Energy-Efficient D2D Communications Underlaying NOMA-Based Networks with Energy Harvesting

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Abstract—This letter investigates the resource allocation problem in device-to-device (D2D) communications underlaying a non-orthogonal multiple access (NOMA)-based cellular network, where both cellular users and D2D users harvest energy from the hybrid access point in the downlink and transmit information in the uplink. We propose a low-complexity iterative algorithm to maximize the energy efficiency of the D2D pair while guaranteeing the quality of service of cellular users. In each iteration, by analyzing the Karush-Kuhn-Tucker conditions, the globally optimal solution can be derived in closed form despite the nonconvexity. Simulation results validate the superiority of the proposed scheme over the existing schemes.

Index Terms—D2D, NOMA, energy harvesting, resource allocation.

I. INTRODUCTION

Device-to-device (D2D) communications have been considered as a promising technology to alleviate the explosive traffic growth in wireless communications. D2D users share the same resources with cellular users (CUs) under control of the cellular network, which can effectively improve the spectral efficiency [1], [2].

The system throughput in D2D underlaying networks is seriously limited by the battery lifetime budget of users. To prolong the lifetime of networks, energy harvesting via radio frequency spectral efficiency [1], [2]. The CUs are multiplexed on the same uplink resource with the underlaid D2D pair. Two-phase transmission with periodicity $T$ is adopted. The CUs and D2D-Tx harvest energy from the HAP with broadcast power $P_0$ in the first phase with allocated time $\tau_c$. During the second phase with allocated time $\tau_i$, all CUs simultaneously transmit information to the HAP while the D2D-Tx communicates with the D2D-receiver (Rx). Obviously, $\tau_c + \tau_i \leq T$ should be satisfied.

Perfect channel state information obtained via pilot sequences is assumed to be known and constant during time $T$. Denote $\mathcal{K} = \{1, \ldots, K\}$ as the set of CUs. In the first phase, the harvested energy of CU $k$ or the D2D-Tx is

$$E^h_u = \tau_e \eta p_0 h_u, \quad \forall u \in \mathcal{K} \cup \{d\},$$

where $h_k$ denotes the channel gain from the HAP to CU $k$, $h_d$ is the channel gain from the HAP to the D2D-Tx, and $\eta$ is the energy transformation efficiency.

In the second phase, the users utilize the energy collected in the first phase to transmit information. Thus, for all users, energy consumption should not exceed the harvested energy. During the first phase, only the constant circuit power is consumed for user $u$, i.e., $p_u^c$. During the second phase, the transmit power and the constant circuit power of user $u$ are denoted by $p_u$ and $p_u^c$, respectively. In practice, $p_u^c \geq p_u^c$ due to the fact that the circuits of information transmission are more complex than that of energy harvesting [5]. Consequently, the energy consumption constraints are given by:

$$\tau_c p_u^c + \tau_i p_u + \tau_i p_u^c \leq E^h_u, \quad \forall u \in \mathcal{K} \cup \{d\}.$$  

In the NOMA-based uplink cellular network, $K$ CUs are multiplexed on the same resource, and SIC is adopted at the receiver (Rx). Obviously, $\tau_c + \tau_i \leq T$ should be satisfied.

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In the NOMA-based uplink cellular network, $K$ CUs are multiplexed on the same resource, and SIC is adopted at the HAP side. With SIC, the information of CUs is decoded in a decreasing order of the channel gain. Let $g_{kB}$ denote the uplink channel gain from CU $k$ to the HAP. Assume that $g_{1B} \geq g_{2B} \geq \cdots \geq g_{KB}$. The SINR of CU $k$ is

$$\text{SINR}_k = \frac{p_k g_{kB} \sum_{j=k+1}^{K} p_j g_{jB} + p_d g_{dB} + \sigma^2}{p_d g_{dB} + \sigma^2}, \quad \forall k \in \mathcal{K} \setminus \{K\},$$

and when $k = K$, the SINR is given by

$$\text{SINR}_K = \frac{p_K g_{kB}}{p_d g_{dB} + \sigma^2},$$
where $g_{4dB}$ is the channel gain from the D2D-Tx to the HAP. Since the D2D pair reuses the resource of CUs, the achievable throughput of the D2D pair is given by
\[ R_d = \tau_l W \log_2 \left( 1 + \frac{p_d g_{4dB}}{\sum_{k=1}^{K} p_k g_{kd} + \sigma^2} \right), \]
where $W$ is the system bandwidth, $g_{kd}$ is the channel gain from CU $k$ to the D2D-Rx, and $g_{4dB}$ is the channel gain between the users of the D2D pair. The total energy consumption $E_d$ of the D2D pair can be given by
\[ E_d = \tau_c p_d^c + \tau_p p_d + \tau_p^c. \]

