= \sum_{i=1}^{N_t} \sum_{l=1, l\neq i}^{N_t} A_i \psi_i^{N_t-2} r^{-1} C_{l,i} \\ &\quad \times \left(r\phi_i e^{-rq_i^2/\phi_i} - r\phi_i + I_2\right)/Q_{l,i}, \end{aligned} \tag{21}$$

where $Q_{l,i} = 1/\psi_i - \phi_i/(\psi_i B_{l,i})$ and I_2 can be evaluated as shown in (22).

Proof: See Appendix C. ∎

B. pdf of the SNR

The following theorem presents the pdf of the SNR. This will be used to compute the moment generating function (MGF) and moments of the SNR.

$$I_2 = rB_{l,i}e^{\frac{Q_{l,i}^2 rB_{l,i}}{4}}\left(-e^{-\frac{r}{B_{l,i}}\left(q_i+\frac{Q_{l,i}B_{l,i}}{2}\right)^2}+e^{-\frac{r}{B_{l,i}}\left(\frac{Q_{l,i}B_{l,i}}{2}\right)^2}\right)$$
$$+rB_{l,i}e^{\frac{Q_{l,i}^2 rB_{l,i}}{4}}\left(-\frac{Q_{l,i}\sqrt{\pi r}}{2\sqrt{-1/B_{l,i}}}\left(\text{erfi}\left(\sqrt{-\frac{r}{B_{l,i}}}\left(q_i+\frac{Q_{l,i}B_{l,i}}{2}\right)\right)-\text{erfi}\left(\sqrt{-\frac{r}{B_{l,i}}}\frac{Q_{l,i}B_{l,i}}{2}\right)\right)\right), \quad (22)$$

where $\text{erfi}(x) = \text{erf}(jx)/j$ is the error function with complex argument defined in [19].

Theorem 3. The pdf of the SNR in (13) is given by

$$f_{SNR}(r) = \sum_{i=1}^{N_t}\sum_{l=1,l\neq i}^{N_t} A_i\psi_i^{N_t-2}C_{l,i} \times\left(-q_i^2 e^{-rq_i^2/\phi_i}+I_4\right)/Q_{l,i}, \quad (23)$$

where I_4 is given in (24).

Proof: See Appendix D. ■

C. MGF

In this subsection we present a new exact closed-form expression for the MGF of the SNR. The MGF can be calculated as in the following theorem.

Theorem 4. The MGF of the SNR in (13) is given by

$$M(s) = \sum_{i=1}^{N_t}\sum_{l=1,l\neq i}^{N_t} \frac{A_i\psi_i^{N_t-2}C_{l,i}}{Q_{l,i}}\left(-\frac{q_i^2}{s+q_i^2/\phi_i}+I_5\right) \quad (25)$$

where $I_5 = I_{51} + I_{52}$. I_{51} can be given as

$$I_{51} = \left(q_i^2+\frac{Q_{l,i}B_{l,i}q_i}{2}-\frac{Q_{l,i}^2B_{l,i}^2}{4}\right)\frac{1}{s+q_i^2/\phi_i}, \quad (26)$$

where we have assumed $s + q_i^2/\phi_i > 0$ and finally, I_{52} is given in (27).

Proof: See Appendix E. ■

Having the MGF in closed-form and employing the well known MGF-based approach for the performance evaluation of digital modulations over fading channels [20], the error performance of a large number of modulation schemes can be studied. Moreover, in the case of binary differential phase shift keying (DPSK), the average BER is directly given by $\frac{1}{2}M(1)$.

D. SNR Moments

In this section we derive the moments of the SNR. The moments are used to compute the capacity results given in Sec. IV.

Theorem 5. The moments of the SNR in (13) are given by

$$E\{r^n\} = \sum_{i=1}^{N_t}\sum_{l=1,l\neq i}^{N_t} \frac{A_iC_{l,i}}{Q_{l,i}\,\psi_i^{2-N_t}}\left(-q_i^2 n!\left(\frac{q_i^2}{\phi_i}\right)^{-n-1}+I_7\right), \quad (28)$$

where r =SNR and $I_7 = I_{71} + I_{72}$. I_{71} can be given as

$$I_{71} = \left(q_i^2+\frac{Q_{l,i}B_{l,i}q_i}{2}-\frac{Q_{l,i}^2B_{l,i}^2}{4}\right)n!\left(\frac{q_i^2}{\phi_i}\right)^{-n-1}, \quad (29)$$

and I_{72} is given in (30).

Proof: See Appendix F. ■

IV. Performance Analysis

In this section, we use the statistical characterization of the SNR to investigate important performance measures of the system: outage probability, SER and ergodic capacity.

A. Outage Probability

The outage probability is the probability that the SNR falls below a given SNR threshold value, r_{th}. The SNR threshold value specifies the minimum SNR requires for acceptable performance. The outage probability of the system can be obtained directly from the cdf of the SNR as

$$P_{out}(r_{th}) = \text{P}[SNR < r_{th}] = F_{SNR}(r_{th}), \quad (31)$$

where $F_{SNR}(\cdot)$ is the cdf of the SNR given in (21).

B. Average SER

For many general modulations, the average SER at a certain SNR can be expressed as [21]

$$P_s = E_\gamma\left\{a_sQ\left(\sqrt{b_s\gamma}\right)\right\}, \quad (32)$$

where γ is the SNR, parameters a_s and b_s are determined by specific constellations, $Q(\cdot)$ is the Gaussian Q-function and $E_X\{\cdot\}$ denotes the expectation over the distribution of X. For example, for BPSK modulation, $a_s = 1$, and $b_s = 2$; and for QPSK modulation, $a_s = 1$, and $b_s = 1$. In [21], (32) is also written as:

$$P_s = a_sE_W\left\{F_{SNR}\left(\frac{W^2}{b_s}\right)\right\} = a_s\int_{-\infty}^{\infty}\frac{e^{-\frac{w^2}{2}}}{\sqrt{2\pi}}F_{SNR}\left(\frac{w^2}{b_s}\right)dw, \quad (33)$$

where W is a ZMCSCG random variable with unit variance. Substituting $F_{SNR}(r)$ in (21) into (33), we have

$$P_s = \sum_{i=1}^{t}\sum_{l=1,l\neq i}^{t} A_i\psi_i^{t-2}C_{l,i}/Q_{l,i}a_s \times\int_{-\infty}^{\infty}\frac{e^{-\frac{w^2}{2}}}{\sqrt{2\pi}}\left(\phi_ie^{-\frac{w^2q_i^2}{b_s\phi_i}}-\phi_i+\frac{b_s}{w^2}I_2\right)dw. \quad (34)$$

Now the average SER can be calculated as

$$P_s = \sum_{i=1}^{t}\sum_{l=1,l\neq i}^{t} A_i\psi_i^{t-2}C_{l,i}/Q_{l,i}a_s \times\left(\phi_i\left(1+\frac{2q_i^2}{b_s\phi_i}\right)^{-1/2}-\phi_i+I_6\right), \quad (35)$$

$$I_4 = \left(q_i^2 + \frac{Q_{l,i}B_{l,i}q_i}{2} - \frac{B_{l,i}^2}{4Q_{l,i}^{-2}}\right)e^{-r\frac{\left(q_i^2+Q_{l,i}B_{l,i}q_i\right)}{B_{l,i}}} - \left(\frac{1}{\sqrt{r}} + \frac{\sqrt{r}B_{l,i}}{2Q_{l,i}^{-2}}\right)\frac{B_{l,i}Q_{l,i}\sqrt{\pi}}{4\sqrt{1/B_{l,i}}}e^{\frac{Q_{l,i}^2 rB_{l,i}}{4}}\mathrm{erf}\left(\sqrt{\frac{r}{B_{l,i}}}\left(q_i + \frac{Q_{l,i}B_{l,i}}{2}\right)\right). \quad (24)$$

$$I_{52} = -\frac{B_{l,i}Q_{l,i}\sqrt{\pi}}{4\sqrt{1/B_{l,i}}}\left(\sqrt{\frac{\pi}{-\mu^2}} - \frac{2}{\sqrt{\pi}\beta_a}\,{}_2F_1\left(\frac{1}{2},1;\frac{3}{2};\frac{\mu_a^2}{\beta_a^2}\right) + B_{l,i}Q_{l,i}^2\left(\frac{1}{-4\mu^2}\sqrt{\frac{\pi}{-\mu^2}} - \frac{1}{3\sqrt{\pi}\beta_a^3}\,{}_2F_1\left(\frac{3}{2},2;\frac{5}{2};\frac{\mu_a^2}{\beta_a^2}\right)\right)\right), \quad (27)$$

where ${}_2F_1(a,b;c;z)$ is the Gauss hypergeometric function, $\mu_a^2 = -s + \frac{Q_{l,i}^2 B_{l,i}}{4}$ and $\beta_a = \sqrt{\frac{1}{B_{l,i}}}\left(q_i + \frac{Q_{l,i}B_{l,i}}{2}\right)$.

