Blind Channel Estimation for MIMO OFDM Systems via Nonredundant Linear Precoding

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***Abstract*—Based on the assumption that the transmitted symbols are independent and identically distributed (i.i.d.), we develop a simple subspace-based blind channel estimation technique for orthogonal frequency-division multiplexing (OFDM) systems by utilizing nonredundant linear block precoding. A novel contribution is that the proposed method can be applied for scenarios, where the number of receive antennas is less than the number of transmit antennas, e.g., multiple-input single-output (MISO) transmissions, in which case the traditional subspace-based methods could not be applied. Further consideration that can eliminate the multidimensional ambiguity in channel estimation under multiple transmitter scenarios is also proposed. The numerical results clearly show the effectiveness of our proposed algorithm.**

***Index Terms*—Blind channel estimation, block precoding, multiple-input multiple-output (MIMO), multiple-input single-output (MISO), orthogonal frequency-division multiplexing (OFDM), subspace, wireless communications.**

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) [1] is a promising candidate for next-generation high-speed wireless multimedia communication systems due to its high data rate, high spectral efficiency, and robustness to frequency-selective channels. On the other hand, an OFDM system combined with multiple antennas at both the transmitter side and the receiver side has attracted considerable attention for its promising capability to combat the multipath fading and increase the system capacity [2].

Since coherent detection in OFDM systems requires reliable channel state information (CSI), channel estimation becomes a critical component for most OFDM systems. A promising family of blind channel estimation method, so called the subspace-based algorithm, has been developed in [3]. For OFDM transmissions, however, the traditional subspace-based method cannot be applied if the number of receive antennas is less than or equal to the number of transmit antennas, since no noise subspace is available.

Based on the assumption that the transmitted symbols are independent and identically distributed to each other, a new type of blind channel estimation method for single-input single-output (SISO) OFDM systems has been proposed in [4]–[6], where a nonredundant linear precoder is used at the transmitter, and the CSI is possessed in all entries of the signal covariance matrix.

In this correspondence, we generalize the precoding method in [4] to multiple-input multiple-output (MIMO) OFDM systems and propose a subspace based approach to increase the performance accuracy. A novel contribution of this generalization is that the channel could be estimated successfully even if the number of transmit antennas goes beyond the traditional limits.We also propose an approach to eliminate the multidimensional ambiguity that is known to exist for channel estimation under multi-transmitter scenarios [7]. Finally, the numerical results are provided to show the effectiveness of our proposed algorithm.

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MATLAB notations for rows and columns are used here. For example, $\mathbf{A}(m,:)$ represents the mth row of the matrix $\mathbf{A}$, and $\mathbf{A}(m_1 : m_2,:)$ represents the submatrix obtained by extracting rows m_1 through m_2 from the matrix $\mathbf{A}$, respectively.

II. Problem Formulation

A. System Model

Let us consider a baseband MIMO OFDM system with N_t transmit antennas and N_r receive antennas. Specifically, if $N_r = 1$, it reduces to a MISO OFDM system. Suppose all the $N_t \times N_r$ channel paths have the memory upper bounded by L, and let $\mathbf{h}_{ij} = [h_{ij,0},\ldots,h_{ij,L}]^T$ denote the equivalent discrete channel response from the ith transmitter to the jth receiver. In each OFDM block, M symbols are transmitted. The cyclic prefix (CP) is added at the front of each transmitted block and is discarded at each received block. As long as the length of the CP is greater than or equal to L, the remaining signal at the jth receiver for the kth block can be represented as [8]

$$\mathbf{x}_j(k) = \sum_{i=1}^{N_t} \bar{\mathbf{H}}_{ij}\mathbf{F}^H\mathbf{s}_i(k) + \bar{\mathbf{n}}_j(k) \tag{1}$$

where $\mathbf{s}_i(k)$ is the $M\times 1$ vector of transmitted symbols; $\bar{\mathbf{n}}_j(k)$ is the $M\times 1$ vector of the unknown white Gaussian noise at the jth receiver with equivalent variance σ_n^2 at each sampling time; $\mathbf{F}$ is the $M \times M$ normalized discrete Fourier transform (DFT) matrix with its (m,q)th entry given by $(1/\sqrt{M})e^{-j2\pi(m-1)(q-1)/M}$, and $\bar{\mathbf{H}}_{ij}$ is the $M \times M$ circulant channel matrix with its (m,q)th entry given by $h^o_{ij,((m-q) \bmod M)}$, where the $M\times 1$ vector $\mathbf{h}^o_{ij}$ is obtained by adding $M-L-1$ zeros at the end of $\mathbf{h}_{ij}$.