Now we are ready to formulate the energy efficiency optimization problem for a NOMA-based cellular network with energy harvesting as a solution of (7) with maximum energy efficiency $EE$
\[ E_d = \tau_c p_d^c + \tau_p p_d + \tau_p^c. \]

By solving problem (7) is equivalent to the problem
\[ \max_{p_c, p_d, \tau_c, \tau_p, \tau_p^c} \frac{\sum_{k=1}^{K} \bar{c}_k}{\sum_{k=1}^{K} \bar{c}_k}. \]

Lemma 1: Solving problem (7) is equivalent to the problem given by
\[ \psi(q) = 0, \]
where the function $\psi(q)$ is defined by
\[ \psi(q) = \max_{p_c, p_d, \tau_c, \tau_p, \tau_p^c} R_d - q E_d. \]

A. Optimal and Feasible Conditions

To solve problem (7), we obtain the following lemmas about the optimal time utilization and power allocation.

Lemma 2: For problem (7), the maximum energy efficiency can always be achieved at $\tau_c + \tau_p = T$.

Proof: Suppose that $\{p_c^*, p_d^*, \tau_c^*, \tau_p^*\}$ is the optimal solution of (7) with maximum energy efficiency $EE^*$, and satisfies $\tau_c^* + \tau_p^* < T$. Then, we construct a new solution $\{p_c^*, p_d^*, \tau_c^*, \tau_p^*\}$, where $\tau_c = \tau_c^* + \tau_p^* = \tau_c^* + \tau_p^* = \tau_c^* + \tau_p^* = T$. With the new solution, constraints (7b)-(7f) are still satisfied and $\bar{E}E = EE^*$ can be verified where $\bar{E}E$ is the new energy efficiency. Therefore, the maximum energy efficiency can always be achieved at $\tau_c + \tau_p = T$.

In the following, we define $\tau = \tau_c$, and $\tau_c$ can be replaced by $T - \tau$ in problems (7) and (9).

Lemma 3: For any optimal solution to problem (7), constraints (7d) must hold with equality.

Proof: Assume that the optimal power of CUs is $p_c^* = [p_1^*, \cdots, p_K^*]$, the optimal power of the D2D-Tx is $p_d^*$ and $\text{SINR}_k > \gamma_k$ is satisfied. With all the other powers of CUs and $p_d^*$ fixed, we reduce the power $p_d^*$ to $p_d^* - \Delta$, $\Delta > 0$ such that $\text{SINR}_k > \gamma_k$. With the new power $p_d^*$, the objective function (7a) is increased with satisfying all constraints of (7), which contradicts that the solution is optimal. Note that there exists one unique solution satisfying (7d) with equality for all CUs since the coefficient matrix and the augmented matrix of corresponding linear equations are row full-rank. Besides, this unique solution is positive and component-wise minimum.

Setting constraints (7d) with equality, the optimal power of CUs can be expressed as
\[ p_c^* = \frac{\gamma_k (p_d g_{4dB} + \sigma^2)}{g_{KB}}, \quad p_d^* = \frac{\gamma_k (\sum_{k=1}^{K} p_j g_{JB} + p_d g_{4dB} + \sigma^2)}{g_{KB}}, \quad \forall k \in K \setminus \{K\}. \]

To solve (11), we introduce
\[ S_k = \sum_{j=k}^{K} p_j^* g_{JB}, \quad \forall k \in K. \]

Plugging (12) into (11) yields
\[ S_k - S_{k+1} = \frac{\gamma_k (S_k + p_d g_{4dB} + \sigma^2)}{g_{KB}}, \quad \forall k \in K \setminus \{K\}. \]

Denoting $A_k = \gamma_k + 1$, $B_k = \gamma_k (p_d g_{4dB} + \sigma^2)$, (13) can be simplified as
\[ S_k = A_k S_{k+1} + B_k, \quad \forall k \in K \setminus \{K\}. \]

We obtain $S_K = B_K$ from (10) and (12). Solving (14) with the recursion method, $S_k$ is given by
\[ S_k = \sum_{j=k}^{K} B_j \prod_{l=k}^{j-1} A_l, \quad \forall k \in K, \]
where we define $\prod_{j=k}^{K} A_l = 1$. Hence, combining (12) and (15) yields
\[ p_c^* = \sum_{j=k}^{K} B_j \prod_{l=k}^{j-1} A_l - \sum_{j=k+1}^{K} B_j \prod_{l=k+1}^{j-1} A_l \]
\[ = C_k p_d + D_k, \]
where $C_k = (\gamma_k g_{4dB} + \gamma_k g_{4dB}) \sum_{j=k+1}^{K} \gamma_j \prod_{l=k+1}^{j-1} (\gamma_l + 1)/g_{KB}$, and $D_k = (\gamma_k g_{4dB} + \gamma_k g_{4dB}) \sum_{j=k+1}^{K} \gamma_j \prod_{l=k+1}^{j-1} (\gamma_l + 1)/g_{KB}$.