$$\begin{aligned} I_{72} = &-\frac{B_{l,i}Q_{l,i}\sqrt{\pi}}{2\sqrt{1/B_{l,i}}}\left(\frac{(v_1-2)!!}{2(-2\mu^2)^{(v_1-1)/2}}\sqrt{\frac{\pi}{-\mu^2}} - \frac{\Gamma\left(\frac{v_1+1}{2}\right)}{\sqrt{\pi}v_1\beta_b^{v_1}}\,{}_2F_1\left(\frac{v_1}{2},\frac{v_1+1}{2};\frac{v_1}{2}+1;\frac{\mu_b^2}{\beta_b^2}\right)\right) \\ &-\frac{B_{l,i}^2Q_{l,i}^3\sqrt{\pi}}{4\sqrt{1/B_{l,i}}}\left(\frac{(v_2-2)!!}{2(-2\mu^2)^{(v_2-1)/2}}\sqrt{\frac{\pi}{-\mu^2}} - \frac{\Gamma\left(\frac{v_2+1}{2}\right)}{\sqrt{\pi}v_2\beta_b^{v_2}}\,{}_2F_1\left(\frac{v_2}{2},\frac{v_2+1}{2};\frac{v_2}{2}+1;\frac{\mu_b^2}{\beta_b^2}\right)\right), \end{aligned} \quad (30)$$

where $v_1 = 2n+1$, $v_2 = 2n+3$, $\mu_b^2 = \frac{Q_{l,i}^2 B_{l,i}}{4}$ and $\beta_b = \sqrt{\frac{1}{B_{l,i}}}\left(q_i + \frac{Q_{l,i}B_{l,i}}{2}\right)$.

$$\begin{aligned} I_6 = &\int_{-\infty}^{\infty}\frac{e^{-\frac{w^2}{2}}}{\sqrt{2\pi}}B_{l,i}e^{\frac{Q_{l,i}^2 w^2 B_{l,i}}{4b_s}}\left(-e^{-\frac{w^2}{B_{l,i}b_s}\left(q_i+\frac{Q_{l,i}B_{l,i}}{2}\right)^2} + e^{-\frac{w^2}{B_{l,i}b_s}\left(\frac{Q_{l,i}B_{l,i}}{2}\right)^2}\right)dw + \int_{-\infty}^{\infty}\frac{e^{-\frac{w^2}{2}}}{\sqrt{2\pi}}e^{\frac{Q_{l,i}^2 w^2 B_{l,i}}{4b_s}} \\ &\times B_{l,i}\left(-\frac{Q_{l,i}w\sqrt{\pi/b_s}}{2\sqrt{-1/B_{l,i}}}\left(\mathrm{erfi}\left(w\sqrt{-\frac{1}{B_{l,i}b_s}}\left(q_i + \frac{Q_{l,i}B_{l,i}}{2}\right)\right) - \mathrm{erfi}\left(w\sqrt{-\frac{1}{B_{l,i}b_s}}\frac{Q_{l,i}B_{l,i}}{2}\right)\right)\right)dw \\ = &-B_{l,i}\frac{1}{\sqrt{2\left(\frac{1}{2}+\frac{q_i^2}{b_s\phi_i}\right)}} + B_{l,i} - \frac{Q_{l,i}B_{l,i}}{2\sqrt{2}b_s}\frac{\left(q_i + \frac{Q_{l,i}B_{l,i}}{2}\right)}{\left(\frac{1}{2}-\frac{Q_{l,i}^2B_{l,i}}{4b_s}\right)}\sqrt{\frac{1}{\left(\frac{1}{2}+\frac{q_i^2}{B_{l,i}b_s}+\frac{Q_{l,i}q_i}{b_s}\right)} + \frac{Q_{l,i}^2B_{l,i}^2}{4b_s\left(\frac{1}{2}-\frac{Q_{l,i}^2B_{l,i}}{4b_s}\right)}} \end{aligned} \quad (36)$$

where I_6 is given in (36).

Using the SNR expression in (13), simplified expressions can found for the following special cases. With channel correlation only, the SNR is given by

$$SNR = \frac{a^2\widetilde{\boldsymbol{u}}^\dagger\boldsymbol{\Lambda}\widetilde{\boldsymbol{u}}}{\sigma_n^2}, \quad (37)$$

and with feedback delay only or channel estimation error only:

$$SNR = \frac{\varpi a^2\widetilde{\boldsymbol{u}}^\dagger\widetilde{\boldsymbol{u}}}{\sigma_n^2 + a^2(1-\varpi)}, \quad (38)$$

where $\varpi = \rho_d^2$ and $\varpi = \frac{\eta_e}{1+\eta_e}$ for the feedback delay only case and channel estimation error only case, respectively. From these expressions, it is clear that correlation alone will not cause an error floor and hence the system has maximum diversity order, N_t. Feedback delay will cause an error floor, giving a diversity order of 0, and estimation error will cause an error floor if it does not reduce with an increase in transmit SNR, a^2. These comments follow from the presence of the a^2 term in the denominator of (38) which means that SNR converges to a finite limit as $a^2 \to \infty$. Also, we observe a greater sensitivity to feedback delay than estimation error if $\rho_d^2 < \frac{\eta_e}{1+\eta_e}$. This follows by equating $\frac{\eta_e}{1+\eta_e}$ to ρ_d^2 which makes the two SNRs equal.

Simplified SER results can be obtained at high transmit SNR. For example, as $a^2 \to \infty$, (38) becomes

$$SNR = \varpi_2\widetilde{\boldsymbol{u}}^\dagger\widetilde{\boldsymbol{u}}, \quad (39)$$

where $\varpi_2 = \frac{\varpi}{1-\varpi}$ and we have assumed that ϖ does not vary with SNR (as in the feedback delay case). Now the SNR is a χ^2 random variable. Hence, using (33), a simplified SER result can be obtained as

$$P_s = a_s\left(1 - \frac{1}{\sqrt{1+\frac{2}{\varpi_2 b_s}}}\Sigma_{k=0}^{N-1}\frac{(2k-1)!!}{k!(\varpi_2 b_s+2)^k}\right), \quad (40)$$

where $(k)!!$ is the double factorial [19].

