Joint Trajectory and Communication Design for Secure UAV Networks

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Abstract—This letter investigates a joint optimization problem of unmanned aerial vehicle (UAV) flight trajectory, downlink transmission power and ground terminals (GTs) association under wiretap channels. Specifically, we consider a scenario where a UAV serves a group of GTs and maximize the minimum secrecy rate to ensure the fairness among GTs. To solve the nonconvex optimization problem, we develop an iterative algorithm based on the successive convex approximation (SCA) and alternating methods. Simulation results demonstrate that the proposed algorithm is effective and can significantly improve the minimum secrecy rate compared with the traditional flight scheme.

Index Terms—UAV communications, joint optimization, trajectory design, physical-layer security.

I. INTRODUCTION

Recently, it is well known that unmanned aerial vehicle (UAV) can be used for diverse applications, such as cargo delivery, aerial imaging, etc [1] [2]. Compared with the ground communication, low-altitude UAV has great advantages such as faster deployment, more flexible reconfiguration and better communication channels for higher probability of the existence of line-of-sight (LOS) links. The UAV flight trajectory was optimized in [3] to maximize the energy efficiency of UAV, subject to the constraints of initial/final locations, minimum/maximum speed as well as acceleration. The UAV trajectory at the edge of multi-cells for data offloading was analyzed in [4] to maximize the sum rate of the edge users served by UAV. In [5], the user achievable rate was maximized for the multi-UAV enabled wireless networks. However, [3]–[5] did not take into account the physical-layer security, and the security of UAV transmission cannot be guaranteed.

On the other hand, physical-layer security has become more and more important in wireless communications [6]. In [7], secure UAV-to-UAV systems were considered, where a group of UAVs try to eavesdrop the transmitted information between the transmitter and the legitimate UAV. The authors in [8] aimed at maximizing the secrecy rate of a communication system where a single UAV transmits data to a legitimate user for the scenario with an eavesdropper. However, only one single legitimate receiver was considered in [7] and [8].

In this letter, we consider the physical-layer security issue in UAV communications and aim to maximize the minimum secrecy rate of ground terminals (GTs) with respect to the UAV flight trajectory, downlink transmission power and GT association. To solve the nonconvex problem, we apply the successive convex approximation (SCA) and alternating methods, and then propose an iterative algorithm with good performance as shown in the simulation results.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

We consider a downlink UAV-enabled wireless communication system with one eavesdropper (Eve) and K GTs denoted by the set \( \mathcal{K} = \{ 1, \cdots, K \} \). The GTs and Eve are located on the ground at zero altitude. The horizontal coordinates of GT \( k (k \in \mathcal{K}) \) and Eve are \( \mathbf{w}_k = [x_k, y_k]^T \) and \( \mathbf{w}_e = [x_e, y_e]^T \), respectively. Assume that the UAV flies at a fixed altitude \( H \) with a flight period \( T \) and serves the GTs via periodic time-division multiple access (TDMA) mode. For ease of exposition, we divide the flight period into \( N \) time slots denoted by the set \( \mathcal{N} = \{ 1, \cdots, N \} \). The time-variant horizontal coordinate of the UAV in time slot \( n \) is denoted by \( \mathbf{q}(n) = [x(n), y(n)]^T \). Since the UAV needs to go back to the initial position at the end of a flight period, the trajectory needs to satisfy

\[
\mathbf{q}[1] = \mathbf{q}[N].
\]

As the maximum speed of UAV is limited, the trajectory of the UAV needs to meet the following constraints:

\[
||\mathbf{q}[n + 1] - \mathbf{q}[n]|| \leq V_{\text{max}} \frac{T}{N}, \quad n = 1, \cdots, N - 1,
\]

where \( V_{\text{max}} \) is the maximum UAV speed. In time slot \( n \), the distance between GT \( k \) and the UAV can be expressed as

\[
d_k[n] = \sqrt{H^2 + ||\mathbf{q}[n] - \mathbf{w}_k||^2}, \quad \forall n \in \mathcal{N}.
\]

For simplicity of analysis, we assume a LOS link from UAV to each GT, where the channel quality is only determined by the transmission distance. Specifically, the channel power gain from the UAV to GT \( k \) in the time slot \( n \) is calculated as

\[
h_{k}[n] = \frac{\rho_0}{H^2 + ||\mathbf{q}[n] - \mathbf{w}_k||^2}, \quad \forall k \in \mathcal{K}, \forall n \in \mathcal{N},
\]