According to (16), constraints (7e) can be rewritten as
\[ p_d \leq P_d^*, \]
where $P_d^* = \min_{k \in K} (P_{d_{\text{max}}} - D_k)/C_k$, $P_{d_{\text{max}}}$. Note that problem (7) is feasible only if $P_d^*$ is non-negative, i.e., $P_{d_{\text{max}}} \geq D_k, \forall k \in K$. Constraints (7f) can be transformed to
\[ \tau_p d + \tau (p_d^* - p_d^* + \eta p_h k) \leq \eta P_0 T h_k - T p_d^*, \]
for all $k \in K$. Consequently, (9) can be simplified as
\[ \min \tau p_d + \tau (p_d^* - p_d^*) + T p_d^* \]
\[ \leq \eta P_0 T h_k - T p_d^*, \]
s.t. $0 \leq \tau \leq T$.
where \( E = \sum_{k=1}^{K} C_k g_{kd}, G = E + g_{dd}, F = \sum_{k=1}^{K} D_k g_{kd} + \sigma^2, H_0 = p_d - p^*_d + \eta P_0 h_d, I_0 = \eta P_0 T h_d - T p_d, H_k = (D_k + p^*_k - p^*_d + \eta P_0 h_d)/C_k, \text{and } I_k = (\eta P_0 T h_d - T p^*_k)/C_k, \forall k \in K. \)

Before solving problem (20), feasibility conditions are given by the following lemma.

**Lemma 4:** The optimization problem (20) is feasible if and only if \( p^*_d \geq 0 \) and \( I_k \geq 0, \forall k \in K \cup \{0\}. \)

Since Lemma 4 can be proved by the contradiction method, the proof is omitted.

**B. Optimal Solution**

To solve nonconvex problem (20), the optimal solution is obtained by analyzing the Karush-Kuhn-Tucker (KKT) conditions. Specifically, the Lagrangian function of (20) is

\[
\mathcal{L}(\tau, p_d, \alpha_1, \alpha_2, \beta_1, \beta_2, \lambda) = -W \log \left( \frac{G p_d + F}{E p_d + F} \right) + q(p_d + p^*_d - p^*_d) - \alpha_1(\tau) + \alpha_2(\tau - T) + \beta_1(-p_d - \beta_2(p_d - P^*_d))^2 + \sum_{k=0}^{K} \lambda_k(\tau p_d + \tau H_k - I_k), \quad (21)
\]

where \( \alpha_1, \alpha_2, \beta_1, \beta_2, \text{ and } \lambda = [\lambda_1, \lambda_2, \cdots, \lambda_K] \geq 0 \) are non-negative dual variables. All locally optimal solutions should satisfy the KKT conditions of problem (20) as follows

\[
\begin{align*}
\frac{\partial \mathcal{L}}{\partial \tau} &= -W \log \left( \frac{G p_d + F}{E p_d + F} \right) + q(p_d + p^*_d - p^*_d) \\
&\quad - \alpha_1 + \alpha_2 + \sum_{k=0}^{K} \lambda_k(p_d + H_k) = 0 \quad (22a) \\
\frac{\partial \mathcal{L}}{\partial p_d} &= \frac{W}{\ln 2} \left( \frac{G}{G p_d + F} + \frac{E}{E p_d + F} \right) + q(T) \\
&\quad - \beta_1 + \beta_2 + \sum_{k=0}^{K} \lambda_k(\tau) = 0 \quad (22b) \\
\alpha_1(\tau) &= \alpha_2(\tau - T) = \beta_1(-p_d - \beta_2(p_d - P^*_d)) = 0 \quad (22c) \\
\lambda_k(\tau p_d + \tau H_k - I_k) &= 0, \quad \forall k \in K \cup \{0\} \quad (22d) \\
(20b) - (20d), \quad \alpha_1, \alpha_2, \beta_1, \beta_2, \lambda, \quad \geq 0, \forall k \in K \cup \{0\}. \quad (22e)
\end{align*}
\]

To obtain the globally optimal solution, we find all feasible solutions to KKT conditions (22) and pick the best one as the final result. It can be easily observed that \( p_d = 0 \) or \( \tau = 0 \) or \( \tau = T \) are not optimal solutions to problem (9), since the achievable throughput of the D2D pair is 0 in this case. Thus, \( p_d = 0 \) or \( \tau = 0 \) or \( \tau = T \) are neither the optimal solutions to problem (20). According to constraints (20b) and (20c), the optimal solution to problem (20) falls into two cases: (1) \( 0 < \tau < T, 0 < p_d < P^*_d \); (2) \( 0 < \tau < T, p_d = P^*_d \). We obtain all solutions that satisfy the KKT conditions in each individual case.