Let

$$\mathbf{H}_{ij} = \mathrm{DFT}(\mathbf{h}_{ij}) = [H_{ij,0}, H_{ij,1},\ldots,H_{ij,M-1}]^T \tag{2}$$

denote the M points DFT of the channel vector $\mathbf{h}_{ij}$. The normalized DFT of the received signal vector from the jth receiver is then represented as

$$\begin{aligned}\mathbf{y}_j(k) &= \mathbf{F}\mathbf{x}_j(k) = \sum_{i=1}^{N_t}\mathbf{F}\bar{\mathbf{H}}_{ij}\mathbf{F}^H\mathbf{s}_i(k) + \mathbf{F}\bar{\mathbf{n}}_j(k)\\ &= \sum_{i=1}^{N_t}\tilde{\mathbf{H}}_{ij}\mathbf{s}_i(k) + \mathbf{n}_j(k)\end{aligned} \tag{3}$$

where $\tilde{\mathbf{H}}_{ij} = \mathrm{diag}\{\mathbf{H}_{ij}\}$ is a diagonal matrix with the diagonal elements obtained from $\mathbf{H}_{ij}$[9]. It can be easily shown that the new random noise vector $\mathbf{n}_j(k)$ has the same statistical distribution as $\bar{\mathbf{n}}_j(k)$.

Let

$$\boldsymbol{\mathcal{H}}_{\mathbf{y}} = \begin{bmatrix}\tilde{\mathbf{H}}_{11} & \ldots & \tilde{\mathbf{H}}_{N_t1}\\ \vdots & \ddots & \vdots\\ \tilde{\mathbf{H}}_{1N_r} & \ldots & \tilde{\mathbf{H}}_{N_tN_r}\end{bmatrix} \tag{4}$$

denote the overall frequency-domain channel matrix with its (j,i)th partitioned block given by $\tilde{\mathbf{H}}_{ij}$. The combined received symbol vector $\mathbf{y}(k)$ can be expressed as

$$\mathbf{y}(k) = [\mathbf{y}_1(k)^T, \mathbf{y}_2(k)^T,\ldots,\mathbf{y}_{N_r}(k)^T]^T = \boldsymbol{\mathcal{H}}_{\mathbf{y}}\mathbf{s}(k) + \mathbf{n}(k) \tag{5}$$

where

$$\mathbf{s}(k) = [\mathbf{s}_1(k)^T, \mathbf{s}_2(k)^T,\ldots,\mathbf{s}_{N_t}(k)^T]^T \tag{6}$$

$$\mathbf{n}(k) = [\mathbf{n}_1(k)^T, \mathbf{n}_2(k)^T,\ldots,\mathbf{n}_{N_r}(k)^T]^T. \tag{7}$$

The source covariance matrix is expressed as

$$\mathbf{R_s} = \mathrm{E}\{\mathbf{s}(k)\mathbf{s}(k)^H\} = \sigma_s^2\mathbf{I}_{MN_t} \tag{8}$$

and σ_s^2 is the transmitted signal power.

B. Nonredundant Precoding

Suppose the symbol from the ith transmitter is precoded, separately, by the $M\times M$ matrix $\mathbf{W}_i$ before the inverse discrete Fourier transform (IDFT) operation. The normalized DFT of the received signal vector from the jth receiver should be rewritten as

$$\mathbf{y}_j(k) = \sum_{i=1}^{N_t}\tilde{\mathbf{H}}_{ij}\mathbf{W}_i\mathbf{s}_i(k) + \mathbf{n}_j(k). \tag{9}$$

III. MIMO Channel Estimation With Matrix Ambiguity

Let $\mathcal{P}$ be the block diagonal matrix with the form

$$\mathcal{P} = \mathrm{diag}\{\mathbf{P}_1, \mathbf{P}_2, \ldots, \mathbf{P}_{N_t}\} \tag{10}$$

where

$$\mathbf{P}_i = \mathbf{W}_i\mathbf{W}_i^H. \tag{11}$$

The signal covariance matrix $\mathbf{R_y}$ is then obtained as

$$\begin{aligned}\mathbf{R_y} &= \mathrm{E}\{\mathbf{y}(k)\mathbf{y}(k)^H\} = \sigma_s^2\mathcal{H}_\mathbf{y}\mathcal{P}\mathcal{H}_\mathbf{y}^H + \sigma_n^2\mathbf{I}_{MN_r}\\ &= \begin{bmatrix}\mathbf{R}_{\mathbf{y},11} & \cdots & \mathbf{R}_{\mathbf{y},1N_r}\\ \vdots & \ddots & \vdots\\ \mathbf{R}_{\mathbf{y},N_r1} & \cdots & \mathbf{R}_{\mathbf{y},N_rN_r}\end{bmatrix}\end{aligned} \tag{12}$$