C. Ergodic Capacity

This subsection considers the ergodic capacity of the beamforming system in correlated Rayleigh channels. The ergodic capacity (in bits/s/Hz) of the system can be evaluated as [15]:

$$\begin{aligned} C &= E_r\left\{\log_2(1+r)\right\} \\ &= \int_0^{\infty}\log_2(1+r)f_{SNR}(r)\,dr. \end{aligned} \quad (41)$$

A closed-form solution for the integral in (41) is difficult to find but it can be evaluated numerically. However, an upper bound for the ergodic capacity can be evaluated which has a closed-form result. According to Jensen's inequality [25], an upper bound on the ergodic capacity in (41) is given by

$$C \le \log_2(1 + E\{r\}), \quad (42)$$

where $E\{r\}$ is the expected value of the SNR or the first moment of the SNR. Therefore, an upper bound for the ergodic

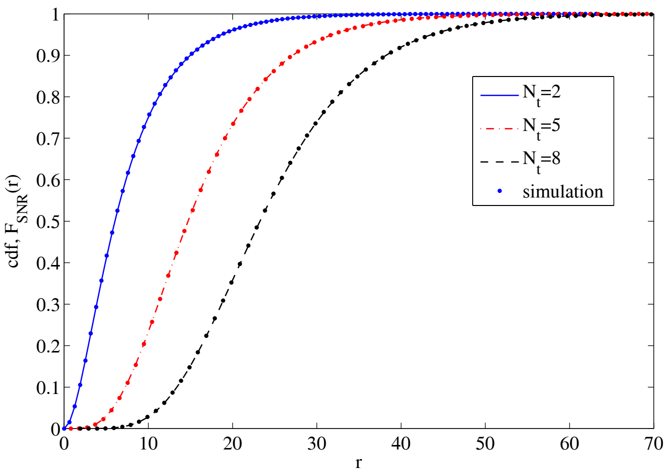

Fig. 2. Analytical and simulated cdfs of the SNR with parameters: $\rho_d = 0.95$, $\eta_e = 20$ dB, $\rho_R = 0.6$, $a^2 = 10$ dB.

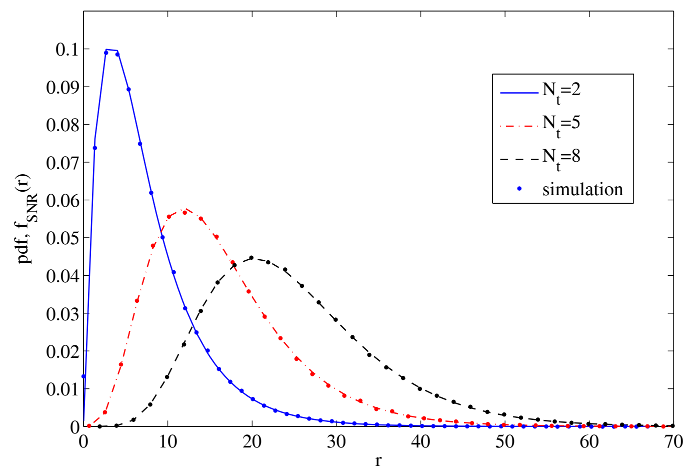

Fig. 3. Analytical and simulated pdfs of the SNR with parameters: $\rho_d = 0.95$, $\eta_e = 20$ dB, $\rho_R = 0.6$, $a^2 = 10$ dB.

capacity can be given as

$$C_U = \log_2(1 + E\{r\}), \tag{43}$$

where $E\{r\}$ can be obtained from (28) as

$$E\{r\} = \sum_{i=1}^{N_t} \sum_{l=1, l\neq i}^{N_t} \frac{A_i \psi_i^{N_t-2} C_{l,i}}{Q_{l,i}} \times \left(-q_i^2 \left(\frac{q_i^2}{\phi_i} \right)^{-2} + I_{7c1} + I_{7c2} \right). \tag{44}$$

In (44), I_{7c1} can be given as

$$I_{7c1} = \left(q_i^2 + \frac{Q_{l,i} B_{l,i} q_i}{2} - \frac{Q_{l,i}^2 B_{l,i}^2}{4} \right) \left(\frac{q_i^2}{\phi_i} \right)^{-2}, \tag{45}$$

and I_{7c2} is given in (46).

V. Numerical and Simulation Results

In all the results given, we have used the correlation matrix, $\boldsymbol{R}$, with elements given by, $\boldsymbol{R}(i,j) = \rho_R^{|i-j|}$, where $0 < \rho_R < 1$ is the transmit correlation coefficient. In this exponential correlation model, bigger ρ_R values represent higher correlation among the transmit antennas. Also, we let $\sigma_n^2 = 1$ and hence a^2 represents the transmit SNR of the system. The channel estimation parameter, η_e is set to 10 dB above the transmit SNR as, in general, the training phase has higher energy than the data transmission [26]. First, in Figs. 2 and 3, we validate the theoretical cdf and pdf given in (21) and (23), respectively, via simulation. The figures show the cdf and pdf of the SNR with different numbers of transmit antennas and it also shows that the theoretical results are in good agreement with the simulations.

Figure 4 shows the system outage performance with $N_t = 2, 3, 4$ for a threshold SNR value, $r_{th} = 0$ dB. The theoretical outage probability was plotted using (31). It can be seen that the theoretical results agree with the simulations in all SNR regimes. From those curves, we can see that the outage performance improves with an increase in the number of antennas at the transmitter due to the diversity improvement.

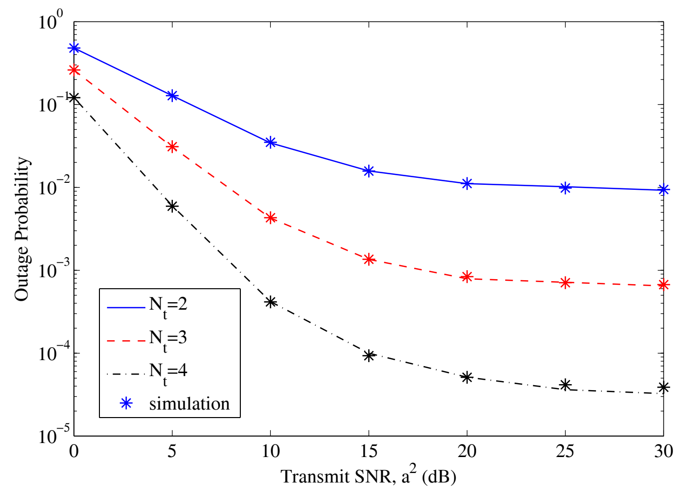

Fig. 4. Analytical and simulated outage probability of the system with parameters: $\rho_d = 0.95$, $\eta_e = a^2 + 10$ dB, $\rho_R = 0.6$.

Figure 5 presents the simulated and analytical average SER of QPSK modulation with $N_t = 2, 3, 4$. Results in the figure confirm the validity and accuracy of our analytical SER result and show that the SER performance increases as N_t increases.

Figure 6 shows the average SER of QPSK modulation with different delay values, $\rho_d = 1.00, 0.995, 0.95, 0.900$. In these results, the channel estimation error is set to be negligible ($\eta_e = 100$ dB) and the channel correlation effect is set to be very small ($\rho_R = 0.001$), thus the results show the effect of ρ_d in the absence of the channel estimation error and the channel correlation. Figure 6 shows that the average SER performance is very sensitive to the feedback delay and the SER performance decreases quite rapidly due to decreases in ρ_d. Also note that the results show an error floor due to feedback delay at high SNR which agrees with (40).