1) **Case** \( 0 < \tau < T, 0 < p_d < P^*_d \)

According to (20d), a feasible region of \( (\tau, p_d) \) consists of \( K + 1 \) linear constraints. In the following, solutions \( (\tau, p_d) \) that satisfy (20d) are classified into: a vertex point, a boundary point and an inner point.

a) If the solution is located in a vertex, at least two of the constraints in (20d) hold with equality. For finite \( K \), we can traverse all couples of \( K + 1 \) constraints to obtain the exact \( \tau \) and \( p_d \). Assume that the \( i \)-th constraint and the \( j \)-th constraint in (20d) hold with equality, the closed form of \( p_d \) and \( \tau \) can be given by

\[
\begin{align*}
p_d &= \frac{I_i H_i - I_j H_j}{I_i - I_j} \quad (23a) \\
\tau &= \frac{I_i - I_j}{H_i - H_j} \quad (23b)
\end{align*}
\]

Considering (22e), we can check whether (23) is a feasible solution to problem (20).

b) If the solution is located in a boundary, we assume that the \( i \)-th constraint in (20d) is the only one that holds with equality. From (22c)-(22d), all dual variables, except \( \lambda_i \), equal to zero. Then, (22) can be simplified as

\[
\begin{align*}
\tau p_d + \tau H_i &= I_i \quad (24a) \\
-W \log \left( \frac{G p_d + F}{E p_d + F} \right) + q(p_d + p^*_d - p^*_d) &= 0 \quad (24b) \\
\tau W \left( \frac{G}{G p_d + F} + \frac{E}{E p_d + F} \right) + q \tau + \lambda_i &= 0 \quad (24c)
\end{align*}
\]

Substituting \( \lambda_i = \frac{W}{\ln 2} \left( \frac{G}{G p_d + F} - \frac{E}{E p_d + F} \right) - q \) from (24c) into (24b) yields

\[
f(p_d) = (p_d + H_i) f'(p_d) + M = 0, \quad (25)
\]

where \( f(p_d) = -W \log \left( \frac{G p_d + F}{E p_d + F} \right) + M \) is a constant defined by \( q(p_d - p^*_d) - H_i \). Defining

\[
g(p_d) = f(p_d) - (p_d + H_i) f'(p_d) + M, \quad (26)
\]

we have \( g'(p_d) = -(p_d + H_i) f''(p_d) < 0 \) since \( f''(p_d) > 0 \). Hence, \( g(p_d) \) is monotonically decreasing, which indicates that (25) has one unique solution. From (25), (26) and (24a), we can obtain

\[
\begin{align*}
p^*_d &= g^{-1}(0) \quad (27a) \\
\tau^* &= \frac{I_i}{H_i + g^{-1}(0)}, \quad (27b)
\end{align*}
\]

where \( g^{-1}(\cdot) \) is the inverse function of \( g(\cdot) \). Whether (27) is a feasible solution should be checked from (22e).

c) If the solution is located inside the feasibility region, all dual variables are equal to zero. With \( \beta_1 = \beta_2 = 0 \) and \( \lambda = 0 \), \( p_d \) can be uniquely obtained from (22b).

Since the objective function (20a) is a linear function of \( \tau \) with given \( p_d \), the optimal \( \tau \) always lies in the maximal or minimal point, which contradicts that the solution is located inside the feasibility region. Hence, the globally optimal solution does not exist in this situation.