with its (b,d)th subblock given by

$$\begin{aligned}\mathbf{R}_{\mathbf{y},bd} &= \sigma_s^2\sum_{i=1}^{N_t}\tilde{\mathbf{H}}_{ib}\mathbf{P}_i\tilde{\mathbf{H}}_{id}^H + \delta(b-d)\sigma_n^2\mathbf{I}_M\\ &= \sigma_s^2\sum_{i=1}^{N_t}(\mathbf{H}_{ib}\mathbf{H}_{id}^H)\odot\mathbf{P}_i + \delta(b-d)\sigma_n^2\mathbf{I}_M\end{aligned} \tag{13}$$

where $\odot$ represents the Hardmard product, and $\delta(\cdot)$ is the Kronecker Delta function.

To proceed, we introduce a clever way by letting all $\mathbf{P}_i$ be the same; namely

$$\mathbf{P} = \mathbf{P}_1 = \cdots = \mathbf{P}_{N_t} = \mathbf{W}\mathbf{W}^H. \tag{14}$$

With this effort, $\mathbf{R}_{\mathbf{y},bd}$ can be rewritten as

$$\mathbf{R}_{\mathbf{y},bd} = \sigma_s^2\left(\sum_{i=1}^{N_t}\mathbf{H}_{ib}\mathbf{H}_{id}^H\right)\odot\mathbf{P} + \delta(b-d)\sigma_n^2\mathbf{I}_M. \tag{15}$$

Consider the following two cases.

1) Case 1: $b\neq d$. In this case, $\mathbf{R}_{\mathbf{y},bd}$ simply has the form of

$$\mathbf{R}_{\mathbf{y},bd} = \sigma_s^2\left(\sum_{i=1}^{N_t}\mathbf{H}_{ib}\mathbf{H}_{id}^H\right)\odot\mathbf{P}. \tag{16}$$

Dividing each entry of $\mathbf{R}_{\mathbf{y},bd}$ by the corresponding entry of $\mathbf{P}$, we obtain

$$\mathbf{Q}_{bd} = \mathbf{R}_{\mathbf{y},bd}\oslash\mathbf{P} = \sigma_s^2\sum_{i=1}^{N_t}\mathbf{H}_{ib}\mathbf{H}_{id}^H \tag{17}$$

where $\oslash$ denotes the elementwise division. Practically, $\mathbf{Q}_{bd}$ is obtained from

$$\mathbf{Q}_{bd} = \mathbf{F}(:,1:L+1)\mathbf{F}^H(\mathbf{R}_{\mathbf{y},bd}\oslash\mathbf{P}). \tag{18}$$

The effect of the item $\mathbf{F}(:,1:L+1)\mathbf{F}^H$ is to enforce the channel length $L+1$ on the time domain. This process is known as denoising [10] which is capable of increasing the estimation accuracy.

2) Case 2: $b=d$. For this case, $\mathbf{R}_{\mathbf{y},bd}$ is written as

$$\mathbf{R}_{\mathbf{y},bd} = \sigma_s^2\left(\sum_{i=1}^{N_t}\mathbf{H}_{ib}\mathbf{H}_{id}^H\right)\odot\mathbf{P} + \sigma_n^2\mathbf{I}_M. \tag{19}$$

On the qth column of $\mathbf{R}_{\mathbf{y},bd}$, except the diagonal element $[\mathbf{R}_{\mathbf{y},bd}]_{qq}$, all the other $M-1$ elements are of the form

$$[\mathbf{R}_{\mathbf{y},bd}]_{mq} = \sigma_s^2\sum_{i=1}^{N_t}H_{ib,m-1}H_{id,q-1}^*, \quad m\neq q \tag{20}$$

where $[\cdot]_{mq}$ denotes the (m,q)th entry of the matrix. Since $\mathbf{P}$ is known as a prior, $\sigma_s^2\sum_{i=1}^{N_t}H_{ib,m-1}H_{id,q-1}^*$ can be calculated as

$$\sigma_s^2\sum_{i=1}^{N_t}H_{ib,m-1}H_{id,q-1}^* = \frac{[\mathbf{R}_{\mathbf{y},bd}]_{mq}}{[\mathbf{P}]_{mq}}, \quad m\neq q. \tag{21}$$