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where $\rho_0$ represents the reference channel power gain at the distance of 1 m. Similarly, the distance between Eve and the UAV in time slot $n$ can be expressed as

$$d_e[n] = \sqrt{H^2 + \|q[n] - w_e\|^2}, \forall n \in \mathcal{N},$$

and the wiretap channel power gain is

$$h_c[n] = \frac{\rho_0}{H^2 + \|q[n] - w_e\|^2}, \forall n \in \mathcal{N}. \quad (6)$$

We define $a_k[n] \in \{0, 1\}$ as the association indicator variable between GT $k$ and the UAV in time slot $n$. If GT $k$ is served by the UAV in time slot $n$, $a_k[n] = 1$; otherwise, $a_k[n] = 0$. We assume that the UAV serves at most one GT in each time slot, i.e.,

$$\sum_{k=1}^{K} a_k[n] \leq 1, \forall n \in \mathcal{N}. \quad (7)$$

Denote $p[n]$ as the UAV downlink transmit power in time slot $n$. When GT $k$ is served by the UAV in time slot $n$, its achievable data rate is

$$R_k[n] = \frac{1}{2} \log_2 \left( 1 + \frac{p[n] h_k[n]}{\sigma^2} \right), \forall k \in \mathcal{K}, \forall n \in \mathcal{N}, \quad (8)$$

where $\sigma^2$ is the additive noise power. The achievable data rate of Eve in the $n$-th time slot is

$$R_e[n] = \frac{1}{2} \log_2 \left( 1 + \frac{p[n] h_e[n]}{\sigma^2} \right). \quad (9)$$

According to [9], the secrecy rate of GT $k$ in time slot $n$ can be expressed as

$$R'_k[n] = a_k[n] \max \{R_k[n] - R_e[n], 0\}. \quad (10)$$

To guarantee the fairness among GTs, we define the minimum secrecy rate of all GTs in a flight period as

$$\eta = \min_{k \in \mathcal{K}} \frac{1}{N} \sum_{n=1}^{N} R'_k[n]. \quad (11)$$

### B. Problem Formulation

In this paper, we aim to optimize the minimum GT secrecy rate with respect to the UAV trajectory, the downlink transmit power and the GT association. The optimization problem is cast as:

$$\max_{\eta, A, P, Q} \eta \quad \text{s.t.} \quad \frac{1}{N} \sum_{n=1}^{N} R'_k[n] \geq \eta, \forall k, \quad (12a)$$

$$\sum_{k=1}^{K} a_k[n] \leq 1, \forall n, \quad (12b)$$

$$a_k[n] \in \{0, 1\}, \forall k, n, \quad (12c)$$

$$|q[n+1] - q[n]| \leq V_{max} \frac{T}{N}, 1 \leq n < N, \quad (12d)$$

$$q[1] = q[N], \quad (12e)$$

$$0 \leq p[n] \leq P_{max}, \forall n, \quad (12f)$$

where $A = \{a_k[n]\}_{k \in \mathcal{K}}, P = \{p[n]\}_{n \in \mathcal{N}}, Q = \{q[n]\}_{n \in \mathcal{N}}$ and $P_{max}$ denotes the maximum downlink transmit power of the UAV.

### III. Proposed Algorithm

Due to the nonconvex and discrete constrains, Problem (12) is a nonconvex problem. Generally, it is difficult to find a globally optimal solution for this problem. In this section, we develop an iterative algorithm to obtain a suboptimal solution with good performance. By applying the SCA and alternating methods, we divide Problem (12) into three subproblems and transform each subproblem to a convex one. Specifically, we first solve the GT association subproblem given UAV transmit power and UAV trajectory. Then we optimize the UAV transmit power given GT association and UAV trajectory. Finally, the UAV trajectory is updated given UAV transmit power and GT association.

#### A. GT Association Optimization

With given transmit power $P$ and trajectory $Q$, we relax the binary variables in (12d) into continuous variables. Then, the GT association problem is

$$\max_{\eta, A} \eta \quad \text{s.t.} \quad \frac{1}{N} \sum_{n=1}^{N} R'_k[n] \geq \eta, \forall k, \quad (13a)$$

$$0 \leq a_k[n] \leq 1, \forall n, \quad (13b)$$

$$0 \leq a_k[n] \leq 1, \forall k, n. \quad (13c)$$

Problem (13) is a standard linear programming and can be efficiently solved by the simplex method. As the association solution of Problem (13) may be fractional, we adopt the rounding method in [5] to further obtain an integer solution.