Figure 7 illustrates the simulated and analytical average SER of QPSK modulation with different transmit correlations, $\rho_R = 0.9, 0.6, 0.1$. In these results, the channel estimation error is set to be negligible ($\eta_e = 100$ dB) and so is the channel delay effect ($\rho_d = 1$). Thus, the results show the effect of the channel correlation in the absence of the channel estimation

$$I_{7c2} = -\frac{B_{l,i}Q_{l,i}\sqrt{\pi}}{2\sqrt{1/B_{l,i}}}\left(\frac{1}{2(-2\mu^2)}\sqrt{\frac{\pi}{-\mu^2}} - \frac{1}{3\sqrt{\pi}\beta_b^3}\,{}_2F_1\left(\frac{3}{2},2;\frac{5}{2};\frac{\mu_b^2}{\beta_b^2}\right)\right) - \frac{B_{l,i}^2Q_{l,i}^3\sqrt{\pi}}{4\sqrt{1/B_{l,i}}}\left(\frac{3}{2(-2\mu^2)^2}\sqrt{\frac{\pi}{-\mu^2}} - \frac{2}{5\sqrt{\pi}\beta_b^5}\,{}_2F_1\left(\frac{5}{2},3;\frac{7}{2};\frac{\mu_b^2}{\beta_b^2}\right)\right). \tag{46}$$

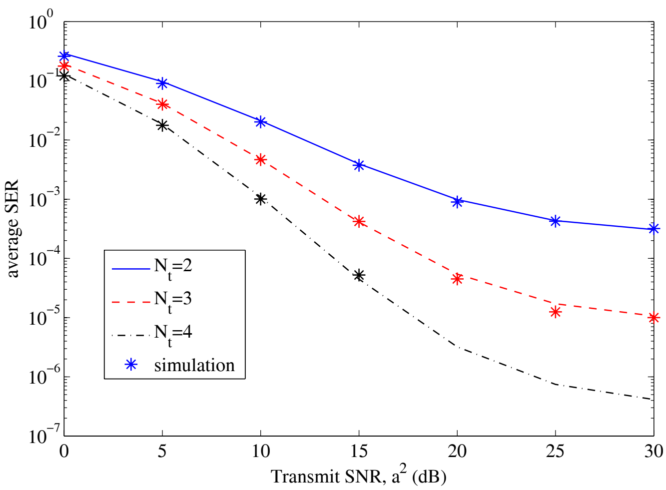

Fig. 5. Analytical and simulated SER with parameters: QPSK, $\rho_d = 0.995$, $\eta_e = a^2 + 10$ dB, $\rho_R = 0.6$.

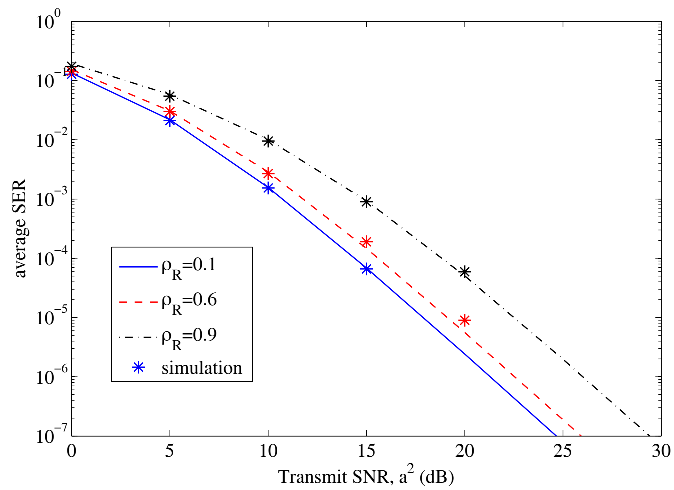

Fig. 7. Analytical and simulated SER for different channel correlation values with parameters: QPSK, $N_t = 3$, $\eta_e = 100$ dB, $\rho_d = 1$.

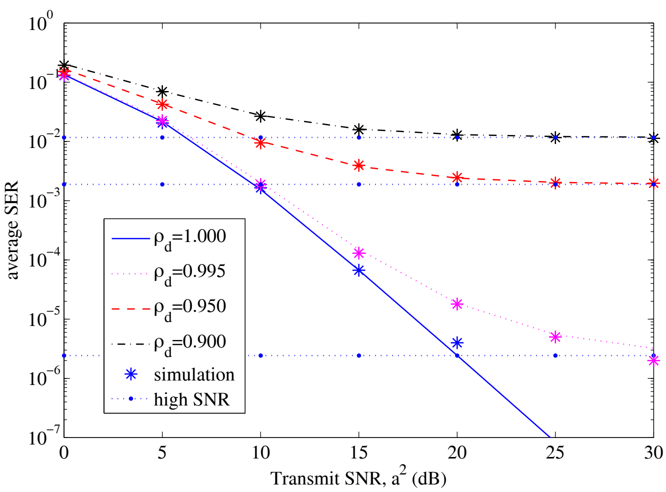

Fig. 6. Analytical and simulated SER for different feedback delays with parameters: QPSK, $N_t = 3$, $\eta_e = 100$ dB, $\rho_R = 0.001$.

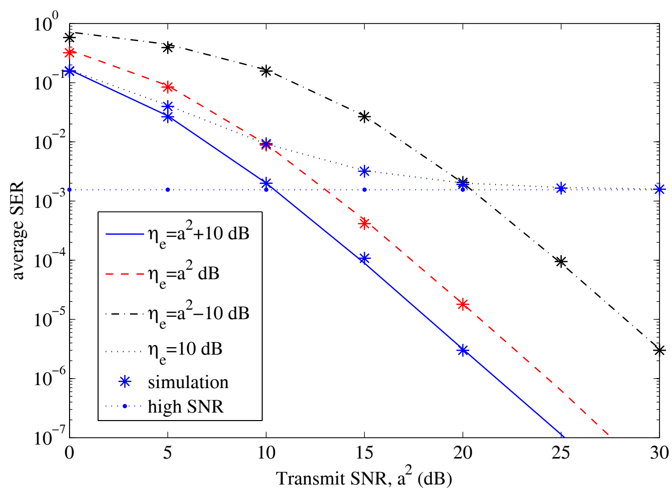

Fig. 8. Analytical and simulated SER for different channel estimation error values with parameters: QPSK, $N_t = 3$, $\rho_R = 0.001$, $\rho_d = 1$.

error and the channel delay effect. We observe that the SER performance gets better with lower transmit correlation as expected and the effect of transmit correlations is not as severe as the channel delay.

Figure 8 shows the average SER of QPSK modulation with different channel estimation errors, $\eta_e = a^2 + 10, a^2, a^2 - 10, 10$ dB. In these results, the channel correlation effect is set to be very small ($\rho_R = 0.001$) and so is the channel delay effect ($\rho_d = 1$). Thus, the results present the effect of the channel estimation error in the absence of the channel correlation and the channel delay effects. As expected, we observe that the SER performance gets better with lower channel estimation error. Note also that when η_e remains the same with an increase in the transmit SNR, the results show an error floor which agrees with (40).

Finally, Fig. 9 validates the ergodic capacity results given in (41) and (42). The figure shows that the theoretical results are in good agreement with the simulations. Note that the result given in (41) does not have a closed-form and is evaluated numerically. The upper bound results for the capacity gives a good approximation for the actual capacity while allowing a closed-form result.

VI. Conclusions

In this paper we investigated the beamforming performance of MISO systems by considering the joint effects of channel

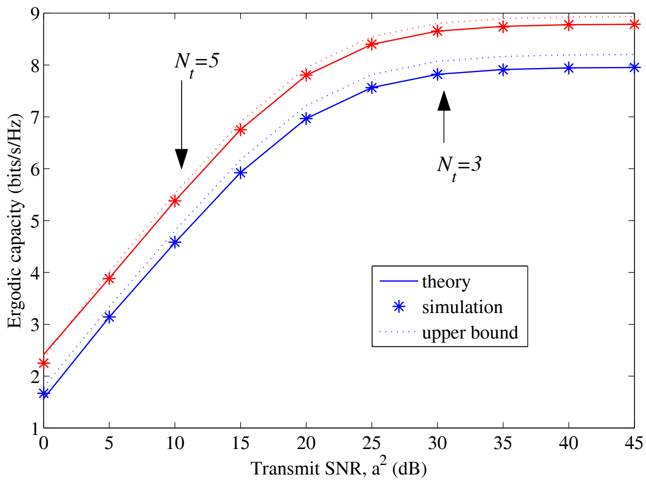

Fig. 9. Analytical and simulated ergodic capacity with parameters: $\rho_R = 0.1$, $\rho_d = 0.995$, $\eta_e = a^2 + 10$ dB.

estimation error and feedback delay under spatial correlation. We derived new exact expressions for the statistics of the SNR which were used to examine the average SER applicable for a large number of modulation schemes. The statististical characterization of the SNR presented in this paper was also used to evaluate important system performance measures: the outage probability and the ergodic capacity of the system. We observed that the average SER performance is very sensitive to feedback delay which causes an error floor. Furthermore, the results showed that higher spatial correlation at the transmit antennas and higher channel estimation error reduces the performance of the system.