However, $\sigma_s^2\sum_{i=1}^{N_t}H_{ib,q-1}H_{id,q-1}^*$ cannot be obtained, since the (q,q)th entry of $\mathbf{R}_{\mathbf{y},bd}$ is corrupted by the unknown noise variance. As long as $M-1\geq L+1$, as usually the case, the following equation holds:

$$\sigma_s^2\sum_{i=1}^{N_t}\mathbf{H}_{ib}H_{id,q-1}^* = \mathbf{F}(:,1:L+1)\mathbf{F}_q^\dagger\mathbf{r}_{bd,q} \tag{22}$$

where $\dagger$ denotes the pseudoinverse, and

$$\begin{aligned}\mathbf{r}_{bd,q} = \Bigg[&\frac{[\mathbf{R}_{\mathbf{y},bd}]_{1q}}{[\mathbf{P}]_{1q}}, \ldots, \frac{[\mathbf{R}_{\mathbf{y},bd}]_{(q-1)q}}{[\mathbf{P}]_{(q-1)q}}, \frac{[\mathbf{R}_{\mathbf{y},bd}]_{(q+1)q}}{[\mathbf{P}]_{(q+1)q}},\\ &\ldots, \frac{[\mathbf{R}_{\mathbf{y},bd}]_{Mq}}{[\mathbf{P}]_{Mq}}\Bigg]^T,\end{aligned} \tag{23}$$

$$\mathbf{F}_q = \begin{bmatrix}\mathbf{F}(1:q-1,1:L+1)\\ \mathbf{F}(q+1:M,1:L+1)\end{bmatrix}. \tag{24}$$

Therefore

$$\begin{aligned}\mathbf{Q}_{bd} &= \left[\sigma_s^2\sum_{i=1}^{N_t}\mathbf{H}_{ib}H_{id,0}^*, \ldots, \sigma_s^2\sum_{i=1}^{N_t}\mathbf{H}_{ib}H_{id,M-1}^*\right]\\ &= \sigma_s^2\sum_{i=1}^{N_t}\mathbf{H}_{ib}\mathbf{H}_{id}^H\end{aligned} \tag{25}$$

can also be obtained for the case $b=d$.

Let

$$\mathcal{Q} = \begin{bmatrix} \mathbf{Q}_{11} & \dots & \mathbf{Q}_{1N_r} \\ \vdots & \ddots & \vdots \\ \mathbf{Q}_{N_r1} & \dots & \mathbf{Q}_{N_rN_r} \end{bmatrix} \tag{26}$$

and define

$$\mathbf{U} = \begin{bmatrix} \mathbf{H}_{11} & \dots & \mathbf{H}_{N_t1} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{1N_r} & \dots & \mathbf{H}_{N_tN_r} \end{bmatrix} \tag{27}$$

as the $N_rM \times N_t$ matrix containing frequency-domain channel response. It can be verified that

$$\mathcal{Q} = \sigma_s^2 \mathbf{U}\mathbf{U}^H = (\sigma_s \mathbf{U})(\sigma_s \mathbf{U})^H. \tag{28}$$

Let $\hat{\mathbf{U}}$ denote the matrix that contains the N_t eigenvectors of $\mathcal{Q}$ that correspond to the largest N_t eigenvalues. From the subspace detection theory [3], $\mathrm{span}(\hat{\mathbf{U}}) = \mathrm{span}(\mathbf{U})$ when $N_t < N_rM$. Therefore, $\hat{\mathbf{U}}$ can be considered as the estimate of $\mathbf{U}$ but with a matrix ambiguity as

$$\hat{\mathbf{U}} = \mathbf{U}\mathbf{T} \tag{29}$$

where $\mathbf{T}$ is an $N_t \times N_t$ unknown matrix. This matrix ambiguity is well known for the blind channel estimation when multiple transmit antennas are used [7].

Since M is greater than 1 for all OFDM systems, the proposed algorithm is applicable to the systems with $N_t > N_r$, e.g., MISO systems. Therefore, a novel contribution is formed by providing a blind channel estimation algorithm for the system where the transmit antennas is greater than the receive antennas.

IV. MIMO Channel Estimation With Scalar Ambiguity

Suppose at the $(kN_t+\tau)$th time interval, $\tau = 1, \dots, N_t$, the symbol block from the ith transmitter is precoded by $\mathbf{W}_{i\tau}$. Then, the corresponding $\mathbf{P}_{i\tau}$ is expressed as

$$\mathbf{P}_{i\tau} = \mathbf{W}_{i\tau}\mathbf{W}_{i\tau}^H, \quad i, \tau = 1, \dots, N_t. \tag{30}$$

Define N_t covariance matrices

$$\mathbf{R}_{\mathbf{y}\tau} = \mathrm{E}\{\mathbf{y}(kN_t+\tau)\mathbf{y}(kN_t+\tau)^H\}, \quad \tau = 1, \dots, N_t. \tag{31}$$

We know from (12) and (13) that the (b,d)th block of $\mathbf{R}_{\mathbf{y}\tau}$ has the form of

$$\begin{aligned} \mathbf{R}_{\mathbf{y}\tau,bd} &= \sigma_s^2 \sum_{i=1}^{N_t} \tilde{\mathbf{H}}_{ib}\mathbf{P}_{i\tau}\tilde{\mathbf{H}}_{id}^H + \delta(b-d)\sigma_n^2 \mathbf{I}_M \\ &= \sigma_s^2 \sum_{i=1}^{N_t} (\mathbf{H}_{ib}\mathbf{H}_{id}^H) \odot \mathbf{P}_{i\tau} + \delta(b-d)\sigma_n^2 \mathbf{I}_M. \end{aligned} \tag{32}$$

Similar steps to estimate the channel response can be carried on as follows.