2) **Case** \( 0 < \tau < T, p_d = P^*_d \)

With \( p_d = P^*_d \), (20a) is a linear function of \( \tau \) where the optimal solution must lie in the boundary of feasible region. In addition, (20c) can be transformed to

\[\tau \leq \frac{I_i}{P^*_d + H_i}, \quad i = \arg \min_{k \in K \cup \{0\}} \left( \frac{I_i}{P^*_d + H_i} \right).\]

Since \( \tau \) is strictly positive, the optimal \( \tau^* \) must satisfy \( \tau^* = \frac{I_i}{P^*_d + H_i} \), if the objective function is monotonically decreasing, i.e., both \(-W \log \left( \frac{G p_d + F}{E p_d + F} \right) + q(P^*_d + p^*_d - p^*_d) < 0 \) and \( 0 < \frac{I_i}{P^*_d + H_i} < T \) are satisfied, the optimal solution is given by

\[
\begin{align*}
p^*_d &= P^*_d \quad (28a) \\
\tau^* &= \min_{k \in K \cup \{0\}} \left( \frac{I_k}{P^*_d + H_k} \right). \quad (28b)
\end{align*}
\]
C. Algorithm and Complexity Analysis

The optimal NOMA-based power control and time allocation (NOMA-OPT) algorithm to solve problem (7) is presented in Algorithm 1. For the proposed algorithm, the major complexity lies in Case 1 in each iteration, where Case 1a involves a complexity of $O(K^2)$ for traversing $K+1$ constraints, and Case 1b has a complexity of $O(K \log_2(1/\epsilon))$ due to the bisection method with a tolerance of $\epsilon$. Thus, the total complexity of the proposed NOMA-OPT is $O(LK^2 + LK \log_2(1/\epsilon))$, where $L$ is the number of iterations.

Algorithm 1: NOMA-OPT

1. Initialization Set the maximum tolerance $\epsilon$ and $q = q_0$.
2. repeat
3. With fixed $q$, obtain the optimal $(\tau^*, p_d^*)$ of (20).
4. Set $q_{pre} = q$, $q = (\tau^* W \log_2 \left( \frac{Gp_d^* + T}{\epsilon p_d^*} \right)) / (\tau^* p_d^* + \tau^* (p_d^* - p_d^*) + T p_d^*)$.
5. until $|q_{pre} - q| \leq \epsilon$.
6. Output $p_d = p_d^*$, $\tau_k = \tau^*$, $\tau_c = T - \tau^*$, $p_k = C_k p_d^* + D_k, \forall k \in K$.

IV. SIMULATION RESULTS

There are three CUs randomly located within the network. The channel gains are modeled as $148.1 + 37.6 \log_{10} D$ [9], where $D$ is the distance measured in kilometers. The other main system parameters are given as $\eta = 0.9$, $T = 10$ s, $\sigma^2 = -174$ dBm/Hz, $W = 20$ KHz, $\epsilon = 10^{-7}$, $P_{\text{max}}^u = 50$ mW, $P_{\text{max}}^r = 10$ mW, $P_{\text{max}}^c = 5$ mW, $\forall u \in K \cup \{d\}$, and $\gamma_k = \gamma_0$, $\forall k \in K$.

Fig. 1 illustrates the convergence behavior of the proposed algorithm corresponding to different values of the maximum transmit power of the HAP while setting $\gamma_0 = 9$. It can be seen from Fig. 1 that no more than four iterations are needed to approach the optimal solution.

We compare the NOMA-OPT with the following three schemes: TDMA-based power and time optimization in [5] where CUs occupy different time slots with the D2D pair underlaying whole information transmission time (labeled as ‘TDMA-OPT’), optimal NOMA-based power control with fixed time allocation in [7] where the time is equally divided between two phases (labeled as ‘NOMA-OPT’) and optimal NOMA-based time allocation with maximum transmit power (labeled as ‘NOMA-OTFP’). For fair comparison, TDMA-based CUs are allocated information transmission durations that ensure the same throughput as NOMA-based CUs.

From Fig. 2, it can be observed that energy efficiency of each scheme decreases with the increased SINR requirement of CUs, since the power of the D2D-Tx is limited as SINR increases. This is mainly because that (i) $P_d$, the upper bound of D2D-Tx’s transmit power, is reduced and (ii) the throughput $R_d$ sharply decreases since $C_k$ and $D_k$ are increasing functions of $\gamma_k$ according to (16). Fig. 2 also illustrates that the proposed scheme is superior over NOMA-OPT scheme and NOMA-OTFP scheme. This is because that NOMA-OPT scheme ignores the trade-off between time for energy harvesting and information transmission, NOMA-OTFP scheme which transmits with maximum power is usually not energy efficient. Furthermore, our scheme also outperforms TDMA-OPT scheme. The reason is that less time is utilized in energy harvesting in TDMA, which leads to low transmit power and low energy efficiency of the D2D pair.

V. CONCLUSION

This letter investigates power control and time allocation problem of a D2D underlaying NOMA-based cellular network with energy harvesting. The optimal energy efficiency of the D2D pair is achieved by solving the KKT conditions. Simulation results show that the proposed scheme yields larger energy efficiency than the existing representative schemes. The resource allocation of a multi-carrier network with multiple D2D pairs is left for future work.

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