#### B. Power Optimization

With fixed GT association $A$ and UAV trajectory $Q$, the UAV power optimization problem is

$$\max_{\eta, P} \eta \quad \text{s.t.} \quad \frac{1}{N} \sum_{n=1}^{N} R'_k[n] \geq \eta, \forall k, \quad (14a)$$

$$0 \leq p[n] \leq P_{max}, \forall k, n. \quad (14b)$$

Note that Problem (14) is nonconvex as the left hand side of constraint (14b) is the difference of two concave functions. To handle constraint (14b), we apply the SCA method where $R_e[n]$ is iteratively approximated by a convex function. Let $p^r[n]$ denote the UAV transmit power in the $r$-th iteration of the SCA method. Due to the fact that $R_e[n]$ is a concave function of $p[n]$, we have

$$R_e[n] = \frac{1}{2} \log_2 \left( 1 + \frac{p[n] h_e[n]}{\sigma^2} \right) \leq \frac{1}{2} \log_2 \left( 1 + \frac{p^r[n] h_e[n]}{\sigma^2} \right) + \frac{h_e[n]}{2 \ln (\sigma^2 + p^r[n] h_e[n])} (p[n] - p^r[n]) \triangleq \hat{R}_e[n]. \quad (15)$$
Replacing $R_e[n]$ with $\hat{R}_e[n]$ in the $r$-th iteration of the SCA method, Problem (14) is approximately rewritten as

$$\max_{\eta} \eta \quad (16a)$$

subject to

$$\frac{1}{N} \sum_{n=1}^{N} a_k[n] \max \left\{ R_k[n] \right. \left. - \hat{R}_e[n], 0 \right\} \geq \eta, \quad \forall k,$$

(16b)

$$0 \leq p[n] \leq P_{\text{max}}, \quad \forall k, n.$$  

(16c)

Problem (16) is a convex problem and can be efficiently solved, e.g., by CVX [10].

### C. UAV Trajectory Optimization

With given GT association $A$ and UAV transmit power $P$, the UAV trajectory optimization problem is formulated as

$$\max_{\eta, \mathbf{Q}, \mathbf{D}} \eta \quad (17a)$$

subject to

$$\frac{1}{N} \sum_{n=1}^{N} R_k[n] \geq \eta, \quad \forall k,$$

(17b)

$$\|q[n+1] - q[n]\| \leq V_{\text{max}} T \frac{T}{N}, \quad \forall n,$$

(17c)

$$q[1] = q[N].$$

(17d)

Problem (17) is a nonconvex problem due to the nonconvex constraint (17b). By introducing slack variables $\mathbf{D} = \{d_e[n]\}$, Problem (17) is equivalent to

$$\max_{\eta, \mathbf{Q}, \mathbf{D}} \eta \quad (18a)$$

subject to

$$\frac{1}{2N} \sum_{n=1}^{N} a_k[n] \left[ \max \left\{ \log_2 \left( 1 + \frac{p[n] h_k[n]}{\sigma^2} \right) \right. \right.$$  

$$- \log_2 \left( 1 + \frac{p[n] \rho_0}{\sigma^2 (H^2 + d_e[n])} \right), 0 \right\} \geq \eta, \quad \forall k,$$

$$d_e[n] \leq \|q[n] - w_e\|^2,$$

$$\|q[n+1] - q[n]\| \leq V_{\text{max}} T \frac{T}{N}, \quad 1 \leq n < N,$$

$$q[1] = q[N].$$

(18b)

(18c)

(18d)

(18e)

Problem (18) is still nonconvex and we apply the SCA technique to solve it. Let $Q^* = q^*[n]$ denote the UAV trajectory in the $r$-th iteration of the SCA method. $R_k[n]$ is a convex function of $\|q[n] - w_k\|^2$, and is thus lower-bounded by

$$R_k[n] = \log_2 \left( 1 + \frac{p[n] h_k[n]}{\sigma^2} \right)$$

$$- \log_2 \left( 1 + \frac{p[n] \rho_0}{\sigma^2 (H^2 + d_e[n])} \right), 0 \right\} \geq \eta, \quad \forall k,$$

$$d_e[n] \leq \|q^*[n] - w_e\|^2,$$

$$\|q[n+1] - q[n]\| \leq V_{\text{max}} T \frac{T}{N}, \quad 1 \leq n < N,$$

$$q[1] = q[N].$$

(18b)