1) Case 1: $b \neq d$. For this case, the (m,q)th entry of $\mathbf{R}_{\mathbf{y}\tau,bd}$ can be expressed as

$$[\mathbf{R}_{\mathbf{y}\tau,bd}]_{mq} = \sigma_s^2 \sum_{i=1}^{N_t} [\mathbf{P}_{i\tau}]_{mq} H_{ib,m-1}H_{id,q-1}^*, \; \tau = 1, \dots, N_t. \tag{33}$$

Note that (33) contains N_t equations of N_t unknown parameters $\sigma_s^2 H_{ib,m-1}H_{id,q-1}^*$, $i = 1, \dots, N_t$. Therefore, the unknown parameters can be obtained as

$$\begin{bmatrix} \sigma_s^2 H_{1b,m-1}H_{1d,q-1}^* \\ \vdots \\ \sigma_s^2 H_{N_tb,m-1}H_{N_td,q-1}^* \end{bmatrix} = \begin{bmatrix} [\mathbf{P}_{11}]_{mq} & \dots & [\mathbf{P}_{N_t1}]_{mq} \\ \vdots & \ddots & \vdots \\ [\mathbf{P}_{1N_t}]_{mq} & \dots & [\mathbf{P}_{N_tN_t}]_{mq} \end{bmatrix}^{-1} \begin{bmatrix} [\mathbf{R}_{\mathbf{y}1,bd}]_{mq} \\ \vdots \\ [\mathbf{R}_{\mathbf{y}N_t,bd}]_{mq} \end{bmatrix}. \tag{34}$$

Note that (34) holds if and only if the inverse term exists. Therefore, we should no longer take the same value for all $\mathbf{P}_{i\tau}$, as did in Section III. Instead, $\mathbf{P}_{i\tau}$ should be designed such that the square matrix in (34) is non-singular. By considering all pairs of (m,q) and properly reorganizing the results coming from (34), we can obtain N_t new matrices

$$\mathbf{Q}_{i,bd} = \sigma_s^2 \mathbf{H}_{ib}\mathbf{H}_{id}^H \quad \text{for} \quad i = 1, \dots, N_t. \tag{35}$$

2) Case 2: $b = d$. For this case, the diagonal entries of $\mathbf{R}_{\mathbf{y}\tau,bd}$ are corrupted by the unknown noise power. Therefore, we may consider only the entries with $m \neq q$. There is

$$[\mathbf{R}_{\mathbf{y}\tau,bd}]_{mq} = \sigma_s^2 \sum_{i=1}^{N_t} [\mathbf{P}_{i\tau}]_{mq} H_{ib,m-1}H_{id,q-1}^* \tag{36}$$

for

$$\tau = 1, \dots, N_t, \quad b = d, \quad m \neq q.$$

With a similar step as in Case 1, we can obtain the value of $\sigma_s^2 H_{ib,m-1}H_{id,q-1}^*$, $i = 1, \dots, N_t$ from (34) for all pairs of (m,q) with $m \neq q$. Then, a new vector can be formed as

$$\begin{aligned} \tilde{\mathbf{r}}_{bd,iq} = [&\sigma_s^2 H_{ib,0}H_{id,q-1}^*, \dots, \sigma_s^2 H_{ib,q-2}H_{id,q-1}^*, \\ &\sigma_s^2 H_{ib,q}H_{id,q-1}^*, \dots, \sigma_s^2 H_{ib,M-1}H_{id,q-1}^*]^T \end{aligned} \tag{37}$$

for each $q = 1, \dots, M$, $i = 1, \dots, N_t$. Note that, if the element $\sigma_s^2 H_{ib,q-1}H_{id,q-1}^*$ (not available since $m \neq q$) is inserted into the qth position of (37), then $\tilde{\mathbf{r}}_{bd,iq}$ becomes $\sigma_s^2 \mathbf{H}_{ib}H_{id,q-1}^*$, which is the M-point DFT of $\sigma_s^2 \mathbf{h}_{ib}H_{id,q-1}^*$. Nevertheless, $\sigma_s^2 \mathbf{h}_{ib}H_{id,q-1}^*$ and $\sigma_s^2 \mathbf{H}_{ib}H_{id,q-1}^*$ can still be obtained from