APPENDIX

A. Joint pdf of Two Quadratic Forms

Consider the two quadratic forms $X = \sum_{i=1}^{N_t} \psi_i X_i$ and $Z = \sum_{i=1}^{N_t} r\phi_i X_i$, where $r\phi_i, \psi_i \geqslant 0$, $X_i \sim \mathrm{Exp}(1)$ and X_i are i.i.d. The characteristic function (CF) of X and Z is defined as [22]

$$\Phi(s,w) = E\{e^{jsX+jwZ}\} = E\{e^{\sum_{i=1}^{N_t}(js\psi_i + jwr\phi_i)X_i}\}$$
$$= \prod_{i=1}^{N_t} E\{e^{(js\psi_i+jwr\phi_i)X_i}\}. \quad (47)$$

Now, using the result in [23] for the CF of exponentially distributed random variable, the expectation in (56) can be calculated as

$$E\{e^{(js\psi_i+jwr\phi_i)X_i}\} = \frac{1}{1-js\psi_i - jwr\phi_i}. \quad (48)$$

Using the partial fraction expansion in [24, p.154], we have

$$\Phi(s,w) = \prod_{i=1}^{N_t} \frac{1}{1-js\psi_i-jwr\phi_i}$$
$$= \prod_{i=1}^{N_t} \frac{1}{1-jwr\phi_i} \prod_{i=1}^{N_t} (1-2js\gamma_i)^{-1}$$
$$= \prod_{i=1}^{N_t} \frac{1}{1-jwr\phi_i} \sum_{i=1}^{N_t} \frac{c_i}{1-2js\gamma_i}, \quad (49)$$

where $\gamma_i = \frac{\psi_i/2}{1-jwr\phi_i}$ and

$$c_i = \gamma_i^{N_t-1} \prod_{l=1, l\neq i}^{N_t} (\gamma_i - \gamma_l)^{-1}. \quad (50)$$

Inverting $\Phi(s,w)$ with respect to s, we have

$$\Phi(w) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi(s,w)e^{-jxs}ds$$
$$= \prod_{k=1}^{N_t} \frac{1}{1-jwr\phi_k} \sum_{i=1}^{N_t} \frac{c_i e^{-x/(2\gamma_i)}}{2\gamma_i} u(x), \quad (51)$$

where $u(x)$ is the unit step function. The result in (51) is obtained by using the following integral from [19]

$$\int_{-\infty}^{\infty} \frac{e^{-jpx}}{\beta - jx} dx = 2\pi e^{-\beta p} u(p). \quad (52)$$

Now, substituting c_i and γ_i into (51), the ith term of (51) can be expressed as

$$\Phi_i(w) = \prod_{k=1}^{N_t} \left(\frac{\psi_i/2}{1-jwr\phi_i}\right)^{N_t-1} \frac{e^{\frac{-x}{2\gamma_i}}(1-jwr\phi_i)}{\psi_i(1-jwr\phi_k)}$$
$$\times \prod_{l=1,l\neq i}^{N_t} \left(\left(\frac{\psi_i/2}{1-jwr\phi_i}\right) - \left(\frac{\psi_l/2}{1-jwr\phi_l}\right)\right)^{-1} u(x)$$
$$= \psi_i^{N_t-2} e^{-x(1-jwr\phi_i)/\psi_i}$$
$$\times \prod_{l=1,l\neq i}^{N_t} ((\psi_i - \psi_l) - jw(\psi_i r\phi_l - \psi_l r\phi_i))^{-1} u(x). \quad (53)$$

Defining $\psi_{i,l} \triangleq \psi_i - \psi_l$ and $\phi_{l,i} \triangleq \psi_i\phi_l - \psi_l\phi_i$, (53) can be rewritten as

$$\Phi_i(w) = \psi_i^{N_t-2} e^{-x(1-jwr\phi_i)/\psi_i} \prod_{l=1,l\neq i}^{N_t} \frac{u(x)}{\psi_{i,l} - jwr\phi_{l,i}}$$
$$= A_i \psi_i^{N_t-2} e^{-x/\psi_i} \prod_{l=1,l\neq i}^{N_t} \frac{e^{jxwr\phi_i/\psi_i} u(x)}{1-jwrB_{l,i}}, \quad (54)$$

where $B_{l,i} = \phi_{l,i}/\psi_{i,l}$ and $A_i = \prod_{l=1,l\neq i}^{N_t} 1/\psi_{i,l}$.

Using the partial fraction expansion in [24, p.154] for the product term in (54) gives

$$\Phi_i(w) = A_i \psi_i^{N_t-2} e^{-x/\psi_i} \sum_{l=1,l\neq i}^{N_t} \frac{r^{-1}C_{l,i}}{r^{-1}B_{l,i}^{-1} - jw} e^{\frac{jxwr\phi_i}{\psi_i}} u(x), \quad (55)$$

where

$$C_{l,i} = B_{l,i}^{N_t-3} \prod_{m=1,m\neq l,m\neq i}^{N_t} (B_{l,i} - B_{m,i})^{-1}. \quad (56)$$

Now, using (55), $\Phi(w)$ in (51) can be given as

$$\Phi(w) = \sum_{i=1}^{N_t} \sum_{l=1,l\neq i}^{N_t} A_i \psi_i^{N_t-2} e^{-x/\psi_i} \frac{r^{-1}C_{l,i} e^{\frac{jxwr\phi_i}{\psi_i}}}{r^{-1}B_{l,i}^{-1} - jw} u(x). \quad (57)$$

To invert $\Phi(w)$ with respect to w, we require the following integrals from [19]

$$\int_{-\infty}^{\infty} \frac{e^{-jpx}}{\beta + jx} dx = \begin{cases} 0 & p > 0 \\ 2\pi e^{\beta p} & p < 0 \end{cases} \quad (58)$$

and

$$\int_{-\infty}^{\infty} \frac{e^{-jpx}}{\beta - jx} dx = \begin{cases} 2\pi e^{-\beta p} & p > 0 \\ 0 & p < 0 \end{cases}, \tag{59}$$

where the integrations are valid when the real part of β is greater than zero. Now, using these integrations, $\Phi(w)$ can be inverted with respect to w to obtain the joint pdf of X and Z given in Theorem 1 as

$$\begin{aligned} f(x,z) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi(w) e^{-jzw} dw \\ &= \sum_{i=1}^{N_t} \sum_{l=1, l\neq i}^{N_t} A_i \psi_i^{N_t-2} e^{-x/\psi_i} r^{-1} C_{l,i} e^{\frac{-1}{rB_{l,i}}\left(\frac{-xr\phi_i}{\psi_i}+z\right)} \\ &\times \begin{cases} u\left(\frac{-xr\phi_i}{\psi_i}+z\right) u(x) & rB_{l,i} > 0 \\ -u\left(\frac{xr\phi_i}{\psi_i}-z\right) u(x) & rB_{l,i} < 0 \end{cases}. \end{aligned} \tag{60}$$

Note that when $rB_{l,i} < 0$, z could have negative values, but we observe that when z takes negative values $f(x,z) = 0$.