(18c)

(18d)

(18e)

Similarly, $\|q[n] - w_e\|^2$ is lower-bounded by $\|q^*[n] - w_e\|^2 + 2 (q^*[n] - w_e)^T (q[n] - q^*[n])$. Then, we reformulate Problem (18) as

$$\max_{\eta, \mathbf{Q}, \mathbf{D}} \eta \quad (20a)$$

subject to

$$\frac{1}{2N} \sum_{n=1}^{N} a_k[n] \left[ \max \left\{ \log_2 \hat{R}_e[n] \right. \right.$$  

$$- \log_2 \left( 1 + \frac{p[n] \rho_0}{\sigma^2 (H^2 + d_e[n])} \right), 0 \right\} \geq \eta, \quad \forall k,$$

$$d_e[n] \leq \|q[n] - w_e\|^2$$

$$+ 2 (q[n] - w_e)^T (q[n] - q^*[n]), \quad \forall n,$$

(20b)

(20c)

(20d)

(20e)

Problem (20) is a convex problem and can be easily solved, e.g., by CVX [10].

#### Algorithm 1: Alternating Procedure for Solving Problem (12)

1. Initialize $P^{(0)}, Q^{(0)}, \eta^{(0)}$, the tolerance $\theta$, the iteration number $l = 0$, and the maximum iteration number $L_{\text{max}}$.
2. repeat
3. With $(P^{(l)}, Q^{(l)})$, obtain $A^{(l+1)}$ by solving Problem (13).
4. With $(A^{(l+1)}, Q^{(l)})$, obtain $P^{(l+1)}$ by solving Problem (16).
5. With $(A^{(l+1)}, P^{(l+1)})$, obtain $Q^{(l+1)}$ by solving Problem (20).
6. Calculate $\eta^{(l+1)}$ by solving Problem (12) and set $l = l + 1$.
7. until $|\eta^{(l+1)} - \eta^{(l)}| \leq \theta$ or $l > L_{\text{max}}$.

#### D. Iterative Algorithm

The alternating procedure for solving Problem (12) is given in Algorithm 1. Since the objective value of Problem (12)
We investigate two scenarios. In the first scenario (Case 1), the initial position of UA V is the coordinate \((100\, \text{m}, 100\, \text{m})\) and the four GTs’ location coordinates are \((\pm 500\, \text{m}, \pm 500\, \text{m})\), respectively. For comparison, we also simulate the traditional scheme that the UA V flies around Eve with a circular trajectory. The radius of the circular flight scheme, our proposed method provides a dramatic increase in the max-min secrecy rate. Besides, a larger flight period yields a UAV flight trajectory of larger perimeter and a higher max-min secrecy rate.

V. Conclusion

In this letter, we have investigated a new type of UAV-enabled wiretap wireless network. Specifically, we maximized the minimum GT secrecy rate with respect to the GT association, the UAV downlink transmit power and the UAV flight trajectory. By using the SCA technique and the alternating method, an efficient and convergent algorithm was proposed. Simulation results showed that compared with the traditional circular flight scheme, our proposed method provides a dramatic increase in the max-min secrecy rate. It is seen from Fig. 1 that Algorithm 1 is convergent and the obtained minimum secrecy rates under Case I are higher than that under Case 2 because of a farther distance from the closest GT to Eve. Fig. 2(a) and Fig. 2(b) present the optimized UAV trajectories under Case 1 \((T = 120, 300\, \text{s})\) and under Case 2 \((T = 120, 150\, \text{s})\), respectively. It is interesting to observe from Fig. 2 that with longer flight periods, the UAV can fly in the trajectories of larger flight perimeter to get away from Eve and achieve higher secrecy rates. The minimum secrecy rates achieved by the traditional circular flight scheme and Algorithm 1 are given in Table I. It can be seen from Table I that our proposed algorithm can significantly improve the minimum secrecy rate compared with the traditional scheme.

### Table I

<table>
<thead>
<tr>
<th>Periods</th>
<th>Traditional scheme</th>
<th>Algorithm 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T = 300, \text{s}) (Case 1)</td>
<td>0.329</td>
<td>0.690</td>
</tr>
<tr>
<td>(T = 120, \text{s}) (Case 2)</td>
<td>0.050</td>
<td>0.280</td>
</tr>
<tr>
<td>(T = 150, \text{s}) (Case 2)</td>
<td>0.051</td>
<td>0.301</td>
</tr>
</tbody>
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### References