$$\sigma_s^2 \mathbf{h}_{ib}H_{id,q-1}^* = \mathbf{F}_q^\dagger \tilde{\mathbf{r}}_{bd,iq}, \tag{38}$$

$$\begin{aligned} \sigma_s^2 \mathbf{H}_{ib}H_{id,q-1}^* &= \sigma_s^2 \mathbf{F}(:, 1:L+1)\mathbf{h}_{ib}H_{id,q-1}^* \\ &= \mathbf{F}(:, 1:L+1)\mathbf{F}_q^\dagger \tilde{\mathbf{r}}_{bd,iq}. \end{aligned} \tag{39}$$

Equation (38) holds as long as $M - 1 \geq L+1$, since $L+1$ elements in DFT vector is enough to recover the time domain vector of length $L+1$. Combining all $\sigma_s^2 \mathbf{H}_{ib}H_{id,q-1}^*$, $q = 1, \dots, M$, we can obtain

$$\begin{aligned} \mathbf{Q}_{i,bd} &= [\sigma_s^2 \mathbf{H}_{ib}H_{id,0}^*, \dots, \sigma_s^2 \mathbf{H}_{ib}H_{id,M-1}^*] \\ &= \sigma_s^2 \mathbf{H}_{ib}\mathbf{H}_{id}^H, \quad i = 1, \dots, N_t \end{aligned} \tag{40}$$

for the case $b = d$.

Define new matrices

$$\mathcal{Q}_i = \begin{bmatrix} \mathbf{Q}_{i,11} & \dots & \mathbf{Q}_{i,1N_r} \\ \vdots & \ddots & \vdots \\ \mathbf{Q}_{i,N_r1} & \dots & \mathbf{Q}_{i,N_rN_r} \end{bmatrix}, \quad i = 1, \dots, N_t. \tag{41}$$

It can be seen that

$$\mathcal{Q}_i = \sigma_s^2 \mathbf{U}_i \mathbf{U}_i^H \tag{42}$$

where

$$\mathbf{U}_i = [\mathbf{H}_{i1}^T, \mathbf{H}_{i2}^T, \dots, \mathbf{H}_{iN_r}^T]^T \tag{43}$$

is the ith column of $\mathbf{U}$ and represents the $N_rM \times 1$ channel response vector from the ith transmitter to all the receivers. Again, $\mathbf{U}_i$ can be obtained from the eigenvector of $\mathcal{Q}_i$ corresponding to its largest eigenvalue. Therefore, by assigning different precoding matrix to different transmitter and taking N_t covariance matrices from different time slots, the multidimensional ambiguity reduces to one scalar ambiguity for each $\mathbf{U}_i$; namely

$$\hat{\mathbf{U}}_i = \alpha_i \mathbf{U}_i \tag{44}$$

where α_i is an unknown complex scalar. Note that, since N_t covariance matrices need to be constructed, the number of the available snapshots seems to be critical to the performance of the algorithm. Roughly speaking, in order to build the reliable covariance matrix comparable to the one in ambiguity-existing case, N_t times more snapshots need to be obtained.

V. Numerical Results

In this section, we examine the performance of the proposed estimator under various scenarios. The three-ray channel model with exponential power delay profile [11]

$$E\{|h_{ij,l}|^2\} = \exp\left(\frac{-l}{10}\right), \quad l = 0, \dots, 2 \tag{45}$$

is used. The phase of each channel ray is uniformly distributed over $[0, 2\pi)$. For all numerical examples, the QPSK symbols are considered, and $M = 64$. All the results are averaged over $N_w = 100$ Monte Carlo runs.

For simulation purpose, we do not use pilot symbols to remove the ambiguity. Instead, the matrix ambiguity is resolved by [12]

$$\mathbf{T} = \min_{\mathbf{T}} \|\mathbf{U} - \hat{\mathbf{U}}\mathbf{T}^{-1}\|_F^2 \tag{46}$$

where $\|\cdot\|_F^2$ denotes the Frobenius norm. The final channel estimate $\mathbf{U}_f$ for data detection is obtained from

$$\mathbf{U}_f = \hat{\mathbf{U}}\mathbf{T}^{-1} = \hat{\mathbf{U}}\hat{\mathbf{U}}^H\mathbf{U}. \tag{47}$$

Similar process is carried on to remove the scalar ambiguity.

A. MIMO OFDM Systems

In the first example, we examine the performance of the proposed algorithm for MIMO OFDM system with two transmit antennas and two receive antennas.