B. Proof of Eq. (20)

Firstly, it can easily be shown that $\psi_i = \eta_e \lambda_i^2/(1+\eta_e\lambda_i)$ and hence,

$$\begin{aligned} \psi_{i,l} &= \frac{\eta_e \lambda_i^2}{(1+\eta_e\lambda_i)} - \frac{\eta_e \lambda_l^2}{(1+\eta_e\lambda_l)} \\ &= \frac{(\lambda_i - \lambda_l)\left(\eta_e(\lambda_i+\lambda_l) + \eta_e^2 \lambda_i \lambda_l\right)}{(1+\eta_e\lambda_i)(1+\eta_e\lambda_l)}. \end{aligned} \tag{61}$$

Substituting $\psi_i = \eta_e \lambda_i^2/(1+\eta_e\lambda_i)$ into β_i and then β_i into ϕ_i, it can be shown that

$$\phi_i = \psi_i \left[\frac{\sigma_n^2}{\rho_d^2 a^2} + \frac{\lambda_i + \eta_e \lambda_i^2 (1-\rho_d^2)}{\rho_d^2 (1+\eta_e\lambda_i)}\right]. \tag{62}$$

Furthermore, substituting (62) into $\phi_{l,i}$, we obtain

$$\phi_{l,i} = -\frac{\psi_i \psi_l \left(1 + (1-\rho_d^2)\eta_e(\lambda_i + \lambda_l + \eta_e \lambda_i \lambda_l)\right)}{\rho_d^2 (1+\eta_e\lambda_i)(1+\eta_e\lambda_l)(\lambda_i - \lambda_l)^{-1}}. \tag{63}$$

Finally, substituting (63) and (61) into $B_{l,i}$, we obtain (20).

C. Proof of Theorem 2

Let

$$g(x,z) \triangleq e^{-x/\psi_i} e^{\frac{-1}{rB_{l,i}}\left(\frac{-xr\phi_i}{\psi_i}+z\right)}, \tag{64}$$

then, using (18), $F_{SNR}(r)$ in (17) can be calculated by

$$\begin{aligned} &F_{SNR}(r) = \mathrm{P}\left[X^2 < Z\right] \\ &= \sum_{i=1}^{N_t} \sum_{l=1, l\neq i}^{N_t} A_i \psi_i^{N_t-2} \frac{C_{l,i}}{r} \int_0^{(rq_i)^2} \int_{z/(rq_i)}^{\sqrt{z}} -g(x,z) dx dz, \end{aligned} \tag{65}$$

where $q_i = \phi_i/\psi_i$. After evaluating the integrals in (65), the cdf can be given as in (21). In (21), I_2 is an integral given by,

$$I_2 = \int_0^{r^2 q_i^2} e^{-Q_{l,i}\sqrt{z} - \frac{z}{rB_{l,i}}} dz. \tag{66}$$

Using the substitution $y = \sqrt{z}$ in (66) gives

$$\begin{aligned} I_2 &= \int_0^{rq_i} 2y e^{-Q_{l,i} y - y^2/(rB_{l,i})} dy \\ &= \int_0^{rq_i} 2y e^{-rB_{l,i}(y + Q_{l,i} rB_{l,i}/2)^2} e^{Q_{l,i}^2 rB_{l,i}/4} dy. \end{aligned} \tag{67}$$

Again, changing the variable to $x = y + Q_{l,i} rB_{l,i}/2$, gives

$$I_2 = 2e^{\frac{Q_{l,i}^2 rB_{l,i}}{4}} \int_{\frac{Q_{l,i} rB_{l,i}}{2}}^{rq_i + \frac{Q_{l,i} rB_{l,i}}{2}} \left(x - \frac{Q_{l,i} rB_{l,i}}{2}\right) e^{-\frac{x^2}{rB_{l,i}}} dx. \tag{68}$$

Then, using the following integrals from [19]

$$\int e^{a\,x^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}\, \mathrm{erfi}\left(\sqrt{a}x\right) \tag{69}$$

and

$$\int x\, e^{a\,x^2} dx = \frac{e^{a\,x^2}}{2a}, \tag{70}$$

I_2 in (21) can be evaluated as shown in (22).

D. Proof of Theorem 3

The pdf of the SNR can be obtained by differentiating the cdf in (21) as

$$f_{SNR}(r) = \sum_{i=1}^{N_t} \sum_{l=1, l\neq i}^{N_t} A_i \psi_i^{N_t-2} \frac{C_{l,i}}{Q_{l,i}} \left(-q_i^2 e^{-rq_i^2/\phi_i} + I_{41}\right), \tag{71}$$

where

$$I_{41} = \frac{d(r^{-1} I_2)}{dr}, \tag{72}$$

can be calculated as in (73). It is found that with the summations in (71), the contribution of the third and the fourth term of I_{41} in (73) is equal to zero. Hence, I_{41} can be simplified to I_4 in (24) and this gives the desired result.

E. Proof of Theorem 4

The MGF of the SNR can be expressed as

$$\begin{aligned} M(s) &= \int_0^{\infty} e^{-sr} f_{SNR}(r)\, dr \\ &= \sum_{i=1}^{N_t} \sum_{l=1, l\neq i}^{N_t} \frac{A_i \psi_i^{N_t-2} C_{l,i}}{Q_{l,i}} \left(-\frac{q_i^2}{s + q_i^2/\phi_i} + I_5\right), \end{aligned} \tag{74}$$

where $I_5 = \int_0^{\infty} e^{-sr} I_4\, dr$ and we assumed $s + q_i^2/\phi_i > 0$. The integral I_5 in (74) can be evaluated in parts such that $I_5 = I_{51} + I_{52}$, where

$$\begin{aligned} I_{51} &= \left(q_i^2 + \frac{Q_{l,i} B_{l,i} q_i}{2} - \frac{Q_{l,i}^2 B_{l,i}^2}{4}\right) \\ &\times \int_0^{\infty} e^{-sr} e^{-r\frac{\left(q_i^2 + Q_{l,i} B_{l,i} q_i\right)}{B_{l,i}}} dr. \end{aligned} \tag{75}$$

Note that $\frac{q_i^2 + Q_{l,i} B_{l,i} q_i}{B_{l,i}} = q_i^2/\phi_i$ so that I_{51} can be calculated as

$$\begin{aligned} I_{51} &= \left(q_i^2 + \frac{Q_{l,i} B_{l,i} q_i}{2} - \frac{Q_{l,i}^2 B_{l,i}^2}{4}\right) \int_0^{\infty} e^{-sr} e^{-r\left(q_i^2/\phi_i\right)} dr \\ &= \left(q_i^2 + \frac{Q_{l,i} B_{l,i} q_i}{2} - \frac{Q_{l,i}^2 B_{l,i}^2}{4}\right) \frac{1}{s + q_i^2/\phi_i}. \end{aligned} \tag{76}$$

$$I_{41} = \left(q_i^2 + \frac{Q_{l,i}B_{l,i}q_i}{2} - \frac{Q_{l,i}^2 B_{l,i}^2}{4}\right) e^{-r\frac{\left(q_i^2+Q_{l,i}B_{l,i}q_i\right)}{B_{l,i}}} - \left(\frac{1}{\sqrt{r}} + \frac{\sqrt{r}B_{l,i}Q_{l,i}^2}{2}\right) \frac{B_{l,i}Q_{l,i}\sqrt{\pi}}{4\sqrt{1/B_{l,i}}} e^{\frac{Q_{l,i}^2 r B_{l,i}}{4}} \operatorname{erf}\left(\sqrt{\frac{r}{B_{l,i}}}\left(q_i + \frac{Q_{l,i}B_{l,i}}{2}\right)\right)$$
$$+ \left(\frac{1}{\sqrt{r}} + \frac{\sqrt{r}B_{l,i}Q_{l,i}^2}{2}\right) \frac{B_{l,i}Q_{l,i}\sqrt{\pi}}{4\sqrt{1/B_{l,i}}} e^{\frac{Q_{l,i}^2 r B_{l,i}}{4}} \operatorname{erf}\left(\sqrt{\frac{r}{B_{l,i}}}\frac{Q_{l,i}B_{l,i}}{2}\right) + \frac{Q_{l,i}^2 B_{l,i}^2}{4}. \tag{73}$$