1) Channel Estimation With Matrix Ambiguity: Firstly, we consider the algorithm with matrix ambiguity. The precoders in [6], although proposed for SISO OFDM systems, will be used here. Specifically

$$[\mathbf{P}]_{mq} = \begin{cases} 1 & m = q \\ p & m \neq q \end{cases} \quad m, q = 1, \dots M, \tag{48}$$

where p is a nonzero value belonging to the region $(-(1/M-1), 1)$.

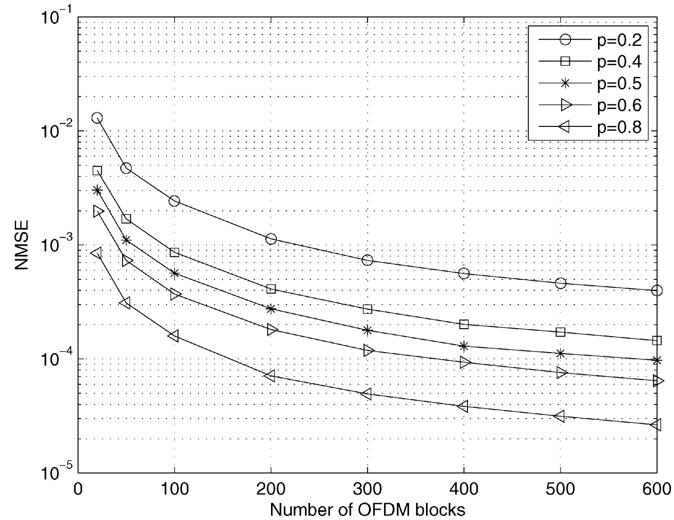

Fig. 1. Channel estimation NMSE for MIMO OFDM versus number of OFDM blocks: with matrix ambiguity.

Because of our assumption on source statistics, the number of received blocks is critical to the performance of the algorithm. We illustrate how the normalized mean-square errors (NMSEs) of the channel estimation vary with different data length in Fig. 1. The signal-to-noise ratio (SNR) is fixed at 10 dB. We see that the value of p is critical to the performance of the proposed algorithm. The larger the p is, the smaller the error will be. However, since a larger p may give an ill-conditioned $\mathbf{P}$, the data detection reliability will be reduced. Therefore, the best value of p cannot be predicted straightforwardly. Moreover, a continuing improvement in estimation accuracy is observed for all p when the number of received OFDM blocks increases. This is quite different from the traditional subspace based method [3], where an error floor is usually met for a fixed SNR even if the number of received OFDM blocks increases.

At the receiver, the minimum mean-square error (MMSE) detection [6] is applied, and the bit error rate (BER) performance of the proposed method for MIMO systems is shown in Fig. 2. The number of the OFDM blocks in this example is taken as 150. It can be seen that $p = 0.2$ performs better at low SNR but is outperformed by $p = 0.4$, $p = 0.5$, $p = 0.6$ at high SNR. However, $p = 0.8$ always gives the worst performance, although it could provide the best channel estimation, as seen from Fig. 1.

2) Channel Estimation With Scalar Ambiguity: We then show the performance results of the proposed method with scalar ambiguity. The total number of received OFDM blocks is taken as 300 such that each covariance matrix is still constructed with 150 samples. The precoding matrices are taken as

$$[\mathbf{P}_{11}]_{mq} = [\mathbf{P}_{22}]_{mq} = \begin{cases} 1.2, & m = q = 1, \dots, \frac{M}{2} \\ 0.8, & m = q = \frac{M}{2}+1, \dots, M \\ \frac{2}{3}, & \text{otherwise} \end{cases},$$

$$[\mathbf{P}_{12}]_{mq} = [\mathbf{P}_{21}]_{mq} = \begin{cases} 0.8, & m = q = 1, \dots, \frac{M}{2} \\ 1.2, & m = q = \frac{M}{2}+1, \dots, M \\ \frac{1}{3}, & \text{otherwise} \end{cases}.$$

The NMSEs versus SNR for $\mathbf{H}_{11}$, $\mathbf{H}_{12}$, $\mathbf{H}_{21}$, and $\mathbf{H}_{22}$ are shown in Fig. 3, separately. We see that the proposed method still works well for estimation with scalar ambiguity.