$$I_{52} = -\frac{B_{l,i}Q_{l,i}\sqrt{\pi}}{4\sqrt{1/B_{l,i}}} \int_0^\infty e^{-sr}\left(\frac{1}{\sqrt{r}} + \frac{\sqrt{r}B_{l,i}Q_{l,i}^2}{2}\right) e^{\frac{Q_{l,i}^2 r B_{l,i}}{4}} \operatorname{erf}\left(\sqrt{\frac{r}{B_{l,i}}}\left(q_i + \frac{Q_{l,i}B_{l,i}}{2}\right)\right) dr. \tag{77}$$

$$I_{52} = -\frac{B_{l,i}Q_{l,i}\sqrt{\pi}}{4\sqrt{1/B_{l,i}}} \int_0^\infty e^{-sy^2}\left(2 + y^2 B_{l,i}Q_{l,i}^2\right) e^{\frac{Q_{l,i}^2 y^2 B_{l,i}}{4}} \operatorname{erf}\left(y\sqrt{\frac{1}{B_{l,i}}}\left(q_i + \frac{Q_{l,i}B_{l,i}}{2}\right)\right) dy. \tag{78}$$

$$I_{72} = -\frac{B_{l,i}Q_{l,i}\sqrt{\pi}}{4\sqrt{1/B_{l,i}}} \int_0^\infty r^n\left(\frac{1}{\sqrt{r}} + \frac{\sqrt{r}B_{l,i}Q_{l,i}^2}{2}\right) e^{\frac{Q_{l,i}^2 r B_{l,i}}{4}} \operatorname{erf}\left(\sqrt{\frac{r}{B_{l,i}}}\left(q_i + \frac{Q_{l,i}B_{l,i}}{2}\right)\right) dr. \tag{79}$$

$$I_{72} = -\frac{B_{l,i}Q_{l,i}\sqrt{\pi}}{4\sqrt{1/B_{l,i}}} \int_0^\infty \left(2y^{2n} + y^{2n+2}B_{l,i}Q_{l,i}^2\right) e^{\frac{Q_{l,i}^2 y^2 B_{l,i}}{4}} \operatorname{erf}\left(y\sqrt{\frac{1}{B_{l,i}}}\left(q_i + \frac{Q_{l,i}B_{l,i}}{2}\right)\right) dy. \tag{80}$$

The integral I_{52} is given in (77). Substituting $y = \sqrt{r}$, we can rewrite I_{52} in (77) as in (78). Now using the following integrals from [19] for $-\mu^2 > 0$, $v > 0$ and $\Re(\beta^2) > \Re(\mu^2)$,

$$\int_0^\infty (1 - \operatorname{erf}(\beta x)) e^{\mu^2 x^2} x^{v-1} dx = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi} v \beta^v} {}_2F_1\left(\frac{v}{2}, \frac{v+1}{2}; \frac{v}{2}+1; \frac{\mu^2}{\beta^2}\right) \tag{81}$$

$$\int_0^\infty e^{\mu^2 x^2} x^{v-1} dx = \frac{(v-2)!!}{2(-2\mu^2)^{(v-1)/2}}\sqrt{\frac{\pi}{-\mu^2}}, \tag{82}$$

we have

$$\int_0^\infty \operatorname{erf}(\beta x) e^{\mu^2 x^2} x^{v-1} dx = \frac{(v-2)!!}{2(-2\mu^2)^{\frac{(v-1)}{2}}}\sqrt{\frac{\pi}{-\mu^2}} - \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi} v \beta^v} {}_2F_1\left(\frac{v}{2}, \frac{v+1}{2}; \frac{v}{2}+1; \frac{\mu^2}{\beta^2}\right). \tag{83}$$

Using the integral (83), I_{52} can be calculated as in Theorem 3.

F. Proof of Theorem 5

The moments of the SNR can be calculated as

$$\begin{aligned} E\{r^n\} &= \int_0^\infty r^n f_{SNR}(r) dr \\ &= \sum_{i=1}^{N_t}\sum_{l=1,l\neq i}^{N_t} \frac{A_i \psi_i^{N_t-2} C_{l,i}}{Q_{l,i}} \\ &\quad \times \left(-\int_0^\infty r^n q_i^2 e^{-rq_i^2/\phi_i} dr + \int_0^\infty r^n I_4 dr\right) \\ &= \sum_{i=1}^{N_t}\sum_{l=1,l\neq i}^{N_t} \frac{A_i \psi_i^{N_t-2} C_{l,i}}{Q_{l,i}} \left(-q_i^2 n! \left(\frac{q_i^2}{\phi_i}\right)^{-n-1} + I_7\right). \end{aligned} \tag{84}$$

where $I_7 = \int_0^\infty r^n I_4 dr$. The integral I_7 in (84) can be evaluated in parts such that $I_7 = I_{71} + I_{72}$, where

$$I_{71} = \left(q_i^2 + \frac{Q_{l,i}B_{l,i}q_i}{2} - \frac{Q_{l,i}^2 B_{l,i}^2}{4}\right) \times \int_0^\infty r^n e^{-r\frac{\left(q_i^2+Q_{l,i}B_{l,i}q_i\right)}{B_{l,i}}} dr. \tag{85}$$

Note that $\frac{q_i^2+Q_{l,i}B_{l,i}q_i}{B_{l,i}} = q_i^2/\phi_i$ so that I_{71} can be calculated as

$$\begin{aligned} I_{71} &= \left(q_i^2 + \frac{Q_{l,i}B_{l,i}q_i}{2} - \frac{Q_{l,i}^2 B_{l,i}^2}{4}\right) \int_0^\infty r^n e^{-r\left(q_i^2/\phi_i\right)} dr \\ &= \left(q_i^2 + \frac{Q_{l,i}B_{l,i}q_i}{2} - \frac{Q_{l,i}^2 B_{l,i}^2}{4}\right) n! \left(\frac{q_i^2}{\phi_i}\right)^{-n-1}. \end{aligned} \tag{86}$$

The integral I_{72} is given in (79). Substituting $y = \sqrt{r}$, we can rewrite I_{72} in (79) as in (80). Now using the integral (83), I_{72} in (80) can be calculated as in Theorem 4.

REFERENCES

[1] J. H. Winters, "The diversity gain of transmit diversity in wireless systems with Rayleigh fading," *IEEE Trans. Veh. Technol.*, vol. 47, pp. 119-123, Feb. 1998.

[2] D. J. Love, R. W. Heath, and T. Strohmer, "Grassmannian beamforming for multiple-input multiple-output wireless systems," *IEEE Trans. Inf. Theory*, vol. 49, pp. 2735-2747, Oct. 2003.

[3] A. Narula, M. J. Lopez, M. D. Trott, and G. W. Wornell, "Efficient use of side information in multiple antenna data transmission over fading channels," *IEEE J. Sel. Areas Commun.*, vol. 16, pp. 1423-1436, Oct. 1998.

[4] Y. Isukapalli and B. D. Rao, "Performance analysis of finite rate feedback MISO systems in the presence of estimation errors and delay," in *Proc. Asilomar Conference Signals, Systems Computers*, Nov. 2007, pp. 1931-1935.

[5] E. Visotsky and U. Madhow, "Space-time transmit precoding with imperfect feedback," *IEEE Trans. Inf. Theory*, vol. 47, Sep. 2001, pp. 2632-2639.

[6] Y. Ma, D. Zhang, A Leith, and Z. Wang, "Error performance of transmit beamforming with delayed and limited feedback," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 1164-1170, Mar. 2009.

[7] N.-S. Kim and Y. H. Lee, "Effect of channel estimation errors and feedback delay on the performance of closed-loop transmit diversity system," in *Proc. IEEE SPAWC 2003*, June 2003, pp. 542-545.

[8] Y. Isukapalli and B. D. Rao, "Packet error probability of a transmit beamforming system with imperfect feedback," *IEEE Trans. Signal Process.*, vol. 58, pp. 2298-2314, Apr. 2010.

[9] J.-Y. Ko and Y.-H. Lee, "Adaptive beamforming with dimension reduction in spatially correlated MISO channels," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 4998-5002, Oct. 2009

[10] M. Kang and M.-S. Alouini, "Impact of correlation on the capacity of MIMO channels," in *Proc. IEEE ICC*, May 2003, pp. 2623-2627.

[11] A. Zanella, M. Chiani, and M. Z. Win, "Performance of MIMO MRC in correlated Rayleigh fading environments," in *Proc. IEEE VTC*, May 2005, pp. 1633-1637.

[12] M. R. McKay, A. J. Grant, and I. B. Collings, "Performance analysis of MIMO-MRC in double-correlated Rayleigh environments," *IEEE Trans. Commun.*, vol. 55, pp. 497-507, Mar. 2007.

[13] P. Nagvanshi, E. Masry, and L. B. Milstein, "Optimum transmit-receive beamforming with noisy channel estimates for correlated MIMO Rayleigh channels," *IEEE Trans. Commun.*, vol. 56, pp. 1869-1880, Nov. 2008.

[14] L. Ding, B. Jiang, and X. Gao, "A simple codebook design algorithm for MISO channels with channel estimation errors," in *Proc. 5th Intl. Conf. Wireless Commun., Networking and Mobile Computing*, Sep. 2009, pp. 1-4.

[15] Y. Isukapalli and B. D. Rao, "Finite rate feedback for spatially and temporally correlated MISO channels in the presence of estimation errors and feedback delay," in *Proc. IEEE GLOBECOM*, Nov. 2007, pp. 2791-2795.

[16] P. Cheng, M. Tao, and W. Zhang, "A new SLNR-based linear precoding for downlink multi-user multi-stream MIMO systems," *IEEE Commun. Lett.*, vol. 14, pp. 1008-1010, Nov. 2010.

[17] E. DallAnese, A. Assalini, and S. Pupolin, "On the effect of imperfect channel estimation upon the capacity of correlated MIMO fading channels," in *Proc. IEEE VTC*, Apr. 2009, pp. 1-5.

[18] X. Zhou, T. Lamahewa, P. Sadeghi, and S. Durrani, "Capacity of MIMO systems: impact of spatial correlation with channel estimation errors," in *Proc. IEEE Int. Conf. Commun. Systems*, Nov. 2008, pp. 817-822.

[19] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th edition. Academic Press, 2007.

[20] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd edition. Wiley, 2005.

[21] S. Chen, W. Wang, X. Zhang, and Z. Sun, "Performance analysis of OSTBC transmission in amplify-and-forward cooperative relay networks," *IEEE Trans. Veh. Technol.*, vol. 59, pp. 105-113, Jan. 2010.

[22] H. Stark and J. W. Woods, *Probability and Random Processes with Application to Signal Processing*, 3rd edition. Prentice-Hall, Inc., 2002.

[23] V. Krishnan, *Probability and Random Processes (Wiley Survival Guides in Engineering and Science)*. Wiley-Interscience, 2006.

[24] N. L. Johnson and S. Kotz, *Distributions in Statistics: Continuous Univariate Distributions*. John Wiley and Sons, 1970, vol. 2.

[25] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. John Wiley and Sons, 1991

[26] M. Duarte, A. Sabharwal, C. Dick, and R. Rao, "Beamforming in MISO systems: empirical results and EVM-based analysis," *IEEE Trans. Wireless Commun.*, vol. 9, pp. 3214-3225, Oct. 2010.

Abdulla Firag (M'09) is a Research Associate with the Department of Electrical and Electronic Engineering, University of Canterbury, New Zealand. He was born in H.A. Thakandhoo, Maldives. He received the B.Eng. degree (with honors) in electrical and electronic engineering from the University of Adelaide, Australia in 1999, and M. Eng. and Ph.D. degrees from the University of Canterbury, New Zealand, in 2003 and 2008, respectively. From February 2000 to June 2001 and July 2003 to June 2004, he was an Electrical and Electronics Engineer at the Maldives Airports Company, Maldives. From July 2004 to February 2006, he was a Project Manager at the Ministry of Communication, Science and Technology, Maldives. From April 2009 to March 2011, he was a Postdoctoral Research Fellow with the Department of Mathematics and Statistics, University of Canterbury, New Zealand. His research interests include statistical analysis and simulation of communication systems, random matrix theory, adaptive equalization for wireless communications, space-time coding, cooperative diversity, and multiple-input-multiple-output (MIMO) wireless relaying systems.

Peter J. Smith (SM'03) received the B.Sc degree in Mathematics and the Ph.D degree in Statistics from the University of London, London, U.K., in 1983 and 1988, respectively. From 1983 to 1986 he was with the Telecommunications Laboratories at GEC Hirst Research Center. From 1988 to 2001 he was a lecturer in statistics at Victoria University, Wellington, New Zealand. Since 2001 he has been a Senior Lecturer and Associate Professor in Electrical and Computer Engineering at the University of Canterbury in New Zealand. His research interests include the statistical aspects of design, modeling and analysis for communication systems, especially antenna arrays, MIMO, cognitive radio and relays.

Himal A. Suraweera (M'07) was born in Kurunegala, Sri Lanka. He received the B.Sc. degree (first class honors) in electrical and electronics engineering, Peradeniya University, Peradeniya, Sri Lanka, in 2001 and the Ph.D. degree from Monash University, Melbourne, Australia, in 2007. He was also awarded the 2007 Mollie Holman Doctoral and 2007 Kenneth Hunt Medals for his doctoral thesis upon graduating from Monash University.

From 2001 to 2002, he was with the Department of Electrical and Electronics Engineering, Peradeniya University, as an Instructor. From October 2006 to January 2007 he was at Monash University as a Research Associate. From February 2007 to June 2009, he was at the center for Telecommunications and Microelectronics, Victoria University, Melbourne, Australia as a Research Fellow. From July 2009 to January 2011, he was with the Department of Electrical and Computer Engineering, National University of Singapore, Singapore as a Research Fellow. He is now a Research Fellow at the Singapore University of Technology and Design, Singapore. His main research interests include relay networks, cognitive radio, MIMO and OFDM.

Dr. Suraweera is an Editor of the IEEE COMMUNICATIONS LETTERS. He serves as a Technical Program Committee Member for international conferences such as ICC 12, GLOBECOM 11 and WCNC 11. He was a recipient of an International Postgraduate Research Scholarship from the Australian Commonwealth during 2003-2006. He received an IEEE COMMUNICATIONS LETTERS exemplary reviewer certificate for 2009.

Arumugam Nallanathan (S'97−M'00−SM'05) is a Reader in Communications at King's College London, United Kingdom. He was an Assistant Professor in the Department of Electrical and Computer Engineering, National University of Singapore, Singapore from August 2000 to December 2007. His research interests include smart grid, cognitive radio and relay networks. In these areas, he has published over 175 journal and conference papers. He is a co-recipient of the Best Paper Award presented at 2007 IEEE International Conference on Ultra-Wideband (ICUWB'2007).

He currently serves on the Editorial Board of IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY and IEEE SIGNAL PROCESSING LETTERS as an Associate Editor. He served as a Guest Editor for *EURASIP Journal of Wireless Communications and Networking*: Special issue on UWB Communication Systems- Technology and Applications. He served as the General Track Chair for the IEEE VTC'2008-Spring, Co-Chair for the IEEE GLOBECOM'2008 Signal Processing for Communications Symposium and IEEE ICC'2009 Wireless Communications Symposium. He currently serves as Co- Chair for the IEEE GLOBECOM'2011 and IEEE ICC'2012 Signal Processing for Communications Symposium and Technical program Co-Chair for IEEE International Conference on Ultra-Wideband'2011 (IEEE ICUWB'2011). He also currently serves as the Vice-Chair for the Signal Processing for Communication Electronics Technical Committee of IEEE Communications Society.