B. MISO OFDM Systems

In this example, we consider the performance results of the proposed algorithm for MISO OFDM system with two transmit and one re-

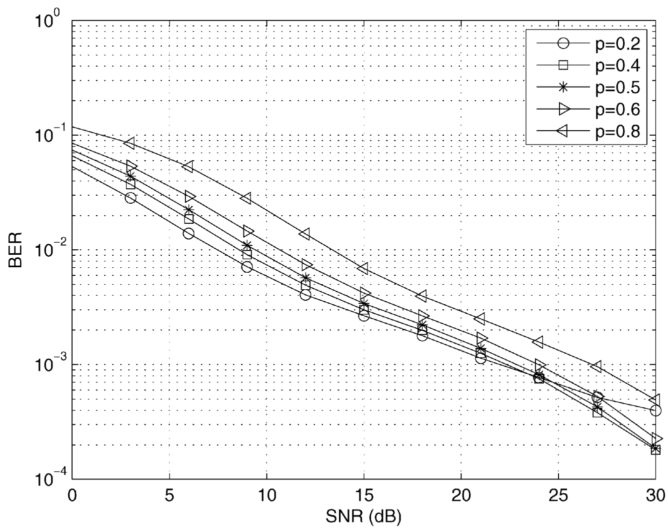

Fig. 2. BER for MIMO OFDM under different p.

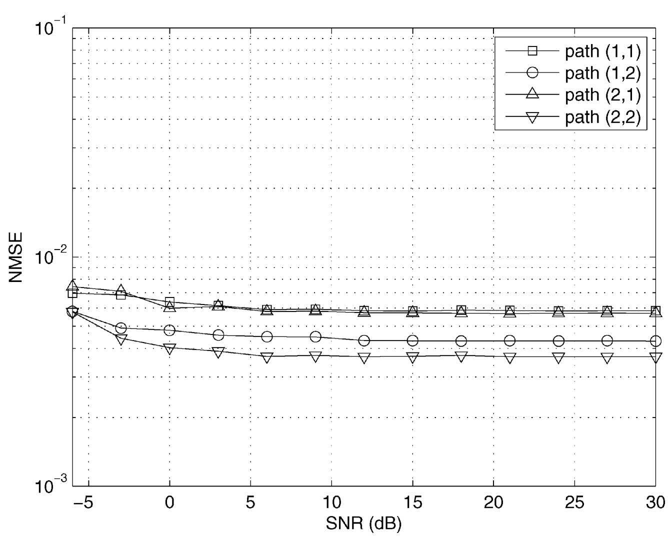

Fig. 3. Channel estimation NMSE for MIMO OFDM versus SNR: with scalar ambiguity.

ceive antennas. Alamouti code [13] is applied at the transmitter and the channel estimation with matrix ambiguity is applied. One hundred and fifty OFDM blocks are received, and the BER performance curves corresponding to different values of p are shown in Fig. 4. We see that the proposed method works well for the MISO case and the data detection is guaranteed. Comparing with those in Fig. 2, where the diversity order for the simulated scenario could be approximated as $N_r - N_t + 1 = 1$ [14], the BER curves in Fig. 4 drop in a much faster rate, since the diversity order with the application of Alamouti code is well known to be 2.

C. Comparison With Existing Methods

The proposed algorithm is shown to outperform the method in [4] when applying to SISO OFDM systems. Due to the space limit, we are unable to provide the comparison here. Interested readers are advised to refer [15].

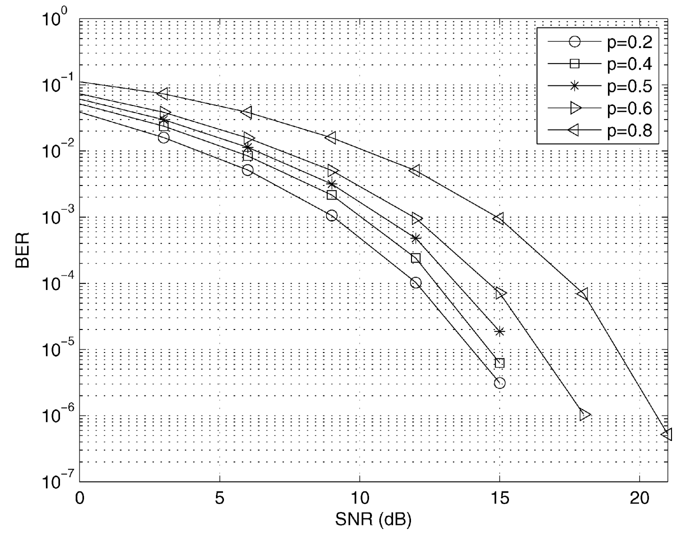

Fig. 4. BER for MISO OFDM with Alamouti code under different p.

VI. CONCLUSION

In this paper, we developed a subspace-based blind channel estimation technique for MIMO OFDM systems using the second order statistical analysis. One novel contribution of the newly proposed method is that it is capable to implement channel estimation even if the number of the transmit antennas is greater than or equal to the number of the receive antennas, where the traditional subspace-based algorithms could not be applied. Both channel estimations with matrix ambiguity and with only scalar ambiguity are considered. Simulation results clearly show the effectiveness of the proposed algorithm under various scenarios.

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